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"Consumption Commitments and Employment Contracts"
Third Version

by

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CONSUMPTION COMMITMENTS AND EMPLOYMENT CONTRACTS*

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Abstract We examine an economy in which the cost of consuming some goods can be reduced by making commitments that reduce flexibility. We show that such consumption commitments can induce consumers with risk-neutral underlying utility functions to be risk averse over small variations in income, but sometimes to seek risk over large variations. As a result, optimal employment contracts will smooth wages conditional on being employed, but may incorporate a possibility of unemployment.

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Consumption Commitments and Employment Contracts

1 Introduction

Suppose a young worker, contemplating future home ownership, marriage and children, faces a choice between two firms. One firm never lays off employees; it responds to adverse economic shocks by reducing the pay of all workers. In contrast, the other firm lays off the most recently hired workers while maintaining the wages of those workers retained. Concerned that a wage reduction at the first firm may force him to scrimp painfully on discretionary expenditures in order to pay the mortgage and feed his family, the worker may prefer the second firm, while holding off on buying a house and starting a family until he has acquired sufficient seniority to preclude layoffs. The two firms may give rise to the same amount of lifetime wage risk, but the latter concentrates this risk in the early years of employment. Because the worker can coordinate his decisions about marriage and mortgages with his (in)vulnerability to income shocks, he may prefer the firm with layoffs and concentrated risk.

Many goods are like housing in this example: they can be consumed more cheaply if one makes commitments that give rise to rigidities in consumption. Owning a house is cheaper (per unit of service) than renting, which is in turn cheaper than living in a hotel. At the same time, the rigidities induced by such *consumption commitments* can exacerbate the effects of income fluctuations. A negative income shock may force a homeowner to go hungry in an attempt to make the payments, incur the costs of selling her house, or default on the mortgage. A renter faces fewer transactions costs and no capital loss, while the hotel guest need only downgrade to a budget motel.[1]

There is ample evidence that commitments affect consumption patterns. Chetty and Szeidl [8] show that households respond to small income shocks by leaving their housing consumption fixed and making relatively large reductions in food expenditures, while responding to larger shocks with more balanced reductions in each. Shore and Sinai [14] show that households vulnerable to moderate income shocks make moderate housing commitments, coupled with precautionary savings that allow them to weather shocks without sacrificing their commitments. Households facing more volatile incomes

[1]Analogous effects can arise even without explicit financial obligations. The expected utility from a vacation home may be jeopardized by negative income shocks, even if there are no further payments to make and the home itself is not at risk.

make more aggressive commitments and save less, understanding that a negative shock may force them to liquidate the commitment.

Worker-consumers who can make consumption commitments have an incentive to coordinate consumption and labor market decisions, matching those times when consumption would be especially vulnerable to income fluctuations with times when income is relatively secure.[2] This raises our basic question: How do consumption commitments affect the optimal employment contracts offered by a firm in a risky market?

In the absence of some market friction, the answer is straightforward. Because consumption commitments give rise to effective risk aversion, the firm will completely insure its workers, subjecting them to neither wage nor employment risk. However, many people do face income risk—otherwise Chetty and Szeidl [8] and Shore and Sinai [14] would have nothing to study—especially risk due to employment shocks. How do consumption commitments affect employment contracts in the presence of some friction that precludes full insurance?

We show that if consumption commitments are sufficiently important and sufficiently costly to reverse, optimal contracts will couple layoffs with wages that are higher and less variable conditional on being employed than they would be without layoffs. Workers who accept such a contract are sometimes immune to layoff risk (e.g., when they have accumulated sufficient seniority), but are also sometimes vulnerable, being laid off if and only if the firm experiences a negative shock. Workers know whether they are vulnerable to being laid off before making their consumption choices, but must make their consumption commitments (if any) before knowing whether they will actually be laid off (if vulnerable). A worker optimally makes few (or in our simplest model, no) consumption commitments when there is positive probability that she will be laid off. In return for this layoff risk, the worker receives higher and more secure wages that better accommodate commitments when she is not at risk.

As we will see in Section 2.4, consumption commitments introduce a nonconcavity into the worker's utility function. Commitments are of relatively little value at low income levels but are more valuable at higher incomes. The ability to tailor consumption commitments to one's vulnerability to layoffs combines with this nonconcavity to make the contract with layoffs attrac-

[2] Several papers make a similar point. Ellingsen and Holden [11] argue that workers who purchase durable goods in expectation of high future wages will make large purchases and then resist lower wages more than they would had their expectations been pessimistic (and hence durable purchases smaller). Ellingsen and Holden [12] analyze a model in which worker indebtedness worsens their bargaining position vis a vis employers.

tive. Eliminating the layoff risk would allow the worker to make better use of consumption commitments in those circumstances when she would otherwise have been vulnerable to layoffs, but comes at the cost of lower wages and greater wage fluctuation when not vulnerable. When consumption commitments are sufficiently important, the commitment-magnified value of increased wages when employment is secure overwhelms the (less burdensome, with few commitments) prospect of a layoff, and the optimal balance features employment risk. Our analysis thus points to a potentially important factor in understanding the coexistence of wage rigidities and employment risk.

Section 2 introduces a model of consumption commitments and employment contracts. Section 3 establishes conditions under which wage smoothing and layoffs are optimal in a simple model, while Section 4 extends the argument to an intertemporal model. Section 5 discusses the results.

2 Consumption Commitments

2.1 The Firm

We consider a firm whose profits are a function of the quantity of worker-consumers $N \in \Re$ that it hires and the realization of a state. Revenue in state 2 (the bad state) is given by the function $f : \Re \to \Re_+$, and in state 1 (the good state) by αf, $\alpha > 1$. The good state occurs with probability p.

We assume that f is twice continuously differentiable on $\Re_+$, with $f' > 0$, $f'' < 0$, $f'(0) = \infty$, and $\lim_{N\to\infty} f'(N) = 0$. We assume that the elasticity

$$\theta(N) \equiv -\frac{f''(N)N}{f'(N)}$$

is bounded below by $\theta^* > 0$. This is the case, for example, for any power function satisfying our assumptions. It is important to our analysis that the marginal product of labor is decreasing in employment. Should complementarities reverse this relationship, our argument would no longer apply.

An employment contract includes the wage rate w_i to be paid in each state $i \in \{1, 2\}$, a quantity n_2 of workers to be "kept on" in the bad state, and a quantity n_1 of workers who are employed only in the good state.[3] The

[3] We assume that all employed workers receive the same payment. This simplifies the calculations, but does not play an important role in the results. It is straightforward to show that if the firm is to lay off workers, it will do so in the bad state, a result already embedded in our specification of n_1 and n_2.

firm's expected payoff is given by

$$p\left(\alpha f(n_1+n_2)-w_1(n_1+n_2)\right)+(1-p)\left(f(n_2)-w_2n_2\right).$$

The firm maximizes this payoff subject to the constraint that the employment contract provides workers with at least their reservation utility. The employment contract must also specify how the workers (if any) to be laid off in the bad state will be selected, a feature that affects the expected payoffs of workers but has no effect on the the firm's payoff. We consider this aspect of an employment contract in Section 3 when examining the interaction between the firm and worker optimization problems.

2.2 Restrictions on Employment Contracts

Some limitation on the wages the firm can pay to workers is critical to our analysis. In the absence of such limitations, the "wage bill" argument of Akerlof and Miyazaki [1] ensures that the optimal labor contract completely insures the worker against risk, featuring no wage fluctuations and no unemployment. Hence, a firm that could perfectly insure workers would do so, leaving us with a model incapable of studying wage or employment risk. Our interest is in how optimal contracts balance wage and unemployment risk in the presence of some friction that precludes perfect insurance.

We build such friction into our model in a particularly simple way—the firm cannot pay a wage w_i in state $i \in \{1,2\}$ that exceeds the marginal product of labor in state i. Our primary interpretation of this constraint is in terms of moral hazard. Payments in excess of marginal products would tempt a firm to fire workers for alleged nonperformance. If it is sufficiently costly to verify performance, contracts with wages in excess of marginal products cannot be sustained. We also assume that the firm cannot make payments to unemployed workers. This assumption is standard in the literature on implicit contracts (beginning with Azariadis [3], Baily [4], and Gordon [13]). Again, if the firm was committed to payments to laid-off workers, the firm would be tempted to simply fire workers rather than lay them off.

A wide variety of other frictions would also give rise to our results.[4] Our argument holds as long as wages have the property that in a full employment contract, wages in the bad state fall short of wages in the good state (see

[4] An absolute prohibition on wage rates in excess of marginal products and payments to laid off workers is clearly unrealistic. We view these stark assumptions as tractable approximations of realistic market frictions, imposing limits on the extent to which wages can exceed marginal products and firms can maintain the incomes of laid-off workers.

note 9 for a qualification), and that reducing bad-state employment allows the firm to maintain a higher wage than would be the case if all workers were retained. The former is a feature of any friction giving rise to the income risk that motivates our work, while a consistent theme in Bewley [5] is that layoffs allow the firm to avoid or attenuate wage reductions. We do not claim that moral hazard considerations are the only force at work. For example, limits on bad-state wages may reflect financial constraints that preclude sustained payments in excess of productivity. However, we must include some constraint on wages, and find moral hazard considerations particularly convenient.

2.3 Worker-Consumers

The worker-consumer (also called either simply a worker or consumer) has a reservation utility, interpreted as the value of alternative market activities, that we denote by $\overline{U} > 0$. The consumer's utility depends on two things: consumption of a good x and consumption of services that can be obtained from either of two other goods, y or z. The consumer has a constant-elasticity-of-substitution utility function over x and $(y+z)$, the level of services she receives from the goods y and z, given by

$$(\gamma x^{\rho} + (1-\gamma)(y+z)^{\rho})^{\frac{1}{\rho}} . \tag{1}$$

The constant-elasticity-of-substitution form for this utility function is not essential to our results, but has the important advantage of allowing us to talk precisely (by varying γ) about the relative importance of the various consumption goods.

The consumer is risk neutral, in the sense that her utility is linear along rays through the origin in the space of feasible consumption bundles:

$$(\gamma(\lambda x)^{\rho} + (1-\gamma)(\lambda y + \lambda z)^{\rho})^{\frac{1}{\rho}} = \lambda \left(\gamma x^{\rho} + (1-\gamma)(y+z)^{\rho}\right)^{\frac{1}{\rho}} .$$

The goods y and z are perfect substitutes representing different ways that the consumer can satisfy her desire for services. We assume that the consumer can purchase either y or z, but not both: she must choose one of the two ways to get the relevant services.[5]

[5]For example, y and z may represent purchased housing and rental housing, which may be good substitutes but which are not easily combined into a single place of residence. We could work with weaker versions of this assumption, with some additional complication, as long as consumption commitments introduce sufficient rigidities in ex post consumption.

Purchasing good y involves a nontrivial commitment, while there is no commitment involved in purchasing z. We model this by assuming that there is an ex ante market (before the state is realized) and an ex post market (after the state has been realized). Commitments to good y must be made in the ex ante market, while trade in z occurs in the ex post market.

Committing to good y (in the ex ante market) entails a fixed cost plus a marginal cost. We normalize prices so that the price of x is one and we normalize units of z and y so that the price of z is one. The cost of committing to good y in the ex ante market is then

$$h(y) = \begin{cases} \beta + \kappa y & \text{if } y > 0 \\ 0 & \text{if } y = 0, \end{cases}$$

where $0 < \kappa < 1$ and $\beta > 0$. Hence, the consumer can purchase the services provided by goods y or z at a cheaper per unit price if she pays the fixed cost of β and purchases in the ex ante market. The nonlinear form of the price of y is meant to capture the idea that securing services via good y is cost effective only if consumption exceeds some minimum level. For example, it is typically not financially attractive to purchase just a little bit of housing. *Ceteris paribus*, there is an advantage to purchasing z rather than y, since purchases in the ex post market can be conditioned on the realized state of the world. This advantage must be weighed against the possible cost saving allowed by commitments to good y.

The commitment to good y in the ex ante market, denoted by $\hat{y}$, can be adjusted in the ex post market after the realization of the state is known, but at a cost per unit different from κ. Additional purchases of y can be made at price $\zeta > 1$, while portions of good y can be sold on the ex post market, at price $\frac{1}{\psi} < \kappa$. Purchases of y in the ex ante market thus come at a lower marginal price than purchases of z, but adjustments to the level $\hat{y}$ are more expensive.[6] We write the price relevant for such a reduction as $\frac{1}{\psi}$ so that larger values of ψ and ζ correspond to more rigid commitments.

Let $[\xi]_+ = \max\{\xi, 0\}$ and $[\xi]_- = \min\{\xi, 0\}$. If a consumer has wage w_i in state i and doesn't face the prospect of being laid off, she has the following utility maximization problem:

$$\max_{x_1,x_2,\hat{y},y_1,y_2,z_1,z_2} p\left(\gamma x_1^\rho + (1-\gamma)(y_1+z_1)^\rho\right)^{\frac{1}{\rho}} + (1-p)\left(\gamma x_2^\rho + (1-\gamma)(y_2+z_2)^\rho\right)^{\frac{1}{\rho}}$$

[6] For example, buying a house with eight-foot ceilings and then increasing the ceiling height to nine feet is more expensive than buying a house with higher ceilings in the first place. Building a house with three bathrooms and then selling one is financially worse than simply not having installed three.

subject to the budget constraints (for $i = 1, 2$)

$$x_i + z_i + h(\hat{y}) + \zeta[y_i - \hat{y}]_+ + \frac{1}{\psi}[y_i - \hat{y}]_- = w_i, \tag{2}$$

and the constraints that the services be purchased via one or the other of y and z, but not both:

$$\hat{y} z_1 = \hat{y} z_2 = 0. \tag{3}$$

Suppose the consumer may be laid off with probability $q > 0$ in state 2. A layoff consigns the consumer to home production, the value of which is normalized to 0 (which may reflect unemployment payments made by the government). Then the consumer's problem is

$$\max_{x_1, x_2, \hat{y}, y_1, y_2, z_1, z_2} p\left[(\gamma x_1^\rho + (1-\gamma)(y_1 + z_1)^\rho)^{\frac{1}{\rho}}\right] + (1-p)\left[(1-q)\left(\gamma x_2^\rho + (1-\gamma)(y_2 + z_2)^\rho\right)^{\frac{1}{\rho}}\right],$$

subject to (2)–(3) and the constraint

$$h(\hat{y}) \leq \frac{1}{\psi}\hat{y}.$$

This last inequality is a "no bankruptcy" constraint, capturing a requirement that a consumer who is laid off must still be able to earn enough (by liquidating the commitment good $\hat{y}$) to meet her fixed payment obligations for the good. Since $\frac{1}{\psi} < \kappa$, this requires a consumer facing layoff risk to set $\hat{y} = 0$ in order to respect the budget constraint in state 2. In other words, consumers at risk of being laid off cannot make commitments. This is no longer the case in Section 4, where a consumer might borrow or save to cover the fixed cost of a commitment. This feature of the static model makes layoffs more costly to workers and thus introduces a bias *against* layoffs.

Because of the no bankruptcy constraint, no individual enters the ex post market having both made a consumption commitment and facing a risk of being laid off. However, consumers face a trade-off between employment risk and the extent to which they can make consumption commitments at the initial stage at which they evaluate the expected utility of a labor contract. Optimal employment contracts are shaped by this initial trade-off, where consumers may find it optimal to accept layoff risk, knowing that this reduces the circumstances in which they can make commitments, in return for more effective commitments when they can make them.

2.4 Commitments and Utility

To gain insight into the consumer's utility maximization problem, we abstract from layoff concerns, fix a commitment level $\hat{y} > 0$, and consider the ex post problem of choosing x and y to

$$\max(\gamma x^{\rho} + (1-\gamma)y^{\rho})^{\frac{1}{\rho}}$$

subject to

$$x + \kappa\hat{y} + \zeta[y-\hat{y}]_{+} + \frac{1}{\psi}[y-\hat{y}]_{-} = I,$$

where I is ex post net income (i.e., realized income minus the fixed commitment cost, or $w_i - \beta$). Studying this ex post problem provides insight into why risk-neutral consumers may, *ex ante*, seek employment risk in exchange for higher wages and lower wage fluctuations when employed.

First, suppose (hypothetically) that the consumer could buy *and* sell good y at price ζ. Figure 1 shows the resulting expansion path, identifying optimal (x, y) bundles for various income levels. This path consists of points such as B, where the consumer's indifference curves are tangent to a budget constraint of the form $I' = x+\zeta y$. Alternatively, suppose the consumer could buy and sell good y at either price κ or price $1/\psi$. Figure 1 again shows the corresponding expansion paths, this time composed of points such as A at which an indifference curve is tangent to a budget line of the form $I'' = x + \kappa y$, or points such as C at which an indifference curve is tangent to a budget line of the form $I''' = x + \frac{1}{\psi}y$.

Now consider the consumer's ex post optimization given commitment to a level of consumption $\hat{y}$, given that additional purchases of good y come at price ζ while sales come at price $1/\psi$. Suppose ex post income I is such that setting $y = \hat{y}$ and spending the remaining income on x yields a point such as A in Figure 1, where the indifference curve is tangent to the budget line $I = x + \kappa\hat{y}$. This bundle is optimal ex ante, given price κ. Ex post, the consumer faces the kinked budget constraint shown in Figure 2 (the dashed lines), since the consumer buys y at the price $\zeta > \kappa$ and sells at price $1/\psi < \kappa$, and hence A remains optimal.

Suppose the consumer has chosen commitment $\hat{y}$ but receives a higher ex post income than that required to purchase bundle A. This higher income would induce increased purchases of y if they could be made at price κ, but small increases in income will not induce additional purchases at the higher price ζ, with the consumer instead spending any additional income on good

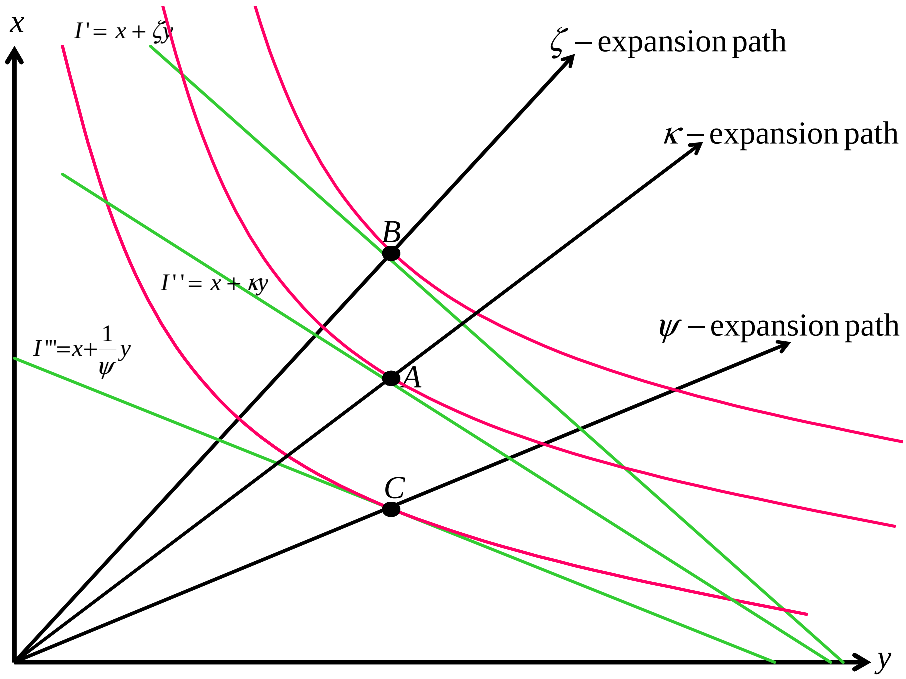

Figure 1: Hypothetical expansion paths in the ex post market, identifying loci of optimal (x, y) pairs as ex post income I varies from 0 to an arbitrarily large level. The ζ-expansion path would be relevant if the price of y were ζ and hence the budget constraint were $I' = x + \zeta y$. The κ-expansion path would be relevant for price κ and budget constraint $I'' = x + \kappa y$, and the ψ-expansion path for price $1/\psi$ and budget constraint $I''' = x + \frac{1}{\psi} y$.

x.[7] This continues until income is sufficiently large to induce consumption bundle B in Figure 2, where the consumer's indifference curve is tangent to a budget line with slope ζ. Thereafter, increases in income prompt the consumer to make adjustments in both goods x and y, expanding along the ζ-expansion path. Analogously, decreases in income first induce the consumer to reduce only the consumption of good x, until reaching a point such as C in Figure 2, where the indifference curve is tangent to a budget line of slope $1/\psi$. Further reductions in income prompt reductions in both x and y, with consumption contracting along the ψ-expansion path.

Figure 3 shows the indirect utility function, denoted by $\tilde{U}$, giving the consumer's (optimal) utility as a function of ex post *gross* income I (i.e., income before incurring the fixed cost β), presuming a commitment $\hat{y}$. The ray

[7]This is the counterpart of Chetty and Szeidl's [8] finding that small income shocks produce no adjustment in housing but large adjustments in food consumption.

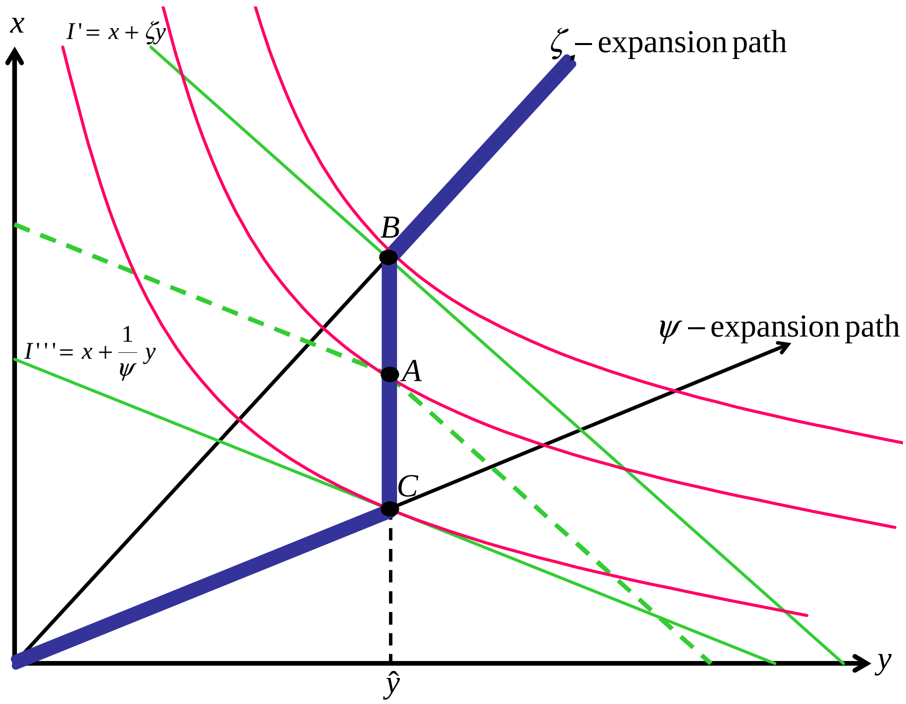

Figure 2: Ex post expansion path (in bold), given commitment $\hat{y}$. Small variations in ex post income (around the level for which $\hat{y}$ is ex ante optimal) prompt changes in x but leave y fixed at $\hat{y}$, giving the vertical portion of the expansion path connecting points B and C. Once income is sufficiently large as to induce consumption bundle B, defined by the tangency of the consumer's indifference curve with a budget line whose slope is given by the price ζ at which the consumer can purchase y in the ex post market, further increases in income induce increases in both x and y, proceeding along the ζ-expansion path. Similarly, once consumption drops to the level consistent with point C, further reductions move the consumer inward along the ψ-expansion path.

marked κ would be the indirect utility function if the consumer purchased services via good y at price κ. This path is linear, since the consumer is risk neutral if allowed to vary x and y freely at prices 1 and κ, but does not pass through the origin, reflecting the fixed cost β. For a given commitment $\hat{y}$, there is an income level $I(\hat{y})$ at which the consumer's unconstrained optimal purchase of good y (at price κ, given fixed cost β) will equal $\hat{y}$, yielding point A in Figure 3 (corresponding to point A in Figures 1 and 2). Realized incomes above this level initially induce increases in the consumption of good x, but leave y unchanged at $\hat{y}$, until reaching point B in Figure 3 (and Figures 1 and 2). At this point, the consumer supplements

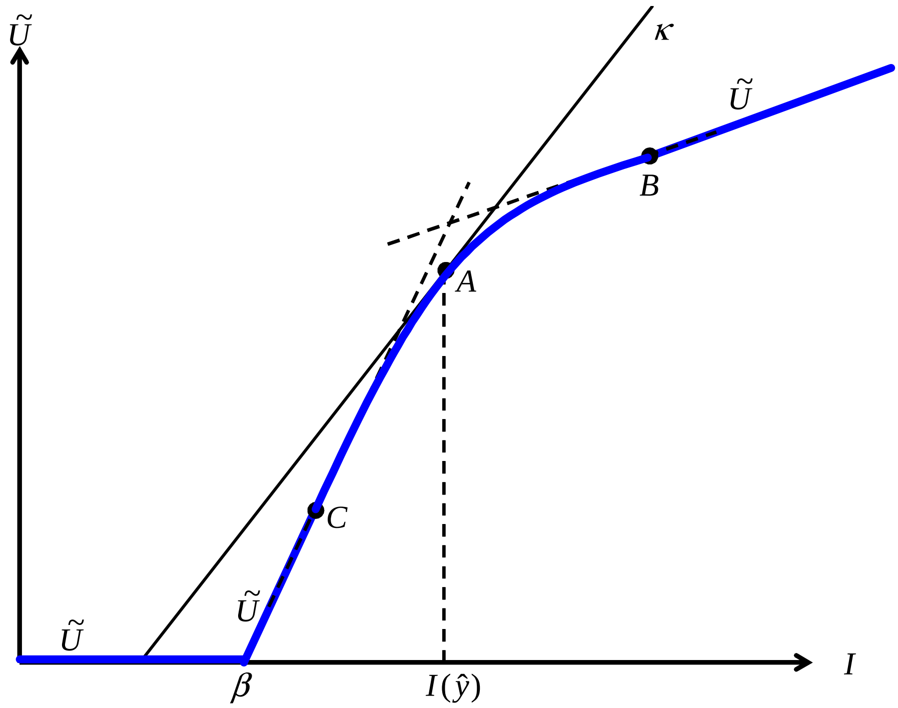

Figure 3: Indirect utility function, labelled $\tilde{U}$, giving utility as a function of ex post gross income I, for commitment $\hat{y}$. The indirect utility function is concave for incomes above β but fails to be concave when incomes $[0, \beta]$ are also considered.

the commitment $\hat{y}$ by purchasing additional quantities of good y at price ζ (as well as additional quantities of good x). The indirect utility function is then again linear, with a flatter slope representing the higher (than κ) price ζ.[8] Similarly, reductions in income below $I(\hat{y})$ initially prompt no reduction in good y, until point C is reached, after which some units of good y are sold at price $\frac{1}{\psi}$ and the indirect utility function is again linear.

The indirect utility function is strictly concave in a neighborhood of $I(\hat{y})$. Having made a commitment $\hat{y}$, the consumer is risk averse over small variations in income, as the cost of adjusting y channels any variation into good

[8] The indirect utility function is differentiable at point B. The extension of this linear segment emanating from point B passes above point A. This segment is part of a linear indirect utility function that would be relevant if the consumer faced prices $(1, \zeta)$ for goods (x, y), with an ex post income subsidized by $(\zeta - \kappa)\hat{y}$, so that the first $\hat{y}$ units of good y can be purchased at price κ, but with no restrictions on purchases of good y. At ex post income $I(\hat{y})$, the consumer can then buy the bundle that gives the utility corresponding to point A, but optimally chooses to purchase less y and more x (given prices $(1, \zeta)$), for a higher utility.

x. However, the indirect utility function is not globally concave, introducing the possibility of risk-seeking behavior over large variations in income. In particular, a mixture of zero utility (being laid off) and the utility corresponding to any point near A is preferred to the utility of the corresponding expected income. The potential attractiveness of such mixtures gives rise to the optimality of employment contracts with layoffs—the firm lays off workers in the bad state, relaxing the marginal product constraint in that state and thereby increasing and smoothing wages for workers when employed.

3 Optimal Layoffs

This section examines optimal employment contracts in a single-period model. This model is designed to retain the flavor of a dynamic model while allowing us to identify the key features of optimal employment contracts with a minimum of clutter. Section 4 extends the analysis to a dynamic model.

3.1 Timing

Events proceed in the following sequence:

1. The firm offers an employment contract (w_1, n_2, w_2, n_2). Workers accept or reject. The optimal contract will provide an expected utility equal to the alternative $\overline{U}$, and workers will choose to accept.

2. Each worker draws an "age": young with probability $n_1/(n_1 + n_2)$ and old with probability $n_2/(n_1 + n_2)$. Young workers are vulnerable to layoffs, that is, they will be laid off in the bad state. This is meant to capture in our static model the features of a dynamic model. In the dynamic model, layoff eligibility will be determined by age. Workers signing a contract know they will be at risk of layoff at some ages (in which case they will make small but not necessarily zero commitments), and that they will be immune from layoffs at other ages (allowing greater use of commitments). In the static model, each worker signing a contract knows that she will be vulnerable to layoff under some age draws (in which case she will make zero commitments) and not vulnerable in others (allowing commitments).

3. Workers make consumption commitments (i.e., choose $\hat{y} > 0$) in the ex ante market or choose not to do so ($\hat{y} = 0$).

4. The state is realized. As is standard in the implicit contracts literature, workers cannot change employers at this point. All workers are retained in state 1, while vulnerable workers are laid off in state 2.

5. Workers who remain employed collect their wage, choose x and either z (in the absence of a commitment) or y (with a commitment). Employed workers consume the resulting bundle while laid-off workers receive the utility of home production.

3.2 Optimal Contracts without Commitments

Our first result is that if the optimal contract does not induce consumption commitments (i.e., all consumers set $\hat{y} = 0$), then the contract features no layoffs and the wage equals the marginal product of labor in each state. The consumer is risk neutral in this case, eliminating any advantage to paying wages that are not equal to marginal products. This in turn removes any incentive for the firm to lay off workers in order to increase marginal products and thus relax wage constraints. Any contract with layoffs is then dominated by a full-employment contract with suitably adjusted wages. Lemma 1 couples this result with obvious sufficient conditions for commitments to be suboptimal.

Lemma 1

(1.1) If the optimal employment contract does not induce commitments, then there are no layoffs and $\alpha f' = w_1 > w_2 = f'$.

(1.2) If either $\kappa > 1$, β is sufficiently large, or γ is sufficiently large, then the optimal contract features no commitments.

Proof.

(1.1) Suppose that the optimal employment contract does not induce commitments and features layoffs (i.e., $n_1 > 0$). If $w_1 = \alpha f'(n_1 + n_2)$, then a marginal reduction in n_1 has no effect on the firm's payoff while increasing consumer utility (by decreasing the layoff probability), introducing slack into the consumer's participation constraint that the firm can exploit to increase its payoff. If $w_1 < \alpha f'(n_1 + n_2)$, then the firm can decrease w_2 and increase w_1, while preserving expected payments to the consumer and expected profits (and hence expected utility, here exploiting the consumer's risk neutrality in the absence of commitments), until $w_1 = \alpha f'(n_1 + n_2)$; at which point n_1 can again be profitably reduced. If there are no layoffs and the expected wage falls short of the expected marginal product in either state, then the firm can profitably increase its employment. Section 6 provides the details of this argument.

(1.2) If $\kappa > 1$, then the cost of buying good 2 via commitments exceeds the cost of making the same purchase on the spot market. Similarly, if β is sufficiently large, the cost $h(\hat{y})$ of buying the quantity $\hat{y}$ via commitments exceeds the cost of purchasing that amount on the spot market for a sufficiently large interval $[0, \hat{y}^*]$ that the consumer makes no commitments. Fixing $\beta > 0$ and κ, if γ is sufficiently large, then the optimal consumption of good y is sufficiently small that the fixed cost β is prohibitive, again ensuring that no commitments are made. ■

3.3 Optimal Contracts with "Flexible Commitments"

If commitments induce no ex post rigidities, we will again have full employment contracts with wages equal to marginal products. Specifically, if we relax our maintained assumption and let $\zeta = \frac{1}{\psi} = \kappa < 1$, so that the level of y can be adjusted ex post without penalty, and if β is sufficiently small (to ensure that commitments are optimal, though this does not require $\beta = 0$), then consumers will make commitments (i.e., will choose $\hat{y} > 0$), but the optimal contract will again feature no layoffs and wages equal to marginal products:

Lemma 2 *If $\zeta = \frac{1}{\psi} = \kappa < 1$ and β is sufficiently small, then the optimal employment contract induces $\hat{y} > 0$, but features no layoffs and $\alpha f' = w_1 > w_2 = f'$.*

Proof. Let $\zeta = \frac{1}{\psi} = \kappa < 1$ and let β be small enough that $\beta + \kappa z_i^* < z_i^*$, $i = 1, 2$, where z_i^* is the optimal state-i consumption of good z when y is unavailable. This ensures that the optimal contract induces the consumer to set $\hat{y} > 0$. Because the consumer is risk neutral ex post, the argument proving Lemma 1.1 then ensures that there are no layoffs and $\alpha f' = w_1 > w_2 = f'$. ■

Hence, consumption commitments potentially affect optimal employment contracts only because commitments make it more difficult to adjust one's consumption in response to employment shocks.

3.4 Optimal Contracts with Commitments

We now return to the ex post rigidities induced by consumption commitments when $\frac{1}{\psi} < \kappa < 1 < \zeta$. The first step toward examining the potential optimality of layoffs is to note that consumption commitments introduce risk aversion over small variations in ex post income (seen in the concavity of the indirect utility function in Figure 3 in a neighborhood of $I(\hat{y})$), causing the firm to optimally smooth wages:

Lemma 3 *When β and κ are sufficiently small, consumers facing no layoff risk make commitments. The optimal employment contract smooths wages, in the sense that $w_2 = f'(n_2)$ and $w_1 < \alpha f'(n_1 + n_2)$.*

Proof. We provide an outline of the argument, leaving the details to Section 6. It is immediate that commitments are optimal for sufficiently small β and κ. Suppose $w_2 = f'$ and $w_1 = \alpha f'$. If there are layoffs, then a marginal reduction in n_1 while preserving n_2 leaves the firm's payoff unaffected, while increasing consumer utility (by reducing the layoff probability), introducing slack in the participation constraint that the firm can exploit to increase its payoff. In the absence of layoffs, we have $w_1 > w_2$, and a marginal reduction in n_2 (holding $n_1 = 0$) again leaves the firm's payoff unchanged, while allowing wage smoothing. It is apparent from Figure 3, along with the optimality of consumption commitments (implying that realized incomes lie in the concave portion of the indirect utility function), that this wage smoothing increases consumer utility. ■

In the good state, the wage falls short of the marginal product of labor ($w_1 < \alpha f'(n_1+n_2)$). If the firm could freely hire workers in an ex post labor market, it would do so until the wage no longer fell short of the marginal product. There is no such equalizing force in the initial labor market. We have assumed in constructing our model that there is no ex post market for workers. This is again a stark but convenient abstraction, capturing the fact that firms and workers can increase the surplus they are to split by making ex ante agreements, tying firms and workers together and thereby limiting the effectiveness of the ex post labor market.

Our basic result shows that if commitments are sufficiently valuable and induce sufficient rigidity in the consumption of good y, optimal contracts will feature layoffs. The following is a special case of Proposition 2 (obtained by setting $\delta = 0$ in Section 4), and we defer proof to the consideration of Proposition 2.

Proposition 1 *For sufficiently small $\beta > 0$, there exist $\bar{\kappa}(\beta) > 0$, $\bar{\gamma}(\beta) > 0$, $\bar{\zeta}(\beta)$ and $\bar{\psi}(\beta)$ such that for all $\kappa < \bar{\kappa}(\beta)$, $\gamma < \bar{\gamma}(\beta)$, $\zeta > \bar{\zeta}(\beta)$ and $\psi > \bar{\psi}(\beta)$, the optimal contract features layoffs.*

Layoffs have two advantages. First, a worker who has made consumption commitments is ex post risk averse. A full employment contract with wages equal to marginal products exposes the worker to risk. The firm has an incentive to offer smoother wage rates, but is constrained in doing so by the marginal product of labor in state 2.[9] Layoffs relax this constraint by reducing state-2 employment and hence increasing the marginal product. Second, consumption commitments magnify the effectiveness of income in generating utility. Even if the firm has perfectly smoothed wages across states (conditional on employment), the consumer may prefer to take on additional employment risk in order to relax the bad-state marginal product constraint on this wage and thus consume more of the commitment good when employed. This is the observation that, in Figure 3, a mixture of zero utility (being laid off) and the utility corresponding to any point near A is preferred to the utility of the corresponding expected income.

The conditions of the proposition ensure that commitments are optimal (β and κ small) and that the rigidities introduced by consumption commitments are relatively severe (γ small and ζ and ψ large), and hence the concave portion of the indirect utility function in Figure 3 is quite concave, making wage smoothing particularly valuable.[10]

Remark 1. As ζ and ψ get arbitrarily large, ex post adjustments in commitments become impossible (the vertical portion of the expansion path in Figure 2 gets arbitrarily large). As Proposition 1 indicates, layoffs are especially likely to be optimal under these circumstances. The preference of consumers to trade some layoff risk for smoother wages conditional on being employed arises not only out of the costs they otherwise incur in making ex post adjustments in their commitments (which are absent if no such adjustments are made), but also out of the large fluctuations in noncommit-

[9]The firm could perfectly smooth wages, without layoffs, by simply reducing the good-state wage to equal the bad-state marginal product: $w_1 = w_2 - f'(n)$. Then the average wage falls short of the average marginal product, and the firm would like to hire more workers. It cannot do so and preserve w_1 and w_2 without pushing the bad-state marginal product below w_2, violating the marginal product constraint. Layoffs again become valuable as a way of relaxing this constraint.

[10]It is immediate from Lemma 2 that layoffs will not be optimal if commitments are not sufficiently rigid.

ment consumption they must otherwise endure to mitigate or avoid ex post adjustments in commitments. ■

Remark 2. Layoffs are potentially optimal in our model because they reduce employment in the bad state, relaxing the marginal-product constraint on wages. Could the firm instead relax this constraint by retaining all of its workers, but having each work fewer hours? Laying off half the workers or halving the time each works may leave the firm with the same effective workforce,[11] but these have quite different effects on the workers. When the firm lays off half its workforce, the remaining workers can be paid as much as $f'(\frac{1}{2}N)$, while if it retains all the workers but cuts their hours by half, each worker can be paid at most half as much, or $\frac{1}{2}f'(\frac{1}{2}N)$, a disadvantage for a policy designed to boost state-2 worker incomes. More generally, retaining all workers but having each work $\lambda < 1$ times full employment would allow the firm to pay up to $\lambda f'(\lambda N)$ per worker. If the elasticity of the production function $-\frac{f''(N)N}{f'(N)}$ is below 1, as is the case if f is a power function, reducing hours would force a *reduction* in payments to workers, exacerbating rather than smoothing ex post payment variations and ensuring that hours reductions would never be part of an optimal contract, even when layoffs could be.[12] ■

Remark 3. Layoffs relax a constraint on the wage the firm can pay. A more effective response would be income fluctuation insurance, offered either by the firm or by a third party, that severs the link between the wage offered by the firm and the payment received by the worker. Our model excludes such insurance and our results will not hold in its presence. We suspect that moral hazard considerations preclude third-party income fluctuation insurance—there is less incentive to actually work once one's income is insured—and preclude the firm's fully insuring workers when laid off (though we have not modelled such factors). Our model incorporates an explicit constraint on the firm's ability to insure against income fluctuations while employed, in the form of a prohibition on wages in excess of marginal products. ■

[11] We ignore here the possibility that output might depend not only on the number of man-hours available, but also the number of workers.

[12] If the elasticity exceeds 1, reducing hours will allow an increase in state-2 payments, though this increase will not be as large as that allowed by layoffs. Of course, reducing hours rather than laying off workers has the advantage of eliminating the possibility of zero wage.

4 An Intertemporal Model

In the one-period model of the previous section, a consumer at risk of being laid off cannot make consumption commitments, for fear of being unable to cover the fixed cost. In practice, a consumer can draw on past savings or borrow against future income to sustain commitments, permitting some commitments even when facing layoff risk. This introduces a feature making layoffs more attractive. At the same time, saving and borrowing allows consumers to smooth wages conditional on employment, attenuating one benefit of layoff contracts. In this section, we extend the analysis to an intertemporal setting that allows saving and borrowing, and again investigate the optimality of layoffs.

4.1 The Firm

The firm is infinitely lived. Workers are potentially employed for two periods. The firm signs a contract with a young worker at the beginning of the worker's tenure with the firm, specifying the wage as a function of the state in each period of employment.

We examine a steady state. In any period, the firm contracts with N workers, employing n_2 workers in the bad state and $N = n_1 + n_2$ in the good state, where n_1 may be zero. We assume that in each period the firm has an equal number of young and old workers. At the beginning of each period the firm hires a set of young workers to replace the old workers of the previous period. The firm can condition a worker's wage on the state, but not the worker's age or the previous-period state. Relaxing this assumption complicates the details of the analysis but does not vitiate the result. The result of this steady state analysis is that the firm's profit maximization problem is very similar to the one it faces in the single-period model of the previous section.

4.2 Worker-Consumers

Each worker-consumer lives for two periods. Let $\hat{y}(j)$ be the consumer's commitment in period j and $h_j(\hat{y}(j))$ be the corresponding cost, where

$$h_1(\hat{y}(1)) = \begin{cases} \beta + \kappa\hat{y}(1) & \text{if } \hat{y}(1) > 0 \\ 0 & \text{if } \hat{y}(1) = 0 \end{cases}$$

$$h_2(\hat{y}(2)) = \begin{cases} \beta + \kappa\hat{y}(2) & \text{if } \hat{y}(2) > 0 \\ 0 & \text{if } \hat{y}(2) = 0. \end{cases}$$

The consumer thus faces no constraints on the ability to adjust the level of the commitment good between periods. Given enough time, people can adjust their consumption of housing services not by incremental changes to their current house, but by moving to a new one.[13] As in the one-period model, the presence of the fixed cost β in the second period captures the fact that once again the commitment is drawn from a technology in which there is a premium on a sufficiently large scale of services.

If the consumer chooses a given level $\hat{y}$ in each period, then $\beta + \kappa\hat{y}$ is paid in each period. One might view the cost of purchased housing as more heavily weighted toward the beginning. We can readily interpret the model as one in which commitment $\hat{y}$ is made in the initial period at cost $(1+\delta)(\beta+\kappa\hat{y})$. Nothing further need be paid if $\hat{y}$ is maintained in the second period, while otherwise the value of the remaining service flow $(\delta(\beta+\kappa\hat{y}))$ must be sold and a new commitment made at cost $h(\hat{y})$ (again, with a transaction cost easily accommodated). Since we have incorporated no capital market imperfections into our model, this is equivalent to the current formulation.

The consumer's utility maximization problem is now

$$\max_{x_i(j),\hat{y}(j),y_i(j),z_i(j),i,j\in\{1,2\}} p(\gamma x_1(1)^\rho + (1-\gamma)(y_1(1)+z_1(1))^\rho)^{\frac{1}{\rho}} + (1-p)(\gamma x_2(1)^\rho + (1-\gamma)(y_2(1)+z_2(1))^\rho)^{\frac{1}{\rho}}$$
$$+\delta[p(\gamma x_1(2)^\rho + (1-\gamma)(y_1(2)+z_1(2))^\rho)^{\frac{1}{\rho}} + (1-p)(\gamma x_2(2)^\rho + (1-\gamma)(y_2(2)+z_2(2))^\rho)^{\frac{1}{\rho}}]$$

where $x_i(j)$, for example, is the quantity of good x consumed in period j in state i, subject to

$$z_i(j)\hat{y}(j) = 0,\ \ i,j = 1,2,$$

and, for each combination of state $i(1)$ and $i(2)$ in periods 1 and 2,

$$\begin{aligned} & x_{i(1)}(1) + z_{i(1)}(1) + h_1(\hat{y}(1)) + \zeta[y_{i(1)}(1) - \hat{y}(1)]_+ \\ & \quad + \delta\left(x_{i(2)}(2) + z_{i(2)}(2) + h_2(\hat{y}(2)) + \zeta[y_{i(2)}(2) - \hat{y}(2)]_+\right) \\ \leq \quad & w_{i(1)}(1) + \delta w_{i(2)}(2) - \frac{1}{\psi}[y_{i(1)}(1) - \hat{y}(1)]_- - \delta\frac{1}{\psi}[y_{i(2)}(2) - \hat{y}(2)]_-, \end{aligned}$$

where $w_i(j)$ is the wage paid in period state i and period j.

As before there are no restrictions on the consumer's ability to tailor x to the period and state. At the beginning of the contract, before learning the first-period state, the consumer has an opportunity to satisfy her service

[13]There may be transaction costs associated with such a move, and other commitment goods such as children may give rise to prohibitive adjustment costs. Adding such costs to the model will only reinforce the rigidities induced by consumption commitments and hence our results.

requirement via commitment, that is to choose a positive level $\hat{y}(1)$ at cost $h_1(\hat{y}_1)$. The consumer can buy additional units of y in the ex post market, but must do so at price ζ. The consumer can reduce consumption of good y below $\hat{y}(1)$, but in the course of doing so can recover only the fraction $\frac{1}{\psi}(\hat{y}(1) - y(1))$ of the cost. This sequence is repeated in the second period, beginning with a new commitment $\hat{y}_2$ made at cost $h_2(\hat{y}_2)$.

Notice that consumers at risk of being laid off are no longer automatically precluded from making commitments. A consumer facing a layoff risk in (only) the first period can borrow from second-period income to cover a first-period commitment should the bad state occur in the first period.

4.3 Timing

Events in each period proceed in the following sequence:

1. The firm offers an employment contract (w_1, n_1, w_2, n_2). Young workers reject or (in equilibrium) accept.

2. If $n_1/(n_1 + n_2) < 1/2$, each young worker draws an "age"–as before, young or old—that makes the worker vulnerable to layoff with probability $2n_1/(n_1+n_2)$ and otherwise not vulnerable. "Second generation" workers are not subject to layoff risk. If $n_1/(n + 1 + n_2) > 1/2$, each first generation worker is vulnerable to layoff, and each second generation worker takes a draw that makes the worker vulnerable to layoff with probability $2(n_1/(n_1 + n_2) - 1/2)$ and otherwise not vulnerable.

3. Workers make consumption commitments (i.e., choose $\hat{y} > 0$) in the "ex ante" market or choose not to do so ($\hat{y} = 0$).

4. The state is realized. All workers are retained in state 1, while vulnerable workers are laid off in state 2.

5. Workers who remain employed collect their wage, choose x and either z (in the absence of a commitment) or y (with a commitment). Employed workers consume the resulting bundle while laid-off workers receive the utility of home production.

We have assumed that layoff priority is based on age, with younger workers being vulnerable to layoffs. However, if the contract calls for only a fraction of the workers of a given age to be laid off, we still require the "age draws" of the static model to fix priority. If we measured age more finely and modeled workers as being employed by the firm for sufficiently many periods, these "age draws" would be unnecessary. As long as the age draws are made before consumers make their consumption commitments, these formulations are equivalent.

Proposition 2 (below) establishes conditions under which a contract with age-based layoffs dominates a full employment contract. This suffices to make our point, since if an optimal contract under our seniority restriction features layoffs, so must an optimal contract without this restriction. However, it leaves open the question of whether seniority is an optimal way to prioritize layoffs. Determining the *optimal* layoff priority requires a richer model, including (among other things) job-specific capital accumulation and imperfect capital markets.

4.4 Equilibrium

Consider first a consumer facing an employment contract with no layoffs. Commitments will be optimal if β and κ are sufficiently small. Given our steady-state assumption, we will have $w_2(1) = w_2(2) = f'(n_2)$. If the optimal contract features no variation at all in the consumer's income, so that $w_1(j) = w_2(j)$, then the consumer would set $\hat{y}(1) = \hat{y}(2)$ and make no transfers between periods. In general, it will be optimal for the firm to smooth the consumer's income by setting $w_1(j) < \alpha f'(n_1 + n_2)$, but not to smooth income perfectly. In this case, a consumer who encounters the bad state in the first period will transfer income from the second period to the first, and a consumer encountering the good state will save some income for the second period. However, the consumer necessarily faces some income risk in the second period, and hence optimally stops short of equalizing first-period expenditures in the good state and the bad state, incurring some risk in the first period in order to smooth the extreme values of the risky second-period income. Borrowing and saving mitigate the risk faced by the consumer, but do not eliminate it.

Once again, layoffs allow the firm to relax the marginal product constraint on the relatively low state-2 wage. Section 6 proves:

Proposition 2 *For sufficiently small $\beta > 0$, there exist $\bar{\kappa}(\beta) > 0$, $\bar{\gamma}(\beta) > 0$, $\bar{\zeta}(\beta)$ and $\bar{\psi}(\beta)$ such that for all $\kappa < \bar{\kappa}(\beta)$, $\gamma < \bar{\gamma}(\beta)$, $\zeta > \bar{\zeta}(\beta)$ and $\psi > \bar{\psi}(\beta)$, the optimal contract features layoffs.*

Remark 4. Our model has only two states and two consumption technologies to allow us to highlight most clearly the relationship between consumption commitments and the optimality of layoffs. Expanding the model beyond two periods requires additional notation, but doesn't alter the qualitative character of our results. Our assumptions that there are only two states and only two consumption technologies make for rather stark equilibria. In the contracts of interest, those unaffected by layoff risks make commitments. Those at risk either make no commitments (in the single-period model) or possibly (in the multiperiod model) make a commitment involving the same fixed cost, though not necessarily the same level of service, as those not at risk. A richer model would allow for shocks of varying sizes and a variety of commitment technologies featuring different trade-offs between fixed and marginal costs, along with many periods. Workers at various stages of their tenure with the firm would face different layoff risks and make commitments of different types and sizes. Optimal employment contracts in such models are shaped by the same forces as in our simpler analysis, but with considerably more complicated details. ■

5 Discussion

Endogenous risk aversion. Consumers who make consumption commitments in our model behave is if they are risk averse over small variations in income, despite their linearly homogeneous utility functions. More generally, the utility functions we can hope to observe are inferred from behavior that is the product of an interaction between preferences and the technology for converting income into consumption. Different technologies may lead us to different and potentially misleading inferences concerning risk aversion. For example, we may infer from consumers' behavior that they are risk neutral, concluding that insurance has no value, while the opening of an insurance market may give rise to both risk-averse behavior and active demand for insurance.[14]

Concentrated risks. Conditional on facing a risk of being laid off, the worker would prefer to concentrate this risk in as few states as possible. In essence, there are economies of scale in bearing risk, inducing workers to lump risks together rather than disperse them.

Habit formation. Our model generates behavior that is similar to that of many habit formation models.[15] Attanasio [2] discusses a typical habit

[14] Chetty [7] also makes this point.

[15] See Deaton [10] and Attanasio [2] for surveys of the habit formation literature and Chetty and Szeidl [9] for an examination of the connection between consumption commit-

formation model which in essence decreases an individual's effective current consumption by a constant times the individual's depreciated aggregate previous consumption, making the individual averse to downward adjustments in consumption. If the force of habit formation is strong enough, it could lead to optimal employment contracts that include layoffs in a manner similar to that shown in this paper.[16]

Morale. Bewley [5, 6] discusses the tendency of employers to insure wage but not layoff risk, in order to avoid detrimental morale effects that especially accompany wage reductions. This differential effect on morale is in turn traced to a convention that wage reductions (but not layoffs) are a violation of fairness or social norms. We agree that adverse morale effects may pose significant barriers to wage reductions. But why are wage reductions devastating for morale, reductions in overtime for hourly employees less so, and appropriately conducted layoffs less so? One possibility is that morale effects reinforce employment practices that are customary, with these practices having become customary because they have economic advantageous linked to their interaction with consumption commitments.[17]

6 Appendix: Details of Proofs

Lemma 1.1. Suppose that the consumer sets $\hat{y} = 0$. Then the consumer is risk neutral and the consumer's indirect utility function can be written as $pw_1 + (1-p)\frac{n_2}{n_1+n_2}w_2$. Attaching multiplier λ to the consumer's participation constraint and multiplier μ to the constraint that $n_1 \geq 0$ (one easily verifies that workers will not optimally be laid off in the good state), while ignoring the constraint that wages not exceed marginal products, the first-order conditions for the firm's profit maximization problem are:

$$n_1 : p(\alpha f'(n_1+n_2) - w_1) - \lambda(1-p)\frac{n_2 w_2}{(n_1+n_2)^2} + \mu = 0 \tag{4}$$

ments and habit formation.

[16]The models of consumption commitments and habit formation exhibit some differences. For example, our model would suggest nontrivial heterogeneity, linked to observable characteristics, across individuals in their aversion to downward adjustments in consumption—an individual who has made consumption commitments will be more averse to income shocks than an individual who has avoided commitments.

[17]Bewley [5, Chapter 13] explains that layoffs have the advantage of focussing adverse effects on those who are no longer with the firm, but also that they have morale effects for the entire workforce that are small compared to those of wage reductions (Chapter 11).

$$n_2 : p(\alpha f'(n_1+n_2) - w_1) + (1-p)(f'(n_2) - w_2) + \lambda(1-p)\frac{n_1 w_2}{(n_1+n_2)^2} = 0 \quad (5)$$

$$w_1 : -p(n_1+n_2) + \lambda p = 0 \quad (6)$$

$$w_2 : -(1-p)n_2 + \lambda(1-p)\frac{n_2}{n_1+n_2} = 0. \quad (7)$$

Assume that $n_1 > 0$, so that there are layoffs, and hence $\mu = 0$. Coupling this with the equality $\lambda = n_1 + n_2$, which we can derive from either of (6) or (7), we can rewrite (4)–(5) as

$$\begin{aligned} p(\alpha f'(n_1+n_2) - w_1) - (1-p)\frac{n_2}{n_1+n_2}w_2 &= 0 \\ p(\alpha f'(n_1+n_2) - w_1) + (1-p)(f'(n_2) - w_2) + (1-p)\frac{n_1}{n_1+n_2}w_2 &= 0. \end{aligned}$$

Substituting the first of these into the second, we have

$$(1-p)\frac{n_2}{n_1+n_2}w_2 + (1-p)(f'(n_2) - w_2) + (1-p)\frac{n_1}{n_1+n_2}w_2 = 0,$$

or $f'(n_2) = 0$, a contradiction. Hence, there must be no layoffs. We can also conclude, from (5) and the constraint that wages not exceed marginal products, that $w_1 = \alpha f'(n_2)$ and $w_2 = f'(n_2)$ which in turn implies that $w_1 > w_2$. ∎

Lemma 3. We proceed quickly through some obvious cases. If both wages fall short of the corresponding marginal products, then the firm could increase profits by hiring more labor at the existing wage rate while preserving the existing probability of a layoff (and hence preserving worker utility). If w_1 equals its marginal product and w_2 falls short of its marginal product, then either (1) $w_2 < w_1$, in which case the firm can increase w_2 and decrease w_1, preserving expected wage payments while preserving or increasing worker utility and leading to a state at which both wages fall short of their marginal products (at which point the firm can increase profits by hiring more labor); or (2) $w_2 \geq w_1$, in which case there must be layoffs in the bad state and the firm can increase profits and consumer utility by hiring more labor in the bad state (and hence reducing the layoff probability). Hence, we must have $w_2 = f'$. If $w_2 > w_1$, then either smoothing wages (if $w_1 < \alpha f'$) or reducing n_1 (if $w_1 = \alpha f'$) again increases consumer utility while preserving the firm's payoff, allowing the firm to exploit the resulting slack in the participation constraint to increase profits. Thus, we must have:

$$\alpha f' \geq w_1 \geq w_2 = f'. \quad (8)$$

The consumer's participation constraint can now be written as

$$\frac{n_2}{n_1+n_2}\left(p\tilde{U}_1(w_1,w_2)+(1-p)\tilde{U}_2(w_1,w_2)\right)+\frac{n_1}{n_1+n_2}pw_1 \geq \bar{U},$$

where $\tilde{U}_i(w_1,w_2)$ is the indirect utility function identifying the consumer's utility when state i is realized, when not vulnerable for layoffs and given wages w_1 and w_2. Notice that in the presence of commitments, both wages are relevant for determining state-i utility and $\tilde{U}_i$ is in general not linear.

Attaching multiplier λ to the participation constraint, η to the constraint $f' - w_2 \geq 0$, and μ to the constraint $n_1 \geq 0$, the first-order conditions for the firm's profit maximization problem are:

$$\begin{aligned} n_1 : \; & p(\alpha f'(n_1+n_2)-w_1) - \lambda\frac{n_2}{(n_1+n_2)^2}(p\tilde{U}_1+(1-p)\tilde{U}_2) \\ & +\lambda p\frac{n_2}{(n_1+n_2)^2}w_1 + \mu = 0 \end{aligned} \tag{9}$$

$$\begin{aligned} n_2 : \; & p(\alpha f'(n_1+n_2)-w_1) + (1-p)(f'(n_2)-w_2) \\ & +\lambda\frac{n_1}{(n_1+n_2)^2}(p\tilde{U}_1+(1-p)\tilde{U}_2) - \lambda p\frac{n_1}{(n_1+n_2)^2}w_1 + \eta f''(n_2) = 0 \end{aligned} \tag{10}$$

$$\begin{aligned} w_1 : \; & -p(n_1+n_2) + \lambda\frac{n_2}{(n_1+n_2)}\left(p\frac{d\tilde{U}_1}{dw_1}+(1-p)\frac{d\tilde{U}_2}{dw_1}\right) \\ & +\lambda p\frac{n_1}{n_1+n_2} = 0 \end{aligned} \tag{11}$$

$$w_2 : -(1-p)n_2 + \lambda\frac{n_2}{(n_1+n_2)}\left(p\frac{d\tilde{U}_1}{dw_2}+(1-p)\frac{d\tilde{U}_2}{dw_2}\right) - \eta = 0. \tag{12}$$

Now suppose first that there are no layoffs, so that $n_1 = 0$. Then (10) becomes:

$$p(\alpha f'(n_2)-w_1) + (1-p)(f'(n_2)-w_2) + \eta f''(n_2) = 0. \tag{13}$$

Now suppose that both wages equal marginal products. Then (13) can be satisfied only if $\eta = 0$. Using $\eta = 0$, we can write (11)–(12) as

$$\begin{aligned} -pn_2 + \lambda\left(p\frac{d\tilde{U}_1}{dw_1}+(1-p)\frac{d\tilde{U}_2}{dw_1}\right) &= 0 \\ -(1-p)n_2 + \lambda\left(p\frac{d\tilde{U}_1}{dw_2}+(1-p)\frac{d\tilde{U}_2}{dw_2}\right) &= 0, \end{aligned}$$

giving

$$\frac{p}{1-p} = \frac{p\frac{d\tilde{U}_1}{dw_1} + (1-p)\frac{d\tilde{U}_2}{dw_1}}{p\frac{d\tilde{U}_1}{dw_2} + (1-p)\frac{d\tilde{U}_2}{dw_2}} = \frac{\frac{d\tilde{U}(w_1,w_2)}{dw_1}}{\frac{d\tilde{U}(w_1,w_2)}{dw_2}} = \frac{p\frac{du(x_1^*,y_1^*)}{dx_1}}{(1-p)\frac{du(x_2^*,y_2^*)}{dx_2}}, \tag{14}$$

where $\tilde{U}(w_1, w_2) = p\tilde{U}_1(w_1, w_2) + (1-p)\tilde{U}_2(w_1, w_2)$ gives expected utility conditional on not being at risk of being laid off, u is the consumer's direct utility function (1), (x_i^*, y_i^*) is the optimal consumption bundle in state i, and the final equality follows from an envelope argument. The outer two terms of this equality, along with $w_1 > w_2$ and hence $\frac{x_1^*}{y_1^*} > \frac{x_2^*}{y_2^*}$, yield a contradiction.

Now suppose that the optimal employment contract features layoffs in state 2. (It is straightforward to exclude the optimality of layoffs in state 1.) Suppose the first weak inequality in (8) is an equality. Since $n_1 > 0$, we have $\mu = 0$. From (9), we then have

$$-\lambda \frac{n_2}{(n_1+n_2)^2}(p\tilde{U}_1 + (1-p)\tilde{U}_2) + \lambda p \frac{n_2}{(n_1+n_2)^2} w_1 = 0.$$

As a result, we have $pw_1 = p\tilde{U}_1 + (1-p)\tilde{U}_2$. This is a contradiction. The maximum utility achieved when making no consumption commitments and faced with wages w_1 in state 1 and 0 in state 2 is pw_1. A consumer who has income $w_2 > 0$ in state 2 and makes no commitments must then receive a higher utility, and a consumer with income w_2 who optimally makes commitments must receive a utility at least as high as the latter, giving the contradiction. Hence, we must have $\alpha f' > w_1 \geq w_2 = f'$. ∎

Proposition 2. We assume that the optimal contract features no layoffs and seek a contradiction. The optimal no-layoff contract must then feature $N = n_2$, $n_1 = 0$, and $\alpha f'(n_2) > w_1 \geq w_2 = f'(n_2)$. The firm's profits are in general given by,

$$p[\alpha f(n_1+n_2) - w_1(n_1+n_2)] + (1-p)[f(n_2) - w_2 n_2],$$

where the assumption that there are no layoffs currently gives $n_1 = 0$. Beginning with the optimal no-layoff contract, we consider an adjustment that decreases n_2, adjusting w_2 so as to preserve equality with the marginal product of labor in the bad state (i.e., $dw_2/dn_2 = f''(n_2)$), increasing w_1 similarly, and adjusting n_1 so as to preserve expected profits. It is a contradiction to show that this adjustment increases consumer utility.

We differentiate with respect to n_2, giving:

$$p\left(\alpha f'(n_1+n_2)-w_1\right)\left[\frac{dn_1}{dn_2}+1\right]+(1-p)[f'(n_2)-w_2]-p\frac{dw_1}{dn_2}(n_1+n_2)-(1-p)\frac{dw_2}{dn_2}n_2=0.$$

Because $f'(n_2) = w_2$, $dw_1/dn_2 = dw_2/dn_2 = f''(n_2)$, and $n_1 = 0$ by assumption, we can rearrange to obtain

$$\frac{dn_1}{dn_2} = -\frac{p[\alpha f'(n_2)-w_1]-f''(n_2)n_2}{p[\alpha f'(n_2)-w_1]}. \tag{15}$$

Let $\tilde{U}(w_1,w_2)$ be the indirect utility function, giving expected utility as a function of the wages w_1 and w_2, conditional on not being at risk of a layoff. This indirect utility is of the form

$$\begin{aligned}\tilde{U}(w_1,w_2) &= p(\tilde{U}_{11}(w_1,w_2)+\delta(p\tilde{U}_{21}(w_1,w_2,1)+(1-p)\tilde{U}_{22}(w_1,w_2,1)))\\ &\quad+(1-p)(\tilde{U}_{12}(w_1,w_2)+\delta(p\tilde{U}_{21}(w_1,w_2,2)+(1-p)\tilde{U}_{22}(w_1,w_2,2))),\end{aligned}$$

where $\tilde{U}_{11}(w_1,w_2)$ is the first-period utility given the good state (with $\tilde{U}_{12}(w_1,w_2)$ in the case of the bad state) and $\tilde{U}_{21}(w_1,w_2,1)$ (for example) is the second-period utility, given that the good state is realized in the second period (the second subscript) and given that the good state was also realized in the first period (the argument 1 in the function). The latter is relevant because the first-period state determines how much the consumer borrows or saves, and hence second-period (state-contingent) income. Let $\tilde{V}$ similarly be the indirect utility function for a consumer at risk of layoff in the first period (only). This function takes a similar form, but differs from $\tilde{U}$ in recognition of the zero income that is now attached to state 2. The consumer's utility is given by:

$$\frac{n_2}{n_1+n_2}\tilde{U}(w_1,w_2)+\frac{n_1}{n_1+n_2}\tilde{V}(w_1,w_2).$$

Differentiating gives (using $dw_1/dn_2 = dw_2/dn_2$)):

$$\begin{aligned}&\frac{dw_2}{dn_2}\left[\frac{\left(\frac{d\tilde{U}(w_1,w_2)}{dw_1}+\frac{d\tilde{U}(w_1,w_2)}{dw_2}\right)n_2+n_1\left(\frac{d\tilde{V}(w_1,w_2)}{dw_1}+\frac{d\tilde{V}(w_1,w_2)}{dw_2}\right)}{n_1+n_2}\right]\\ &+\tilde{U}(w_1,w_2)\left(\frac{n_1+n_2-n_2\left(\frac{dn_1}{dn_2}+1\right)}{(n_1+n_2)^2}\right)+\tilde{V}(w_1,w_2)\left(\frac{\frac{dn_1}{dn_2}(n_1+n_2)-n_1\left(\frac{dn_1}{dn_2}+1\right)}{(n_1+n_2)^2}\right).\end{aligned}$$

Using the facts that $w_2 = f'(n_2)$, $n_1 = 0$, and $dw_2/dn_2 = f''(n_2)$, we have a contradiction if

$$\left(\frac{d\tilde{U}}{dw_1}+\frac{d\tilde{U}}{dw_2}\right)\frac{f''(n_2)n_2}{f'(n_2)}+\left(\frac{\tilde{U}(w_1,w_2)}{f'(n_2)}\right)\left(-\frac{dn_1}{dn_2}\right)+\left(\frac{\tilde{V}(w_1,w_2)}{f'(n_2)}\right)\frac{dn_1}{dn_2}<0,$$

or

$$-\left(\frac{d\tilde{U}}{dw_1}+\frac{d\tilde{U}}{dw_2}\right)\theta<\frac{dn_1}{dn_2}\left(\frac{\tilde{U}(w_1,w_2)}{f'(n_2)}-\frac{\tilde{V}(w_1,w_2)}{f'(n_2)}\right).$$

Using (15), this is

$$p[\alpha f'(n_2)-w_1]\left(\frac{d\tilde{U}}{dw_1}+\frac{d\tilde{U}}{dw_2}\right)\theta\ >\ \left(\frac{\tilde{U}(w_1,w_2)}{f'(n_2)}-\frac{\tilde{V}(w_1,w_2)}{f'(n_2)}\right)(p[\alpha f'(n_2)-w_1]-f''(n_2)n_2),$$

or, using the fact that the consumer's expected utility $\tilde{U}(w_1, w_2)$ must equal the reservation wage (given our working hypothesis of no layoffs),

$$p[\alpha-\frac{w_1}{f'(n_2)}]\left(\frac{d\tilde{U}}{dw_1}+\frac{d\tilde{U}}{dw_2}\right)\theta>\left(\frac{\overline{U}}{f'(n_2)}-\frac{\tilde{V}(w_1,w_2)}{f'(n_2)}\right)(p(\alpha-\frac{w_1}{f'(n_2)})+\theta).$$

Now fix β and κ sufficiently small that the consumer makes commitments, and hold β fixed while letting κ decrease. As κ and γ get small, $\overline{U}/f'(n_2)$ is bounded (because the firm optimally sets $f'(n_2) \geq \beta$, to ensure the feasibility of consumption commitments) while $\frac{d\tilde{U}}{dw_1}+\frac{d\tilde{U}}{dw_2}$ approaches infinity (because small κ allows increases in w_2 to yield ever larger increases in $\hat{y}$, the marginal utility of which remain large as γ gets small). Noting that θ is by assumption bounded away from zero, the inequality (and contradiction) thus holds if $\alpha - w_1/f'(n_2)$ is positive and bounded away from zero. It is positive by (the counterpart for the two-period model of) Lemma 3. We then note that $w_1/f'(n_2)$ approaches one, for fixed β and κ, as γ gets small and ζ and ψ get large, since in the limit increments in the state-1 wage are worthless. We then need only set γ sufficiently small and ζ and ψ sufficiently large.[18]

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[18] We see the two basic requirements of our argument in this final derivation. First, $\alpha - w_1/f'(n_2) > 0$, which ensures that the firm can be compensated for reducing n_2 and increasing w_2 by a profitable (finite) increase in n_1, will hold whenever the bad-state wage falls short of the good state wage. Given $w_2 < w_1$, the consumer risk aversion induced by commitments then ensures that the optimal contract smooths wages, giving $\alpha - w_1/f'(n_2) > 0$. Second, $\theta > 0$ indicates that reducing bad-state employment allows the firm to increase the bad-state wage. This would be replaced by an analogous but possibly different condition if the link between state-2 employment and wage reflected some friction other than our constraint $w_2 \leq f'(n_2)$.

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