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"Efficient Search on the Job and the Business Cycle"

by

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Efficient Search on the Job and the Business Cycle

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Abstract

We build a directed search model of the labor market in which workers' transitions between unemployment, employment, and across employers are endogenous. We prove the existence, uniqueness and efficiency of a recursive equilibrium with the property that the distribution of workers across employment states does not affect the agents' values and strategies. Because of this property, we are able to compute the equilibrium outside the non-stochastic steady-state. We use a calibrated version of the model to measure the effect of productivity shocks on the US labor market. We find that productivity shocks generate procyclical fluctuations in the rate at which unemployed workers become employed and countercyclical fluctuations in the rate at which employed workers become unemployed. Moreover, we find that productivity shocks generate large countercyclical fluctuations in the number of vacancies opened for unemployed workers and even larger procyclical fluctuations in the number of vacancies created for employed workers. Overall, productivity shocks alone can account for 80 percent of unemployment volatility, 30 percent of vacancy volatility and for the nearly perfect negative correlation between unemployment and vacancies.

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1 Introduction

1.1 Motivation

From December 1969 to November 1970, the US unemployment rate increased from 3.5 to 6.1 percent. During this period, unemployment increased partly because the workers' transition rate from unemployment to employment[1] (henceforth, the UE rate) dropped from 51 to 44 percent per month. In part, unemployment increased because the workers' transition rate from employment to unemployment (henceforth, the EU rate) increased by 30 percent. Similarly, during the 1960, 1973, 1981 and 1990 recessions, the unemployment rate increased because of both a significant decline in the UE rate and a significant surge in the EU rate.

From January 2001 to November 2001, the US vacancy rate fell by approximately 30 percent. During this period, the number of workers moving from unemployment to employment declined from 3.25 to 3.23 millions per month (a 1 percent decline). In contrast, the number of workers moving from one employer to the other declined from 4.5 to 3.7 millions per month (a 17 percent decline).[2] More generally, over the period between January 1994 and June 2006, the correlation between the vacancy rate and the workers' flow from unemployment to employment is -0.3, while the correlation between the vacancy rate and the workers' flow across employers is 0.49.

The first set of observations suggests that, in order to study the cyclical fluctuations of the unemployment rate, an economist should use a model in which both the UE and the EU rates are endogenous. The second set of observations suggests that, in order to study the fluctuations of the vacancy rate, an economist should use a model in which the hiring flow of both unemployed and employed workers is endogenous. When taken together, these observations suggest that, in order to study the dynamics of the labor market at the business cycle frequency, an economist should use a model that endogenizes

[1] In Section 5.1, the reader will find the definitions of the workers' transition rate from unemployment to employment, from employment to unemployment, and from employer to employer. Moreover, he will find the definitions of the unemplyoment and the vacancy rate.

[2] We measure the number of workers moving from unemployment to employment as the product between civilian unemployment and the workers' transition rate from unemployment to employment (as defined in Section 5.1). Similarly, we measure the number of workers moving from one employer to the other as the product between civilian employment and the workers' trasition rate from employer to employer (as defined in Section 5.1).

the UE rate, the EU rate, and the rate at employed workers move from one employer to the other (henceforth, EE rate).

1.2 Summary

In the first part of this paper, we construct a search-theoretic model of the labor market in which the workers' transitions between employment, unemployment and across different employers are all endogenous. In the second part of the paper, we calibrate the model in order to match the fundamental features of worker's turnover in the US labor market. In the last part of the paper, we use the calibrated model to measure the contribution of aggregate productivity shocks to the cyclical volatility of US unemployment, vacancies and other labor market variables over the period 1951 (I) - 2006 (II).

In our model, the labor market is populated by ex-ante homogeneous workers—each endowed with one indivisible unit of labor—and ex-ante homogeneous firms—each operating a production technology that turns labor into final goods. Moreover, in our model labor market, trade is the outcome of a search-and-matching process. In particular, firms choose how many vacancies to open and how much to offer to the workers who fill them. Then, workers choose how much to demand for filling a vacancy. Finally, some of the workers and the firms who agree on the terms of trade successfully match and begin to produce the final good. We assume that the productivity of a match is the sum of an aggregate and an idiosyncratic component.

In equilibrium, firms are indifferent between opening different types of vacancies, because the vacancies that offer more generous terms of trade attract more workers and, hence, are easier to fill. Workers, however, have strict preferences over different types of vacancies. In particular, unemployed workers prefer to search for vacancies that offer less generous terms of trade and are easier to find (because they attract fewer workers), while employed workers prefer to search for vacancies that offer more generous terms of trade but are harder to find. Similarly, workers who are employed in more productive jobs prefer to search for vacancies that offer better terms of trade. In equilibrium, an employed worker becomes unemployed when the idiosyncratic component of the productivity of his match falls below an endogenous job-destruction threshold.

When a positive shock to the aggregate component of labor productivity hits the economy, firms have the incentive to open more vacancies per worker at all different terms of trade. In response to the increase in the vacancy/worker ratio, unemployed workers and (on average) employed workers search for vacancies that not only offer more generous terms of trade, but are also easier to find. Also, when a positive shock to the aggregate component of productivity hits the economy, workers and firms find it optimal to keep some of the matches that previously they would have destroyed. Overall, a positive shock to the aggregate component of productivity tends to increase the UE and the EE rates, and to decrease the EU rate.

In the second part of the paper, we calibrate the model. In particular, we calibrate the parameters that describe the search technology so that the workers' average transition rates between employment, unemployment and across employers are the same in the model as in the data. We calibrate the stochastic process for the idiosyncratic component of productivity to approximate the empirical distribution of workers across different tenure lengths. Finally, we calibrate the stochastic process for the aggregate component of productivity so that the average productivity of labor has the same statistical properties in the model and in the data.

In the third part of the paper, we use the calibrated model to measure the contribution of aggregate productivity shocks to the cyclical fluctuations of the US labor market. We find that aggregate productivity shocks account for 40 percent of the observed volatility of the UE rate, and for approximately all of the observed volatility of the EU rate. As a result, aggregate productivity shocks alone can account for more than 80 percent of the observed volatility of unemployment. Moreover, we find that productivity shocks generate countercyclical fluctuations in the number of vacancies created for unemployed workers and larger procyclical fluctuations in the number of vacancies created for employed workers. Overall, aggregate productivity shocks alone can account for more than 30 percent of the cyclical volatility of the vacancy rate and for their nearly perfectly negative correlation with unemployment. In light of these findings, we conclude that aggregate productivity shocks may well be the fundamental cause of labor market volatility in the postwar US.

In the last part of the paper, we measure the contribution of aggregate productivity shocks to the cyclical volatility of the US labor market using a version of our model in which the UE rate is endogenous, but the EU and EE rates are exogenous because matches are constrained to be homogeneous and workers are constrained to search off the job. As it turns out, the constrained version of our model coincides with the canonical search model formulated by Pissarides (1985, 2000) and Shimer (2005). We find that, when an economist uses the constrained model, he not only ignores the effect of productivity shocks on the EU rate, but he also underestimates the effect of productivity shocks on the UE rate because he mismeasures the elasticity of the matching function with respect to vacancies. Moreover, when an economist uses the constrained model, he not only ignores the effect of productivity shocks on the number of vacancies created for employed workers, but he also mismeasures the effect of productivity shocks on the number of vacancies created for unemployed workers. Finally, when an economist uses the constrained model, he underestimates the magnitude of productivity shocks in the postwar US. For all of these reasons, he incorrectly concludes that aggregate productivity shocks account for less than 10 percent of the cyclical volatility of unemployment and for less than 20 percent of the cyclical volatility of vacancies. These findings confirm our initial conjecture that, in order to understand the behavior of unemployment and vacancies over the business cycle, an economist needs a model in which not only the UE, but also the EU and the EE rates are endogenous.

1.3 Related Literature

In this paper, we develop the first stochastic model of the labor market in which workers search on and off the job, and the search process is directed (i.e. workers can choose whether to search for vacancies that offers more or less generous terms of trade) rather than random (i.e. workers meet all the vacancies with the same probability). For this model, we establish two useful properties of the equilibrium. First, we prove that the equilibrium is such that the agents' values, strategies and the vacancy/applicant ratio depend on the state of the economy only through the realization of the aggregate productivity shock, and not through the multi-dimensional distribution of workers across employment states (namely, unemployment and employment at different jobs). Because of this property, solving the equilibrium requires solving a system of functional equations in which the unknown functions are only one-dimensional. Second, we prove that, as long as employment contracts are complete, the equilibrium allocation coincides with the solution to the social planner's problem. Because of this property, we are able to provide a rich analytical characterization of the equilibrium allocation. While other papers had previously identified these two properties in steady-state models of directed search on the job (respectively, Shi 2006 and Moen and Rosen 2006), our paper is the first to establish them in a stochastic environment.

In models of random search on the job, the equilibrium does not have these attractive properties (see Mortensen 1994, Pissarides 1994, Burdett and Mortensen 1998, Barlevy 2002, Postel-Vinay and Robin 2002, Burdett and Coles 2003, Menzio 2005, Nagypál 2007). First, in these models, the equilibrium allocation does not coincide with the solution to the social planner's problem. Second, in these models, the equilibrium is (in general) such that the agents' values, strategies and the vacancy/applicant ratio depend on the entire distribution of workers across employment states.[3] Therefore, solving the equilibrium of these models outside of steady-state requires solving a system of functional equations in which the unknown functions have at least as many dimensions as the number of different employment states in which workers can be.

In this paper, we construct the first measure of the contribution of aggregate productivity shocks to the cyclical volatility of US unemployment, vacancies and transition rates that is based on a model in which the UE, EU and EE rates are endogenous. Mortensen and Pissarides (1994), Mortensen (1994), Barlevy (2002), and Ramey (2007) develop search-theoretic models of the business cycle in which the EU and the UE rates are endogenous. In these models, as in ours, workers move from employment to unemployment when the idiosyncratic component of productivity of their job falls below an

[3] In models of directed search on the job, workers who are in different employment states choose to search for vacancies that offer different terms of trade. Therefore, in these models, the distribution of workers across employment states has no effect on the firm's expected benefit from creating a particular type of vacancy and, in turn, on the equilibrium vacancy/worker ratio, and on the workers' values and strategies. In contrast, in models of random search on the job, workers who are in different employment states search for the same vacancies. Therefore, as long as different workers have different reservation values when they meet a prospective employer, the distribution will affect the firm's expected benefit from creating a vacancy and, in turn, the equilibrium vacancy/worker ratio, and the workers' values and strategies.

endogenous destruction threshold. Moreover, as in our model, a positive shock to the aggregate component of productivity tends to lower the endogenous destruction threshold and, hence, the EU rate. However, none of these papers measures the contribution of aggregate productivity shocks to the cyclical volatility of both the UE and the EU rates. In fact, Barlevy (2002) and Ramey (2007) calibrate the parameters of their models so that aggregate productivity shocks account for all of the observed volatility of the EU rate. In Mortensen and Pissarides (1994) and Mortensen (1994), the parameters of the model are chosen so that aggregate productivity shocks account for all of the unemployment volatility that is observed in the data.

Mortensen (1994), Pissarides (1994), Nagypál (2007) and Ramey (2007) develop search-theoretic models of the business cycle in which the UE and the EE rates are endogenous. All of these papers conclude that, if an economist uses a model that abstracts from employer-to-employer transitions, he does not significantly underestimate the contribution of productivity shocks to the volatility of the UE rate. In this paper, we reach a very different conclusion. In particular, we find that, if an economist uses a model that abstracts from EE transitions, he underestimated the elasticity of the matching function with respect to vacancies by more than 60 percent. For this reason, he underestimates the contribution of aggregate productivity shocks to the volatility of the UE rate by more than 70 percent. While other papers had already noticed this potential source of bias in the estimation of the elasticity of the matching function (e.g. Petrongolo and Pissarides 1999, Menzio 2005, or Nagypál 2007), our paper is the first to measure its extent.

2 The Model

2.1 Physical Environment

Time is discrete and continues forever. The economy is populated by a continuum of workers with measure one and by a continuum of firms with positive measure. Each worker has the von Neumann-Morgenstern utility function $\sum_{t=0}^{\infty} \beta^t c_t$, where $c_t \in \mathbb{R}$ is the worker's consumption in period t and $\beta \in (0,1)$ is the discount rate. Each firm has the von Neumann-Morgenstern utility function $\sum_{t=0}^{\infty} \beta^t \pi_t$, where $\pi_t \in \mathbb{R}$ is the firm's profit in period t. In this economy, the labor market is organized in a continuum of submarkets indexed by $x \in \mathbb{R}$, where x denotes the value offered to a worker in that submarket (explained further below). In submarket x, the ratio between the number of jobs that are vacant and the number of workers who are searching is denoted by $\theta(x) \in \mathbb{R}_+$. We refer to $\theta(x)$ as the *tightness* of submarket x.[4]

At the beginning of each period, the state of the economy can be summarized by the triple $(y, u, g) \equiv \psi \in \Psi$. The first element of ψ denotes the aggregate component of labor productivity, $y \in Y = \{y_1, y_2, ...y_{N_y}\}$, where $N_y \geq 2$. The second element denotes the measure of workers who are unemployed, $u \in [0, 1]$. The last element is a function $g : Z \to [0, 1]$, with $g(z)$ denoting the measure of workers who are employed at a job with idiosyncratic productivity $z \in Z = \{z_1, z_2, ...z_{N_z}\}$, where $N_z \geq 2$.[5] Clearly, $u + \sum_i g(z_i) = 1$.

Each period is divided into four stages: separation, search, matching and production. During the first stage, an employed worker becomes unemployed with probability $\tau \in [\delta, 1]$, where τ is determined by the worker's labor contract. The lower bound on τ denotes the probability of exogenous job destruction, $\delta \in (0, 1)$.

During the second stage, a worker gets the opportunity of searching for a job with a probability that depends on his recent employment history. In particular, if the worker was unemployed at the beginning of the period, he can search with probability $\lambda_u \in [0, 1]$. If the worker was employed at the beginning of the period and did not lose his job during the separation stage, he can search with probability $\lambda_e \in [0, 1]$. If the worker lost his job during the separation stage, he cannot search. Conditional on being able to search, the worker chooses which submarket to visit. Also, during the second stage, a firm chooses how many vacancies to create and where to locate them. The cost of maintaining a vacancy for one period is $k > 0$. Both workers and firms take the tightness $\theta(x)$ parametrically.[6]

[4] In submarkets that are not visited by any workers, $\theta(x)$ is an out-of-equilibrium conjecture that helps determine equilibrium behavior.

[5] Note that the assumption that Y and Z are finite sets is not necessary for establsihing any of the theoretical results in this paper. We make this assumption only to simplify the notation.

[6] That is, workers and firms treat the tightness $\theta(x)$ just like households and firms treat prices in a Walrasian Equilibrium.

During the third stage, the workers and the vacancies in submarket x come together through a frictional matching process. In particular, a worker finds a vacant job with probability $p(\theta(x))$, where $p : \mathbb{R}_+ \to [0,1]$ is a twice continuously differentiable, strictly increasing, strictly concave function which satisfies the boundary conditions $p(0) = 0$, $p(\bar{\theta}) = 1$. Similarly, a vacancy finds a worker with probability $q(\theta(x))$, where $q : \mathbb{R}_+ \to [0,1]$ is a twice continuously differentiable, strictly decreasing function such that $q(\theta) = \theta^{-1}p(\theta)$, $q(0) = 1$, and $\lim_{\theta\to\infty} q(\theta) = 0$. The properties of the functions p and q are meant to capture the realistic feature that, the tighter is the submarket, the higher is the probability that a worker finds a vacancy and the lower is the probability that a vacancy finds a worker.

When a worker meets a firm in submarket x, he is offered an employment contract which gives him the lifetime utility x if he accepts it. If the worker rejects the firm's offer (an event that does not occur along the equilibrium path), he returns to his previous employment position. If the worker accepts the offer, he first leaves his previous employment position to enter his new employment relationship with the firm. Then, the worker and the firm draw the the idiosyncratic productivity $\tilde{z} \in Z$ of their match, where $\tilde{z}$ is a random variable with a density function $f : Z \to [0,1]$. The idiosyncratic component of productivity is constant throughout the duration of the match.

During the last stage, an unemployed worker produces and consumes b units of output. A worker employed at a job z produces $y + z$ units of output and consumes w of them, where w is specified by the worker's labor contract. At the end of the last stage, nature draws next period's aggregate productivity $\hat{y}$ from the probability distribution $\phi(\hat{y}|y)$, $\phi : Y \times Y \to [0,1]$. Throughout this paper, the caret indicates variables or functions in the next period.

2.2 Contractual Environment

The literature has considered a variety of assumptions about the contractual environment in models of search on the job. For example, Burdett and Coles (2003), Stevens (2004) and Shi (2006) assume that a labor contract is a wage/tenure profile. Burdett and Mortensen (1998), Delacroix and Shi (2006) and Shimer (2006) assume that a contract is a wage that remains constant throughout the employment relationship. Barlevy (2002) and Nagypál (2007) assume that a contract can only prescribe the current wage and is renegotiated in every period. In this paper, we depart from the existing literature, and assume that employment contracts are complete. That is, the contracts prescribe the wage, the separation strategy, and the worker's on-the-job search strategy as a function of the entire history of the match. While the assumption of complete contracts is strong, it is a useful a benchmark that should be studied before considering alternative assumptions.

To specify the contracts, let the history of a match be a vector $\{z; y^t\} \in Z \times Y^t$, where z is the match-specific component of productivity and $y^t = \{y_1, y_2, ...y_t\}$ is the sequence of realizations of the aggregate component of productivity since the inception of the match.[7] An employment contract $\underline{a} \in A^{N_z}$ is an allocation $\{w_t, \tau_t, n_t\}_{t=0}^{\infty}$. The first element of $\underline{a}$ denotes the wage as a function of the worker's tenure t and the history of the match $\{z; y^t\}$, where $w_t : Z \times Y^t \to \mathbb{R}$. The second element denotes the separation probability as a function of the tenure t and the history $\{z, y^{t+1}\}$, where $\tau_t : Z \times Y^{t+1} \to [\delta, 1]$. The last element denotes the submarket where the worker searches while on the job as a function of the tenure t and the history $\{z, y^{t+1}\}$, where $n_t : Z \times Y^{t+1} \to \mathbb{R}$.

In the remainder of the paper, we let $\underline{a}(z; y^t) \in A$ denote the allocation prescribed by the employment contract $\underline{a}$ after the history $\{z; y^t\}$ is realized. Note that $\underline{a}(z; y^t)$ is equal to $\{w_t(z; y^t), \tau_t(z; y^t, \hat{y}), n_t(z; y^t, \hat{y})\} \cup \underline{a}(z; y^t, \hat{y})$.

3 Conditions and Definition of Equilibrium

In this paper, we are interested in recursive equilibria in which the agents' values, optimal decisions, and the market tightness depend on the aggregate state of the economy $\psi = (y, u, g)$ only through y and not through the multi-dimensional distribution of workers across employment states. In such equilibria, we can write the tightness in submarket x as $\theta(x; y)$, instead of $\theta(x; \psi)$, when the aggregate component of productivity is y. Moreover,

[7] In general, a complete contract should specify w, τ, and n as functions of the match-specific component of productivity z and the sequence of realizations of the aggregate state of the economy since the inception of the match, $\psi^t = \{\psi_1, \psi_2, ...\psi_t\}$. However, in this paper, we are interested in equilibria in which the tightness function $\theta(x)$ depends on the aggregate state of the economy $\psi = (y, u, g)$ only through y and not through the entire distribution of workers across employment states. In these equilibria, the history $\{z; y^t\}$ provides enough contingencies for a contract to be efficient.

we can denote $U(y)$ as the lifetime utility of an unemployed worker when the aggregate component of productivity is y. Similarly, $W(z;y|a)$ denotes the lifetime utility of a worker who is employed at a job with idiosyncratic productivity z and whose contract prescribes the allocation a. $J(z;y|a)$ denotes the lifetime profits of the firm that employs him. The lifetime utilities U, W, and J are measured at the beginning of the production stage.

3.1 Worker's Value of Searching

Consider a worker who has received the opportunity to look for a job at the beginning of the search stage. If the worker visits submarket x, he succeeds in finding a job with probability $p(\theta(x;y))$, and he fails with probability $1-p(\theta(x;y))$. If he succeeds, he enters the production stage in a new employment relationship which gives him the lifetime utility x. If he fails, he enters the production stage in the same employment position that he previously held, which gives him the lifetime utility v. Therefore, conditional on visiting submarket x, the worker's lifetime utility at the beginning of the search stage is $v+p(\theta(x;y))\,(x-v)$. Conditional on choosing x optimally[8], the worker's lifetime utility is $v+D(v;y)$, where

$$D(v;y) = \max_x p(\theta(x;y))(x-v). \tag{R1}$$

Denote $m(v;y)$ as the solution for x of the maximization problem in (R1).

3.2 Worker's Value of Unemployment

Consider an unemployed worker at the beginning of the production stage. In the current period, the worker produces and consumes b units of output. In the next period, the worker enters the search stage without a job and has the opportunity to look for one with probability λ_u. Therefore, the worker's lifetime utility $U(y)$ is equal to

$$U(y) = b + \beta\mathbb{E}[U(\hat{y}) + \lambda_u D(U(\hat{y});\hat{y})]. \tag{R2}$$

[8] This qualification is relevant. When the worker is unemployed, he chooses x to maximize his lifetime utility. However, when the worker is employed, he chooses x according to the prescriptions of his labor contract, rather than to maximize his lifetime utility.

Throughout this paper, $\mathbb{E}$ denotes the conditional expectation on $\hat{y}$, calculated with the distribution $\phi(\hat{y}|y)$.

3.3 Joint Value of a Match

Consider a matched pair of a firm and a worker at the beginning of the production stage. The history of their match is $\{z, y^t\}$. Let $a = \{w, \tau, n\} \cup \hat{a}$ denote the allocation prescribed by their employment contract after the history $\{z; y^t\}$ has realized.

In the current period, the worker consumes w units of output. During the next separation stage, the worker loses his job with probability τ, and keeps it with probability $1-\tau$. In the first case, the worker enters the search stage unemployed and does not have the opportunity to look for a new job. In the second case, the worker enters the search stage employed and, with probability λ_e, he has the opportunity to look for an alternative job in submarket n. Therefore, the worker's lifetime utility $W(z; y|a)$ is equal to

$$\begin{aligned} W(z; y|a) = \; & w + \beta\mathbb{E}\{\tau(\hat{y})U(\hat{y}) + [1-\tau(\hat{y})]W(z; \hat{y}|\hat{a}(\hat{y}))\}+ \\ & +\beta\mathbb{E}\left\{[1-\tau(\hat{y})]\lambda_e p(\theta(n(\hat{y}); \hat{y}))\left[n(\hat{y}) - W(z; \hat{y}|\hat{a}(\hat{y}))\right]\right\}. \end{aligned} \tag{R3}$$

In the current period, the firm's profit is $y+z-w$. During the next separation stage, the firm loses the worker with probability τ. During the next matching stage, the firm loses the worker with probability $(1-\tau)\lambda_e p(\theta(n))$. The probability that the firm keeps the worker until the next production stage is $(1-\tau)\,(1-\lambda_e p(\theta(n)))$. Therefore, the firm's lifetime profits $J(z; y|a)$ are equal to

$$J(z; y|a) = y + z - w + \beta\mathbb{E}\left\{[1-\tau(\hat{y})]\left[1-\lambda_e p(\theta(n(\hat{y}); \hat{y}))\right] J(z; \hat{y}|\hat{a}(\hat{y}))\right\}. \tag{R4}$$

Now, consider the hypothetical problem of choosing the allocation a in order to maximize the sum of the worker's lifetime utility and the firm's lifetime profits from the match. As we prove in the appendix, the maximized joint value of the match $V(z; y)$ is

$$\begin{aligned} V(z; y) = \max_{w,\tau,n} \; & y + z + \beta\mathbb{E}\{\tau(\hat{y})U(\hat{y}) + [1-\tau(\hat{y})]\,V(z; \hat{y})\}+ \\ & +\beta\lambda_e\mathbb{E}\{[1-\tau(\hat{y})]\,p(\theta(n(\hat{y}); \hat{y}))[n(\hat{y}) - V(z; \hat{y})]\}, \\ & w \in \mathbb{R}, \quad \tau : Y \rightarrow [\delta, 1]\,, \quad n : Y \rightarrow \mathbb{R}. \end{aligned} \tag{R5}$$

From equation (R5), we can immediately derive the properties of the allocation

$a^*(z;y) = \{w_t^*, \tau_t^*, n_t^*\}_{t=0}^{\infty}$ that maximizes the joint value of the match. At the separation stage, $a^*(z;y)$ specifies that the worker and the firm should voluntarily break up if and only if the sum of their values is greater when they are apart than when they are together. That is, $\tau_{t-1}^*(y^t) = 1$ iff $U(y_t)$ is greater than $V(z;y_t) + \lambda_e D(V(z;y_t), y_t)$, and $\tau_t^*(y_t) = \delta$ otherwise. At the search stage, the allocation specifies that the worker should visit the submarket that maximizes the product between the probability of finding a job and the worker's and firm's joint value from finding a job, i.e. $n_{t-1}^*(y^t) = m(V(z;y_t);y_t)$. Finally, since the wage is just a transfer from the firm to the worker and both parties are risk neutral, the allocation may specify any $\{w_t^*\}_{t=0}^{\infty}$. Therefore, the allocation $a^*(z;y)$ may attain any division of the joint value of the match $V(z;y)$ between the firm and the worker.

3.4 Firm's Value of a Meeting

When a firm meets a worker in submarket x, it chooses an employment contract that maximizes its expected profits subject to providing the worker with the lifetime utility x. Formally, the firm solves the problem

$$\begin{array}{ll} \max_{\underline{a} \in A^{N_z}} & \sum_i J(z_i;y|\underline{a}(z_i))f(z_i), \\ \text{s.t.} & \sum_i W(z_i;y|\underline{a}(z_i))f(z_i) = x. \end{array} \tag{R6}$$

What is the solution to (R6)? First, consider a generic contract $\underline{a}$. Conditional on any realization z of the idiosyncratic component of productivity, the firm's profits $J(z;y|\underline{a}(z))$ cannot be greater than the difference between the maximized joint value of the match, $V(z;y)$, and the worker's lifetime utility, $W(z;y|\underline{a}(z))$. Therefore, if the contract $\underline{a}$ provides the worker with the expected lifetime utility x, the firm's expected profits cannot be greater than $\sum_i V(z_i;y)f(z_i) - x$. Next, consider the contract $\underline{a}^* = \{a^*(z_i;y)\}_i$. Conditional on any realization z of the idiosyncratic component of productivity, the firm's profits $J(z;y|a^*(z;y))$ are equal to the difference between the maximized joint value of the match, $V(z;y)$, and the worker's lifetime utility, $W(z;y|a^*(z;y))$. Therefore, for the appropriate selection of wages, the contract $\underline{a}^*$ provides the worker with the expected lifetime utility x and the firm with the expected profits $\sum_i V(z_i;y)f(z_i) - x$. These observations lead to the following proposition.

Proposition 1 *(Optimal Contract) (i) The firm's value from meeting a worker in submarket* x *is* $\sum_i V(z_i; y)f(z_i) - x$. *(ii) Any employment contract that solves the firm's problem (R6) prescribes the allocation: (a)* $n_{t-1}(z; y^t) = m(V(z; y_t); y_t)$, *for all* $\{z; y^t\} \in Z \times Y^t$, $t = 1, 2, ...$; *(b)* $\tau_{t-1}(z; y^t) = d(z; y_t)$, *for all* $\{z; y^t\} \in Z \times Y^t$, $t = 1, 2, ...$, *where* $d(z; y) = 1$ *iff* $U(y) > V(z; y) + \lambda_e D(V(z; y); y)$ *and* $d^*(z; y) = \delta$ *otherwise.*

Proof. In Appendix B. ■

In the remainder of the paper, we are going to describe the prescriptions of the optimal employment contract with the policy functions $\{d(z; y), m(\upsilon; y)\}$, rather than with the sequence $\{\tau_t, n_t\}_{t=0}^{\infty}$.

3.5 Market Tightness

During the search stage, a firm chooses how many vacancies to create and where to locate them. The firm's benefit of creating a vacancy in submarket x is the product between the probability of meeting a worker, $q(\theta(x; y))$, and the value of meeting a worker, $\sum_i V(z_i; y)f(z_i) - x$. The firm's cost of creating a vacancy in submarket x is k. When the benefit is strictly smaller than the cost, the firm's optimal policy is to create no vacancies in x. When the benefit is strictly greater than the cost, the firm's optimal policy is to create infinitely many vacancies in x. And when the benefit and the cost are equal, the firm's profits are independent from the number of vacancies it creates in submarket x.

In any submarket that is visited by a positive number of workers, the tightness $\theta(x; y)$ is consistent with the firm's optimal creation strategy if and only if

$$q(\theta(x; y)) \left[\sum_i V(z_i; y)f(z_i) - x\right] \leq k, \tag{R7}$$

and $\theta(x; y) \geq 0$, with complementary slackness. In any submarket that workers do not visit, the tightness $\theta(x; y)$ is consistent with the firm's optimal creation strategy if and only if $q(\theta(x; y)) \cdot [\sum_i V(z_i; y)f(z_i) - x]$ is smaller or equal than k. Following most of the literature on directed search (e.g. Acemoglu and Shimer 1999, Shi 2006, Menzio 2007), we restrict attention to equilibria in which the tightness $\theta(x; y)$ satisfies condition (R7) in all submarkets.[9]

3.6 Laws of Motion

From the optimal policy functions, we can compute the probability that a worker transits from one employment state to the other. First, consider a worker who is unemployed at the beginning of the period. Let $\theta_u(y)$ denote $\theta(m(U(y);y);y)$. Then, at the end of the period, the worker is still unemployed with probability $1-\lambda_u p(\theta_u(y))$, and he is employed at job of type $\hat{z}$ with probability $\lambda_u p(\theta_u(y))f(\hat{z})$. Next, consider a worker who is employed at a job of type z at the beginning of the period. Let $\theta_z(z;y)$ denote $\theta(m(V(z;y);y);y)$. Then, at the end of the period, the worker is unemployed with probability $d(z;y)$. He is employed at a job of type $\hat{z} \neq z$ with probability $[1-d(z;y)]\ \lambda_e p(\theta_z(z;y))f(\hat{z})$, and at a job of type z with probability $[1-d(z;y)]\ \{1-\lambda_e p(\theta_z(z;y))[1-f(z)]\}$.

From these transition probabilities, we can compute the laws of motion for the measure of unemployed workers and for the measure of workers employed at each idiosyncratic productivity z. In particular, the measure of workers who are unemployed at the end of the period is:

$$\hat{u} = u(1-\lambda_u p(\theta_u(y))) + \textstyle\sum_i d(z_i;y)g(z_i). \tag{R8}$$

Similarly, the measure of workers who, at the end of the period, are employed at a job with idiosyncratic productivity z is:

$$\hat{g}(z) = h(\psi)f(z) + (1-d(z;y))(1-\lambda_e p(\theta_z(z;y)))g(z). \tag{R9}$$

The function $h(\psi)$ denotes the measure of workers who are hired during the matching stage and is given as follows:

$$h(\psi) = u\lambda_u p(\theta_u(y)) + \textstyle\sum_i (1-d(z_i;y))\lambda_e p(\theta_z(z_i;y))g(z_i).$$

[9] This restriction is made without loss in generality. To see why, consider an equilibrium in which submarket x_0 is not visited by any workers and its tightness $\theta(x_0)$ is such that $\theta(x_0) > 0$ and $q(\theta(x_0))[\sum_i V(z_i;y)f(z_i) - x_0] < k$. Then, modify the equilibrium by replacing $\theta(x_0)$ with $\tilde{\theta}(x_0)$, where $\tilde{\theta}(x_0)$ is the tightness of submarket x_0 that satisfies condition (R7). In this modified equilibrium, the workers' search strategy is unchanged because $\tilde{\theta}(x_0)$ is smaller than $\theta(x_0)$. In this modified equilibrium, the firms' creation startegy is unchanged because $q(\tilde{\theta}(x_0))\,[\sum_i V(z_i;y)f(z_i) - x_0]$ is smaller than k.

3.7 Definition of Equilibrium

The previous paragraphs motivate the following definition of equilibrium.

DEFINITION 1: *A Block Recursive Equilibrium (BRE) consists of a market tightness function* $\theta^* : \mathbb{R} \times Y \to \mathbb{R}_+$*; a search value function* $D^* : \mathbb{R} \times Y \to \mathbb{R}$*, and policy function* $m^* : \mathbb{R} \times Y \to \mathbb{R}$*; an unemployment value function* $U^* : Y \to \mathbb{R}$*; a match value function* $V^* : Z \times Y \to \mathbb{R}$*; a separation function* $d^* : Z \times Y \to \mathbb{R}$*; and the laws of motion* $\hat{u}^* : \Psi \to [0, 1]$*, and* $\hat{g}^* : Z \times \Psi \to [0, 1]$ *for unemployment and employment. These functions satisfy the following requirements:*

(i) *For all* $x \in \mathbb{R}$ *and all* $\psi \in \Psi$*,* θ^* *satisfies the functional equation (R7);*

(ii) *For all* $V \in \mathbb{R}$ *and all* $\psi \in \Psi$*,* D^* *satisfies the functional equation (R1), and* m^* *is the associated optimal policy function;*

(iii) *For all* $\psi \in \Psi$*,* U^* *satisfies the functional equation (R2);*

(iv) *For all* $z \in Z$ *and all* $\psi \in \Psi$*,* V^* *satisfies the functional equation (R6), and* d^* *is the associated optimal policy function;*

(v) *For all* $\psi \in \Psi$*,* $\hat{u}^*$ *and* $\hat{g}^*$ *satisfy the equations (R8) and (R9).*

4 Existence and Efficiency of Equilibrium

In this section, we prove existence, uniqueness and efficiency of a Block Recursive Equilibrium. To this aim, we first formulate the problem of the social planner and characterize its solution. Next, we prove that, if a Block Recursive Equilibrium exists, then it generates the same allocation that solves the planner's problem. Moreover, we prove that a BRE can always be built from the solution to the planner's problem. We conclude the section by providing a qualitative characterization of the equilibrium in and out of steady state.

4.1 Social Planner's Problem

At the beginning of the period, the social planner observes the state of the economy $\psi = \{y, u, g\}$. At the separation stage, he chooses the destruction probability $d(z)$ for matches with idiosyncratic productivity z, $d : Z \to [\delta, 1]$. At the search stage, he chooses the tightness θ_u for the submarket where he sends unemployed workers to look for jobs, $\theta_u \in \mathbb{R}_+$, and the tightness $\theta_z(z)$ for the submarket where he sends workers employed on jobs of type z to look for better jobs, $\theta_z : Z \to \mathbb{R}_+$. The choices of d, θ_u and θ_z determine the distribution of workers across employment states at the production stage and, hence, at the beginning of next period. The social planner's objective is to maximize the sum of current and future aggregate consumption discounted at the rate β. Denote the planner's value function as $s^0(\psi)$. The planner's problem is

$$\begin{aligned} & s^0(\psi) = \max_{d,\theta_u,\theta_z} F(d, \theta_u, \theta_z|\psi) + \beta \mathbb{E} s^0(\hat{\psi}) \\ \text{s.t.} \quad & \hat{u} = u\left[1 - \lambda_u p(\theta_u)\right] + \textstyle\sum_i d(z_i) g(z_i), \\ & \hat{g}(z) = h(\psi) f(z) + \left[1 - d(z)\right]\left[1 - \lambda_e p(\theta_z(z))\right] g(z), \\ & h(\psi) = \lambda_u p(\theta_u) u + \lambda_e \textstyle\sum_i \left[1 - d(z_i)\right] p(\theta_z(z_i)) g(z_i), \end{aligned} \tag{P1}$$

where F is the current period's aggregate consumption given by

$$F(d, \theta_u, \theta_z|\psi) = \hat{u}b + \textstyle\sum_i (y + z_i)\hat{g}(z_i) - k[\lambda_u u \theta_u + \lambda_e \sum_i (1 - d(z_i)) g(z_i) \theta_z(z_i)].$$

The planner's value function $s^0(\psi)$ is linear in both the measure u of workers who are unemployed and the measure $g(z)$ of workers who are employed at jobs with idiosyncratic productivity z. That is,

$$s^0(\psi) = s_u^0(y) u + \textstyle\sum_i s_z^0(z_i; y) g(z_i). \tag{P2}$$

The coefficient $s_u^0(y)$ can be interpreted as the difference between the present value of output produced by a worker who is currently unemployed and the present value of output invested in creating vacancies for him. Similarly, the coefficient $s_z^0(z; y)$ can be interpreted as the present value of net output produced by a worker who is currently employed at a job of type z. In line with basic economic intuition, the coefficient $s_z^0(z; y)$ is increasing in z. These properties of the planner's value function are established in the following proposition.

Proposition 2 *(Social Planner's Problem) (i) The value of the plan $s^0 : \Psi \to \mathbb{R}$ is the unique solution to the functional equation (P1). (ii) There exist functions $s_u^0 : Y \to \mathbb{R}$ and $s_z^0 : Z \times Y \to \mathbb{R}$ such that the value of the plan $s^0(y, u, g)$ is equal to $s_u^0(y)u + \sum_i s_z^0(z_i; y)g(z_i)$. (iii) The function $s_z^0(z_i; y)$ is non-decreasing in z.*
Proof. In Appendix C. ■

The planner's assignment of vacancies to the submarket with unemployed workers is optimal only if

$$k \geq p'(\theta_u)\{y - b + \beta\mathbb{E}[\textstyle\sum_i s_z^0(z_i; \hat{y})f(z_i) - s_u^0(\hat{y})]\} \tag{P3}$$

and $\theta_u \geq 0$, with complementary slackness. This condition is easy to understand. The left hand side of (P3) is the cost of assigning an extra vacancy to the submarket with unemployed workers. The right hand side of (P3) is the expected benefit from such an extra vacancy, given by the product of two terms. The first term, $p'(\theta_u)$, is the number of unemployed workers who find a job because of the extra vacancy. The second term is the difference between the present value of net output produced by an employed and an unemployed worker, measured at the production stage. Notice that, since the left hand side is independent from θ_u and the right hand side is strictly decreasing, the optimality condition (P3) admits a unique solution in each aggregate state ψ. Moreover, since (P3) depends on the aggregate state of the economy only through y, the optimal policy is a function $\theta_u^0 : Y \to \mathbb{R}_+$.

The planner's assignment of vacancies to the submarket with workers who are employed at jobs of type z is optimal only if

$$k \geq p'(\theta_z(z))\{-z + \beta\mathbb{E}[\textstyle\sum_i s_z^0(z_i; \hat{y})f(z_i) - s_z^0(z; \hat{y})]\} \tag{P4}$$

and $\theta_z(z)$, with complementary slackness. The interpretation of the optimality condition (P4) is similar to that of (P3), except that the extra vacancy is assigned to a submarket populated by workers who are employed at jobs with idiosyncratic productivity z rather than unemployed. As it is the case for (P3), the optimality condition (P4) admits a unique solution for $\theta_z(z)$ in each aggregate state ψ. Moreover, since (P4) depends on the aggregate state of the economy ψ only through y, the optimal policy is a function $\theta_z^0 : Z \times Y \to \mathbb{R}_+$.

The planner's choice of the destruction probability for matches with idiosyncratic productivity z is optimal if and only if $d(z) = 1$ whenever

$$\begin{aligned} b + \beta \mathbb{E} s_u^0(\hat{y}) > \quad & -\lambda_e k \theta_z^0(z;y) + \left[1 - \lambda_e p(\theta_z^0(z;y))\right] \left[y + z + \beta \mathbb{E} s_z^0(z;\hat{y})\right] + \\ & + \lambda_e p(\theta_z^0(z;y)) \left\{ y + \beta \mathbb{E} \left[\textstyle\sum_i s_z^0(z_i;\hat{y}) f(z_i) \right] \right\}, \end{aligned} \tag{P5}$$

and $d(z) = \delta$ otherwise. The interpretation of this condition is straightforward. The left hand side of (P5) is the present value of net output produced by a worker who is unemployed at the beginning of the production stage. The right hand side of (P5) is the present value of net output produced by a worker who is employed at a job with idiosyncratic productivity z at the beginning of the search stage. Clearly, the optimality condition (P5) admits only one solution for $d(z)$ in each aggregate state ψ. Moreover, since (P5) depends on the aggregate state of the economy ψ only y, the optimal policy is a function $d^0 : Z \times Y \to [\delta, 1]$.

Finally, the derivative of the social planner's value function with respect to the measure of unemployed workers is:

$$\begin{aligned} s_u^0(y) = \quad & -k\lambda_u \theta_u^0(y) + \left[1 - \lambda_u p(\theta_u^0(y))\right] \left[b + \beta \mathbb{E} s_u^0(\hat{y})\right] + \\ & + \lambda_u p(\theta_u^0(y)) \left\{ y + \beta \mathbb{E} \left[\textstyle\sum_i s_z^0(z_i;y_+) f(z_i) \right] \right\}. \end{aligned} \tag{P6}$$

Similarly, the derivative of the social planner's value function with respect to the measure of workers employed at jobs of type z is:

$$\begin{aligned} s_z^0(z;y) = \quad & d^0(z;y) \left[b + \beta \mathbb{E} s_u^0(\hat{y})\right] - \left[1 - d^0(z;y)\right] k \lambda_e \theta_z^0(z;y) + \\ & + \left[1 - d^0(z;y)\right] \left[1 - \lambda_e p(\theta_z^0(z;y))\right] \left[y + z + \beta \mathbb{E} s_z^0(z;\hat{y})\right] + \\ & + \left[1 - d^0(z;y)\right] \lambda_e p(\theta_z^0(z;y)) \left\{ y + \beta \mathbb{E} \left[\textstyle\sum_i s_z^0(z_i;\hat{y}) f(z_i) \right] \right\}. \end{aligned} \tag{P7}$$

4.2 Equilibrium Allocation

Denote with $\{D^*, m^*, U^*, V^*, d^*, \theta^*\}$ a Block Recursive Equilibrium. The market tightness function $\theta^*(x;y)$ is derived from the equilibrium condition (R7). In particular, let $\tilde{x}(y)$ denote the difference between the firm's and worker's joint value of a match and the cost of a vacancy, i.e. $\tilde{x}(y) \equiv \sum_i V^*(z_i;y) f(z_i) - k$. In all of the submarkets where workers are offered less than $\tilde{x}(y)$, the equilibrium tightness is strictly positive and such that the firm's benefit from opening a vacancy is equal to the cost. As the lifetime utility offered to the workers approaches $\tilde{x}(y)$, the equilibrium tightness converges towards zero. In all of the submarkets where workers are offered more than $\tilde{x}(y)$, $\theta^*(x;y)$ is equal to zero. Formally, the equilibrium market tightness is:

$$\theta^*(x;y) = \begin{cases} q^{-1}\left(k/(\sum_i V^*(z_i;y)f(z_i) - x)\right) & \text{if } x \leq \tilde{x}(y), \\ 0 & \text{if } x > \tilde{x}(y). \end{cases} \tag{E1}$$

The search policy function $m^*(v;y)$ satisfies the equilibrium condition (R1). That is, $m^*(v;y)$ maximizes the product between the worker's probability of finding a job, i.e. $p(\theta^*(x;y))$, and the worker's value of taking the job and leaving his previous employment position, i.e. $x - v$. Equation (E1) implies that the worker's probability of finding a job is zero in all submarkets $x > \tilde{x}(y)$. Equation (E1) also implies that, in all submarkets $x \leq \tilde{x}(y)$, the worker's value of a job is equal to the difference between the worker's and firm's joint value of a match and the firm's expected cost of creating a match, i.e. $x = \sum_i V^*(z_i;y)f(z_i) - k/q(\theta^*(x;y))$. Therefore, the search policy function is:

$$m^*(v;y) \in \arg\max_x \{-k\theta^*(x;y) + p(\theta^*(x;y)) \left[\sum_i V^*(z_i;y)f(z_i) - v\right]\}. \tag{E2}$$

In equilibrium, whenever an unemployed worker has the opportunity to search, he visits submarket $m^*(U^*(y);y)$. Let $\theta_u^*(y)$ denote the tightness of this submarket. In equilibrium, whenever a worker employed at a job with idiosyncratic productivity z has the opportunity to search, he visits submarket $m^*(V^*(z;y);y)$. Let $\theta_z^*(z;y)$ denote the tightness of this submarket. From equation (E2), it follows that the tightness $\theta_u^*(y)$ satisfies the condition

$$k \geq p'(\theta_u^*(y)) \left[\sum_i V^*(z_i;y)f(z_i) - U^*(y)\right] \tag{E3}$$

and $\theta_u^*(y) \geq 0$, with complementary slackness. Similarly, from equation (E2), it follows that the tightness $\theta^*(z;y)$ satisfies the condition

$$k \geq p'(\theta^*(x;y)) \left[\sum_i V^*(z_i;y)f(z_i) - V^*(z;y)\right] \tag{E4}$$

and $\theta_z^*(z;y) \geq 0$, with complementary slackness.

In equilibrium, the lifetime utility of an unemployed worker is $U^*(y)$ at the beginning of the production stage. Let $s_u^*(y)$ denote the lifetime utility of an unemployed worker at the beginning of the separation stage, i.e. $s_u^*(y) = U^*(y) + \lambda_u D(U^*(y); y)$. In equilibrium, the worker's and firm's joint value of a match is $V^*(z; y)$ at the beginning of the production stage. Let $s_z^*(z; y)$ denote the worker's and firm's joint value of a match at the beginning of the separation stage, i.e. $s_z^*(z; y)$ equals the sum between $d^*(z; y) \cdot U^*(z; y)$ and $(1 - d^*(z; y))[V^*(z; y) + \lambda_e D^*(V^*(z; y); y)]$. Then, the equilibrium condition (R2) implies that

$$\begin{aligned} s_u^*(y) = \;& -k\lambda_u \theta_u^*(y) + [1 - \lambda_u p(\theta_u^*(y))] \left[b + \beta \mathbb{E} s_u^*(\hat{y})\right] + \\ & + \lambda_u p(\theta_u^*(y)) \left\{y + \beta \mathbb{E}\left[\textstyle\sum_i s_z^*(z_i; \hat{y}) f(z_i)\right]\right\}. \end{aligned} \tag{E5}$$

And the equilibrium condition (R5) implies that

$$\begin{aligned} s_z^*(z; y) = \;& d^*(z; y) \left[b + \beta \mathbb{E} s_u^*(\hat{y})\right] - [1 - d^*(z; y)]\, k\lambda_e \theta_z^*(z; y) + \\ & + [1 - d^*(z; y)] \left[1 - \lambda_e p(\theta_z^*(z; y))\right] \left[y + z + \beta \mathbb{E} s_z^*(z; \hat{y})\right] + \\ & + [1 - d^*(z; y))\lambda_e p(\theta_z^*(z; y))] \left\{y + \beta \mathbb{E}\left[\textstyle\sum_i s_z^*(z_i; \hat{y}) f(z_i)\right]\right\}. \end{aligned} \tag{E6}$$

where $d^*(z; y)$ is equal to 1 if

$$\begin{aligned} b + \beta \mathbb{E} s_u^*(\hat{y}) > \;& -\lambda_e k \theta_z^*(z; y) + [1 - \lambda_e p(\theta_z^*(z; y))] \left[y + z + \beta \mathbb{E} s_z^*(z; \hat{y})\right] + \\ & + \lambda_e p(\theta_z^*(z; y)) \left\{y + \beta \mathbb{E}\left[\textstyle\sum_i s_z^*(z_i; \hat{y}) f(z_i)\right]\right\}, \end{aligned} \tag{E7}$$

and $d^*(z; y) = \delta$, otherwise.

At this point, the reader may have recognized that the equilibrium objects $\{d^*, \theta_u^*, \theta_z^*, s_u^*, s_z^*\}$ satisfy the same system of equations that is satisfied by the solution to the social planner's problem $\{d^0, \theta_u^0, \theta_z^0, s_u^0, s_z^0\}$. This system of equations admits only one solution. Therefore, any Block Recursive Equilibrium is efficient. Moreover, the equations (E3)–(E7) are not only necessary for a Block Recursive Equilibrium, but they are also sufficient. Therefore, an equilibrium can always be constructed from the solution to the social planner's problem. We summarize these findings as the paper's main theoretical result.

Theorem 3 *(Existence, Uniqueness and Efficiency) (i) A Block Recursive Equilibrium exists. (ii) Let $\{D^*, m^*, U^*, V^*, d^*, \theta^*\}$ be a Block Recursive Equilibrium. Let $\theta_u^*(y)$ denote $\theta^*(m^*(U^*(y); y); y)$, and let $\theta_z^*(z; y)$ denote $\theta^*(m^*(V^*(z; y); y); y)$. Then, the equilibrium allocation $\{\theta_u^*, \theta_z^*, d^*\}$ is equal to the social planner's allocation $\{\theta_u^0, \theta_z^0, d^0\}$.*
Proof: In the Appendix D. ■

The efficiency of the equilibrium is an intuitive result. Complete contracts guarantee that, whenever an employed worker has to make a choice, he takes into account the effect of his decision on the profits of his current employer. Moreover, directed search guarantees that the worker's value in submarket x is equal to the joint value of a match to the worker and his prospective employer net of the cost of creating a match in a submarket with tightness $\theta(x; y)$.

A surprising result is the existence of an equilibrium in which the agents' value and policy functions and the market tightness function do not depend on the distribution of workers across employment states. Given the equivalence between the equilibrium allocation and the plan, we can provide some intuition for this result by looking at the social planner's problem.

For example, consider the planner's choice of θ_u. The cost of assigning $\theta_u u$ vacancies to the submarket visited by unemployed workers is $k\theta_u$ per worker. This cost does not depend on the distribution of workers across employment states because the technology for creating vacancies is linear. For each unemployed worker, the probability of becoming employed is $p(\theta_u)$. This probability does not depend on the number of workers who are unemployed because the matching process between vacancies and applicants features constant returns to scale. This probability does not depend on the number of workers who are in other employment states, because different workers visit different submarkets. Finally, the additional output produced by each unemployed worker who becomes employed is independent from the workers' distribution because the production technology is linear in labor (both at home and in the market). Since the planner's objective function is independent from the distribution of workers across employment states, so are the optimal policy function $\theta_u^0(y)$ and the value function $s_u^0(y)$. The reader should notice that, for the previous argument to hold, it is critical that different workers search in different submarkets. That is, it is critical that search is directed.

4.3 Characterization of Equilibrium

Now, we are in the position to characterize the equilibrium of our model economy. Equation (E3) implies that the tightness of the submarket visited by an unemployed worker is an increasing function of the difference between the value of a new match, i.e.

$\sum V^*(z_i; y) f(z_i)$, and the value of unemployment, i.e. $U^*(y)$. Equation (E4) implies that the tightness of the submarket visited by an employed worker is an increasing function of the difference between the value of a new match and the value of his current match. Since the value of a match is increasing in the idiosyncratic component of its productivity, $\theta_z^*(z; y)$ is a decreasing function of z.

Equation (E7) characterizes the workers' transitions from employment to unemployment. In particular, an employed worker becomes unemployed with probability 1 if the value of his match at the beginning of the separation stage is smaller than the value of unemployment. Otherwise, he becomes unemployed with probability δ. Since the value of a match is strictly increasing in the idiosyncratic component of productivity, there exists a $z^{eu}(y)$ such that $d^*(z; y) = 1$ for all $z < z^{eu}(y)$ and $d^*(z; y) = \delta$ for all.$z \geq z^{eu}(y)$.

Even though we are not able to characterize analytically the relationship between $\{d^*, \theta_u^*, \theta_z^*\}$ and y, we can easily compute it. For the parameter values in Table 2, the difference between the value of a match and the value of unemployment is increasing in the aggregate component of productivity. On the one hand, this implies that the tightness of the submarket visited by unemployed workers is an increasing function of y. On the other hand, this implies that the probability that a worker employed at a job of type z is a decreasing function of y.

For the parameter values in Table 2, the difference between the value of a new match and the value of a match with a relatively low idiosyncratic productivity is increasing in y. The difference between the value of a new match and a relatively high productivity match is decreasing in y. Therefore, the effect that a positive shock to aggregate productivity has on the tightness of the submarket visited by an employed worker depends on the quality of his job.

5 Calibration

We begin this section by describing the dataset that we are going to use to calibrate our model. This dataset includes all the information used by Shimer (2005) to calibrate the textbook search model of Pissarides (1985). However, since our model has more parameters than Pissarides', the dataset contains additional information about the job-to-job transition rate and the tenure distribution. In the second part of the section, we describe and motivate the calibration strategy. In particular, we explain why we can recover the distribution of idiosyncratic productivities from the tenure distribution. In the last part of the section, we report the results of the calibration.

5.1 Data

We measure quarterly productivity as the CPS output per worker in the non-farm business sector, and unemployment as a 3-month average of the CPS monthly rate of unemployment in the civilian population. We construct the cyclical component of these two variables as the difference between the log of the raw data and an HP trend (with smoothing parameter 1600). Over the period between 1951(I) and 2006(II), the average of our measure of productivity is 82 (100 being productivity in 1992) and the average of our measure of unemployment is 5.6 percent. Over the same period, the cyclical components of productivity and unemployment move together. However, cyclical unemployment is more than 10 times as volatile as productivity. These and other statistics are reported in Table 1.

We measure the rate at which employed workers become unemployed (the EU rate) as well as the rate at which unemployed workers become employed (the UE rate) using the methodology developed by Shimer (2005). Specifically, we measure the EU rate in month t as $h_t^{eu} = u_{t+1}^s/(1-u_t)$, where u_{t+1}^s is the CPS short-term unemployment rate in month $t+1$, and u_t is the CPS unemployment rate.[10] We measure the UE rate in month t as $h_t^{ue} = 1-(u_{t+1}-u_{t+1}^s)/u_t$. Then, we construct the quarterly transition rates by taking 3-month averages of h_t^{eu} and h_t^{ue}. Over the period between 1951(I) and 2006(II), the average EU rate is 2.6 percent, and the average UE rate is 45 percent. Over this period, the cyclical component of the EU rate is positively correlated with cyclical unemployment and it is approximately 60 percent as volatile. The cyclical component of the UE rate is negatively correlated with unemployment and it is approximately 65

[10]The CPS defines the short-term unemployment rate as the ratio between the number of civilians who have been unemployed for 0 to 4 weeks and the civilian labor force. However, with the 1994 redesign of the CPS, there has been a change in the measurement of the duration of unemployment. As discussed in Elsby, Michaels and Solon (2007), the change in the measurement can be corrected by multiplying the official short-term unemployment by 1.15 in each month from February 1994 on.

percent as volatile.

The rate at which workers move from employer to employer is measured by Nagypál (2008) from the CPS microdata. Specifically, she measures the EE rate in month t as $h_t^{ee} = f_t^{ee}/e_t$, where f_t^{ee} is the number of workers who are employed at different firms in months t and $t+1$, and e_t is the number of workers who are employed in month t. Over the period between 1994(I) and 2006(II), the average EE rate is 2.9 percent. Over the same period, the cyclical component of the EE rate is negatively correlated with cyclical unemployment and it is approximately 30 percent as volatile. Prior to 1994, Nagypál's measure of the EE rate cannot be constructed because the CPS did not collect data on job-to-job transitions.

We measure vacancies with the Conference Board Help-Wanted Index. Over the period 1951(I)-2006(II), the contemporaneous correlation between cyclical vacancies and cyclical unemployment is -.92. Over the same period, the standard deviation of cyclical vacancies is 10 percent higher than the standard deviation of cyclical unemployment.

Finally, in order to calibrate the probability distribution of the match-specific component of productivity, we use information about the duration of employment relationships in the US labor market. In particular, we use the measure of the distribution of workers across tenure lengths that Diebold, Neumark and Polsky (1997) have constructed from the 1987 CPS tenure supplement. This tenure distribution is plotted in Figure 1.

5.2 Calibration Strategy

With the data described in the previous paragraphs, we need to calibrate the household's preferences $\{b, \beta\}$, the search technology $\{\lambda_u, \lambda_e, p, \delta\}$, and the production technology $\{k, Z, f, Y, \phi\}$. For the sake of simplicity, we restrict attention to job-finding probability functions of the form $p(\theta) = \min\{1, \theta^\gamma\}$, $\gamma \in (0,1)$. We also restrict the distribution of the idiosyncratic component of productivity to be a 1,000 point approximation of a Weibull distribution with mean μ_z, scale σ_z, and shape α_z.[11] The aggregate component of productivity obeys a 3-state Markov process with unconditional mean μ_y, standard

[11] The Weibull density function is:

$$f(z) = \frac{\alpha_z}{\sigma_z}\left(\frac{z-\mu_z}{\sigma_z} + \Gamma\left(\frac{1}{\alpha_z}+1\right)\right)^{\alpha_z - 1} \exp\left[-\left(\frac{z-\mu_z}{\sigma_z} + \Gamma\left(\frac{1}{\alpha_z}+1\right)\right)^{\alpha_z}\right],$$

deviation σ_y, and autocorrelation ρ_y. Without loss of generality, we normalize μ_y to 1 and μ_z to 0.

We choose one month as the length of a model period. We set β so that the annual interest rate in the model is 5 percent. We set the vacancy cost k, the scale parameter in the distribution function of the idiosyncratic component of productivity σ_z, and the search probability λ_e so that the average UE, EU and EE rates are the same in the model as in the data (see Table 1). We set the search probability λ_u to 1 because it is difficult to identify it separately from k and λ_e.

Our strategy for calibrating the remaining parameters is less standard and deserves some discussion. In the model, the parameter γ determines the elasticity of the UE rate with respect to the tightness of the submarket visited by unemployed workers, θ_u. Moreover, since a disproportionate number of vacancies are created in this submarket, the parameter γ is positively correlated with the elasticity of the UE rate with respect to the ratio between *total* vacancies and unemployment. Therefore, even without data on θ_u, we are able to identify γ from the coefficient of $\log(v/u)$ in the regression of $\log h^{ue}$.

In the model, the shape parameter in the density function of idiosyncratic productivity, α_z, and the exogenous separation rate, δ, affect the shape of the hazard/tenure profile, i.e., the probability that a worker leaves his job as a function of tenure. A higher α_z reduces the skewness of the probability distribution of the match-specific component of productivity. In turn, this tends to reduce the hazard rate at short tenures (1 to 2 years) and to increase it at medium tenures (2 to 4 years). In contrast, a higher δ increases the hazard rate at all tenures, including long ones (more than 4 years). Therefore, we are able to identify both α_z and δ by minimizing the distance between the tenure distribution generated by the model and its empirical counterpart.[12]

In the model, the ratio between the productivity of labor at home and in the market is $b/(y + \sum_i z_i g(z_i))$. In the US economy, Hall and Milgrom (2008) estimate the ratio between labor productivity at home and in the market to be 71 percent. Therefore, we

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where Γ is the gamma function.With this distribution, we will be able to use α_z in matching the tenure distribution and σ_z in matching the EU rate. In contrast, if $f(z)$ is the normal or the lognormal distribution, one parameter (i.e., the standard deviation) is forced to serve both roles in the calibration.

[12] Moscarini (2003) uses the same strategy and the same data to calibrate an on-the-job search model in which workers and firms receive noisy signals about the unobservable quality of their match.

can identify the parameter b by equating the productivity ratio in the model and in the data[13]. Finally, we choose σ_y and ρ_y so that the average productivity of labor has the same standard deviation and autocorrelation in the model and in the data.

5.3 Calibration Outcomes

Column a in Table 2 contains the results of our calibration. Most notably, we find that employed workers have the opportunity of searching the labor market nearly as often as unemployed workers ($\lambda_e = 0.83$, $\lambda_u = 1$). Yet, the rate at which employed workers move from one employer to the other is 20 times smaller than the rate at which unemployed workers become employed because the latter seek jobs that offer less generous terms of trade and are easier to find.

We also find that there is a great deal of uncertainty about the productivity of a new match. At the ninetieth percentile of the probability distribution $f(z)$, the productivity of a match is twice as large as at the tenth percentile. However, because the survival probability of a match is endogenous, not all of this uncertainty translates into dispersion in the cross-sectional productivity distribution $g(z)$. At the ninetieth percentile of $g(z)$, the productivity of a match is only 1.3 times as large as at the tenth percentile. This process of endogenous selection also creates a large wedge between the expected productivity of a new match and the average of the cross-sectional productivity distribution. In particular, the expected productivity of a new match, $\mu_y + \sum z_i f(z_i)$, is equal to 1, while the cross-sectional average productivity of a match, $apl = (1-u)^{-1} \sum (\mu_y + z_i) g(z_i)$, is 1.37.

6 Business Cycle Analysis

In this section, we use the calibrated model to measure the contribution of aggregate productivity shocks to the cyclical volatility of US unemployment, vacancies and other

[13] Hagedorn and Manovskii (2008) set b so that the average cost of recruiting a worker is the same in the model and in the data. Given this calibration target, Hagedorn and Manovskii find that the relative productivity of labor at home and in the market is approximately 90 percent. If we were to set the productivity ratio to 0.90 rather than 0.71, our model would predict an even larger response of unemplyoment and vacancies to a given shock to the aggregate component of productivity.

labor market variables. Then, we compare these measurements with those that an economist would obtain if he were to use a version of the model in which the EU and the EE rates are exogenous. From this comparison, it will be clear that, in order to properly measure the contribution of shocks to the cyclical volatility of the US labor market, an economist needs a model in which not only the UE, but also the EU and EE rates are endogenous. These two measurement exercises are carried out in the second and third part of the section. In the first part of the section, as a preliminary step, we use the calibrated model to measure the response of the US labor market to a 1 percent increase in the aggregate component of productivity.

6.1 Response to a Productivity Shock

In order to study the response of the labor market to a 1 percent increase in the aggregate component of productivity, we first compute the Block Recursive Equilibrium of our calibrated model. Then, we feed into the model the sequence of realizations of aggregate component of productivity $\{y_t\}$, where $y_t = \mu_y$ for all $t \leq 9,000$ and $y_t = 1.01 \cdot \mu_y$ for all $t > 9,000$. Finally, we calculate the percentage change in unemployment, vacancies and other labor market variables in response to the increase in the aggregate component of productivity.

The firm's and worker's joint value of a match increases when the productivity shock hits the economy. In response to the increase in the value of a match, firms open more vacancies per applicant in every submarket x. In response to the increase in the labor market tightness, unemployed workers choose to visit submarkets in which vacancies offer more generous terms of trade and the probability of trade is higher. Similarly, employed workers choose, on average, to visit submarkets in which both the terms-of-trade and the probability of trade are higher. Therefore, the UE and the EE rate increase. In contrast, the EU rate decreases because the increase in the aggregate component of productivity induces workers and firms to keep matches that previously they would have destroyed. Since the rate at which workers flow out of unemployment decreases and the rate at which workers flow into unemployment increases, the unemployment rate unambiguously falls. More precisely, a 1 percent increase in the aggregate component of productivity leads to a 2 percent increase in the UE rate, a 6 percent increase in the EE rate, a 6 percent decrease in the EU rate, and an 8 percent decrease in the unemployment rate (see Figure 3).

When the productivity shock hits the economy, firms open more vacancies for each unemployed worker. However, since the number of workers who are unemployed falls so much, firms end up opening fewer vacancies for this group of workers. Similarly, when the productivity shock hits the economy, firms create more vacancies for each employed worker. Since the number of employed workers increases, firms increase the number of vacancies opened for this second group of workers. Overall, vacancies increase by approximately 3 percent (see Figure 4).

When the positive shock to the aggregate component of productivity hits the economy, the distribution of employed workers across jobs with different match-specific productivities is subject to two opposing forces. On the one hand, the increase in aggregate productivity induces firms and workers to keep some low productivity matches that they would have previously destroyed. This first force tends to worsen (in the stochastic dominance sense) the distribution of match-specific productivities. On the other hand, in response to the shock, workers employed at low-productivity jobs search in tighter submarkets. This second force tends to improve the distribution of match-specific productivities. Figures 5 and 6 show that the first force dominates the second one.

In Figure 5, we plot the impulse response function of the fraction of workers employed at jobs with idiosyncratic productivity z lower than 0.23 (i.e., the 10th percentile of the ergodic distribution of match-specific productivities), greater than 0.23 and lower than 0.29 (i.e., the 20th percentile of the ergodic distribution of match-specific productivities), and greater than 0.29. In Figure 5, we see that the fraction of workers employed at the least productive class of jobs increases by more than 2 percent; the fraction of workers employed at the intermediate class of jobs increases by 0.5 percent; and the fraction of workers employed at the most productive jobs decreases by 0.5 percent. Overall, the average of the distribution of match-specific productivities falls by 0.4 percent in response to the shock. As a consequence, a 1 percent increase in the aggregate component of productivity does not increase the average productivity of labor by $1 \cdot (\mu_y/apl) = 0.73$

percent, but only by 0.65 percent (see Figure 6).

6.2 Productivity Shocks and Business Cycles

How much of the cyclical fluctuations in the US labor market are driven by aggregate productivity shocks (henceforth, y-shocks)? In order to answer this question, we compute the Block Recursive Equilibrium of our calibrated model. Then, we draw a realization of the calibrated stochastic process for the aggregate component of productivity y, and we compute the quarterly time series of unemployment, vacancies and other labor market variables.[14] Finally, we pass the log of these series through an HP-filter with smoothing parameter 1600.

Table 3 contains a statistical summary of our simulated data. The first lesson that we draw from these tables is that y-shocks generate fluctuations in the EU transition rate that are negatively correlated with the fluctuations in the average productivity of labor and are approximately 8.5 times as large. In addition, y-shocks generate fluctuations in the UE transition rate that are positively correlated with average productivity fluctuations and are 3 times as large. As a result, unemployment moves in the opposite direction of average productivity and it is 10.5 times more volatile.

The second lesson that we draw from Table 3 is that y-shocks generate fluctuations in the number of vacancies created for unemployed workers that are positively correlated with the fluctuations in unemployment and are 0.65 times as volatile. Also, y-shocks generate fluctuations in the number of vacancies created for employed workers that are negatively correlated with the fluctuations in unemployment and are 1.1 times as volatile. Overall, total vacancies move in the opposite direction of unemployment and are approximately 0.4 times as volatile.

By comparing Tables 1 and 3, we find that aggregate productivity shocks account for 40 percent of the UE rate volatility that is observed in the US economy over the period 1951(I) - 2006(II); and they account for approximately all of the observed volatility of the EU transition rate. Overall, aggregate productivity shocks alone can account for more than 80 percent of the observed unemployment volatility. Moreover, we find that y-shocks

[14] Since the model is monthly, we measure the quarterly time series of unemployment, vacancy and transition rates by taking 3-months averages of the monthly rates generated by the model.

account for more than 30 percent of the volatility of vacancies and for the nearly perfectly negative correlation between unemployment and vacancies (i.e., the Beveridge curve). Finally, we find that y-shocks can precisely reproduce the matrix of correlations between unemployment, vacancies and the workers' transition rates across different employment states. In light of these findings, we conclude that aggregate productivity shocks may well be the fundamental source of cyclical fluctuations in the US labor market.

However, aggregate productivity shocks cannot be the only cause of the US business cycles. First of all, y-shocks alone generate a counterfactually strong correlation between average labor productivity and other labor market variables (e.g. unemployment, vacancies, etc.). Second, y-shocks alone generate too much unemployment volatility through fluctuations in the EU rate and too little of it through fluctuations in the UE rate. Finally, y-shocks leave more than half of the observed volatility of vacancies unexplained.

6.3 Comparisons with the Canonical Search Model

At the beginning of this paper, we conjectured that, if an economist wants to properly measure the contribution of aggregate productivity shocks to the cyclical fluctuations of unemployment and vacancies, he should use a model in which not only the UE rate, but also the EU and the EE rates are endogenous.

In order to test this conjecture, we add the constraints $\sigma_z = 0$ and $\lambda_e = 0$ to our model. The first constraint states that the idiosyncratic component of productivity is the same for all matches and, hence, it implies that the EU transition rate is exogenous. The second constraint states that employed workers do not have the opportunity of searching for better jobs and, hence, it implies that the EE transition rate is exogenous. As it turns out, the constrained version of our model coincides with the canonical search model of Pissarides (1985, 2000), and Shimer (2005). We then calibrate the constrained version of our model using the same targets that we used in Section 5.2, with the obvious exclusion of the EE transition rate and the tenure distribution. The results of this calibration are reported as column b in Table 2. Finally, we solve for the Block Recursive Equilibrium of the constrained model, we draw a realization for the stochastic process of y and we compute the time series for unemployment, vacancies and other labor market variables.

The results of this simulation are reported in Table 4.

According to the constrained model, y-shocks generate fluctuations in the unemployment rate that are negatively correlated with the fluctuations in the average productivity of labor and are 0.6 times as volatile. Also, according to the constrained model, y-shocks generate fluctuations in the vacancy rate that are positively correlated with the fluctuations in the average productivity of labor and are 2.5 times as volatile. By comparing these statistics with those reported in Table 3, we conclude that, if an economist uses a version of our model in which the EU and the EE rates are exogenous (i.e., if an economist uses the canonical search model), he is going to dramatically underestimate the fraction of the cyclical volatility of unemployment and vacancies that is caused by aggregate productivity shocks.

Next, we want to understand why the canonical search model and ours produce such different estimates of the contribution of aggregate productivity shocks to the cyclical fluctuations of vacancies and unemployment. First, in our model, when a positive productivity shock hits the economy, the EU transition rate falls because workers and firms become less selective about the idiosyncratic productivity of the matches that they are willing to keep. In the canonical search model, when a positive productivity shock hits the economy, the EU transition rate does not change because all matches are constrained to be identical. For this reason, the same productivity shock tends to generate a smaller decline in unemployment in the canonical search model than in ours (see Figures 3 and 7).

Second, in our model, a positive shock to the aggregate component of productivity leads to a decline in the number of vacancies that are created for unemployed workers and to an increase in the number of vacancies that are created for employed workers. In contrast, in the canonical search model, a positive y-shock leads to an increase in the number of vacancies that firms open for unemployed workers, because the unemployment rate decreases much less than in our model. Moreover, in the canonical search model, a positive y-shock does not affect the number of vacancies created for employed workers because $\lambda_e = 0$. Therefore, the canonical search model distorts in opposite directions the estimates of the effect that a y-shock has on the number of vacancies created for unemployed and employed workers. As a result, the canonical search model distorts only marginally the estimated effect of a y-shock on the total vacancy rate (see Figures 4 and 8).

Third, in our model, a 1 percent increase in the aggregate component of productivity does not increase the average productivity of labor by $1 \cdot (\mu_y/apl) = 0.73$ percent, but only by 0.65 percent because workers and firms become less selective about the quality of the matches that they are willing to keep. In the canonical search model, a 1 percent increase in the aggregate component of productivity translates into a 0.73 percent increase in average productivity because all matches are identical. Since both models are calibrated to match the empirical volatility of the average productivity of labor, y-shocks are approximately 12 percent smaller in the canonical model than in ours. In turn, smaller y-shocks generate smaller fluctuations in unemployment and vacancies.

Fourth, the effect of productivity shocks in the two models differs because the calibrated elasticity of the job-finding probability is different[15]. That is, the two models have different values of the parameter γ in the job-finding probability function $p(\theta) = \min\{\theta^\gamma, 1\}$. In both models, the calibrated value of γ is such that the elasticity of the UE rate with respect to the vacancy/unemployment ratio is the same in the model as in the data, namely 0.22. Therefore, in both models, the calibrated value of γ is equal to $0.22 \cdot [\Delta \log(v/u)/\Delta \log \theta_u]$, where θ_u is the tightness of the submarket visited by unemployed workers. In our model, because the number of vacancies created for employed workers moves together with θ_u, $\Delta \log(v/u)$ is greater than $\Delta \log \theta_u$. As a result, the calibrated value of γ is 0.65. In the canonical model, because workers are not allowed to search on the job, v/u is equal to θ_u and so γ is equal to 0.22. In turn, a smaller γ implies that the EU rate (and, consequently, the unemployment rate) is less responsive to a given shock to the aggregate component of productivity.

From this discussion, it is clear that, in order to properly measure the contribution of y-shocks to the cyclical fluctuations of the US labor market, an economist needs to

[15] Petrongolo and Pissarides (2001) prove that, if the number of employed job-seekers is procyclical, the coefficient of $\log(v/u)$ in the regression of $\log(h^{ue})$ provides a downward biased estimate of the elasticity γ of the job-finding probability function with respect to vacancies. Based on this theoretical argument, Menzio (2005) and Nagypál (2007) simulate their models of on-the-job search by using a value of γ that is higher than the coefficient of $\log(v/u)$ in the regression of $\log(h^{ue})$. However, unlike in this paper, neither Menzio (2005) nor Nagypál (2007) attempt to calibrate the value of γ.

endogenize both the EU and the EE rate along with the UE rate. For example, if an economist uses a version of our model in which the UE and the EU rates are endogenous, but the EE rate is exogenous (because λ_e is constrained to be 0), he underestimates the elasticity of the job-finding probability with respect to the vacancy/applicant ratio. For this reason, he underestimates the contribution of y-shocks to the volatility of the UE rate and, consequently, of the unemployment rate. Moreover, he ignores the effect that y-shocks have on the number of vacancies created for employed workers. For this reason, he incorrectly concludes that y-shocks generate fluctuations in unemployment and vacancies that are positively correlated[16].

7 Conclusions

In the first part of this paper, we have built a directed search model of the labor market in which the workers' transitions between employment, unemployment and across employers are endogenous. For this model, we have proved existence, uniqueness and efficiency of a recursive equilibrium with the property that the distribution of workers across different jobs is a state variable which does not affect the agents' value and policy functions, or the tightness function. Because of this property, the computation of the efficient equilibrium is as simple as the computation of the equilibrium of a model without heterogeneity.

In the second paper of this paper, we have calibrated our model to match the features of workers' turnover in the US labor market over the period 1951(I)-2006(II). Then, we have used the calibrated model to measure the effect of aggregate productivity shocks on the volatility of unemployment and vacancies. We have found that aggregate productivity shocks alone account for approximately 50 percent of the cyclical fluctuations in the UE transition rate and for all of the cyclical fluctuations in the EU transition rate. As a result, productivity shocks alone can explain more than 80 percent of the cyclical volatility of unemployment. We have found that productivity shocks generate large countercyclical fluctuations in the number of vacancies created for unemployed workers and larger procyclical fluctuations in the number of vacancies created for employed workers. Overall, productivity shocks alone can account for 30 percent of the cyclical

[16] All the details about this measurement exercise are available upon request.

volatility of vacancies, as well as for the strong negative correlation between vacancies and unemployment.

By comparing these measurements with those derived using the canonical search model of Pissarides (1985), we have vindicated our initial conjecture. That is, in order to properly assess the effect of productivity shocks on unemployment and vacancies, an economist needs a model, such as ours, in which the workers' transitions between employment, unemployment and across employers are all endogenous.

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A Joint Value of a Match

The definition of $V(z;y)$ is

$$V(z;y) = \max_{a \in A}[W(z;y|a) + J(z;y|a)]. \tag{A1}$$

First, notice that the allocation $a = \{w, \tau, n\} \cup \hat{a}$ belongs to the set A if and only if $w \in \mathbb{R}$, $\tau : Y \to [\delta, 1]$, $n : Y \to \mathbb{R}$, and $\hat{a} : Y \to A$. Second, notice that the worker's lifetime utility $W(z;y|a)$ is equal to the RHS of equation (R2) and the firm's lifetime profits $J(z;y|a)$ are equal to the RHS of equation (R3). In light of these observations, we can rewrite (A1) as

$$\begin{aligned} V(z;y) = \; & \max_{w,\tau,n,\hat{a}} \; y + z + \beta \mathbb{E}\left\{\tau(\hat{y})U(\hat{y}) + [1-\tau(\hat{y})]\lambda_e p(\theta(n(\hat{y});\hat{y}))n(\hat{y})\right\} + \\ & + \beta \mathbb{E}\left\{[1-\tau(\hat{y})][1-\lambda_e p(\theta(n(\hat{y});\hat{y}))]\left[J(z;\hat{y}|\hat{a}(\hat{y})) + W(z;\hat{y}|\hat{a}(\hat{y}))\right]\right\}, \\ & w \in \mathbb{R}, \; \tau : Y \to [\delta, 1], \; n : Y \to \mathbb{R}, \; \hat{a} : Y \to A. \end{aligned} \tag{A2}$$

Now, notice that both the probability that the match survives during the separation stage, i.e. $1 - \tau(\hat{y})$, and the probability that the match survives during the search stage, i.e. $1 - \lambda_e p(\theta(n(\hat{y});\hat{y}))$, are non negative numbers. In light of this observation, we can rewrite (A2) as

$$\begin{aligned} V(z;y) = \; & \max_{w,d,n} y + z + \beta \mathbb{E}\left\{\tau(\hat{y})U(\hat{y}) + [1-\tau(\hat{y})]\lambda_e p(\theta(n(\hat{y});\hat{y}))n(\hat{y})\right\} + \\ & + \beta \mathbb{E}\{[1-\tau(\hat{y})][1-\lambda_e p(\theta(n(\hat{y});\hat{y}))] \max_{\hat{a} \in A}\left[J(z;\hat{y}|\hat{a}) + W(z;\hat{y}|\hat{a})\right]\}, \\ & w \in \mathbb{R}, \; \tau : Y \to [\delta, 1], \; n : Y \to \mathbb{R}. \end{aligned} \tag{A3}$$

Finally, notice that the maximum of the sum between the worker's continuation utility $W(z;\hat{y}|\hat{a})$ and the firm's continuation profits $J(z;\hat{y}|\hat{a})$ is equal to $V(z;\hat{y})$. Therefore, (A3) is equal to equation (R5) in the main text. ■

B Proof of Proposition 1

Let the contract $\underline{a}$ be a feasible choice for the firm's problem (R6). First, notice that, for any realization z_i of the idiosyncratic component of productivity, the contract $\underline{a}$ prescribes an allocation $\underline{a}(z_i)$ which may not necessarily maximize the joint value of the match, i.e. $W(z_i;y|\underline{a}(z_i)) + J(z_i;y|\underline{a}(z_i))$ is smaller than or equal to $V(z_i;y)$. Second, notice that, since $\underline{a}$ is feasible, it provides the worker with the lifetime utility x, i.e.

$\sum_i W(z_i; y|\underline{a}(z_i))f(z_i) = x$. In light of these observations, it follows that the contract $\underline{a}$ provides the firm with the following profits:

$$\begin{aligned} \textstyle\sum_i J(z_i; y|\underline{a}(z_i))f(z_i) \quad &\leq \textstyle\sum_i V(z_i; y)f(z_i) - \sum_i W(z_i; y|\underline{a}(z_i))f(z_i) = \\ &= \textstyle\sum_i V(z_i; y)f(z_i) - x. \end{aligned} \tag{A4}$$

Let $\underline{a}^*$ denote the contract $\{w_t^*, \tau_t^*, n_t^*\}_{t=0}^{\infty}$ that has the following properties: (a) $\tau_{t-1}^*(z; y^t) = 1$ iff $U(y_t) > V(z; y_t) + \lambda_e D(V(z; y_t); y_t)$ and $\tau_{t-1}^*(z; y^t) = \delta$ otherwise, for all $\{z; y^t\} \in Z \times Y^t$, $t = 1, 2, ...$; (b) $n_{t-1}^*(z; y^t) = m(V(z; y_t); y_t)$, for all $\{z; y^t\} \in Z \times Y^t$, $t = 1, 2, ...$; (c) $w_t^*(z; y^t)$ is such that $\sum_i W(z_i; y|\underline{a}^*(z_i))f(z_i) = x$. First, notice that, for any realization z_i of the idiosyncratic component of productivity, the contract $\underline{a}^*$ prescribes an allocation $\underline{a}^*(\mathrm{z}_i)$ which maximizes the joint value of the match. Second, notice that $\underline{a}^*$ provides the worker with the lifetime utility x. In light of these two observations, it follows that the contract $\underline{a}^*$ provides the firm with the following profits:

$$\begin{aligned} \textstyle\sum_i J(z_i; y|\underline{a}^*(z_i))f(z_i) \quad &= \textstyle\sum_i V(z_i; y)f(z_i) - \sum_i W(z_i; y|\underline{a}^*(z_i))f(z_i) = \\ &= \textstyle\sum_i V(z_i; y)f(z_i) - x. \end{aligned} \tag{A5}$$

The contract $\underline{a}^*$ is a feasible choice for the firm's problem (R6), and it provides the firm with more profits than any other feasible choice. Hence, it is optimal.

Finally, the reader can easily verify that, if a contract $\{w_t, \tau_t, n_t\}_{t=0}^{\infty}$ solves the firm's problem (R6), then it maximizes the joint value of the match. Hence, the contract $\{w_t, \tau_t, n_t\}_{t=0}^{\infty}$ prescribes that (a) $\tau_{t-1}(z; y^t) = 1$ iff $U(y_t) > V(z; y_t) + \lambda_e D(V(z; y_t); y_t)$ and $\tau_{t-1}(z; y^t) = \delta$ otherwise, for all $\{z; y^t\} \in Z \times Y^t$, $t = 1, 2, ...$; (b) $n_{t-1}(z; y^t) = m(V(z; y_t); y_t)$, for all $\{z; y^t\} \in Z \times Y^t$, $t = 1, 2, ...$ ■

C Proof of Proposition 2

(i) Let Ψ denote the set $Y \times [0, 1]^{N(z)+1}$. Let $C(\Psi)$ denote the set of bounded continuous functions $r : \Psi \to \mathbb{R}$, with the sup norm. Define the operator T on $C(\Psi)$ by

$$\begin{aligned} &(Tr)(\psi) = \max_{d,\theta_u,\theta_z} F(d, \theta_u, \theta_d|\psi) + \beta\mathbb{E}\left[r(\hat{\psi})\right] \\ \text{s.t.} \quad &\hat{u} = u\left[1 - \lambda_u p(\theta_u)\right] + \textstyle\sum_i d(z_i)g(z_i), \\ &\hat{g}(z) = h(\psi)f(z) + \left[1 - d(z)\right]\left[1 - \lambda_e p(\theta_z(z))\right]g(z), \\ &d : Z \to [\delta, 1],\ \theta_u \in [0, \bar{\theta}],\ \theta_z : Z \to \left[0, \bar{\theta}\right]. \end{aligned} \tag{A6}$$

For each $r \in C(\Psi)$ and $\psi \in \Psi$, the problem in (A6) is to maximize a continuous function over a compact set. Hence the maximum is attained and the argmax is non-empty. Since both F and r are bounded, Tr is also bounded; and since F and r are continuous, it follows from the Theorem of the Maximum (see Stokey, Lucas and Prescott 1989, page 62) that Tr is also continuous. Hence, the operator T maps $C(\Psi)$ into itself.

Since the operator T satisfies the remaining hypotheses of Blackwell's sufficient conditions for a contraction (see Stokey, Lucas and Prescott 1989, page 54), it follows that T has a unique fixed point $\tilde{s} \in C(\Psi)$. And since $\lim_{t\to\infty} \beta^t \tilde{s}(\psi) = 0$ for all $\psi \in \Psi$, it follows that the fixed point $\tilde{s}$ is equal to the value of the plan s^0.

(ii) Let $L(\Psi)$ denote the set of bounded continuous functions $r : \Psi \to \mathbb{R}$ that are linear in the measure u of unemployed workers as well as in the measure $g(z)$ of workers employed at jobs with idiosyncratic productivity z, i.e.

$$r(\psi) = r_u(y)u + \textstyle\sum_i r_z(z_i; y)g(z_i).$$

Given a function r in $L(\Psi)$, consider the problem (A6). For each $\psi \in \Psi$, the necessary condition for the optimality of θ_u is:

$$k \geq p'(\theta_u)\{y - b + \beta\mathbb{E}[\textstyle\sum_i r_z(z_i; \hat{y})f(z_i) - r_u(\hat{y})]\} \tag{A7}$$

and $\theta_u \geq 0$, with complementary slackness. Since the function $p'(\theta)$ is strictly decreasing in θ, there is at most one θ_u that satisfies condition (A7). Hence the optimum is unique. Since (A7) depends on ψ only through y, the optimal policy is a function $\tilde{\theta}_u : Y \to [0, \overline{\theta}]$.

For each $\psi \in \Psi$, the necessary condition for the optimality of $\theta_z(z)$ is:

$$k \geq p'(\theta_z(z))\{-z + \beta\mathbb{E}[\textstyle\sum_i r_z(z_i; \hat{y})f(z_i) - r_z(z; \hat{y})]\} \tag{A8}$$

and $\theta_z(z) \geq 0$, with complementary slackness. Since $p'(\theta)$ is strictly decreasing in θ, there is at most one $\theta_z(z)$ that satisfies condition (A8). Hence the optimum is unique. Since (A8) depends on ψ only through y, the optimal policy is a function $\tilde{\theta}_z : Z \times Y \to [0, \overline{\theta}]$.

For each $\psi \in \Psi$, the necessary and sufficient condition for the optimality of d is $d(z) = 1$ if

$$\begin{aligned} b + \beta\mathbb{E}[r_u(\hat{y})] > \quad & -\lambda_e k\theta_z(z) + [1 - \lambda_e p(\theta_z(z))]\,[y + z + \beta\mathbb{E}r_z(z; \hat{y})] + \\ & +\lambda_e p(\theta_z(z))\,\{y + \beta\mathbb{E}\,[\textstyle\sum_i r_z(z_i; \hat{y})f(z_i)]\}, \end{aligned} \tag{A9}$$

and $d(z) = \delta$ otherwise. Since (A9) does not depend on d, there is exactly one d that satisfies condition (A8). Since (A9) depends on ψ only through y, the optimal policy is a function $\tilde{d} : Z \times Y \to [\delta, 1]$.

Define the function $\tilde{r}_u : Y \to \mathbb{R}$ by

$$\begin{aligned} \tilde{r}_u(y) = \quad & -k\lambda_u\tilde{\theta}_u(y) + \left[1 - \lambda_u p(\tilde{\theta}_u(y))\right] [b + \beta \mathbb{E} r_u(\hat{y})] + \\ & + \lambda_u p(\tilde{\theta}_u(y)) \left\{y + \beta \mathbb{E}\left[\textstyle\sum_i r_z(z_i; \hat{y}) f(z_i)\right]\right\}. \end{aligned} \tag{A10}$$

And define the function $\tilde{r}_z : Z \times Y \to \mathbb{R}$ by

$$\begin{aligned} \tilde{r}_z(z; y) = \quad & \tilde{d}(z; y) [b + \beta \mathbb{E} r_u(\hat{y})] - [1 - \tilde{d}(z; y)] k\lambda_e\tilde{\theta}_z(z; y) + \\ & + [1 - \tilde{d}(z; y)] \left[1 - \lambda_e p(\tilde{\theta}_z(z; y))\right] [y + z + \beta \mathbb{E} r_z(z; \hat{y})] + \\ & + [1 - \tilde{d}(z; y)] \lambda_e p(\tilde{\theta}_z(z; y)) \left\{y + \beta \mathbb{E}\left[\textstyle\sum_i r_z(z_i; \hat{y}) f(z_i)\right]\right\}. \end{aligned} \tag{A11}$$

It is then immediate that

$$(Tr)(\psi) = \tilde{r}_u(y)u + \textstyle\sum_i \tilde{r}_z(z_i; y) g(z_i).$$

Hence, the operator T maps $L(\Psi)$ into itself. Since $L(\Psi)$ is a closed subset of $C(\Psi)$, it follows that the fixed point s^0 of the operator T belongs to $L(\Psi)$ (see Stokey, Lucas and Prescott 1989, page 52).

(iii) Let $M(\Psi)$ denote the set of functions $r : \Psi \to \mathbb{R}$ such that $r \in L(\Psi)$ and $r_z : Z \times Y \to \mathbb{R}$ is non decreasing in z. Given a function $r \in M(\Psi)$, let $\tilde{r}$ denote Tr. As we proved in part (ii), the function $\tilde{r}$ belongs to the set $L(\Psi)$. Also as we proved in part (ii), the derivative $\tilde{r}_z(z; y)$ is equal to (A10). Using the optimality conditions (A7)–(A9), we can rewrite (A10) as

$$\begin{aligned} \tilde{r}_z(z, y) = \quad & b + \beta \mathbb{E} r_u(y_+) + \max_{d \in [\delta, 1]} \{(1 - d)[y + z - b + \beta \mathbb{E}[r_z(z, \hat{y}) - r_u(\hat{y})]] \\ & + (1 - d) \lambda_e \max_{\theta \in \mathbb{R}_+} [-k\theta + p(\theta)[-z + \beta \mathbb{E}[\textstyle\sum_i r_z(z, \hat{y}) f(z_i) - r_z(z, \hat{y})]]]\}. \end{aligned}$$

Since $r_z(z; y)$ is non decreasing in z, it follows that $\tilde{r}_z(z_2; y) \geq \tilde{r}_z(z_1; y)$ for all $z_2 \geq z_1$. Hence, the operator T maps the set $M(\Psi)$ into itself. Since $M(\Psi)$ is a closed subset of $L(\Psi)$, it follows that the fixed point s^0 belongs to $M(\Psi)$ as well. ■

D Proof of Theorem 3

(i) We want to prove that a Block Recursive Equilibrium exists. To this aim, we first construct a supposed equilibrium $\{D^*, m^*, U^*, V^*, d^*, \theta^*\}$ from the solution to the social planner's problem. Then, we verify that the putative equilibrium satisfies conditions (i)–(iv) in Definition 1.

In the supposed equilibrium, the worker's value from unemployment $U^*(y)$ is set equal to $b + \beta \mathbb{E} s_u^0(\hat{y})$, where s_u^0 is the derivative of the social planner's value function s^0 with respect to the unemployment rate. The firm's and worker's joint value from a match $V^*(z; y)$ is set equal to $y + z + \beta \mathbb{E} s_z^0(z; \hat{y})$, where s_z^0 is the derivative of the social planner's value function with respect to $g(z)$. The market tightness function $\theta^*(x; y)$ is set equal to $q^{-1}(k/(\sum_i V^*(z_i; y) f(z_i) - x))$ for all $x \leq \tilde{x}(y)$; and $\theta^*(x; y)$ is set equal to zero for all $x > \tilde{x}(y)$. Finally, the worker's search value function $D^*(\upsilon; y)$ and policy function $m^*(\upsilon; y)$ are set equal to the maximum and the maximizer of $p(\theta^*(x; y))\ (x - \upsilon)$.

By construction, the market tightness function θ^* satisfies the equilibrium condition (i). Also by construction, the worker's search value D^* and policy m^* satisfy the equilibrium condition (ii). As proved in the main text, whenever conditions (i) and (ii) are satisfied, we have that

$$m^*(\upsilon; y) \in \arg\max_x \{-k\theta^*(x; y) + p(\theta^*(x; y))[\textstyle\sum_i V^*(z_i; y) f(z_i) - \upsilon]\}, \qquad \text{(A12)}$$

and $D^*(\upsilon; y)$ is the maximum of the problem in (A12). Hence the tightness $\theta_u^*(y)$ of the submarket visited by unemployed workers satisfies the optimality condition (E3); and the tightness $\theta_z^*(z; y)$ of the submarket visited by employed workers satisfies the optimality condition (E4). Since $U^*(y)$ is equal to $b + \beta \mathbb{E} s_u^0(\hat{y})$ and $V^*(z; y)$ is equal to $y + z + \beta \mathbb{E} s_z^0(z; \hat{y})$, the tightness $\theta_u^*(y)$ also satisfies the necessary condition (P3) for the optimality of the solution to the social planner's problem. Since (P3) admits only one solution, $\theta_u^*(y)$ is equal to $\theta_u^0(y)$. Similarly, we can prove that $\theta_z^*(z; y)$ is equal to $\theta_z^0(z; y)$ and that $d^*(z; y)$ is equal to $d^0(z; y)$.

Since $\theta_u^0(y)$ is equal to $\theta_u^*(y)$, the envelope condition (P6) can be written as

$$s_u^0(u) = U^*(y) + \lambda_u D^*(U^*(y); y). \qquad \text{(A13)}$$

In turn, (A13) implies that $U^*(y)$ is equal to

$$U^*(y) = b + \beta \mathbb{E} s_u^0(\hat{y}) = b + \beta \mathbb{E}[U^*(\hat{y}) + \lambda_u D^*(U^*(\hat{y}); \hat{y}))]. \tag{A14}$$

Hence $U^*(y)$ satisfies the equilibrium condition (iii). Similarly, we can prove that the firm's and worker's joint value from a match $V^*(z; y)$ satisfies the equilibrium condition (iv).

(ii) We want to prove that any equilibrium is efficient. To this aim, let $\{D^*, m^*, U^*, V^*, d^*, \theta^*\}$ denote a Block Recursive Equilibrium. Let $s_u^*(y)$ denote the worker's value of unemployment at the beginning of the separation stage, i.e. $U^*(y) + \lambda_u D^*(U^*(y); y)$. Let $s_z^*(z; y)$ denote the firm's and worker's joint value of a match at the beginning of the separation stage, i.e. $V^*(z; y) + \lambda_e D^*(V^*(z; y); y)$. Let $\theta_u^*(y)$ denote the tightness of the submarket visited by unemployed workers, i.e. $\theta_u^*(y) = \theta^*(m^*(U^*(y); y); y)$. And let $\theta_z^*(z; y)$ denote the tightness of the submarket visited by workers who are employed at jobs with idiosyncratic productivity z, i.e. $\theta_z^*(z; y) = \theta^*(m^*(V^*(z; y); y); y)$.

Define the function $r : \Psi \to \mathbb{R}$ as $r_u(y)u + \sum r_z(z; y)g(z_i)$, where $r_u(y)$ is equal to $s_u^*(y)$ and $r_z(z; y)$ is equal to $s_z^*(z; y)$. Given the function r, consider the problem (A6). For each $(y, u, g) \in \Psi$, the optimal market tightness $\tilde{\theta}_u(y)$ satisfies the condition

$$k \geq p'(\tilde{\theta}_u(y))\{y - b + \beta \mathbb{E}[\textstyle\sum_i r_z(z_i; \hat{y}) f(z_i) - r_u(\hat{y})]\} \tag{A15}$$

and $\tilde{\theta}_u(y) \geq 0$, with complementary slackness. Since $r_z(z_i; \hat{y}) = s_z^*(z_i; \hat{y})$ and $r_u(\hat{y}) = s_u^*(\hat{y})$, $\tilde{\theta}_u(y)$ also satisfies condition (E4). Since (E4) admits only one solution, $\tilde{\theta}_u(y)$ is equal to $\theta_u^*(y)$. Similarly, we can prove that the optimal tightness $\tilde{\theta}_z(z; y)$ is equal to $\theta_z^*(z; y)$. And we can prove that the optimal job destruction probability $\tilde{d}(z; y)$ is equal to $d^*(z; y)$.

Define the function $\tilde{r} : \Psi \to \mathbb{R}$ as Tr. As we proved in Proposition 2, $\tilde{r}$ belongs to the set $L(\Psi)$. As we also proved in Proposition 2, the derivative $\tilde{r}_u(y)$ is equal to

$$\begin{aligned} \tilde{r}_u(y) = \quad & -k\lambda_u \tilde{\theta}_u(y) + \left[1 - \lambda_u p(\tilde{\theta}_u(y))\right] [b + \beta \mathbb{E} r_u(\hat{y})] + \\ & + \lambda_u p(\tilde{\theta}_u(y)) \left\{y + \beta \mathbb{E}\left[\textstyle\sum_i r_z(z_i; \hat{y}) f(z_i)\right]\right\}. \end{aligned} \tag{A16}$$

Since $r_z(z_i; \hat{y}) = s_z^*(z_i; \hat{y})$, $r_u(\hat{y}) = s_u^*(\hat{y})$ and $\tilde{\theta}_u(y) = \theta_u^*(y)$, the right hand side of (A16) is equal to the right hand side of (E5). Hence $\tilde{r}_u(y)$ is equal to $s_u^*(y)$. Similarly, we can prove that $\tilde{r}_z(z;y)$ is equal to $s_z^*(z_i;y)$. Taken together, these two observations imply that

$$(Tr)(\psi) = s_u^*(y)u + \textstyle\sum_i s_z^*(z_i;y)g(z_i) = r(\psi). \tag{A17}$$

Since it is a fixed point of the operator T, r is equal to the social planner's value function s^0. And the policy $\{\tilde{\theta}_u, \tilde{\theta}_z, \tilde{d}\} = \{\theta_u^*, \theta_z^*, d^*\}$ is equal to the solution to the social planner's problem $\{\theta_u^0, \theta_z^0, d^0\}$. ■

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Table 1: U.S. Quarterly Data, 1951:I–2006:II

		u	v	h^{ue}	h^{eu}	h^{ee}	apl
Average		.056	63.9	.452	.026	.029	84.2
Relative Std		12.2	13.5	7.56	7.03	4.15	1
Quarterly Acr		.873	.905	.820	.692	.595	.761
Unemployment	u	1	-.919	-.920	.777	-.631	-.250
Vacancies	v	—	1	.907	-.784	.661	.410
UE Rate	h^{ue}	—	—	1	-.677	.664	.258
EU Rate	h^{eu}	—	—	—	1	-.289	-.480
EE Rate	h^{ee}	—	—	—	—	1	.173
Average Prod	apl	—	—	—	—	—	1

Source: Own calculations using data from the BLS.

Table 2: Calibration Outcomes

	Description	(a) Baseline	(b) P85	Target
β	discount rate	.996	.996	real interest rate
b	home productivity	.987	.987	home/mkt prod.
λ_u	off the job search prob.	1	1	normalization
λ_e	on the job search prob.	.833	—	EE rate
γ	elasticity of p wrt θ	.650	.220	reg. coef. of v/u on h^{ue}
k	vacancy cost	1.77	2.84	UE rate
δ	destruction prob.	.011	.027	tenure distribution
μ_z	average idios. prod.	0	.371	normalization
σ_z	scale idios. prod.	1.17	—	EU rate
α_z	shape idios. prod.	4	—	tenure distribution
μ_y	average agg. prod.	1	1	normalization
σ_y	std. agg. prod.	1.52	1.36	std. average prod.
ρ_y	autocorr. agg. prod.	0.76	0.76	std. average prod.

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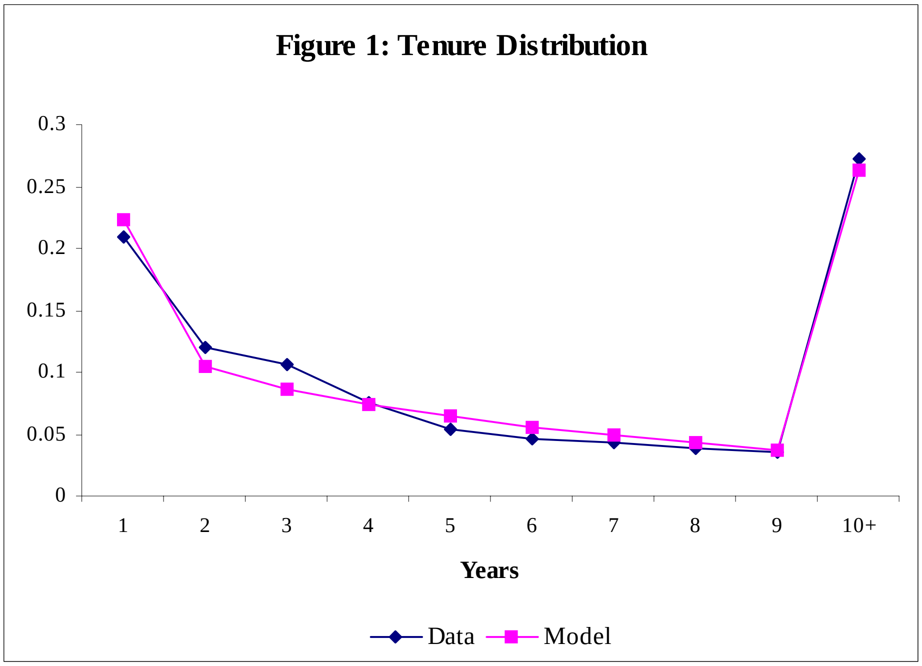


Figure 1: Tenure Distribution
0.3
0.25
0.2
0.15
0.1
0.05
0
1
2
3
4
5
6
7
8
9
10+
Years
Data
Model


Figure 2: Job Distribution
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.7
-0.5
-0.3
-0.1
0.1
0.3
0.5
0.7
z
new jobs
all jobs

.

Figure 3: Unemployment and Hazard Dynamics

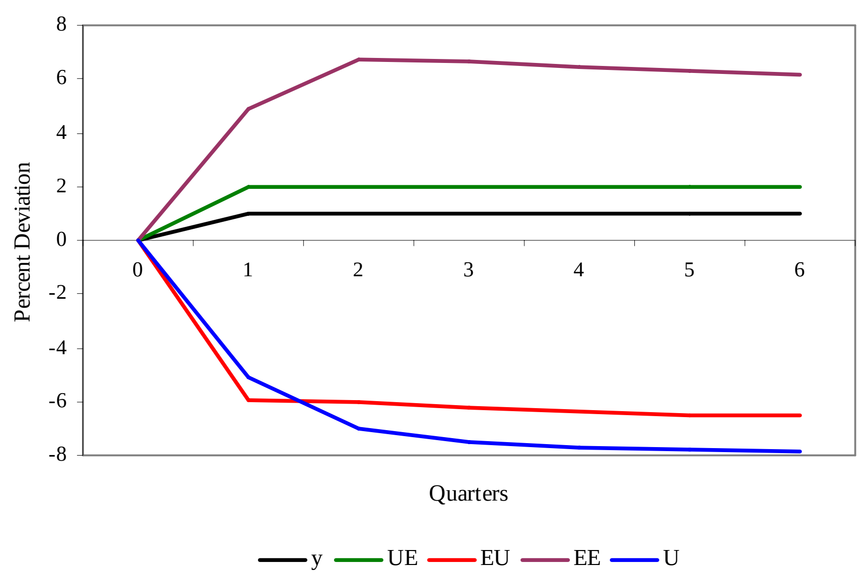
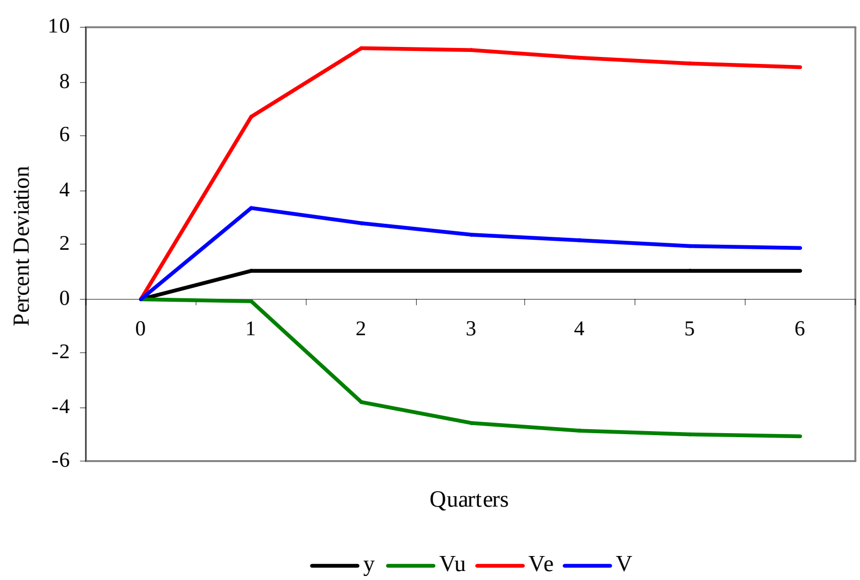


Figure 4: Vacancies Dynamics



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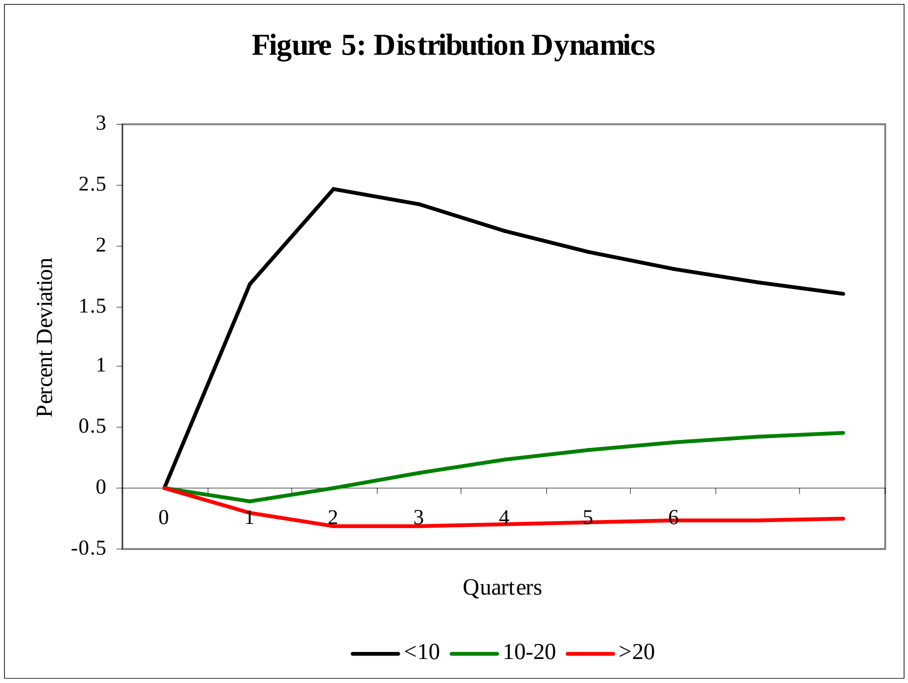
Figure 5: Distribution Dynamics
Percent Deviation
3
2.5
2
1.5
1
0.5
0
-0.5
0
1
2
3
4
5
6
Quarters
<10
10-20
>20

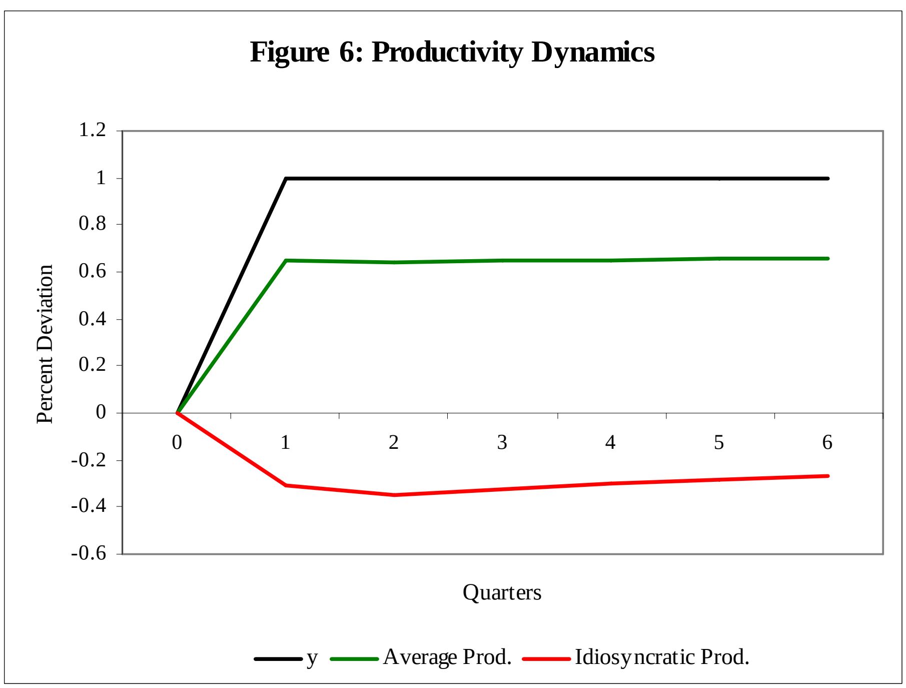
Figure 6: Productivity Dynamics
Percent Deviation
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
0
1
2
3
4
5
6
Quarters
y
Average Prod.
Idiosyncratic Prod.

.

Table 3: Productivity Shocks

		u	v	v^u	v^e	h^{ue}	h^{eu}	h^{ee}	apl
Relative Std		10.5	4.06	6.74	11.7	2.98	8.79	8.66	1
Quarterly Acr		.837	.650	.771	.792	.775	.762	.792	.778
Unemployment	u	1	-.812	.877	-.974	-.969	.971	-.970	-.971
Vacancies	v	—	1	-.458	.890	.909	-.894	.895	.901
Vac for Un	v^u	—	—	1	-.747	-.746	.749	-.786	-.756
Vac for Emp	v^e	—	—	—	1	.990	-.957	.999	.988
UE Rate	h^{ue}	—	—	—	—	1	-.970	.988	.999
EU Rate	h^{eu}	—	—	—	—	—	1	-.954	-.972
EE Rate	h^{ee}	—	—	—	—	—	—	1	.986
Average Prod	apl	—	—	—	—	—	—	—	1

Table 4: Productivity Shocks in P85

		u	v	h^{ue}	h^{eu}	h^{ee}	apl
Relative Std		.667	2.78	.742	0	—	1
Quarterly Acr		.826	.726	.770	1	—	.771
Unemployment	u	1	-.946	-.974	0	—	-.974
Vacancies	v	—	1	.994	0	—	.994
UE Rate	h^{ue}	—	—	1	0	—	.999
EU Rate	h^{eu}	—	—	—	1	—	0
EE Rate	h^{ee}	—	—	—	—	—	—
Average Prod	apl	—	—	—	—	—	1

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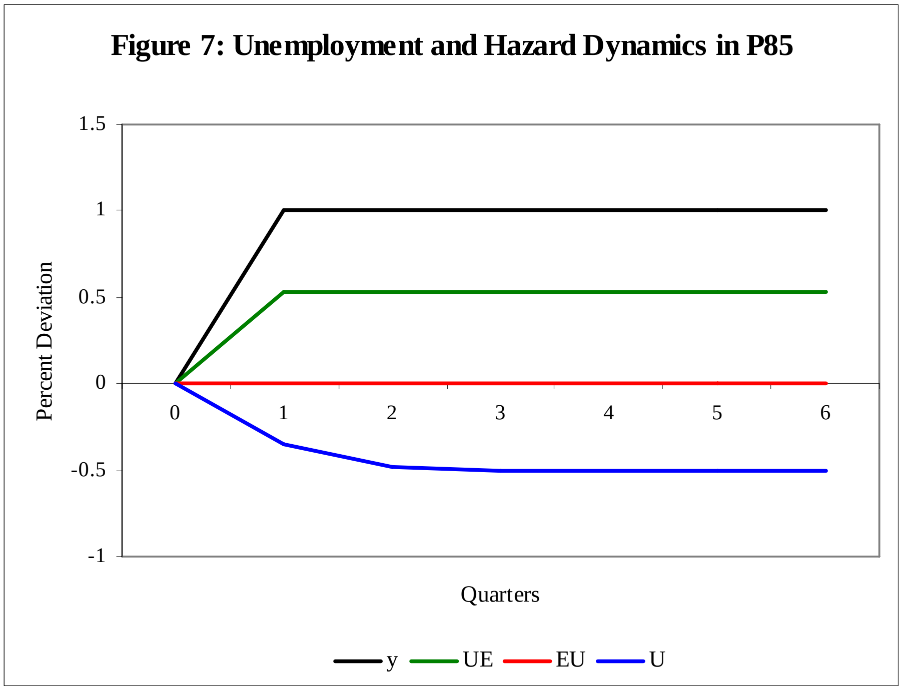
Figure 7: Unemployment and Hazard Dynamics in P85
1.5
1
0.5
0
-0.5
-1
Percent Deviation
0
1
2
3
4
5
6
Quarters
y
UE
EU
U

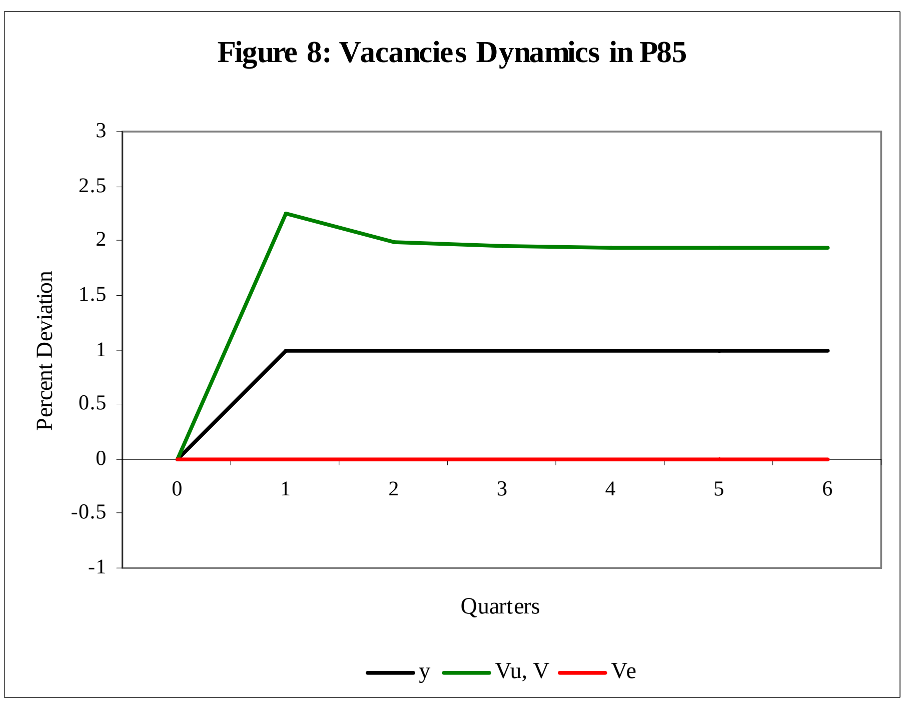
Figure 8: Vacancies Dynamics in P85
3
2.5
2
1.5
1
0.5
0
-0.5
-1
Percent Deviation
0
1
2
3
4
5
6
Quarters
y
Vu, V
Ve