PIER Working Paper 08-008

"External Economies of Scale and Comparative Advantage"

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http://ssrn.com/abstract=1097569

EXTERNAL ECONOMIES OF SCALE AND COMPARATIVE ADVANTAGE

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Abstract We investigate the interplay, in international trade, between comparative advantage and increasing returns to scale that are external to the firm. We focus especially on "advantage reversals," where the country with a comparative-cost disadvantage in producing a good nevertheless is able to export it because of the economies of large-scale production. We examine trade policy in such a situation, looking especially at whether that policy should aim at basic policy-regime change.

Koji Shimomura was a delightful person, a warm friend, a dependable follower of sumo, and a thoroughly responsible academic. We miss him. Here we address a subject that he would have appreciated. We return to the implications of increasing returns to scale for the gains from trade and for the role of trade policy. This topic was pioneered by Frank Graham (1923).

I Introduction

International trade theory identifies three possible reasons for countries to trade: *comparative advantage* (to exploit differences), *economies of scale* (to concentrate more productively on fewer activities), and *imperfect competition* (to expose firms to more competition). This paper discusses the relation between the first two, with a focus on Graham's concerns about the role of policy.

In recent decades treatments of international trade featuring economies of scale and imperfect competition have become commonplace. But another, earlier, literature has addressed the former with little or no attention given to the latter.[1] Ethier (1982) addresses *external* economies of scale (without being all that clear about it), but, except briefly, ignores comparative-advantage differences and trade policy (beyond the trade vs. autarky choice). Ruffin (2007) addresses *internal* economies of scale (without being all that clear about it) and comparative-cost differences, and looks explicitly at trade policy. The present paper targets the ground between those two papers, addressing the interplay between external scale economies and comparative advantage.

A few words about why such an investigation may be of some relevance to reality. We utilize a two-sector model in which one sector is characterized by external increasing returns to scale. It is

natural to think of this sector as manufacturing in general, with individual firms rendered more efficient as a result of the infrasturcture, commercial law, industrial labor force, etc. consequent to a large manufacturing sector, that is, to more industrialization. Such a model could supply a natural framework for analyzing the widespread policy choices, in the 1950s and 60s, by developing countries about whether to open their economies or to rely instead on extensive import-substitution policies. And the framework could also be useful to analyze the choices, in the late 1980s and extending to the current time, by these same countries about the wisdom of basic policy reform (though in this case, ignoring international economies of scale could be questionable).

These constitute two of the four international economic policy events of the past sixty years that are of fundamental significance. Also, in both cases, the policy question was not about marginal changes in trade policy, but about regime change. So, in what follows, we investigate whether or not the role of trade policy implied by the theory is to manage regime change.

II The Model

Suppose two countries (home and foreign), and two goods: *A* and *B* (the good with increasing returns to scale). We employ the familiar Ricardian model of technology, modified by the introduction of scale economies in one sector.

The home economy

The production side of the home economy is as follows.

$$L = L_A + L_B.$$

$$L_A = A.$$

$$k(L_B)^\alpha = B, \tag{1}$$

where $\alpha > 1$.

One unit of A is defined as the quantity that can be produced by one unit of home labor. We assume that all markets are perfectly competitive. Were B-production characterized by constant returns to scale, the relative price P of B in terms of A would equal $1/k$ in autarky.

The scale economies in the B sector are external to the firm. That is, the representative firm i faces the production function $B_i = [kL_B{}^{\alpha-1}]L_{Bi}$ where L_{Bi} denotes the labor employed by firm i and L_B denotes the sum of all L_{Bi}. The individual firm, unable to influence the overall allocation of resources, takes its labor productivity $[kL_B{}^{\alpha-1}]$ as parametric [see Chipman (1970)]. Summing the production function over all firms gives the aggregate B production function **(1)**.

Assumption 1 *The economies of scale in the production of* B *are* external *to the firm.*

Take home labor as the numeráire, so that $w = 1 = P_A$ if A is being produced. The zero-profit condition for B is $P_B B = L_B$. In view of **(1)**, these zero-profit conditions yield the following supply function for B, if both sectors are active.

$$P_S = k^{-1/\alpha} B^{(1-\alpha)/\alpha} \tag{2}$$

where P_S denotes the supply price of B in terms of A.

The demand side is described by:

$$D_B = (\gamma/2)P^{-\varepsilon},$$

with the demand for A therefore the residual: $D_A = L - (\gamma/2)P^{1-\varepsilon}$. This in turn generates the inverse demand function

$$P_D = ([2/\gamma]D_B)^{-1/\varepsilon} \tag{3}$$

where P_D denotes the relative demand price.

Autarky equilibrium

In autarky equilibrium, $P_S = P_D$ and $B = D_B$, so that **(2)** and **(3)** determine that equilibrium. Also, if both goods are produced, the labor allocated to the B sector is:

$$L_B = a_0 \, k^{(\varepsilon-1)/(\alpha+\varepsilon-\alpha\varepsilon)} \equiv L_0 \qquad \textbf{(4)}$$

where $a_0 = (\gamma/2)^{1/(\alpha+\varepsilon-\alpha\varepsilon)}$. If $L \geq L_0$, $L_B = L_0$; otherwise $L_B = L$.

Assume the Marshallian disequilibrium adjustment process:

$$dB/dt = P_D - P_S.$$

The local stability condition for this adjustment process [see Ethier (1982)] is $\alpha/(\alpha-1) > \varepsilon$.

The foreign economy

Analogously for the foreign economy,

$$L^* = L_A^* + L_B^*.$$

$$L_A^* = A^*.$$

$$k^*(L^*_B)^\alpha = B^*.$$

A unit of foreign labor is defined as the amount required to produce a unit of good A.

The demand side is:

$$D^*_B = (\gamma/2)P^{-\varepsilon}.$$

Assumption 2 $k < k^*$.

In this sense the foreign economy has a comparative-cost advantage in B. Otherwise the two economies are identical, except for size.

Assumption 3 $\alpha/(\alpha-1) > \varepsilon$.

Proposition 1 *Assumption 3 ensures that each country possesses a Marshallian-stable autarky*

equilibrium.

Assumption 3 implies that the elasticity of demand for output is smaller than the numerical elasticity of supply with respect to the price; thus, a smaller output will push the demand price higher than the supply price, which implies firms will want to expand output back to the equilibrium level.

Assumption 3 can also be written as $\alpha + \varepsilon > \alpha\varepsilon$. Thus it places an upper bound on the amount of elasticity (*i. e.*, the magnitudes of α and ε) in the model. If elasticities are high enough to violate the assumption, scale economies are sufficient to allow the home country to have the cost advantage in autarky, despite having the smaller k. We accordingly describe the world as *comparative-cost dominant* if Assumption 3 holds so that stability obtains, and as *scale-dominant* if the assumption fails, implying instability.

If both goods are produced in the autarky equilibrium, the labor allocated to the B sector is given by:

$$L_B{}^* = a_0 \, (k^*)^{(\varepsilon-1)/(\alpha\varepsilon-\alpha-\varepsilon)} \equiv L_0{}^*. \tag{5}$$

If $L^* \geq L_0{}^*$, $L_B{}^* = L_0{}^*$; otherwise $L_B{}^* = L^*$.

III Advantage Reversals

Turn now to free trade between the home and foreign economies. We first consider whether there is an equilibrium in which the home country is the sole supplier of B in the world market despite its cost disadvantage — an *advantage reversal*. If this were so, and if the home country also produces some A,

$$B = \gamma \, P^{-\varepsilon}$$

$$B = k(L_B)^\alpha$$

$$L_B = PB.$$

An advantage-reversal equilibrium

These are three equations in three variables (P, B, L_B) and so can be solved in terms of the parameters. In particular, we have the solution $L_B = f(\gamma, \alpha, \varepsilon, k)$.

Assumption 4 $L > f(\gamma, \alpha, \varepsilon, k)$.

This assumption ensures that the home economy is large enough so that, in equilibrium, it is able to supply the entire world demand for B. In such an equilibrium, the home economy would produce both goods and export B, while the foreign economy would produce only A.

With the home economy producing both goods, its supply price is still given by **(2)**. As $B^* = 0$, the foreign supply price is infinite. Finally, the world demand price is now

$$P_D = ([1/\gamma][D_B + D_B^*])^{-1/\varepsilon}. \tag{6}$$

In such an equilibrium, $P_D = P_S < P_S^*$ and $B = D_B + D_B^*$. Then **(2)** and **(6)** determine the equilibrium. The Marshallian stability condition for this equilibrium is easily confirmed to be $\alpha/(\alpha-1) > \varepsilon$, and thus ensured by Assumption 3.

With the home supply curve unchanged as the negative relation **(2),** and world demand greater at every P than was the home demand in autarky, the world relative price of B must be lower than it was in the home country in autarky. Since home income in terms of A equals L, home income in terms of B must be higher in the free-trade equilibrium than in autarky. Thus the home country is better off in the advantage-reversal equilibrium than in autarky, despite exporting the good in which it has a comparative-cost disadvantage — economies of scale dominate comparative advantage. It is possible to construct examples where the foreign economy loses from trade, but

they appear to be extreme.[2]

Proposition 2 *If Assumptions 1 – 4 hold, there exists a Marshallian-stable, advantage-reversal equilibrium in which the home country exports good* B *and the foreign produces only good* A. *In this equilibrium the home country is better off than in autarky.*

External vs. internal economies of scale

Assumption 1 is critical for the allocation of resources described in the previous subsection actually to describe an equilibrium. With $B^* = 0$ and with foreign firms treating labor productivity as parametric, P_S^* is necessarily infinite, and therefore greater than P_S. But if the scale economies are internal, labor productivity is not parametric, and P_S^* is then determined by what it would cost the foreign economy to supply the quantity $(D_B + D_B^*)$, if it can do so:

$$P_{S^*} = (k^*)^{-1/\alpha} \, B^{(1-\alpha)/\alpha}. \tag{7}$$

Then Assumption 2 (that $k^* > k$) implies that $P_S^* < P_S$, so that this allocation of resources, with $B > 0 = B^*$, is not an equilibrium.

Proposition 4 *Assumption 1 is necessary for the existence of the advantage-reversal equilibrium, whenever* the foreign economy could instead supply the world demand for good B.

IV Alternative Equilibria where the Foreign Economy Supplies the Entire *B* Market

Increasing returns to scale tend to spawn multiple equilibria, so we consider additional possibilities. As we are interested in the consequences of an advantage-reversal equilibrium, we continue to impose Assumption 4, but we shall vary L^* to consider all alternative relative sizes of the two countries.

A large foreign economy

If the foreign economy is large enough, it, too, can supply the entire world demand for B.

Assumption 5a $L^* \geq f(\gamma, \alpha, \varepsilon, k^*)$.

This assumption allows an allocation of resources where the foreign economy produces both goods and exports B. The home economy produces only A.

With the foreign economy producing both goods, its supply price is given by **(7)**. As $B = 0$, the home supply price is infinite, given that the scale economies are external to the firm. Finally, the world demand price is still given by **(6)**. Thus **(6)** and **(7)** determine an equilibrium. In such an equilibrium, $P_D = P_S^* < P_S$ and $B^* = D_B + D_B^*$. The Marshallian stability condition for this equilibrium is again easily confirmed to be $\alpha/(\alpha-1) > \varepsilon$, and so again ensured by Assumption 3. Trade causes P to fall in both countries, so both the home and foreign countries are better off in this equilibrium than in autarky.

Proposition 5 *If Assumptions 1 – 4 and 5a hold there exist both an advantage-reversal equilibrium and another equilibrium in which the foreign economy exports good* B *and the home produces only good* A. *Both equilibria are Marshallian stable, and in each the country exporting* B *gains from trade.*

This case is treated in Section III of Ruffin (2007), except that the advantage-reversal equilibrium is not present, as shown earlier.

Since both equilibria are stable, which obtains is a matter of serendipity. Also the B exporter is better off than in autarky in each of the two equilibria. However, the advantage-reversal case gives only scale-economy gains from trade, whereas the non-reversal equilibrium generates both these and comparative-advantage gains. Thus the latter equilibrium is better in this sense.

Regime change

As shown in Ruffin (2007), stability conditions preclude the home country from using conventional tariffs for its own benefit. But there could be a scope for foreign tariff policy. Suppose the world finds itself in the advantage-reversal equilibrium. Let the foreign country implement a prohibitive tariff on B. Suppose that foreign B producers have lower costs than home producers. At this point, the foreign economy begins to export B, its tariff thereby becoming redundant, and the world converges to the non-reversal equilibrium. So consider whether, in autarky, foreign B producers do indeed have the cost advantage. Because the scale economies are external to the B firm, it takes $k L^{\alpha-1}$ as parametric (in autarky, this is also the productivity of a unit of A foregone). Then foreign B producers will have an autarkic cost advantage over their counterparts in the home economy if and only if

$$k L_0^{\,\alpha-1} \;<\; k^* L_0^{*\,\alpha-1} \; .$$

Utilizing **(4)** and **(5)** this can be written as

$$a_0^{\,\alpha-1} \left[(k^*)^{1/(\alpha + \varepsilon - \alpha\varepsilon)} - k^{\,1/(\alpha + \varepsilon - \alpha\varepsilon)} \right] > 0$$

which, in light of Assumption 2 (that $k^* > k$), will hold if and only if the world is comparative-cost dominant. That is, if the stability condition (Assumption 3) is met. This gives the following.

Proposition 6 *With Assumptions 1 – 4, and 5a, foreign tariff policy can "flip" the world from the advantage-reversal case to the non-reversal case, presumably a social improvement.*

The potential role for trade policy, in this case is just to enable regime change. Note that home policy cannot be used to engineer a "flip."

A smaller foreign economy: size reversals

Next consider an equilibrium in which the foreign economy just supplies the entire world demand

for *B*, producing no *A*.

$$B = \gamma P^{-\varepsilon} \tag{8}$$

$$B = k^* (L^*)^\alpha \tag{9}$$

$$w^* L^* = PB \tag{10}$$

$$w^* \geq 1. \tag{11}$$

Constraint **(11)** says that in equilibrium positive profit cannot be made in the *A* sector; it would be satisfied with equality were some *A* being produced. Equations **(8)**, **(9)** and **(10)** can be solved for

$$L^* = F^* (w^*; \gamma, \varepsilon, \alpha, k^*). \tag{12}$$

Let f^* denote $f(\gamma, \varepsilon, \alpha, k^*)$. Then, by definition of f,

$$f^* = F^* (1; \gamma, \varepsilon, \alpha, k^*).$$

If L^* exceeds f^*, the extra foreign labor is used to produce *A* for local consumption. The foreign economy supplies the world market for *B*, as discussed in the previous subsection, with $w^* = 1$.

Next, consider a reduction in L^* below f^*. Use **(8)**, **(9)** and **(10)** to calculate:

$$dw^*/dL^* = [(\alpha\varepsilon - \alpha - \varepsilon)] \, a_1 \, L^{*(\alpha\varepsilon - \alpha - 2\varepsilon)/\varepsilon} \tag{13}$$

where $a_1 = (\gamma^{1/\varepsilon} /\varepsilon) k^{*(\varepsilon-1)/\varepsilon}$, a positive constant. A reduction of L^* below f^*, with the foreign economy continuing to supply the world market for *B* (*i. e.*, **(8)** – **(10)**), will be consistent with **(11)** if and only if $dw^*/dL^* < 0$. From **(13)**, this will be true if and only if Assumption 3 holds. The basic intuition is that "too much" elasticity (*i. e.*, high values of α and ε) mean that a fall in L^* causes a rise in *PB*, so that **(10)** requires w^* to fall, violating **(11)**. The fact that $dw^*/dL^* < 0$ is in itself interesting: It says that the stability condition trumps economies of scale in the impact of the labor supply on wages. One might think that it would be possible with increasing returns to scale for a smaller economy to lower wages; but the stability condition rules this out.

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Furthermore, the relation between w^* and L^* in **(11)** is clearly monotonic, from **(13)**. Thus if the foreign economy can supply the entire world B market with a L^* slightly below f^*, it can do so with *any L^** below f^*. Since in such equilibria the smaller country is the supplier of the good with increasing returns to scale, we describe them as *size reversals*. Indeed, if L^* is small enough, $w^*k > k^*$. Assumption 1 would be critical for such an equilibrium, since home B firms would capture the market if they could internalize the scale economies.

Proposition 7 *If Assumptions 1 – 4 and 5b hold, but 5a does not, there exist both an advantage-reversal equilibrium and a size-reversal equilibrium in which the home economy produces only A and the foreign economy only B. Both equilibria are stable, and in the size-reversal equilibrium the foreign country gains from trade.*

Regime change

Size reversals are a characteristic of a comparative-cost dominant world. But this is just the context in which the foreign economy can use a prohibitive tariff on B to engineer regime change from an advantage-reversal equilibrium. The new regime will feature a non-reversal equilibrium if the foreign economy is relatively large, and a size-reversal equilibrium if it is relatively small.

Proposition 8 *If Assumptions 1 – 4 hold, the foreign economy can use a prohibitive tariff to flip from an advantage-reversal equilibrium to a non-reversal equilibrium, if Assumption 5a holds, or to a size-reversal equilibrium, if 5a fails.*

V Alternative Equilibria where the Foreign Economy Supplies Part of the *B* Market

Now consider the possibility that the foreign economy is not large enough to supply the world

demand for B by itself, so that if it does produce only B and export it, it must compete with a home B sector.

A knife edge

An equilibrium in which the foreign economy specializes in the production of B and supplies part of the world market is described as follows.

$$B + B^* = \gamma\, P^{-\varepsilon}$$

$$B = k(L_B)^\alpha, \qquad B^* = k^*(L^*)^\alpha$$

$$L_B = PB, \qquad w^*\, L^* = PB^*$$

$$w^* \geq 1.$$

Given w^*, these equations determine P, B, B^*, L_B and L^*. In particular, they can be solved for

$$L^* = G^*\,(w^*;\, \gamma,\, \varepsilon,\, \alpha,\, k,\, k^*). \qquad\qquad (14)$$

Define $L_1^* = G^*\,(1;\, \gamma,\, \varepsilon,\, \alpha,\, k,\, k^*)$, and let L_B^0 solve $(L_B^0/L^*)^{\alpha-1} = k^*/k$.

Assumption 5b $\quad L^* = L_1^*$

If this holds, we have an equilibrium in which the home country specializes in B and exports it, $w^* = 1$, and $L_B = L_B^0$. Costs in the two countries are identical, with the home economy having a B sector just larger enough than the foreign's to offset exactly its comparative disadvantage. But w^* is bounded below by unity, since in a competitive equilibrium the A sector cannot offer positive profit opportunities. Thus any shock that increases B in the home country will initiate a cumulative movement to the advantage-reversal outcome: The equilibrium will be unstable.

Proposition 9 *If Assumptions 1 – 4 and 5b hold there exist both an advantage-reversal equilibrium and a non-reversal equilibrium. The latter is unstable.*

A small foreign economy

Next consider a foreign economy smaller than the knife-edge size.

Assumption 5c $L_1^* > L^*$.

This assumption allows an allocation of resources whereby the home economy would produce both goods and import B. The foreign economy would produce only B. In such an allocation, scale economies neutralize exactly the foreign comparative advantage: $w^* (L_B/L_1^*)^{\alpha-1} = k^*/k$, and $w^* > 1$. The supply price is now given by **(2)**, where B denotes the home country's supply, and the home demand is

$$B + M = (\gamma/2)\, P^{-\varepsilon} \tag{15}$$

where M denotes the home country's import of B from the foreign country. Now, **(2)** and **(15)** imply

$$B + M = (\gamma/2)\, (k^{-1/\alpha}\, B^{(1-\alpha)/\alpha}]^{-\varepsilon}. \tag{16}$$

Differentiating **(16)** implicitly,

$$dB/dM = -\,1/[1 + \varepsilon([B + M]/M)([1 - \alpha]\alpha) < 0.$$

Thus the insertion of imports M from the foreign country causes home production B to fall. Thus P rises at home, and the home country loses. The magnitude of the price change also depends on the parameters, so the foreign country may either gain or lose.

Proposition 10 *If Assumptions 1 − 4 and 5c hold there exist both an advantage-reversal equilibrium and a non-reversal equilibrium in which the home economy produces both goods and the foreign economy produces only* B. *In the non-reversal equilibrium, the home country loses from trade.*

Both equilibria are stable, so it is again a matter of serendipity which obtains. In this case one

of the countries (but not both) will be able to gain from a tariff policy. Section IV of Ruffin (2007) considers this case, and shows that the home economy can gain from a tariff in the non-reversal equilibrium.

Regime change

In a comparative-cost dominant world (Assumption 3 holds), the foreign economy can again use a prohibitive tariff to flip from an advantage-reversal equilibrium to the non-reversal situation. In view of the various possible combinations of gains and losses, it is possible that the foreign economy might indeed wish to engineer a regime change. (Of course, if both countries lose from trade, it is in both countries' interest to retreat to autarky — yet another example of regime change).

VI Concluding Remarks

We have investigated the relationship between external economies of scale and comparative advantage regarding the welfare consequences of international trade. The notable feature is the possibility of an advantage reversal — where the effects of scale economies overwhelm those of comparative-cost differences. The existence of such an equilibrium implies a distinct role for trade policy, namely that such policy could be used to engineer a regime change. But this existence — and thus this role — is crucially sensitive to whether the scale economies are internal or external to the firm.

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Footnotes

1. Important contributions include Matthews (1950), Jones (1968), Kemp (1969), Melvin (1969), Panagariya (1981), and Kemp and Shimomura (2000). Helpman (1984) provides a valuable survey.

2. See Helpman (1984) for a more detailed related argument.