



Penn Institute for Economic Research  
Department of Economics  
University of Pennsylvania  
3718 Locust Walk  
Philadelphia, PA 19104-6297  
[pier@econ.upenn.edu](mailto:pier@econ.upenn.edu)  
<http://www.econ.upenn.edu/pier>

## ***PIER Working Paper 07-004***

“A Model of Money with Multilateral Matching”, Second Version

by

Manolis Galenianos and Philipp Kircher

<http://ssrn.com/abstract=957791>

# A Model of Money with Multilateral Matching\*

Manolis Galenianos<sup>†</sup>

Philipp Kircher<sup>‡</sup>

Pennsylvania State University

University of Pennsylvania

First Draft: December 2005

This Version: November 2006

## Abstract

We develop a model of monetary exchange that avoids several common criticisms of the recent microfoundations literature. First, rather than random matching, we assume that buyers know the location of all sellers, and hence the process of finding a partner is deterministic, although trade is still stochastic since the number of buyers visiting a given seller is random. Second, given multilateral matching, rather than bargaining, we assume that goods are allocated according to second-price auctions. Third, given this mechanism, we do not have to assume agents can observe each other's money holdings or preferences, as is necessary for tractability with bargaining. A novel result is that homogeneous buyers hold different amounts of money, leading to equilibrium price dispersion. We find the closed-form solution for the distribution of money holdings. We characterize equilibrium and efficient monetary policy.

## 1 Introduction

In recent years a large literature has developed which models trading frictions explicitly to deliver environments with micro-foundations for the existence of fiat money (see Kiyotaki and Wright (1993) for the canonical model). While successful in addressing many issues, some

---

\*We thank Randy Wright for his support and encouragement. We benefitted from the comments of Ed Green, Guillaume Rocheteau and Ruilin Zhou, as well as conference and seminar participants. Earlier versions of this paper were circulated under the titles “Directed Multilateral Matching in a Monetary Economy” and “Dispersion of Money Holdings and Efficiency.”

<sup>†</sup>Corresponding author. e-mail: [manolis@psu.edu](mailto:manolis@psu.edu)

<sup>‡</sup>e-mail: [kircher@econ.upenn.edu](mailto:kircher@econ.upenn.edu)

of the assumptions that are commonly used to describe decentralized trade have attracted criticism. First, trade is usually modeled to occur in random bilateral meetings between the agents in the economy. ‘Random’ refers to the fact that an agent wishing to, say, purchase a certain commodity cannot just visit a seller of the good in order to trade; instead, he has to search for trading partners, potentially meeting with people who cannot supply the good that he wants. The randomness of the matching process is at odds with the presumption that economic agents are generally aware, or can easily learn, where some commodity is traded and therefore mitigate these search costs (Howitt (2005)). Furthermore, the assumption of pairwise meetings results in a bilateral monopoly and people typically use bargaining as a way of determining the terms of trade (e.g. see Trejos and Wright (1995) and Shi (1995); exceptions include Green and Zhou (1998, 2002)). Since the outcome of the bargaining game becomes intractable in the presence of private information, it is usually assumed that the money holdings and preferences of the agents are observable. To the extent that complete information affects the agents’ incentives of holding money, an environment with private information may have different implications (such as policy recommendations).

In this paper we address these issues by modeling decentralized trade in a very different way which yields new insights while keeping with the spirit of the money search literature. In every period the agents who want to consume (buyers) know where the seller of each commodity is located and hence the actual process of finding a trading partner is deterministic. Frictions are introduced by assuming that buyers can only visit one seller at a time and they cannot coordinate their decisions of which location to go to. In other words, a buyer visits one of the sellers who produce his desired commodity, but the particular seller is picked at random among the producers of that good. This results in stochastic demand realizing at a given seller’s location. The additional frictions of anonymity and lack of double coincidence of wants make fiat money essential for trade in these meetings.

The second innovation of this paper is to exploit the multilateral nature of the matching process to formulate an alternative notion of price formation based on auctions rather than bargaining. We assume that sellers have some capacity constraints in the sense that they cannot serve all possible levels of demand that they may face. To simplify matters, we fix the supply of each seller at a single indivisible good. In this setting, a natural way to allocate the good without resorting to strong informational assumptions is to use an auction. In this paper, we assume that a second-price auction takes place.<sup>1</sup> This mechanism lends itself to the natural interpretation of intra-buyer competition for the good: the buyers that visit

---

<sup>1</sup>In the conclusions, we discuss the possibility of using different auctions. Our conjecture is that any standard auction yields the same qualitative results, but the second price auction is much easier to analyze. An alternative way to deal with private information is to allow sellers to post price-quantity menus, as in Faig and Jerez (2006) or Ennis (2006).

the same seller try to outbid each other, as in an ascending bid auction, whose outcome is identical to a second-price auction. The result is that the ‘wealthiest’ buyer purchases the good and pays the second highest money holdings to the seller. It is important to note that a buyer may hold less money than his actual valuation for the good, i.e. he may be willing but unable to spend more, as in a budget constrained auction.<sup>2</sup>

To introduce the explicit choice of liquidity for the agents and to keep our model tractable we embed the idea presented above in the framework of Lagos and Wright (2005), henceforth LW. That is, agents have periodic access to a Walrasian market where they trade in a competitive way and they can re-balance their portfolios of fiat money (i.e., choose their liquidity level). Furthermore, as in LW, we assume that the agents have quasi-linear preferences in the Walrasian market which implies that there are no wealth effects in the demand for money. As a result the trading history of an agent does not affect his choice of how much cash to hold.

In our model, the agents face randomness in consumption because demand conditions are stochastic: a buyer can always visit a seller of his desired commodity, but it may turn out that the price is above the buyer’s liquidity due to high demand. This is in contrast to models of bilateral matching where consumption uncertainty is typically due to the randomness of the matching process, i.e. due to whether a trading partner is found or not. Therefore, the trade-off faced by the buyer when deciding his money holdings is the following. Bringing more money allows him to outbid more of his potential competitors, leading to a higher probability of consuming. On the other hand, carrying fiat money is costly because the value of any unspent balances depreciates due to inflation and discounting.

Our main result is that identical buyers choose to bring different amounts of money to the search market and we derive the closed-form solution for the unique distribution of money holdings. The intuition behind this result is not hard to see: if all buyers held the same amount of money, then a deviant bringing infinitesimally more would always win the good and hence enjoy discretely higher probability of consuming for negligible additional cost. As a result, in equilibrium buyers with the same valuation for the good are indifferent between holding a range of possible money balances. Furthermore, dispersion in money leads to dispersion in prices since the sale price depends on how many buyers visit a particular seller.<sup>3</sup> The mixed strategies of the buyers can be purified by introducing type heterogeneity. In section 6 we consider two cases: the buyers have different valuations for the good or they

---

<sup>2</sup>See Che and Gale (1998). One contribution of our paper is to endogenize the budget of buyers.

<sup>3</sup>Dispersion of money holdings is typically a feature of models where an agent’s liquidity *does* depend on his history of trades, e.g. Molico (2006), Green and Zhou (1998, 2002), or Camera and Corbae (1999). Price dispersion is also an equilibrium outcome of the monetary model of Head and Kumar (2005) where it results from informational asymmetries among buyers.

have different productivity levels in the Walrasian market, both of which lead to pure strategy equilibria in term of the buyers' decision of how much money to hold.

To examine how output is affected by the inflation rate, we introduce an entry decision on the side of the sellers. This allows us to evaluate the welfare properties of our model. The main result is that entry is suboptimal except for the case where the money supply contracts at the rate of time preference – the Friedman rule. This happens because the value of fiat money depreciates over time when the inflation rate exceeds the Friedman rule and hence buyers bring less money than their valuation for the sellers' good. As a result sellers receive less on average than their social contribution to the match and therefore fewer sellers enter to the market than at the optimum. At the Friedman rule holding fiat money is costless, leading buyers to bring balances equal to their valuation of the good because the intra-buyer competition dominates.

Related monetary models include Julien, Kennes and King (in press) which considers directed search and auctions in a setting with indivisible money; Corbae, Temzelides and Wright (2003) which considers cooperative directed matching with bilateral meetings; Goldberg (in press) which constructs a monetary model with indivisible money and goods where buyers can direct their search to the sellers of their desired commodity. Related non-monetary models include Camera and Selcuk (2005) in which the trading price is sensitive to local demand conditions due to possible renegotiations, and Satterthwaite and Shneyerov (in press) which analyzes convergence to efficiency with vanishing frictions in an environment with multilateral matching and auctions.

The rest of the paper is organized as follows. Section 2 describes the model and proves some preliminary results. The following section solves the buyer's problem and derives the equilibrium distribution of money balances, while section 4 describes the entry decision of sellers. Section 5 examines the efficiency properties of inflation. Section 6 considers a number of extensions. Section 7 touches on robustness issues and concludes.

## 2 The Model

Time is discrete and runs forever. Each period is divided in two subperiods, following LW: a Walrasian market characterized by competitive trading and a search market characterized by trading frictions that are modeled explicitly. There is a continuum of infinitely-lived agents who belong to one of two different types, called *buyers* and *sellers* (types  $b$  and  $s$ , respectively). The difference is that while both types produce and consume in the Walrasian market, in the search market a buyer can only consume and a seller can only produce. Meetings in the search market occur between subsets of the population in a way described

in detail below and they are characterized by two main frictions. First, all meetings are assumed to be anonymous which precludes credit. Hence all trades have to be *quid pro quo*. Second, there is no double coincidence of wants, as is clear from the assumptions on agents' types: some agents can only produce while others can only consume. Therefore, the agents cannot use barter to exchange goods. These frictions mean that a medium of exchange is essential for trade (see Kocherlakota (1998) and Wallace (2001)).

There is a single storable object, fiat money, which can be used as a medium of exchange in the search market. The stock of money at time  $t$  is given by  $M_t^S$  and it is perfectly divisible. The money stock changes at gross rate  $\gamma$ , so that  $M_{t+1}^S = \gamma M_t^S$ , and new money is introduced, or withdrawn if  $\gamma < 1$ , via lump sump transfers to all agents in the Walrasian market. We focus on policies with  $\gamma \geq \beta \delta$ , where  $\beta \delta$  is the discount factor as discussed below, as it is easy to check that there is no equilibrium otherwise. Furthermore, to examine what happens when the rate of money growth is exactly equal to the discount factor (the Friedman rule) we take the limit of equilibria as  $\gamma \rightarrow \beta \delta$ .

We denote the measure of buyers and sellers by  $B$  and  $S$ , respectively. Let  $W_t^j(m)$  be the value of an agent of type  $j \in \{b, s\}$  who enters the Walrasian market at time  $t$  holding  $m$  units of money. His instantaneous utility depends on consumption,  $x$ , and hours of work,  $h$ . We assume that preferences are quasi-linear and take the form  $U(x) - h$ , where an hour of work produces one unit of the consumption good  $x$ . Furthermore, we assume that  $U'(x) > 0$  and  $U''(x) < 0$  for all  $x$  and the Inada conditions  $\lim_{x \rightarrow 0} U'(x) = \infty$ ,  $\lim_{x \rightarrow \infty} U'(x) = 0$ . Let  $\beta$  be the discount rate between the Walrasian and search markets and denote the value of carrying  $m'$  dollars to the search market of period  $t$  by  $V_t^j(m')$ . The agent's value function in the Walrasian market at time  $t$  is

$$\begin{aligned} W_t^j(m) &= \max_{x, h, m'} \{U(x) - h + \beta V_t^j(m')\} \\ \text{s.t. } x &\leq h + \phi_t(\hat{T}_t + m - m'), \end{aligned} \tag{1}$$

where  $\phi_t$  is the value of money in consumption terms and  $\hat{T}_t$  is the nominal monetary transfers to (or from) the agent, i.e.  $\hat{T}_t = (\gamma - 1) M_{t-1}/(B + S)$ .<sup>4</sup>

It will prove useful to solve some of the non-monetary decisions of the agents at this stage so that we can concentrate on the more interesting choices relating to money holdings later on. Substituting the constraint into equation (1) with equality gives

$$W_t^j(m) = \phi_t (m + \hat{T}_t) + \max_{x, m'} \{U(x) - x - \phi_t m' + \beta V_t^j(m')\}.$$

---

<sup>4</sup>We only examine equilibria with  $h > 0$  and hence we are ignoring the non-negativity constraints on  $h$ . One can impose conditions on the primitives to guarantee that this holds, as shown in LW.

Note that the quasi-linearity of preferences simplifies the problem of the agent significantly by eliminating wealth effects: current balances,  $m$ , do not have any effect on the decisions of consumption or future money balances. Furthermore, our assumptions on  $U(\cdot)$  ensure that  $U'(x^*) = 1$  is both a necessary and sufficient condition for the optimal choice of  $x$ .

As a result the problem can be further simplified to

$$W_t^j(m) = \phi_t (m + \hat{T}_t) + U^* + \max_{m'} [-\phi_t m' + \beta V_t^j(m')], \quad (2)$$

for  $j \in \{b, s\}$ , where  $U^* = U(x^*) - x^*$  and  $U'(x^*) = 1$ .

The choice of future money balances is more involved and we first need to describe the search market in order to see the relevant incentives.

The search market operates as follows. First, each potential seller decides whether to incur utility cost  $K$  in order to enter the search market. We interpret  $K$  as a production cost that has to be undertaken prior to matching with buyers. The seller can choose which good to produce out of the set of possible goods  $\{1, \dots, G\}$ . For the main part of the paper we examine the special case of  $G = 1$  which gives all the relevant intuition, while the case of a general  $G$  is analyzed in section 6.1. We assume that the production (entry) cost endows the seller with a single indivisible unit of the good. Furthermore, each good is perishable, can be transferred at zero cost, and the utility to the seller of consuming his own good is zero while the utility a buyer receives is given by  $u > 0$ . Indivisibility aside, the other assumptions about the goods are tailored so that the seller's reservation price is zero. Having a strictly positive reservation price does not significantly change our results but it complicates the analysis. It is therefore examined in section 6.2 by introducing a transaction cost for the seller although giving sellers positive utility from consuming their own good leads to similar results.

Sellers that enter the search market set up their shop at some physical location. There is a continuum of locations each accommodating at most one seller. We denote the buyer-seller ratio at time  $t$  by  $\lambda_t$ . We assume that the measure of potential sellers,  $S$ , is large enough so that  $\lambda_t$  is determined by an indifference condition for entry. Furthermore, it will prove convenient to normalize the measure of buyers to 1. This implies that the measure of sellers who choose to enter the search market is given by  $1/\lambda_t$ . We continue the analysis by taking  $\lambda_t$  to be a parameter and we consider the effects of entry in section 4.

Next, matching occurs between buyers and sellers. We model this in a different way from most of the literature. We assume that buyers can see all the locations that are populated with sellers and therefore they can visit a seller for sure. In this sense, the process of finding

a trading partner is deterministic even with a single type of good.<sup>5</sup> Nonetheless, the fact that this is a large market prevents buyers from coordinating with each other about what location to visit. We capture this inherent lack of coordination by assuming that every buyer chooses at random which one out of all the available sellers to visit. This assumption is crucial because it implies that the number of buyers that visit a particular seller is a random variable, and hence demand is stochastic, while supply is fixed at one unit. Therefore, the good may get rationed and some of the buyers may end up not consuming. Before describing the allocation process, note that we have urn-ball matching and so the number of buyers follows a Poisson distribution with parameter  $\lambda_t$ .<sup>6</sup> This matching function exhibits constant returns to scale and therefore the buyer-seller ratio is the only relevant statistic.

The way the good is allocated is a further innovation of this paper: we assume that a second-price auction takes place. The underlying idea is that buyers make price bids that can be matched by other buyer who have visited the same location, like an ascending bid (or, second-price) auction with the seller accepting any non-negative bids since his reservation price is zero. All buyers have the same valuation for the good, but they may hold different amounts of money. As a result, the buyer with most money (or, one of the buyers with the highest money holdings picked at random in the case of a tie) buys the good and the price that he pays is equal to the money holdings of his ‘richest’ competitor. If a single buyer appears at some location then he gets the good at a price of zero. This mechanism balances demand and supply at the lowest price that clears the market, i.e. the lowest price such that exactly one unit of the good is demanded. An innovation with respect to bilateral matching is that the presence of potentially many buyers at the same location means that the seller receives a positive share of the surplus even though he has no bargaining power.

The incentives to hold money are now clear: holding more money increases the probability of consuming since it allows the buyer to outbid more of his potential competitors; on the other hand it is costly because the value of any unspent money balances depreciates over time due to discounting and inflation. It is also worth noting that the reason why an agent may not spend his fiat money is very different than in most of the monetary search literature. In this paper, the amount of money that a buyer ends up spending depends on how many

---

<sup>5</sup>In the case of multiple types of goods, the buyers can visit the seller who has the good they desire, making matching non-random along this dimension. In the single-good case, it is the fact that the buyer can deterministically match with some seller that distinguishes our matching technology from the usual random-matching models. However, we want to abstract from the repeated game effects that may arise if a buyer meets with the same seller at every search market. One way to do this is to assume sellers populate a random location every time they enter; alternatively, we can assume that it is not the same sellers that enter the market each period.

<sup>6</sup>Suppose  $k$  buyers are allocated randomly across  $l$  sellers. The number of buyers that visit a given seller follows a binomial distribution with probability  $1/l$  and sample size  $k$ . As  $k, l \rightarrow \infty$  keeping  $k/l = \lambda$  the distribution converges to a Poisson distribution with parameter  $\lambda$ .



other buyers visit the same location and on how much money they hold. Hence, despite the fact that a buyer is matched with some seller with probability one he does not in general spend all of his money. In most of the rest of the literature, however, the cost of liquidity arises from the fact that the agent holding fiat money may fail to meet someone whose good he wants to buy due to the randomness of the matching process.

We now turn to characterizing the buyers' value function. Whether a buyer transacts, and at what price, depends on how many other buyers have visited the same location and on how much money they hold. Hence we need to introduce some more notation to describe the money holdings of all buyers: aggregating the choices of  $m'$  across buyers gives the distribution of money holdings at the end of Walrasian trading (or equivalently the beginning of the search market) which we denote by  $\hat{F}_t(\cdot)$ .

Let  $V_t^b(m, n)$  denote the expected payoff of a buyer who carries  $m$  dollars and meets  $n$  other buyers at the location that he visits. Poisson matching implies that the probability that he meets exactly  $n$  competitors is given by  $P_n^t = \lambda_t^n e^{-\lambda_t} / n!$ . The value of entering the search market with  $m$  dollars is thus given by

$$V_t^b(m) = \sum_{n=0}^{\infty} P_n^t V_t^b(m, n). \quad (3)$$

To calculate  $V_t^b(m, n)$ , let  $m_{(n)}$  be the highest money holdings among the  $n$  competitors that the current buyer faces. If  $m < m_{(n)}$  the buyer with  $m$  dollars does not transact and he keeps all his money for the next Walrasian market. If  $m > m_{(n)}$  the buyer with  $m$  dollars buys the good and pays  $m_{(n)}$  to the seller. If  $m = m_{(n)}$  there is a tie and the good is allocated at random to one of the buyers with  $m_{(n)}$  dollars, who then transfers his full money holdings to the seller. Therefore,  $m_{(n)}$  is the only statistic needed in order to calculate the buyer's payoff when matched with  $n$  competitors.

The money holdings of each competitor is a random draw from  $\hat{F}_t(\cdot)$  since buyers are allocated at random across sellers. Hence, the highest money holdings among the  $n$  other buyers is the highest order statistic among  $n$  *iid* draws from  $\hat{F}_t(\cdot)$  which is distributed according to  $\hat{F}_t(\cdot)^n$ .<sup>7</sup> Furthermore, observe that the probability that two randomly chosen buyers hold exactly the same amount of money is strictly positive only if  $\hat{F}_t(\cdot)$  has a mass point at that level. To denote this possibility, we define  $\mu_t(m) \equiv \hat{F}_t(m) - \hat{F}_t(m^-)$ , where  $\hat{F}_t(m^-) = \lim_{\tilde{m} \nearrow m} \hat{F}_t(\tilde{m})$ . This definition implies that  $\mu_t(m) > 0$  if and only if there is a mass point at  $m$ . Conditional on all competitors holding weakly less than  $m$  dollars, the number of competing buyers (out of  $n$ ) who have exactly  $m$  dollars follows a binomial distri-

---

<sup>7</sup>This is a standard result from statistics. For instance, see Hogg and Craig (1994).

bution with sample size  $n$  and probability  $\mu_t(m)/\hat{F}_t(m)$ .<sup>8</sup> Let  $q_t^{nk}(m)$  denote the probability that  $k$  out of the other  $n$  buyers hold exactly  $m$  dollars conditional on none of them having more than  $m$  dollars. If there is no mass point at  $m$ , then  $q_t^{n0}(m) = 1$  and  $q_t^{nk}(m) = 0$  for  $k \geq 1$ .

The value of meeting  $n$  competitors at time  $t$  when holding  $m$  dollars is given by

$$\begin{aligned}
V_t^b(m, n) &= [1 - \hat{F}_t(m)^n] \delta W_{t+1}^b(m) + \\
&\int_0^{m^-} [u + \delta W_{t+1}^b(m - \tilde{m})] d\hat{F}_t(\tilde{m})^n + \\
&\hat{F}_t(m)^n \sum_{k=1}^n q_t^{nk}(m) \left[ \frac{u + \delta W_{t+1}^b(0)}{k+1} + \frac{k \delta W_{t+1}^b(m)}{k+1} \right], \tag{4}
\end{aligned}$$

where  $\delta$  is the discount factor between the search and Walrasian markets. The term in the first square brackets gives the probability that at least one competitor holds *strictly more* money than  $m$  dollars, which means that the current buyer does not purchase the good and he keeps his money for next period's Walrasian market. The second term denotes the expected payoff when all other buyers hold *strictly less* money and hence the buyer with  $m$  dollars gets the good and pays the amount that his 'richest' competitor holds. The integral gives the instantaneous utility from consuming the good,  $u$ , plus the continuation value after accounting for the capital loss due to the payment. Finally, if none of the competitors bring more than  $m$  dollars but  $k \geq 1$  of them hold exactly  $m$  dollars, then with probability  $1/(k+1)$  the current buyer gets the good, consumes, and continues to the next Walrasian market without any money; with probability  $k/(k+1)$  he does not purchase and he keeps all his money for the next period. It should be clear from this discussion that the probability of a purchase is discontinuous at  $m$  if  $\hat{F}_t(\cdot)$  has a mass point at that level. Moreover, as mentioned above, the last term of equation (4) drops out if there is no mass point at  $m$ . This completes the description of the buyer's problem.

We now turn to the sellers. First, note that sellers can derive no benefit from holding money and therefore they carry no money to the search market. A seller can choose whether to enter the search market or not. If he does, he gives up  $K$  units of utility and may earn some revenues which he can spend in the following Walrasian market. Let  $\hat{\Pi}_t$  be the seller's expected revenues if he enters the search market at time  $t$ . Note that  $\hat{\Pi}_t$  is a sufficient statistic for the value of entry due to the linearity of  $W_t^s(\cdot)$ . If the seller chooses not to enter

---

<sup>8</sup>Conditional on all buyers holding weakly less than  $m$  dollars, the money holdings of an agent is a random draw from  $\hat{F}_t(\cdot)$  truncated at, but including,  $m$ . Hence, the probability that the result of any draw is exactly equal to  $m$  is given by  $\mu_t(m)/\hat{F}_t(m)$ . The binomial distribution follows since there are  $n$  draws.

he continues to the following Walrasian market without money. As a result, the seller's value of the search market is given by

$$V_t^s = \max\{-K + \delta W_{t+1}^s(\hat{\Pi}_t), \delta W_{t+1}^s(0)\}.$$

In equilibrium sellers are indifferent between the two options which means that, using equation (2), the following condition has to hold for all  $t$ :

$$\delta \phi_{t+1} \hat{\Pi}_t = K. \quad (5)$$

To determine  $\hat{\Pi}_t$ , note that the price a seller receives is equal to the second highest money holdings among the buyers that show up in his location. When  $n$  buyers visit a particular seller, the second highest order statistic is distributed according to  $\hat{F}_t^{(n-1, n)}(m) = n \hat{F}_t(m)^{n-1} [1 - \hat{F}_t(m)] + \hat{F}_t(m)^n$  (Hogg and Craig (1994)). The probability that  $n$  buyers show up is given by  $P_n^t$ , and we define the distribution of prices by  $\hat{G}_t(m) \equiv \sum_{n=1}^{\infty} P_n^t \hat{F}_t^{(n-1, n)}(m)$ . In other words,  $\hat{G}_t(m)$  denotes the probability that a seller receives no more than  $m$  dollars in the search market at time  $t$ , after summing over all the possible number of buyers. Therefore, the expected revenues of a seller at  $t$  are given by  $\hat{\Pi}_t = \int_0^{\infty} \tilde{m} d\hat{G}_t(\tilde{m})$ .

Last, we need to define market clearing in the money market. Since sellers have zero balances, the demand for money at time  $t$  is given by the amount that buyers want to hold, i.e.  $M_t^D = \int_0^{\infty} \tilde{m} d\hat{F}_t(\tilde{m})$ . At time  $t$ , the money market is in equilibrium if

$$M_t^D = M_t^S. \quad (6)$$

Turning to the equilibrium definition, note that there is always an equilibrium where fiat money is not valued, as is common in monetary models. Throughout this paper we concentrate attention on monetary equilibria with the property  $\phi_t > 0 \forall t$ , and statements about non-existence of an equilibrium refer to monetary equilibria. Furthermore, we only examine stationary equilibria in the sense that real variables remain constant over time. In particular, we restrict attention to equilibria where  $\lambda_t = \lambda_{t+1}$  and the real demand for money does not change, i.e.  $\phi_t M_t^D = \phi_{t+1} M_{t+1}^D$ . Since the money supply grows at a constant rate  $\gamma$  and  $M_t^D = M_t^S$  the latter condition implies that  $\phi_t = \gamma \phi_{t+1}$ . An equilibrium is defined as follows.

**Definition 2.1** *An equilibrium is a list  $\{W_t^j, V_t^j, \hat{F}_t, \phi_t, \lambda_t\}$  where  $W_t^j$  and  $V_t^j$  are the value functions,  $\hat{F}_t$  is the distribution of money holdings at the beginning of the search market at  $t$ ,  $\phi_t$  is the price of money at  $t$ , and  $\lambda_t$  is the buyer-seller ratio at  $t$  such that the following*

conditions are satisfied for all  $t$ .

1. *Optimality:* given  $\phi_t$ , any  $m' \in \text{supp}\hat{F}_t$  solves (2).
2. *Market Clearing:* equation (6) holds.
3. *Free entry:* equation (5) holds.
4. *Monetary Equilibrium and Stationarity:*  $\phi_t = \gamma \phi_{t+1} > 0$  and  $\lambda_t = \lambda_{t+1}$ .

At this stage we should remark that there are two important decisions that agents make in our environment: buyers choose how much money to hold and sellers choose whether to enter. More specifically, a buyer takes as given  $\lambda_t$ ,  $\phi_t$ , and other buyers' decisions in order to pick his optimal holdings. Aggregating across buyers, this yields  $\hat{F}_t(\cdot)$  as a function of  $\lambda_t$  and  $\phi_t$ . In equilibrium, the price of money is such that money demand equals  $M_t^S$ , which pins down  $\phi_t$  as a function of the buyer-seller ratio and the supply of money. Last, free entry of sellers gives  $\lambda_t$  as a function of the cost of entry. We proceed to characterize the buyers' decisions in the next section for a given  $\lambda_t$ . The entry of sellers is examined in section 4.

### 3 The Buyers' Problem

At the beginning of every period the problem of the individual buyer is to choose the optimal money holdings, taking as given the choices of all other agents and the price of money. It is immediate that the utility of consumption puts an upper bound on the range of the optimal fiat money decision. Let  $m_t^*$  be such that a buyer is indifferent between spending  $m_t^*$  to consume or keeping the full amount for the next Walrasian market. This amount exists since  $u < \infty$ , and it is defined by the following equation:  $u + \delta W_{t+1}^b(0) = \delta W_{t+1}^b(m_t^*) \Rightarrow m_t^* = u/(\delta \phi_{t+1})$ . It is easy to verify that in equilibrium a buyer never brings more than  $m_t^*$  to the search market, since any additional amount is not spent and hence it simply depreciates. Letting  $\underline{m}_t$  and  $\bar{m}_t$  denote the infimum and supremum, respectively, of the support of  $\hat{F}_t(\cdot)$  this discussion implies that  $0 \leq \underline{m}_t \leq \bar{m}_t \leq m_t^*$ .

We can reformulate the buyer's problem on a period-by-period basis as

$$\max_{m \in [0, m_t^*]} -\phi_t m + \beta V_t^b(m), \quad (7)$$

taking  $\hat{F}_t(\cdot)$  and  $\phi_t$  as given (stationarity implies that knowing the price of money for some  $t$  pins down the whole path of prices). The first proposition describes some properties that the distribution  $\hat{F}_t(\cdot)$  has to satisfy in equilibrium.

**Proposition 3.1** *In equilibrium  $\hat{F}_t(\cdot)$  is non-atomic on  $[0, m_t^*)$ , the support of  $\hat{F}_t(\cdot)$  is connected, and the infimum of the support is 0.*

Proof: Suppose that  $\hat{F}_t(\cdot)$  has a mass point at some  $\tilde{m} \in [0, m_t^*)$  and recall that equation (4) implies that the probability of buying is discontinuous at  $\tilde{m}$ . Purchasing the good for  $\tilde{m}$  dollars gives positive net utility (since  $\tilde{m} < m_t^*$ ) and hence  $V_t^b(\tilde{m}) < V_t^b(\tilde{m}^+)$ . Since the cost of bringing infinitesimally more money is negligible it is clear that bringing  $\tilde{m} + \epsilon$  yields strictly higher payoff than  $\tilde{m}$  and therefore in equilibrium a buyer never brings  $\tilde{m}$  yielding a contradiction.

Suppose that there is no buyer whose money holdings belong to some interval  $(m_1, m_2)$ , with  $\underline{m}_t \leq m_1 < m_2 \leq \bar{m}_t$ . We now show that the buyer with  $m_1$  dollars is strictly better off. The reason is that the  $m_1$ -buyer trades in exactly the same events as the buyer with  $m_2$  dollars since they both outbid exactly the same competitors (except when there is a mass point at  $m_2$  which can only occur if  $m_2 = m_t^*$ ; however in that case the additional transactions that the  $m_2$ -buyer can perform do not yield any utility gains since he is indifferent between keeping his money or consuming). It is therefore easy to verify that  $V_t^b(m_2) = V_t^b(m_1) + \phi_{t+1} (m_2 - m_1)$ . Examining the initial decision of how much money to hold, we have that  $-\phi_t m_1 + \beta V_t^b(m_1) - [-\phi_t m_2 + \beta V_t^b(m_2)] = (m_2 - m_1) [\phi_t - \beta \delta \phi_{t+1}]$  which is strictly positive since  $\phi_t = \gamma \phi_{t+1}$  and  $\gamma > \beta \delta$ . This means that choosing to carry  $m_1$  dollars gives higher value than holding  $m_2$  which cannot hold in equilibrium.

Last, a buyer bringing  $\underline{m}_t$  dollars can only transact when he does not meet any competitors, in which case the price he pays is equal to 0. This means that  $V_t(\underline{m}_t) = V_t(0) + \delta \phi_{t+1} \underline{m}_t$ , which implies that  $\underline{m}_t > 0$  cannot occur in equilibrium for the same reason as above. *QED*

The reason why  $\hat{F}_t(\cdot)$  is non-atomic in its interior is straightforward to see (the next proposition proves that there is no mass point at the upper boundary either). If there is a mass point in the distribution of money holdings, then it is very likely to meet some buyer holding exactly that amount of money. In that case, a buyer who brings infinitesimally more money faces a discretely higher probability of winning the auction for negligible additional cost. Therefore, this buyer enjoys a higher expected payoff which cannot happen in equilibrium.<sup>9</sup>

One important implication of this result is that the optimal decision of buyers is correspondence valued: there is a range of values of  $m$  that, in equilibrium, yield the same expected payoff and therefore buyers are willing to randomize over them. Furthermore,  $V_t(m)$  is *not* strictly concave, but rather it has to be linear in the domain of solutions, as can be seen from

---

<sup>9</sup>The logic of this proof is similar to the no mass point proof of Burdett and Judd (1983) and Burdett and Mortensen (1998) though the context is very different.

equation (7).

For the remainder of the paper we only consider  $\hat{F}_t(\cdot)$  that are continuous on  $[0, m_t^*)$  with  $\hat{F}_t(0) = 0$  and  $\text{supp}\hat{F}_t = [0, \bar{m}_t]$ . As a result, we can rewrite equations (3) and (4) as follows:

$$V_t^b(m) = \delta W_{t+1}^b(m) + \sum_{n=0}^{\infty} P_n^t \{u \hat{F}_t(m)^n - \delta \phi_{t+1} \int_0^m \tilde{m} d\hat{F}_t(\tilde{m})^n\}. \quad (8)$$

This expression is very intuitive: the first term is the value that the buyer can guarantee himself without a purchase; inside the braces, the first term is the probability of buying the good times the instantaneous utility of consumption while the second term is the expected capital loss from a purchase. Note that we have not accounted for the event where the buyer has  $m = m_t^*$  dollars and he meets another buyer holding exactly the same amount, which can occur since we have not ruled out the possibility of a mass point at  $m_t^*$ . However, in that event the price is  $m_t^*$  which means that the buyer is indifferent between buying the good or continuing with all his money. Therefore, the value of holding  $m_t^*$  is still given by equation (8).

We now turn to the explicit characterization of the solution to the buyer's problem. In equilibrium,  $-\phi_t m + \beta V_t^b(m)$  has to be constant on  $[0, \bar{m}_t]$ . Our strategy is to construct  $\hat{F}_t(\cdot)$  so that this condition holds.

**Proposition 3.2** *In equilibrium, the distribution of money holdings is uniquely defined by*

$$\hat{F}_t(m) = \frac{1}{\lambda_t} \log\{1 - e^{\lambda_t} [\gamma/(\beta \delta) - 1] \log[1 - \frac{\delta \phi_t m}{\gamma u}]\}. \quad (9)$$

Furthermore,  $\bar{m}_t < m_t^*$ .

Proof: Equation (7) implies that  $V_t^{b'}(m) = \phi_t/\beta$  for  $m \in [0, \bar{m}_t]$ . For  $V_t^b(\cdot)$  to be differentiable, any equilibrium  $\hat{F}_t(\cdot)$  has to be differentiable on  $(0, \bar{m}_t)$ . We start by assuming differentiability and we then verify that our solution satisfies this property.

Taking the derivative of (8) with respect to  $m$  we get (using Leibniz's rule and noting that  $m$  does not enter the integrand)

$$\begin{aligned} V_t^{b'}(m) &= \delta \phi_{t+1} + \sum_{n=0}^{\infty} P_n^t \{u n \hat{F}_t(m)^{n-1} \hat{F}_t'(m) - \delta \phi_{t+1} m n \hat{F}_t(m)^{n-1} \hat{F}_t'(m)\} \\ &= \delta \phi_{t+1} + (u - \delta \phi_{t+1} m) \hat{F}_t'(m) \sum_{n=0}^{\infty} P_n^t n \hat{F}_t(m)^{n-1} \\ &= \delta \phi_{t+1} + (u - \delta \phi_{t+1} m) \hat{F}_t'(m) \lambda_t e^{-\lambda_t (1-\hat{F}_t(m))}, \end{aligned} \quad (10)$$

where the last step follows from the fact that  $n \sim Po(\lambda_t)$ .

Equating (10) with  $\phi_t/\beta$  and rearranging yields the following differential equation:

$$\lambda_t \hat{F}'_t(m) e^{\lambda_t \hat{F}_t(m)} = e^{\lambda_t} \frac{\delta \phi_{t+1} i_t}{u - \delta \phi_{t+1} m},$$

where  $i_t \equiv \phi_t/(\phi_{t+1} \beta \delta) - 1$  is the nominal interest rate at  $t$ . Integrating both sides over  $m$  and using the initial condition  $\hat{F}_t(0) = 0$  yields (9), recalling that  $\phi_t = \phi_{t+1} \gamma$ .

The maximum money balances,  $\bar{m}_t$ , can be calculated by using  $\hat{F}_t(\bar{m}_t) = 1$ :

$$\bar{m}_t = \frac{u}{\delta \phi_{t+1}} (1 - e^{-\frac{1-e^{-\lambda_t}}{i_t}}). \quad (11)$$

Since  $m_t^* = u/(\delta \phi_{t+1})$  it is clear that all buyers bring less money than  $m_t^*$  and hence there is no mass point in the distribution of money holdings. *QED*

The next step is to close the buyers' side of the model by finding the equilibrium price of money,  $\phi_t$ , which equates the demand of money with exogenous supply  $M_t^S$ .

**Proposition 3.3** *There is a unique equilibrium price  $\phi_t^*$  such that  $M_t^D = M_t^S$ .*

Proof: Using the expressions derived in the previous proposition, we can define money demand at  $t$  as a function of  $\phi_t$ ,  $M_t^D(\phi_t)$ . We first prove that money demand decreases monotonically in the price of money by showing that the money distribution that results from a low  $\phi_t$  first order stochastically dominates the one that results from high  $\phi_t$ . Using equation (9), some algebra shows that

$$\partial \hat{F}_t(m)/\partial \phi_t = \frac{e^{\lambda_t} i_t \delta m}{\lambda_t^2 \{1 - e^{\lambda_t} i_t \ln[1 - (\delta \phi_t m)/(\gamma u)]\} [\gamma u - \delta \phi_t m]} > 0,$$

which implies that the proportion of buyers holding no more than  $m$  dollars increases with  $\phi_t$  and hence  $\partial M_t^D/\partial \phi_t < 0$ .

To complete the proof we need to show that  $M_t^D(\infty) < M_t^S < M_t^D(0)$  for some arbitrary  $M_t^S$ . Note that  $\lim_{\phi_t \rightarrow \infty} \bar{m}_t = 0 \Rightarrow \lim_{\phi_t \rightarrow \infty} M_t^D(\phi_t) = 0$ . Also,  $\lim_{\phi_t \rightarrow 0} \bar{m}_t = \infty$  and  $\lim_{\phi_t \rightarrow 0} \hat{F}_t(m) = 0, \forall m < \bar{m}_t$  imply that  $\lim_{\phi_t \rightarrow 0} M_t^D(\phi_t) = \infty$ . *QED*

**Corollary 3.1** *The price of money is determinate. The buyer-seller ratio,  $\lambda_t$ , uniquely determines the distribution of money holdings of buyers.*

To simplify notation, we now redefine all variables in real terms. We express a dollar in terms of its consumption value in the search market. Using equation (8) it is clear that  $m$

dollars are worth  $z_t = \delta \phi_{t+1} m$  units of utility at time  $t$ , i.e. we convert the  $m$  dollars to utility terms at the price of the following Walrasian market (when they can next be used) and discount that utility to present search market terms. Together with the stationarity condition  $\phi_t = \gamma \phi_{t+1}$ , this implies that the real value of any unspent balances depreciates at rate  $\gamma$ : in period  $t + 1$  the  $m$  dollars are worth  $z_{t+1} = \delta \phi_{t+2} m = z_t/\gamma$ . Similarly, we denote real transfers by  $T_t = \delta \phi_{t+1} \hat{T}_t$  and the expected real revenues by  $\Pi_t = \delta \phi_{t+1} \hat{\Pi}_t$ . Last, note that since we are in a stationary environment we can dispense with the time subscript.

Making the relevant substitutions into our value functions we obtain

$$W^b(z) = (z + T) \gamma/\delta + U^* + \max_{z'} \{-z' \gamma/\delta + \beta V^b(z')\} \quad (12)$$

$$V^b(z) = \delta W^b(z/\gamma) + \sum_{n=0}^{\infty} P_n \{u F(z)^n - \int_0^z \tilde{z} dF(\tilde{z})^n\} \quad (13)$$

$$W^s(z) = (z + T) \gamma/\delta + U^* + \beta V^s \quad (14)$$

$$V^s = \max\{\delta W^s(\Pi/\gamma) - K, \delta W^s(0)\}. \quad (15)$$

We can also rewrite the distribution of real balances as

$$F(z) = \frac{1}{\lambda} \ln\{1 - e^\lambda i \ln(1 - \frac{z}{u})\}, \quad (16)$$

where  $i = \gamma/(\delta \beta) - 1$  and define the distribution of real revenues  $G(\cdot)$  accordingly. Furthermore, this means that the highest real money holdings are given by

$$\bar{z} = u (1 - e^{-\frac{1-e^{-\lambda}}{i}}). \quad (17)$$

Note that  $\bar{z} < u$  as long as  $i > 0$ . As  $\gamma \rightarrow \beta \delta$  and the rate of money growth approaches the Friedman rule,  $i \rightarrow 0$ . This implies that  $F(z) \rightarrow 0$  for any  $z < \bar{z}$  and the distribution of real balances collapses to a mass point at  $u$ . Moreover, at the Friedman rule  $\bar{z} = u$  which means that the real balances of *every* buyer is equal to his valuation for the good.

Figure 1 shows the density of real money holdings for different levels of the interest rate and  $\lambda = 1$ . At very high interest rates the density is decreasing. In the intermediate range it is U-shaped. For low interest rate it is increasing.

## 4 The Sellers' Problem

In the previous section we established that the buyer-seller ratio uniquely determines the buyers' distribution of real money holding and hence expected real profits. Therefore, from



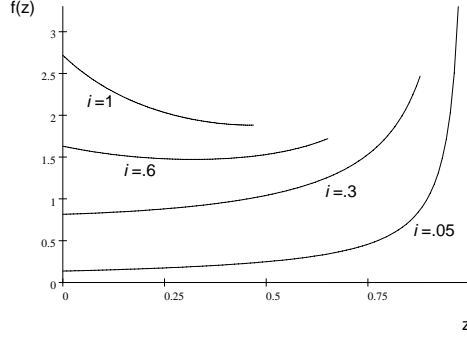


Figure 1: Density of real money balances for different levels of the interest rate  $i$  and  $u = 1$ ,  $\lambda = 1$ .

now on we write  $\Pi(\lambda)$ . In equilibrium, free entry requires that  $\Pi(\lambda) = K$ . We first characterize the distribution of prices and prove that if an equilibrium exists then there is a unique  $\lambda^*$  satisfying the free entry condition. We then show that an equilibrium exists if and only if the inflation rate is below a threshold value. The uniqueness of equilibrium is interesting because typically there are multiple stationary equilibria in models with bilateral matching and bargaining (Rocheteau and Wright (2005)). We elaborate on the reasons that lead to this difference in results at the end of the section.

**Proposition 4.1** *The distribution of prices is given by*

$$G(z) = (1 + \lambda - \lambda F(z)) e^{-\lambda (1-F(z))}, \quad (18)$$

where  $F(z)$  is defined in (16).

Proof: To get the distribution of prices recall that  $G(z) = \sum_{n=0}^{\infty} P_n F^{(n-1,n)}(z)$  and  $F^{(n-1)}(z) = n F(z)^{n-1} [1 - F(z)] + F(z)^n$ . Equation (18) follows after some algebra. *QED*

**Proposition 4.2** *If an equilibrium exists, then it is unique.*

Proof: To prove uniqueness, it is sufficient to show that  $\partial\Pi(\lambda)/\partial\lambda > 0$ . We show that  $\partial G(z)/\partial\lambda < 0$  which means that the distribution of prices for high  $\lambda$  first order stochastically dominates the one for a low  $\lambda$ . This implies that the expected price (profits) is strictly higher when there are more buyers per seller in the search market. Using equation (16), one can

show that

$$\begin{aligned}\frac{\partial F(z)}{\partial \lambda} &= -\frac{F(z)}{\lambda} + \frac{-e^{-\lambda} i \ln(1 - z/u)}{\lambda [1 - e^{-\lambda} i \ln(1 - z/u)]} \\ &= \frac{1}{\lambda} [1 - F(z) - e^{-\lambda} F(z)].\end{aligned}\tag{19}$$

The last step is to note that

$$\begin{aligned}\frac{\partial G(z)}{\partial \lambda} &= -\lambda (1 - F(z)) [1 - F(z) - \lambda \frac{\partial F(z)}{\partial \lambda}] e^{-\lambda (1 - F(z))} \\ &= -\lambda (1 - F(z)) e^{-\lambda} < 0,\end{aligned}\tag{20}$$

where the second equality results from inserting equation (19). This completes the proof. *QED*

It is easy to check that  $\lim_{\lambda \rightarrow 0} \Pi(\lambda) = 0$ . Therefore, if  $\lim_{\lambda \rightarrow \infty} \Pi(\lambda) > K$  the (unique) equilibrium exists.

**Proposition 4.3** *Given  $K$ , an equilibrium exists if and only if  $\gamma < \bar{\gamma}(K)$ , where  $\bar{\gamma}(K)$  is defined by  $K = u (1 - e^{-1/\bar{i}})$  and  $\bar{i} = \bar{\gamma}(K)/(\beta \delta) - 1$ .*

Proof: As  $\lambda \rightarrow \infty$  a seller is visited by some buyer for sure. Furthermore, the seller's revenues converge to the highest money holdings. Recalling that  $\bar{z} = u (1 - e^{-\frac{1 - e^{-\lambda}}{i}})$ ,

$$\lim_{\lambda \rightarrow \infty} \Pi(\lambda) = \lim_{\lambda \rightarrow \infty} \bar{z} = u (1 - e^{-1/\bar{i}}).$$

Noting that the maximum profits are decreasing in the inflation rate ( $\partial \Pi(\infty)/\partial \gamma < 0$ ) and that  $\Pi(\infty) = K$  if and only if  $\gamma = \bar{\gamma}(K)$  completes the proof. *QED*

As mentioned above, a result of this section that is worth commenting on is the fact that the expected profits of sellers increase monotonically in the buyer-seller ratio,  $\lambda$ , which leads to uniqueness of equilibrium. This happens because the distribution of prices resulting from a high value of  $\lambda$  first order stochastically dominates the one resulting from a low value, as seen in figure 2.<sup>10</sup> Higher revenues, however, do not occur because every buyer brings more money when faced with more competition. In fact, some buyers may choose to hold less money than before as can be seen in the left graph of figure 2, where buyers at the bottom end of the distribution of money holdings choose to hold *less* money as  $\lambda$  increases from 1 to 5. However, even if some of the buyers bring less money, there are more buyers around which

---

<sup>10</sup>Note that the figure shows the density part of the distribution of prices and it omits the mass point at a zero price which results when zero or one buyers show up.

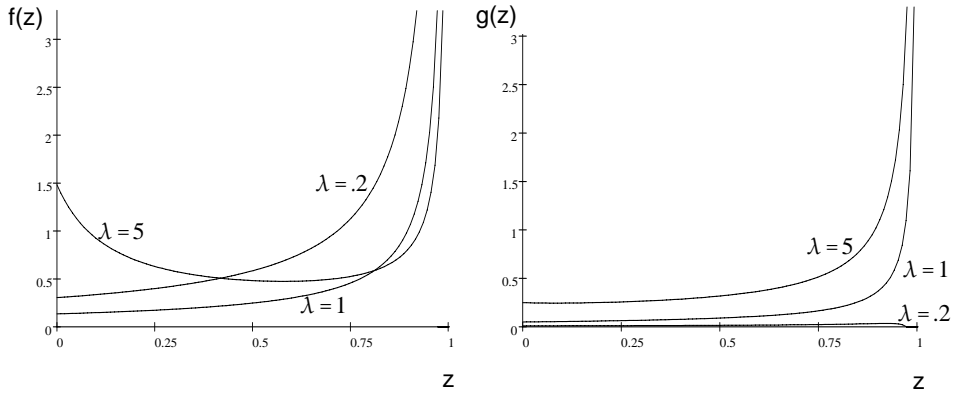


Figure 2: Density of real balances and prices for different levels  $\lambda$  and  $u = 1$ ,  $i = 0.05$ .

pushes the distribution of prices upwards. It is therefore the multilateral nature of matching that leads to the uniqueness result.

In contrast, Rocheteau and Wright (2005) find multiplicity of equilibria in the model with bilateral matching and bargaining. The reason why expected profits do not increase monotonically in the buyer-seller ratio in their framework is the following: a buyer anticipates that he is less likely to be matched with a seller the higher is  $\lambda$ . Therefore, any money the buyer may hold is more likely to depreciate rather than to be spent and hence he brings lower balances to the market. As a result, although a seller can find a buyer with greater probability, he now receives less money per match. Whether expected profits (probability of trade times revenues) increase in the buyer-seller ratio or not depends on parameter values, leading to generic multiplicity of equilibria.

## 5 Efficiency and Inflation

In this section we examine the effects of inflation on efficiency. Since every meeting between a seller and some buyers results in a purchase, the question of interest is whether the efficient number of sellers enter into the market. We show that efficiency is attained only when the inflation rate is at the Friedman rule, i.e. the stock of money decreases at the rate of time preference.

We start by solving for the optimal level of entry. A planner chooses  $\lambda$  to maximize the surplus in the search market. In other words, he maximizes the following objective function:

$$\mathcal{W} = \frac{(1 - e^{-\lambda}) u}{\lambda} - \frac{K}{\lambda}. \quad (21)$$

The first term gives the total number of sellers ( $1/\lambda$ ), times the probability that a seller trades ( $1 - e^{-\lambda}$ ), times the surplus that is generated from a trade ( $u$ ). The second term gives the total production cost of  $1/\lambda$  sellers.

Setting the first order conditions with respect to  $\lambda$  to zero yields

$$(1 - e^{-\lambda^P} - \lambda^P e^{-\lambda^P}) u = K. \quad (22)$$

It is easy to check that the second derivative is negative, hence equation (22) is both necessary and sufficient. The planner's optimal buyer-seller ratio is denoted by  $\lambda^P$ .

Equation (22) suggests that the market should operate (i.e.  $\lambda < \infty$ ) whenever  $u > K$ . In other words, there is an immediate inefficiency whenever the inflation rate is high enough to prevent any entry to the search market. Now consider the case where an equilibrium does exist. The buyer-seller ratio is determined by the free entry condition  $\Pi(\lambda) = K$  and therefore efficiency is attained only if expected profits are given by  $\Pi(\lambda) = (1 - e^{-\lambda} - \lambda e^{-\lambda}) u$ . It turns out that the sellers' expected profits are equal to the amount that leads to efficient entry only when the inflation rate is at the Friedman rule. To see this, recall that as  $\gamma \rightarrow \beta \delta$  the real balances of all buyers are equal to their valuation of the good,  $u$ . As a result, a seller appropriates the full surplus of a match if two or more buyers show up and he receives zero in the complementary case. The probability of the former event is given by  $1 - P_0 - P_1 = 1 - e^{-\lambda} - \lambda e^{-\lambda}$  and hence the expected revenues of the seller are equal to the left hand side of (22) leading to efficient entry of sellers. Last, if the inflation rate is more than the Friedman rule, then the sellers appropriate a strictly lower part of the surplus and therefore entry is suboptimal. The following proposition summarizes our results.

**Proposition 5.1** *The level of entry is efficient at the Friedman rule. When  $\gamma > \beta \delta$  entry is suboptimal.*

Proof: See above. *QED*

## 6 Extensions

We now consider several extensions to the basic framework developed above: multiple goods, positive transaction cost for sellers, buyers that are heterogeneous in the utility they derive from consumption, and buyers that are heterogeneous in terms of their productivity. Our equilibrium notion remains the same, with the obvious adjustments for the additional elements that we introduce.

## 6.1 Multiple Goods

There are  $G$  goods and every active seller has to decide which good to produce before entering the search market. Let  $S_g$  denote the number of sellers who enter the search market to sell good  $g$ . Each buyer learns which good he prefers to consume for the period before choosing his money holdings: consuming the preferred good yields  $u$  and consuming any other good yields zero.<sup>11</sup> A buyer wants to consume good  $g \in \{1, \dots, G\}$  with probability  $Q_g$ , where  $\sum_{g=1}^G Q_g = 1$ . This means that the number of buyers preferring good  $g$  each period is given by  $B_g = B Q_g$ . Since buyers can see the produce of each seller, they visit a location where their desired good is for sale. We retain the assumption that buyers cannot coordinate and thus randomly choose from one of those sellers. Therefore, all sellers who have good  $g$  face the same buyer-seller ratio, which we label  $\lambda_g$ .

Since sellers can choose which good to sell, the expected profits of producing any good have to be equal in equilibrium. Furthermore, recall that profits can be summarized by, and are a strictly increasing function of, the buyer-seller ratio that a seller faces. This implies that in equilibrium the sellers of all goods face exactly the same queue length and hence the distribution of money holdings is the same across the buyers of all goods. This completes the description of the multiple goods case.

In contrast to the standard random search models such as Kiyotaki and Wright (1993), the number of goods does not affect our results. The reason is that agents can observe the location and produce of sellers, and therefore always target their search to those producers that sell their desired items. Randomness does not arise from mismatch between buyers and sellers but rather from the fact that each seller supplies only a limited number of items and therefore might not serve all buyers.

## 6.2 Positive Marginal Cost

Next we investigate an environment where sellers incur marginal cost  $c \in (0, u)$  when they transfer the good to some buyer, in addition to the production cost  $K$ .<sup>12</sup> The main difference is that now the sellers have a positive reservation price. This makes it costly for buyers to participate in the search market because they need to bring a minimum amount of money in order to have a chance of purchasing, as opposed to being able to buy even if they bring zero dollars, which was the case in section 3. Since that amount may remain unspent, and hence

---

<sup>11</sup>Nothing substantial would change if instead we introduced a good-specific utility of consumption  $u_g$ . Note, however, that the average amount of money that an agent brings depends on the utility of the good that he wants to consume that period.

<sup>12</sup>If sellers had positive consumption value for the good, the analysis would be practically identical. Therefore, we do not consider that case explicitly.

lose value, this introduces a positive participation cost. We proceed the analysis by first fixing the number of sellers that are in the market to some  $\bar{S}$  and analyzing the participation decision of buyers. We then look at the entry problem of sellers.

Let  $r$  be the minimum real price that the seller is willing to receive for the good. A seller is indifferent between selling the good or not at exactly  $r = c$ . It is immediate that bringing balances in  $(0, r)$  is dominated by bringing zero, since a positive but insufficient amount cannot be used for any purchase and hence it simply depreciates. We label the buyers that bring  $r$  real dollars or more to the market as *effective* buyers and denote their measure by  $\bar{B}$ . The effective buyer-seller ratio is then given by  $\lambda_E = \bar{B}/\bar{S}$  and any (monetary) equilibrium has  $\lambda_E \in (0, \infty)$ . The decision problem of effective buyers is almost identical to the one in section 3, and leads to a distribution of money holdings as characterized in the following proposition.

**Proposition 6.1** *Consider  $c \in (0, u)$ . In equilibrium the distribution of money holdings of effective buyers is non-atomic in  $[r, z^*)$  and it is given by  $F(z) = \frac{1}{\lambda_E} \ln\{1 - e^{\lambda_E} [\gamma/(\beta \delta) - 1] \ln[\frac{u-z}{u-c}]\}$ . Also,  $\bar{z} < z^*$ .*

Proof: The proof is similar to propositions 3.1 and 3.2 and is therefore omitted. *QED*

We turn to the buyers' decision to participate in the search market. We want to compare the expected payoffs of participating in the search market versus staying out. Since all effective buyers earn the same expected payoffs regardless of how much money they carry, a simple way to characterize their value is to consider a buyer holding  $r$ . This buyer only purchases if there is no competitor at the location he visits (which occurs with probability  $e^{-\lambda_E}$ ) in which case he spends all of his money. Otherwise, he continues to the next Walrasian market with  $r$  dollars. Therefore, this buyer participates in the market only if

$$\begin{aligned} \beta \delta W^b(0) &\leq -r \gamma/\delta + \beta [e^{-\lambda_E} (u + \delta W^b(0)) + (1 - e^{-\lambda_E}) \delta W^b(r/\gamma)] \Leftrightarrow \\ i c &\leq e^{-\lambda_E} (u - c), \end{aligned} \tag{23}$$

recalling that  $\delta W^b(r/\gamma) = r + \delta W^b(0)$  and  $r = c$ . This condition puts an upper bound on the buyer-seller ratio, as a function of the inflation rate:

$$\bar{\lambda}_E(\gamma) \leq -\log\left(\frac{c i}{u - c}\right). \tag{24}$$

It is obvious that as the inflation rate increases, the buyers are willing to participate in the market only if they face less competition from each other. Note that when  $\gamma \geq \beta \delta u/c$ , the right-hand side of (24) is non-positive, hence there is no trade in the search market.

This occurs because it is too costly for buyers to bring even the minimum amount required by sellers to produce (this effect is similar to the case where no sellers enter into the search market, as described in section 4). If the inflation rate is below that threshold, the number of buyers is determined by the indifference condition that results from setting (23) to equality, for a given number of sellers  $\bar{S}$ .

Turning to the entry decision of sellers, note that their profits are still characterized by the expressions derived in section 4. The only difference is that now the upper bound for profits is given by  $\Pi(\bar{\lambda})$  which is strictly lower than  $\lim_{\lambda \rightarrow \infty} \Pi(\lambda)$ . As a result, the set of  $K$ 's that can support trading in the search market is strictly smaller than in section 4. A similar investigation as in section 5 easily reveals that the market attains efficient entry if and only if the long-run monetary growth rate is at the Friedman rule.

### 6.3 Heterogeneous Valuations: Discrete Types

Consider the case of two types of buyers who differ in how much they enjoy consuming the good of the search market. In particular, share  $\alpha_H$  are high type buyers and they receive  $u_H$  when consuming; the complementary proportion,  $\alpha_L \equiv 1 - \alpha_H$ , are low types and receive  $u_L \in (0, u_H)$ . We show that every low type buyer holds less money than any high type buyer. We then characterize the distributions of money balances of each type.

Let  $F_j(\cdot)$  denote the distribution of real balances and  $Z_j \equiv \text{supp}F_j(\cdot)$  denote the support of that distribution for an agent of type  $j \in \{L, H\}$ . Then,  $F(z) = \alpha_H F_H(z) + \alpha_L F_L(z)$  gives the unconditional distribution of money balances.

**Proposition 6.2** *Consider an environment in which share  $\alpha_H$  of consumer has consumption utility  $u_H$  while share  $1 - \alpha_H$  has utility  $u_L < u_H$ . In equilibrium, real money distributions are characterized by*

$$F_L(z) = \frac{1}{\lambda_L} \log\{1 - e^\lambda i \ln(1 - \frac{z}{u_L})\}, \quad \forall z \in Z_L = [0, \bar{z}_L] \quad (25)$$

$$F_H(z) = \frac{1}{\lambda_H} \log\{1 - e^{\lambda_H} i \ln(\frac{u_H - z}{u_H - \bar{z}_L})\} \quad \forall z \in Z_H = [\bar{z}_L, \bar{z}_H], \quad (26)$$

where  $\bar{z}_L = u_L (1 - e^{-e^{-\lambda_H}(1-e^{-\lambda_L})/i})$  and  $\bar{z}_H = u_H (1 - e^{-(1-e^{-\lambda_H})/i}) + \bar{z}_L e^{-(1-e^{-\lambda_H})/i}$ .

Proof: Let  $\bar{z}$  be the highest balances of any agent. An argument similar to proposition 3.1 shows that  $\text{supp}F = [0, \bar{z}]$  and  $F(\cdot)$  is non-atomic on  $[0, \bar{z}]$ , if  $\bar{z} < z^*$ . While we cannot guarantee that  $F(\cdot)$  is differentiable, an argument similar to the one in section 3 shows that  $F_j(\cdot)$  is differentiable in the interior of  $Z_j$ . As a result we can meaningfully evaluate the first order conditions of buyers.

Buyer optimization implies that  $V'_j(z) = \gamma/(\beta \delta)$  when  $z \in Z_j$  for both types. The derivative of  $V_j(\cdot)$  can be evaluated on the interiors of  $Z_L$  and  $Z_H$ . Therefore, for  $z \in \text{int}(Z_L) \cup \text{int}(Z_H)$

$$\begin{aligned} V'_j(z) &= \frac{\gamma}{\delta} + \sum_{n=0}^{\infty} P_n \{u_j n F'(z) F^{n-1}(z) - n z F'(z) F^{n-1}(z)\} \\ &= \frac{\gamma}{\delta} + (u_i - z) \lambda F'(z) e^{-\lambda (1-F(z))}. \end{aligned} \quad (27)$$

It is now easy to see that  $V'_H(z) > V'_L(z)$  for  $z \in \text{supp}F(\cdot)$ . As a result, all low type buyers hold less money than any high type buyer. Therefore, the support of the two distributions are non-overlapping and adjacent, i.e.  $Z_L = [0, \bar{z}_L]$  and  $Z_H = [\bar{z}_L, \bar{z}]$ , and  $F(\cdot)$  is not differentiable at  $\bar{z}_L$ .

To replicate the analysis of proposition 3.2, note that the search market value functions for the two types of agents are given by

$$V_L(z) = \delta W^L(z) + e^{-\lambda_H} \sum_{n=0}^{\infty} P_n^L \{u_L F_L(z)^n - \int_0^z \tilde{z} dF_L(\tilde{z})^n\} \quad (28)$$

$$V_H(z) = \delta W^H(z) + \sum_{n=0}^{\infty} P_n^H \{u_H F_H(z)^n - \int_0^z \tilde{z} dF_H(\tilde{z})^n\}, \quad (29)$$

where  $P_N^j$  is the probability that  $n$  buyers of type  $j$  visit the same seller for  $j \in \{L, H\}$ . Note that a low type buyer has a chance to buy only if no high type buyers appear at the location he visits, which occurs with probability  $e^{-\lambda_H}$ . Also, it does not matter to a high type whether any low types are visiting his location, since they hold less money with probability one.

We can now replicate the analysis of section 3 to arrive at the explicit characterization of the distributions of real money balances for the two types. The initial conditions for the two distributions are  $F_L(0) = 0$ ,  $F_H(\bar{z}_L) = 0$  and  $F_L(\bar{z}_L) = 1$ , which leads to (25) and (26). *QED*

## 6.4 Heterogeneous Valuations: Continuous Types

Turning to continuous types, suppose that the buyers' utility is distributed according to  $u \sim H(\cdot)$  which is continuous,  $\text{supp}H(\cdot) = [\underline{u}, \bar{u}]$ , and  $0 < \underline{u} < \bar{u} < \infty$ . To solve this case, we assume that there exists a money demand function  $z(u)$  such that the optimal strategy for a buyer of type  $\hat{u}$  is to bring  $z(\hat{u})$ , when all other buyers use the same demand function.<sup>13</sup> The discussion in the previous section shows that a buyer who values the good more brings more

<sup>13</sup>This is similar to a bidding function in auction theory.



money to the market and we therefore restrict attention to money demand functions that are strictly increasing. It is also straightforward to show that  $z(u)$  has to be continuous in any equilibrium, for the same reasons why  $F(\cdot)$  is continuous. We now proceed to characterize  $z(u)$  which in turn pins down  $F(\cdot)$ .

**Proposition 6.3** *Consider an environment where  $u \sim H(\cdot)$ . The equilibrium money demand function  $z(\cdot)$  is characterized by*

$$z(u) = u - \int_{\underline{u}}^u e^{-\lambda (1-H(u))/i} - \int_{\underline{u}}^u e^{-\lambda(H(u)-H(\tilde{u}))/i} d\tilde{u}. \quad (30)$$

The distribution of real money holdings is then given by  $F(z) = H(\zeta(z))$ , where  $\zeta(\cdot)$  is the inverse of the money demand function.

Proof:  $F(z(u)) = H(u)$  since  $z(u)$  is strictly increasing. As a result, in equilibrium a buyer ‘beats’ all competitors with lower valuations and loses from buyers who value the good more. Therefore, when examining potential deviations for the buyer, we look at the cases where he ‘pretends’ to be of a different type. Hence, given  $z(u)$ , the problem of a type- $u$  buyer when choosing his money holdings is

$$\max_{\hat{u}} -z(\hat{u}) \gamma/\delta + \beta [\delta W(z(\hat{u})) + \sum_{n=0}^{\infty} P_n \{u H(\hat{u})^n - \int_0^{\hat{u}} z(\tilde{u}) dH(\tilde{u})^n\}]. \quad (31)$$

The first order conditions of this problem have to equal zero at  $\hat{u} = u$  for the buyer to bring the amount prescribed by the money demand function. In other words

$$\begin{aligned} -z'(u) \gamma/\delta + \beta [z'(u) + \sum_{n=0}^{\infty} P_n \{u n H'(u) H(u)^{n-1} - z(u) n H'(u) H(u)^{n-1}\}] &= 0 \Rightarrow \\ -z'(u) i + (u - z(u)) \lambda H'(u) e^{-\lambda (1-H(u))/i} &= 0. \end{aligned} \quad (32)$$

Furthermore, equation (32) has to hold for all  $u$ . This means that  $z(u)$  is defined by the following first order linear differential equation:

$$z'(u) + z(u) \lambda H'(u) e^{-\lambda (1-H(u))/i} = u \lambda H'(u) e^{-\lambda (1-H(u))/i}.$$

We can multiply both side with the integrating factor  $e^{v(u)}$ , where

$$\begin{aligned} v(u) &= \int \lambda H'(\tilde{u}) e^{-\lambda (1-H(\tilde{u}))/i} d\tilde{u} \\ &= e^{-\lambda (1-H(u))/i}. \end{aligned}$$

The left-hand side is then given by  $d[z(u) e^{v(u)}]/du$ , and we can use the fundamental theorem of calculus to arrive at the explicit formulation for the equilibrium money demand function given in (30).  $F(z) = H(\zeta(z))$  follows from strict monotonicity of the bidding function. *QED*

Equation (30) gives a unique and explicit solution for the money demand function for a given buyer-seller ratio, given  $H(\cdot)$ . This in turn determines the distribution of money holdings. In the reverse direction, knowledge of the distribution of money holdings and the buyer-seller ratio can be used to infer about the distribution of heterogeneity in the population. The distribution of market prices is still governed by (18), so the distribution of prices can also be used in this regard.

## 6.5 Heterogeneous Productivity

Consider the case where buyers have different productivity in the Walrasian market, while having the same valuation for the search market good. This case is interesting because it allows us to talk about ‘richer’ and ‘poorer’ agents depending on their productive capacity. We show that the dispersion of money holdings is identical as in the case where agents are heterogeneous in terms of their valuations.

In particular, consider an economy  $A$  where buyers are heterogeneous in terms of their productivity in the Walrasian market, while having identical valuation  $u$  for the search market good. A buyer of type  $\psi$  who provides  $h$  units of labor in the Walrasian market produces  $\psi h$  units of output. Let  $D(\cdot)$  denote the distribution of buyer types and assume that it is continuous,  $\text{supp}D(\cdot) = [\underline{\psi}, \bar{\psi}]$ , and  $0 < \underline{\psi} < \bar{\psi} < \infty$ . We compare the outcomes of this economy ( $A$ ) to the economy analyzed in section 6.4 (call this economy  $B$ ). In economy  $B$  buyers are homogeneous in terms of productivity but they are heterogeneous in terms of their valuation for the search market good. Let  $H(\cdot)$  denote the distribution of valuations and assume that it is continuous. Here we show that this preceding analysis also yields insights into the case of heterogeneous productivity. That is, insights about economy  $A$  can be gained by analyzing economy  $B$  for an appropriate choice of the distribution  $H(\cdot)$ .

**Proposition 6.4** *The equilibrium distribution of money holdings of an economy  $A$  is identical to the equilibrium money distribution of economy  $B$  when the distribution of valuations is given by  $H(y) = D(y/u)$  for all  $y$ , where  $u$  is the utility of the search market good in economy  $A$ .*

Proof: The value for a buyer of type  $\psi$  the value of entering the market with  $z$  real dollars

is given by

$$\begin{aligned} W^\psi(z) &= \max_{x,h,z'} \{U(x) - h + \beta V^\psi(z')\} \\ \text{s.t. } x &\leq \psi h + (z + T - z') \gamma/\delta. \end{aligned}$$

Substituting the constraint inside the objective function we get

$$W^\psi(z) = (z + T) \gamma/(\delta \psi) + U_\psi^* + \max_{z'} \{-z' \gamma/(\delta \psi) + \beta V^\psi(z')\},$$

where  $U_\psi^* = U(x_\psi^*) - x_\psi^*$  and  $x_\psi^*$  is determined by  $U'(x_\psi^*) = 1/\psi$ . Note that  $W^\psi(z)$  is still linear in  $z$  but the slope now depends on  $\psi$ .

The value of entering the search market can now be constructed as in (8)

$$V^\psi(z) = \delta W^\psi(z/\gamma) + \sum_{n=0}^{\infty} P_n \{u F(z)^n - \int_0^z \tilde{z}/\psi dF(\tilde{z})^n\}.$$

where now  $\delta W^\psi(z/\gamma) = z/\psi + \delta W^\psi(0)$ . It will prove to define  $\tilde{W}^\psi(z) = \psi W^\psi(z)$  and  $\tilde{V}^\psi(z) = \psi V^\psi(z)$  to obtain the system of equations

$$\begin{aligned} \tilde{W}^\psi(z) &= (z + T) \gamma/\delta + \psi U_\psi^* + \max_{z'} \{-z' \gamma/\delta + \beta \tilde{V}^\psi(z')\} \\ \tilde{V}^\psi(z) &= z + \delta \tilde{W}^\psi(0) + \sum_{n=0}^{\infty} P_n \{\psi u F(z)^n - \int_0^z \tilde{z} dF(\tilde{z})^n\}. \end{aligned}$$

Thus, the problem resembles the one in which the valuations in the search market are given by  $\psi u$ , except that  $\psi U_\psi^*$  depends on  $\psi$ . However,  $\psi U_\psi^*$  is a constant and hence it does not affect the monetary trade-offs. Therefore, the problem reduces to having homogeneous productivity and a distribution of valuations  $H(\cdot)$  which is given by  $H(y) = D(y/u)$  for all  $y$ . Since the buyers' monetary decisions are identical in both economies, the profits for firms are also identical and free entry yields identical buyer-seller-ratios. *QED*

The transformation underlying the proof establishes that an observer of the search market cannot distinguish between heterogeneity in productivity or heterogeneity in valuations for search market consumption. Additional information of Walrasian market consumption, however, allows such a distinction. Different tastes for consumption of the search market good but homogeneous productivity imply identical Walrasian market consumption for all agents. Heterogeneity in terms of productivity but homogeneity in taste imply differences in Walrasian market consumption, and a positive correlation between Walrasian market con-

sumption and search market consumption.

## 7 Conclusions

We develop a monetary model that avoids some of the most common criticisms of modern monetary theory: random matching and observability of money holdings and preferences in bargaining. In our environment, the process of finding a trading partner for the buyer is deterministic and the main friction is that buyers cannot coordinate with each other, which sometimes leads to multiple buyers visiting the same seller. Multilateral matching allows us to use second-price auctions to determine the terms of trade, without needing to make additional assumptions on the information structure of the model. Most of characterization results for the equilibrium are novel. The incentives that buyers face when choosing their money holdings are very different due to the fact that they are in direct competition with each other for the good. This results in dispersion of money holdings which leads to price dispersion. The characterization of the unique equilibrium is tractable and the distribution of money holdings and prices admits a closed-form solution. Moreover, we show that efficient entry is attained only when the inflation rate is equal to the Friedman rule.

We conjecture that our key qualitative results –money dispersion and efficiency at the Friedman rule– obtain when the sellers use any standard auction that is renegotiation proof in the sense that the reserve price equals the sellers’ cost. While a formalization of this statement is left for future work, the intuition is very similar to the case of second-price auctions. In any standard auction it is only the highest bidder who obtains the good (or one of the high-bidders in the case of a tie). The possibility of a mass point in the money distribution can be disproved in much the same way as before: the potential gains of being able to bid more than a mass of competitors are discrete while the additional costs are negligible, regardless of the specifics of how the price is determined. Furthermore, at the Friedman rule agents can bring money that are equal to their valuation without incurring a cost. In this case there are no liquidity considerations and we are in the usual auction setting where there is revenue equivalence between the second-price auction (which yields efficient entry, as demonstrated in section 5) and any other standard auction. As a result, entry is efficient in other auction settings as well. Away from the Friedman rule, agents shade their money holdings down, and hence they transfer fewer real resources to the seller leading to suboptimal entry.

## References

- [1] Burdett, Kenneth, and Kenneth Judd. "Equilibrium Price Dispersion." *Econometrica*, 51 (1983), 955-969.
- [2] Burdett, Kenneth, and Dale T. Mortensen. "Wage Differentials, Employer Size, and Unemployment." *International Economic Review*, (1998), 257-273.
- [3] Camera, Gabriele, and Dean Corbae. *Money and Price Dispersion*. *International Economic Review* 40 (1999), 985-1008.
- [4] Camera, Gabriele, and Cemil Selcuk. *Price Dispersion with Directed Search*. Mimeo (2005).
- [5] Che, Yeon-Koo, and Ian Gale. *Standard Auctions with Financially Constrained Bidders*. *Review of Economic Studies* 65 (1998), 1-21.
- [6] Corbae, Dean, Ted Temzelides, and Randall Wright. *Directed Matching and Monetary Exchange*. *Econometrica* 71 (2003), 731-756.
- [7] Ennis, Huberto. *Search, Money, and Inflation under Private Information*. Mimeo (2006).
- [8] Faig, Miguel and Belen Jerez. *Inflation, Prices, and Information in Competitive Search*. *Advances in Macroeconomics* 6 (1) (2006).
- [9] Goldberg, Dror. *Money with Partially Directed Search*. *Journal of Monetary Economics*, *forthcoming*.
- [10] Green, Edward J. and Ruilin Zhou. *A Rudimentary Matching Model with Divisible Money and Prices*. *Journal of Economic Theory* 81 (1998), 252-271.
- [11] Green, Edward J. and Ruilin Zhou. *Dynamic Monetary Equilibrium in a Random Matching Economy*. *Econometrica* 70 (2002), 929-969.
- [12] Head, Allen, and Alok Kumar. *Price Dispersion, Inflation, and Welfare*. *International Economic Review* 46 (2005), 533-572.
- [13] Hogg, Robert V., and Allen Craig. *Introduction to Mathematical Statistics*, 5th edition (1994). Prentice Hall.
- [14] Howitt, Peter. *Beyond Search: Fiat Money in Organized Exchange*. *International Economic Review* 46 (2005), 405-429.

- [15] Julien, Benoit, John Kennes and Ian King. *Bidding for Money*, Journal of Economic Theory (forthcoming).
- [16] Kiyotaki, Nobuhiro, and Randall Wright. *A Search-Theoretic Approach to Monetary Economics*. American Economic Review 83 (1993), 63-77.
- [17] Kocherlakota, Narayana. *Money is Memory*. Journal of Economic Theory 81 (1998), 232-251.
- [18] Lagos, Ricardo, and Randall Wright. *A Unified Framework for Monetary Theory and Policy Analysis*. Journal of Political Economy (2005), 463-484.
- [19] Molico, Miguel. *The Distribution of Money and Prices in Search Equilibrium*. International Economic Review 47 (2006), 701-722.
- [20] Rocheteau, Guillaume, and Randall Wright. *Money in Search Equilibrium, in Competitive Equilibrium, and in Competitive Search Equilibrium*. Econometrica 73 (2005), 175-202.
- [21] Satterthwaite, Mark, and Artyom Shneyerov. *Dynamic Matching, Two-Sided Information, and Participation Costs: Existence and Convergence to Perfect Competition*. Econometrica (forthcoming).
- [22] Shi, Shouyong. *Money and Prices: A Model of Search and Bargaining*. Journal of Economic Theory 67 (1995), 467-496.
- [23] Trejos, Alberto, and Randall Wright. *Search, Bargaining, Money, and Prices*. Journal of Political Economy 103 (1995), 118-141.
- [24] Wallace, Neil. *Whither Monetary Economics?* International Economic Review 42 (2001), 847-870.