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"Consumption Commitments and Employment Contracts"

by

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CONSUMPTION COMMITMENTS AND EMPLOYMENT CONTRACTS*

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Abstract We examine an economy in which the cost of consuming some goods can be reduced by making commitments that reduce flexibility. We show that such consumption commitments can induce consumers with risk-neutral underlying utility functions to be risk averse over small variations in income, but sometimes to seek risk over large variations. As a result, optimal employment contracts will smooth wages conditional on being employed, but may incorporate a possibility of unemployment.

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Consumption Commitments and Employment Contracts

1 Introduction

An individual whose purchases involve future financial obligations is likely to have a different attitude toward income risk than one who chooses more flexible arrangements. Negative income shocks can have serious financial consequences for the holder of a mortgage, for example, perhaps even inducing foreclosure. Renting instead of buying provides some insulation from income shocks: failing to make the rent may force one to move, but with lower transactions costs and without the risk of a capital loss. One could achieve even more flexibility by living in hotels, adjusting the quality as needed in response to income fluctuations. Analogous effects can arise even without explicit financial obligations. The expected utility from a vacation home may be jeopardized by negative income shocks, even if there are no further payments to make and the home itself is not at risk.

We refer to choices that give rise to rigidities in consumption as *consumption commitments*. Consumption commitments can be valuable. Purchasing a house typically gives greater benefit per dollar spent than does renting an apartment.[1] Hotels are yet more expensive than apartments. Owning a vacation home near a ski resort can be cheaper than regularly renting.

Consumption commitments can introduce elements of risk aversion into the behavior of people whose underlying preferences are risk neutral. More important for this paper, vulnerability to income shocks is a consequence of consumption decisions, giving rise to utility gains from coordinating consumption and labor market decisions by matching those times that consumption would be especially vulnerable to income fluctuations with times that income is secure.[2] We show that this coordination can induce "economies of scale" in risk bearing. A consumer who makes relatively few commit-

[1] Not only are there tax benefits associated with home ownership, but purchasing allows one to make idiosyncratic capital improvements that increase the utility of the housing, while renting typically entails a premium because of moral hazard concerns.

[2] Several papers make a similar point. Ellingsen and Holden [8] analyze a model in which workers make purchases of durable goods based on expectations about future wages. When those expectations are high, workers will make large purchases, and then will resist lower wages more than they would have had they had more pessimistic expectations (and consequently purchased fewer durables). Ellingsen and Holden [9] analyze a model in which worker indebtedness worsens their bargaining position vis a vis employers. Chetty [5] demonstrates empirically the importance of commitments in estimating risk aversion.

ments, perhaps in response to a relatively risky income, may find the cost of bearing *additional* risk relatively low. An individual faced with a given amount of lifetime income uncertainty then may rationally choose to concentrate as much risk as possible into his or her early years, in return for a relatively secure income (and more consumption commitments) in later years. The risk aversion induced by consumption commitments may thus be coupled with risk seeking preferences over making such commitments. As a result, optimal employment contracts may smooth wages conditional on unemployment, but feature employment risk.

Section 2 introduces a model of consumption commitments and employment contracts. Section 3 establishes conditions under which wage smoothing and layoffs are optimal in a simple model, while Section 4 extends the argument to an intertemporal model. Section 5 discusses the results.

2 Consumption Commitments

2.1 The Firm

We consider a firm whose profits are a function of the quantity of worker-consumers $N \in \Re$ that it hires and the realization of a state. Revenue in state 2 (the bad state) is given by the function $f : \Re \to \Re_+$, and in state 1 (the good state) by αf, for $\alpha > 1$. The good state occurs with probability p.

We assume that f is twice continuously differentiable on $\Re_+$, with $f' > 0$, $f'' < 0$, $f'(0) = \infty$, and $\lim_{N\to\infty} f'(N) = 0$. We assume that the elasticity

$$\theta(N) \equiv -\frac{f''(N)N}{f'(N)}$$

is bounded below by $\theta^* > 0$. This is the case, for example, for any power function satisfying our assumptions.

We represent the workers hired by the firm as the interval $[0, N]$, with $j \in [0, N]$ being an index that identifies different workers. An employment contract specifies the wage rate to be paid in each state $i \in \{1, 2\}$, denoted by w_i, and specifies which of the N employees are to be employed in that state, with the remainder (if any) being laid off.

Let $\chi_2(j)$ be a (measurable) indicator function equal to 1 if employee j is employed in state 2 and equal to 0 if j is laid off. Let

$$\begin{aligned} n_1 &= N - n_2 \\ n_2 &= \int_0^N \chi_2(j)dj. \end{aligned}$$

Hence, n_2 is the quantity of workers "kept on" in the bad state, and n_1 the quantity of workers who are employed if the state is good and laid off otherwise. The firm's revenues in state 1 are then given by $\alpha f(n_1 + n_2)$ and in state 2 by $f(n_2)$. Embedded in this notation is the assumption, sacrificing no generality, that if the firm is to lay off workers, it will do so in the bad state.

We assume the firm faces three constraints in writing employment contracts. First, the firm cannot make payments to unemployed workers. We interpret this as reflecting moral hazard considerations. Unemployment compensation provides insufficient incentives to undertake home production or seek alternatives.

Second, the firm cannot pay a wage w_i in state $i \in \{1, 2\}$ that exceeds the marginal product of labor in state i. This again reflects moral hazard. Making payments in excess of marginal products makes it tempting for the firm to fire workers for alleged nonperformance.

Some limitation on the amount the firm can pay to workers is essential to our analysis. In its absence, an argument analogous to the "wage bill" argument of Akerlof and Miyazaki [1] would ensure that the optimal labor contract completely insures the worker against risk, featuring no wage fluctuations and no unemployment. Our interest is in studying how optimal contracts balance wage and unemployment risk, in the presence of some imperfection that potentially forces the contract to contain some risk.

Finally, we assume that all workers who remain employed at the firm after the state of nature is realized receive the same wage. This simplifies the calculations, but does not play an important role in the results.

The firm's payoff is given by

$$p\left(\alpha f(n_1 + n_2) - w_1(n_1 + n_2)\right) + (1 - p)\left(f(n_2) - w_2 n_2\right).$$

The firm maximizes this payoff subject to the constraint that the employment contract provides workers with at least their reservation utility.

2.2 Worker-Consumers

The worker-consumer (also called either a worker or consumer) has a reservation utility, interpreted as the value of alternative market activities, that we denote by $\overline{U} > 0$. The consumer's utility depends on two things: consumption of a good x and consumption of services that can be obtained from either of two other goods, y or z. The consumer has a constant-elasticity-of-substitution utility function over x and $(y + z)$, the level of services he receives from the goods y and z, given by

$$(\gamma x^\rho + (1-\gamma)(y+z)^\rho)^{\frac{1}{\rho}}\,. \tag{1}$$

The constant-elasticity-of-substitution form for this utility function is not essential to our results, but has the important advantage of allowing us to talk precisely (by varying γ) about the relative importance of the various consumption goods.

The consumer is risk neutral, in the sense that his utility is linear along rays through the origin:

$$(\gamma(\lambda x)^\rho + (1-\gamma)(\lambda y + \lambda z)^\rho)^{\frac{1}{\rho}} = \lambda\left(\gamma x^\rho + (1-\gamma)(y+z)^\rho\right)^{\frac{1}{\rho}}.$$

The goods y and z are perfect substitutes representing two different ways that the consumer can satisfy his desire for services. For example, y and z may represent purchased housing and rental housing. We assume that the consumer can purchase either y or z, but not both: he must choose one of the two ways to get the relevant services.[3]

In a traditional model in which all goods have linear prices, whichever of goods y and z had the higher price would be irrelevant. We depart from the traditional model in two ways. First, we assume purchasing good y involves a nontrivial commitment, while there is no such commitment involved in purchasing z. Formally, we model this by assuming that there is an ex ante market (before the state is realized) and an ex post market (after the state has been realized) for the services provided by purchases of y and z. Commitments to good y must be made in the ex ante market, while trade in z occurs after the state is realized. *Ceteris paribus*, there is an advantage to purchasing z rather than y, since purchases in the ex post market can be conditioned on the realized state of the world. Second, we assume that committing to good y (in the ex ante market) entails a fixed cost plus a marginal cost. We normalize prices so that the price of x is one and we normalize units of z and y so that the price of z is one. The cost of committing to good y in the ex ante market is

$$h(y) = \begin{cases} \beta + \kappa y & \text{if } y > 0 \\ 0 & \text{if } y = 0, \end{cases}$$

[3]For example, rented and purchased housing are not easily combined into a single place of residence. We could work with weaker versions of this assumption, with some additional complication, as long as consumption commitments introduce sufficient rigidities in ex post consumption.

where $0 < \kappa < 1$ and $\beta > 0$. Hence, the consumer can purchase the services provided by goods y or z at a cheaper per unit price if he pays the fixed cost of β and purchases in the ex ante market. The nonlinear form of the price of y is meant to capture the idea that securing services via good y is cost effective only if consumption exceeds some minimum level. For example, purchased housing may be financially attractive only above a given threshold level of services.

The commitment to good y in the ex ante market, denoted by $\hat{y}$, can be adjusted in the ex post market after the realization of the state is known, but at a cost per unit different from κ. Additional purchases of y can be made at price $\zeta > 1$, while portions of good y can be sold on the ex post market, at price $\frac{1}{\psi} < \kappa$. Purchases of y in the ex ante market thus come at a lower marginal price than purchases of z, but adjustments to the level $\hat{y}$ are more expensive. For example, buying a house with eight-foot ceilings and then increasing the ceiling height to nine feet is more expensive than buying a house with higher ceilings in the first place. Building a house with three bathrooms and then selling one is financially worse than simply not having installed three. We write the price relevant for such a reduction as $\frac{1}{\psi}$ so that larger values of ψ and ζ correspond to more rigid commitments.

Let $[\xi]_+ = \max\{\xi, 0\}$ and $[\xi]_- = \min\{\xi, 0\}$. If a consumer has wage w_i in state i and doesn't face the prospect of being laid off, he has the following utility maximization problem:

$$\max_{x_1,x_2,\hat{y},y_1,y_2,z_1,z_2} p\left(\gamma x_1^\rho + (1-\gamma)(y_1+z_1)^\rho\right)^{\frac{1}{\rho}} + (1-p)\left(\gamma x_2^\rho + (1-\gamma)(y_2+z_2)^\rho\right)^{\frac{1}{\rho}}$$

subject to the budget constraints (for $i = 1, 2$)

$$x_i + z_i + h(\hat{y}) + \zeta[y_i - \hat{y}]_+ + \frac{1}{\psi}[y_i - \hat{y}]_- = w_i,$$

and the constraints that the services be purchased via one or the other of y and z, but not both:

$$\hat{y}z_1 = \hat{y}z_2 = 0.$$

Suppose the consumer may be laid off with probability $q > 0$ in state 2. A layoff consigns the consumer to home production, with value 0. Then the consumers' problem is

$$\max_{x_1,x_2,\hat{y},y_1,y_2,z_1,z_2} p\left(\gamma x_1^\rho + (1-\gamma)(y_1+z_1)^\rho\right)^{\frac{1}{\rho}} + (1-p)\left[(1-q)\left(\gamma x_2^\rho + (1-\gamma)(y_2+z_2)^\rho\right)^{\frac{1}{\rho}}\right],$$

subject to the budget constraints

$$\begin{aligned} x_i + z_i + h(\hat{y}) + \zeta[y_1 - \hat{y}]_+ + \frac{1}{\psi}[y_1 - \hat{y}]_- &= w_i \\ h(\hat{y}) &\leq \frac{1}{\psi}\hat{y} \end{aligned}$$

and the constraints:

$$\hat{y}z_1 = \hat{y}z_2 = 0.$$

The second budget constraint is a "no bankruptcy" constraint, capturing a requirement that a consumer who is laid off must still be able to earn enough (by liquidating the commitment good $\hat{y}$) to meet her fixed payment obligations for the good. Since $\frac{1}{\psi} < \kappa$, this effectively requires the consumer to set $\hat{y} = 0$ in order to respect the budget constraint in state 2—consumers at risk of being laid off cannot make commitments. We relax this assumption in Section 4, noting for the moment only that this assumption introduces a bias *against* layoffs.

2.3 Commitments and Utility

To gain insight into the consumer's utility maximization problem, fix a commitment level $\hat{y} > 0$ and consider the ex post problem of choosing x and y to

$$\max(\gamma x^\rho + (1-\gamma)y^\rho)^{\frac{1}{\rho}}$$

subject to

$$x + \kappa\hat{y} + \zeta[y - \hat{y}]_+ + \frac{1}{\psi}[y - \hat{y}]_- = I,$$

where I is ex post net income (i.e., realized income minus the fixed commitment cost, or $w_i - \beta$).

Figure 1 shows a representative budget line, marked $I = x + \kappa y$, and the corresponding expansion path, given by the ray marked κ, identifying optimal consumption as I varies. This would be the appropriate expansion path if the consumer had paid the fixed cost β and could now freely alter the choices of good x and good y, at marginal price κ, in response to variations in ex post income. Of course, this expansion path is doubly irrelevant: if the consumer knew her ex post net income would be small, she would not make a commitment, and once the commitment is made, she cannot adjust the quantity of good y at price κ. However, this construction is a helpful benchmark in assessing the consumer's ex post consumption. In particular, if the there is no uncertainty about the consumer's ex post income and the

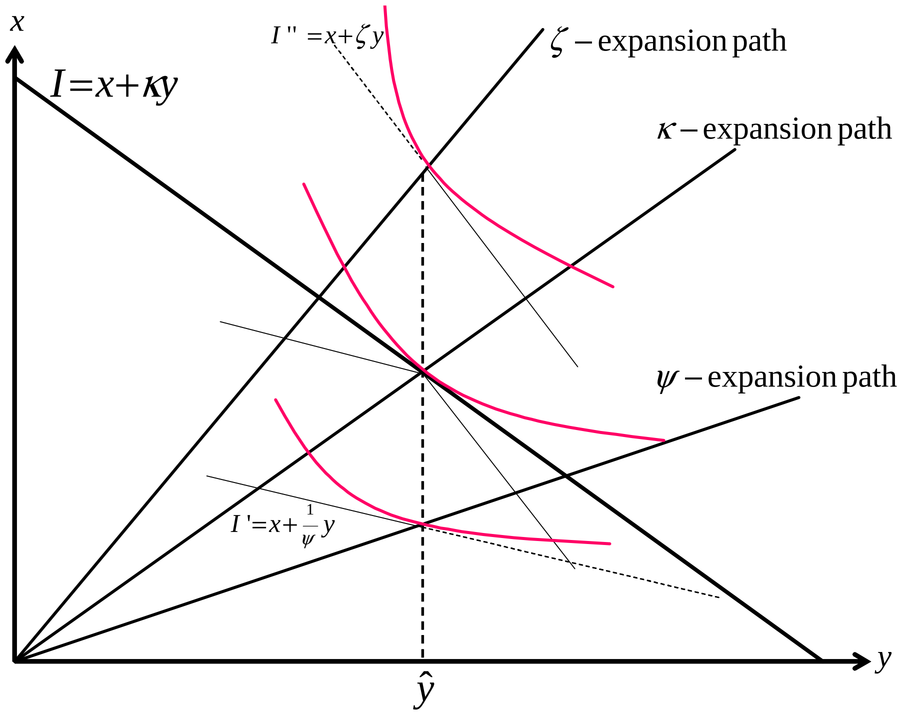

Figure 1: Expansion paths in the ex post market, identified by the corresponding good-2 price.

consumer optimally makes commitment $\hat{y}$, then the resulting consumption bundle will be given by the intersection of a budget constraint of the form $I = x + \kappa y$ with the ray κ.

Now suppose the consumer has committed to $\hat{y}$, and consider the optimal consumption bundle as ex post income varies. The consumer faces a series of budget constraints that are kinked at the quantity $\hat{y}$. Reducing the quantity y below $\hat{y}$ is accomplished by selling y at price $\frac{1}{\psi} < \kappa$, giving a flatter budget constraint for reductions in the commitment good. One such budget constraint is shown, given by $I' = x + \frac{1}{\psi}y$ for some I'. Increasing y above $\hat{y}$ is accomplished by buying good y at price ζ, giving a steeper budget line (with one such line given by $I'' = x + \zeta y$).

The rays marked ζ and ψ are the expansion paths relevant if the consumer could purchase unlimited amounts of good y at prices ζ and $\frac{1}{\psi}$ in the ex post market (again, thinking of the consumer as having paid the fee β), respectively. The consumer's ex post expansion path, given the rigidities induced by a commitment $\hat{y}$, is shown in Figure 2. If the consumer's ex post net income happens to hit just the right level, the consumption bun-

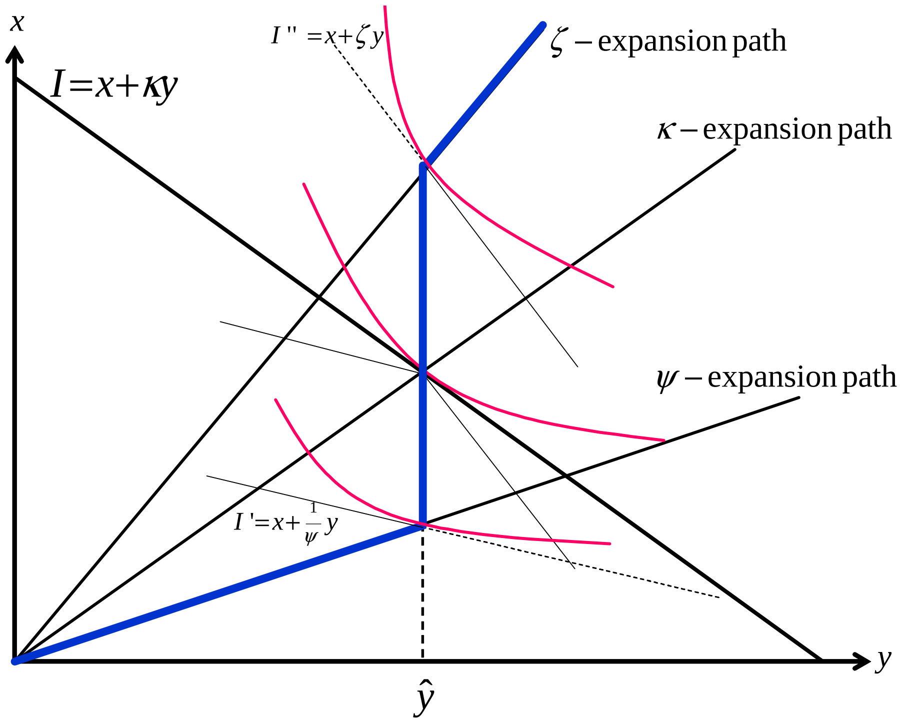

Figure 2: Ex post expansion path, given the consumer has incurred the fixed cost and chosen commitment $\hat{y}$.

dle is on the expansion path κ, with $y = \hat{y}$. In this case, the commitment to $\hat{y}$ poses no constraints. Moderately higher incomes prompt no change in consumption of the commitment good and increased consumption of the noncommitment good, until hitting the expansion path ζ, at which point the consumer proceeds along this path. Similarly, reductions in income prompt reductions in x until hitting and following path ψ.

Figure 3 shows the indirect utility, denoted by $\tilde{U}$, as a function of ex post gross income I (i.e., income before incurring the fixed cost β), presuming a commitment $\hat{y}$. The ray marked κ would be the indirect utility function if the consumer purchased services via good y, at price κ. This path is linear, since the consumer is risk neutral if allowed to vary x and y freely at prices 1 and κ, but does not pass through the origin, reflecting the fixed cost β. For a given commitment $\hat{y}$, there is an income level $I(\hat{y})$ at which the consumer's unconstrained optimal purchase of good y (at price κ, given fixed cost β) will equal $\hat{y}$, yielding point A in Figure 3. Realized incomes above this level, given commitment $\hat{y}$, initially induce increases in the consumption of good x, but leave y unchanged, until reaching point B in Figure 3.

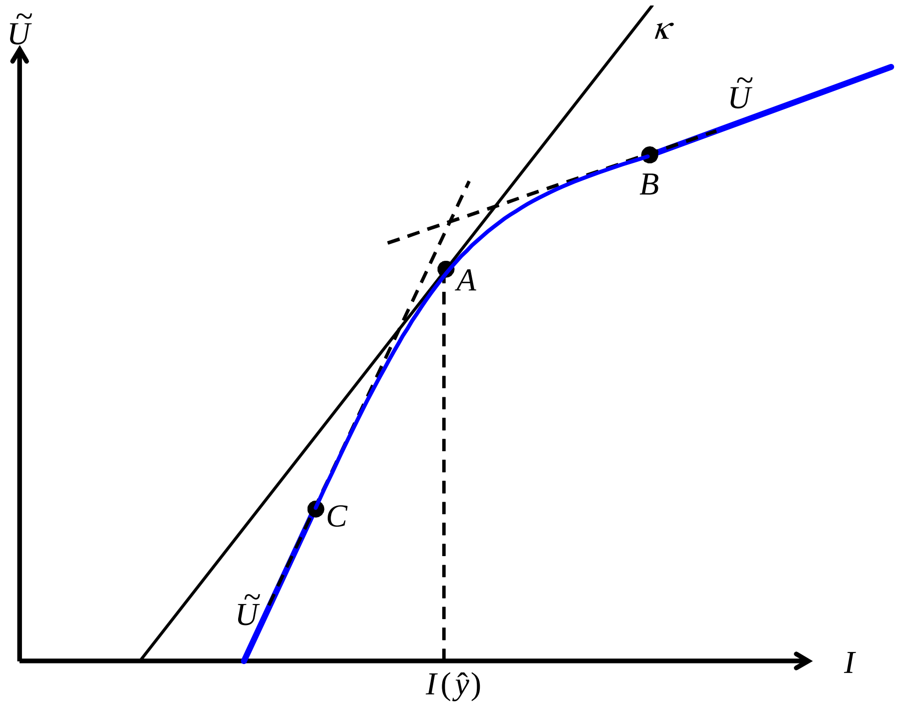

Figure 3: Indirect utility function, giving utility as a function of ex post gross income I, for commitment $\hat{y}$.

At this point, the consumer supplements the commitment $\hat{y}$ by purchasing additional quantities of good y at price ζ (as well as additional quantities of good x). The indirect utility function is then again linear, with a flatter slope representing the higher (than κ) price ζ.[4] Similarly, reductions in income below $I(\hat{y})$ initially prompt no reduction in good y, until point C is reached, after which some units of good y are sold at price $\frac{1}{\psi}$ and the indirect utility function is again linear. Putting these together, the indirect utility function is strictly concave in a neighborhood of $I(\hat{y})$. Having made a commitment $\hat{y}$, the consumer is risk averse over small variations in income, as the cost of adjusting y channels any variation into good x. However, the

[4]Notice that the indirect utility function is differentiable at point B. The extension of this linear segment emanating from point Bpasses above point A. This segment is part of a linear indirect utility function that would be relevant if the consumer faced prices $(1, \zeta)$ for goods (x, y), with an ex post income subsidized by $(\zeta - \kappa)\hat{y}$, so that the first $\hat{y}$ units of good y can be purchased at price κ, but with no restrictions on purchases of good y. At ex post income $I(\hat{y})$, the consumer can then buy the bundle that gives the utility corresponding to point A, but optimally chooses to purchase less y and more x (given prices $(1, \zeta)$), for a higher utility.

indirect utility function is not globally concave, even weakly, introducing the possibility of risk-seeking behavior over large variations in income. In particular, there are circumstances in which the consumer would welcome lotteries that put positive probability on zero income.

3 Optimal Layoffs

This section examines optimal employment contracts in a single-period model (extended to multiple periods in Section 4). This allows us to identify the key features of optimal employment contracts with a minimum of clutter.

Our first result is that if consumers do not make consumption commitments (i.e., set $\hat{y} = 0$), then there are no layoffs and the wage equals the marginal product of labor in each state. The consumer is risk neutral in this case, removing any advantage from paying wages that are not equal to marginal products, while any contract with layoffs is dominated by a full-employment contract with suitably adjusted wages. Lemma 1 couples this result with obvious sufficient conditions for commitments to be suboptimal.

Lemma 1

(1.1) If the optimal employment contract does not induce commitments, then there are no layoffs and $\alpha f' = w_1 > w_2 = f'$.

(1.2) If either $\kappa > 1$, β *is sufficiently large, or* γ *is sufficiently large, then the optimal contract features no commitments.*

Proof.

(1.1) Suppose that the optimal employment contract does not induce commitments and features layoffs (i.e., $n_1 > 0$). If $w_1 = \alpha f'(n_1 + n_2)$, then a marginal reduction in n_1 has no effect on the firm's payoff while increasing consumer utility (by decreasing the layoff probability), introducing slack into the consumers' participation constraint that the firm can exploit to increase its payoff. If $w_1 < \alpha f'(n_1 + n_2)$, then the firm can decrease w_2 and increase w_1, while preserving expected payments to the consumer and expected profits (and hence expected utility, here exploiting the consumer's risk neutrality in the absence of commitments), until $w_1 = \alpha f'(n_1 + n_2)$; at which point n_1 can again be profitably reduced. If there are no layoffs and the expected wage falls short of the expected marginal product in either state, then the firm can profitably increase its employment. Section 6 contains details.

(1.2) If $\kappa > 1$, then the cost of buying good 2 via commitments exceeds the cost of making the same purchase on the spot market. Similarly, if

β is sufficiently large, the cost $h(\hat{y})$ of buying the quantity $\hat{y}$ via commitments exceeds the cost of purchasing that amount on the spot market for a sufficiently large interval $[0, \hat{y}^*]$ that the consumer makes no commitments. Fixing $\beta > 0$ and κ, if γ is sufficiently large, then the optimal consumption of good y is sufficiently small that the fixed cost β is prohibitive, again ensuring that no commitments are made. ■

The rigidities associated with consumption commitments are the only force in this model pushing consumers away from risk neutrality. Specifically, if we relax our maintained assumption and let $\zeta = \frac{1}{\psi} = \kappa < 1$, so that the level of y can be adjusted ex post without penalty, and if β is sufficiently small (but not necessarily zero), then consumers will make commitments (i.e., will choose $\hat{y} > 0$), but the optimal contract will again feature no layoffs and wages equal to marginal products:

Lemma 2 *If $\zeta = \frac{1}{\psi} = \kappa < 1$ and β is sufficiently small, then the optimal employment contract induces $\hat{y} > 0$, but features no layoffs and $\alpha f' = w_1 > w_2 = f'$.*

Proof. Let $\zeta = \frac{1}{\psi} = \kappa < 1$ and let β be small enough that $\beta + \kappa z_i^* < z_i^*$, $i = 1, 2$, where z_i^* is the optimal state-i consumption of good z when y is unavailable. This ensures that the optimal contract induces the consumer to set $\hat{y} > 0$. Because the consumer is risk neutral ex post, the argument proving Lemma 1.1 then ensures that there are no layoffs and $\alpha f' = w_1 > w_2 = f'$. ■

We now return to the ex post rigidities induced by consumption commitments when $\frac{1}{\psi} < \kappa < 1 < \zeta$. We consider two ways in which layoffs can be arranged. In the first, they are *random.* If proportion q of the consumers must be laid off in state 2, then every consumer faces a layoff probability of q, and the layoff draws are made after the state is realized. In the other, layoffs are *concentrated.* Proportion q of the consumers are again drawn to be laid off should state 2 occur, with each consumer equally likely to be drawn, but with the draws made before the ex ante market opens. Unlike random layoffs, concentrated layoffs allow workers to know prior to making consumption decisions whether they are at risk of being laid off. Concentrated layoffs in this setting reappear in our intertemporal model in the form of a seniority-based layoff scheme.

Random layoffs preclude consumption commitments, since the budget constraint would be violated if a consumer who had made a commitment is laid off. Concentrated layoffs allow consumers who are not at risk of a layoff to make commitments. This gives the latter an advantage:

Lemma 3 *If the optimal contract involves layoffs, then there exists a contract with concentrated layoffs that maximizes the firm's payoff. If some consumer strictly prefers to make consumption commitments (set $\hat{y} > 0$) under such a contract, no random-layoff contract maximizes the firm's payoff.*

Proof. Let layoffs be random. The consumer then necessarily sets $\hat{y} = 0$. Switching to concentrated layoffs while preserving other aspects of the employment contract (including the layoff probability) preserves the firm's payoff and does not eliminate any consumption possibilities available to the consumer, ensuring that the consumer (and hence the firm) cannot fare worse under concentrated layoffs. Concentrated layoffs must be strictly better for the consumer (and hence the firm) if, when not at risk of being laid off, the consumer strictly prefers commitments. ■

The first step toward examining the potential optimality of layoffs is to note that consumption commitments introduce risk aversion over small variations in ex post income, causing the firm to optimally smooth wages:

Lemma 4 *When β and κ are sufficiently small, consumers make commitments. The optimal employment contract then smooths wages, in the sense that $w_2 = f'(n_2)$ and $w_1 < \alpha f'(n_1 + n_2)$.*

Proof. We provide an outline of the argument, leaving the details to Section 6. It is immediate that commitments are optimal for sufficiently small β and κ. Suppose $w_2 = f'$ and $w_1 = \alpha f'$. If there are layoffs, then a marginal reduction in n_1 while preserving n_2 leaves the firm's payoff unaffected, while increasing consumer utility (by reducing the layoff probability), introducing slack in the participation constraint that the firm can exploit to increase its payoff. In the absence of layoffs, we have $w_1 > w_2$, and a marginal reduction in n_2 (holding $n_1 = 0$) again leaves the firm's payoff unchanged, while allowing wage smoothing. It is apparent from Figure 3, along with the optimality of consumption commitments (implying that realized incomes fall in the concave portion of the indirect utility function), that this wage smoothing increases consumer utility. ■

Our basic result builds on the ex post risk aversion introduced by consumption commitments to show that if commitments are sufficiently valuable and induce sufficient rigidity in the consumption of good y, optimal contracts will feature layoffs. The following is a special case of Proposition 2 (obtained by setting $\delta = 0$ in Section 4), and we defer proof to the consideration of Proposition 2.

Proposition 1 *For sufficiently small $\beta > 0$, there exist $\bar{\kappa}(\beta) > 0$, $\bar{\gamma}(\beta) > 0$, $\bar{\zeta}(\beta)$ and $\bar{\psi}(\beta)$ such that for all $\kappa < \bar{\kappa}(\beta)$, $\gamma < \bar{\gamma}(\beta)$, $\zeta > \bar{\zeta}(\beta)$ and $\psi > \bar{\psi}(\beta)$, the optimal contract features layoffs.*

Layoffs are potentially optimal because a worker who has made consumption commitments is ex post risk averse. The firm responds by smoothing the consumer's wages across states, but is constrained in doing so by the marginal product of labor in state 2. Layoffs relax this constraint by reducing state-2 employment and hence increasing the marginal product. The conditions of the proposition ensure that commitments are optimal (β and κ small) and that the rigidities introduced by consumption commitments are relatively severe (γ small and ζ and ψ large), and hence the concave portion of the indirect utility function in Figure 3 is quite concave, making wage smoothing particularly valuable.[5] Of course, this wage smoothing comes at the cost of possibly being laid off, a large negative income shock. As Figure 3 shows, however, the consumer may actually seek risk involving this outcome.

Remark. Layoffs are potentially optimal in our model because they reduce employment in the bad state, relaxing the marginal-product constraint on wages. Could the firm instead relax this constraint by retaining all of its workers, but having each work fewer hours? Laying off half the workers or halving the time each works may leave the firm with the same effective workforce,[6] but these have quite different effects on the workers. When the firm lays off half its workforce, the remaining workers can be paid as much as $f'(\frac{1}{2}N)$, while if it retains all the workers but cuts their hours by half, each worker can be paid at most half as much, or $\frac{1}{2}f'(\frac{1}{2}N)$, a disadvantage for a policy designed to boost state-2 worker incomes. More generally, retaining all workers but having each work $\lambda < 1$ times full employment would allow the firm to pay up to $\lambda f'(\lambda N)$ per worker. If the elasticity of the production function $-\frac{f''(N)N}{f'(N)}$ is below 1, as is the case if f is a power function, reducing

[5]It is immediate from Lemma 2 that layoffs will not be optimal if commitments are not sufficiently rigid.

[6]We ignore here the possibility that output might depend not only on the number of man-hours available, but also the number of workers.

hours would force a *reduction* in payments to workers, exacerbating rather than smoothing ex post payment variations and ensuring that hours reductions would never be part of an optimal contract, even when layoffs could be.[7] ■

Remark. Layoffs allow ex post incomes to be smoothed. A more effective response would be income fluctuation insurance, offered either by the firm or by a third party. Our results will not hold in the presence of such insurance. We suspect that moral hazard considerations preclude third-party income fluctuation insurance, and preclude the firm's fully insuring workers when laid off (though we have not modelled these factors). We have incorporated an explicit constraint on the firm's ability to insure against income fluctuations while employed, in the form of a prohibition on wages in excess of marginal products. ■

4 An Intertemporal Model

The model analyzed in the previous section illustrates how an optimal contract can include layoffs to smooth wages. Since it is a one-period model, it precludes an obvious method for smoothing wages, namely consumer borrowing and saving. In this section, we describe how the analysis can be extended to an intertemporal setting.

4.1 The Firm

The firm is infinitely-lived. Workers are potentially employed for two periods. The firm signs a contract with a young worker at the beginning of the worker's tenure with the firm, specifying the wage as a function of the state, in each period of employment.

We examine a steady state. In any period, the firm contracts with N workers, employing n_2 consumers in the bad state and $N = n_1 + n_2$ in the good state, where n_1 may be zero. We assume that in each period the firm has an equal number of young and old workers. At the beginning of each period the firm hires a set of young workers to replace the old workers of the previous period. Layoffs, if they occur, are concentrated, meaning that only young workers are at risk (unless more than half of the workers are laid

[7] If the elasticity exceeds 1, reducing hours will allow an increase in state-2 payments, though this increase will not be as large as that allowed by layoffs. Of course, reducing hours rather than laying off workers has the advantage of eliminating the possibility of zero wage.

off), who once again take draws for their layoff eligibility at the beginning of the period. We assume that the firm can condition a worker's wage on the state, but not the worker's age or the previous-period state. Relaxing this assumption complicates the details of the analysis but does not vitiate the result. The result of this steady state analysis is that the firm's profit maximization problem is very similar to the one it faces in the single-period model of the previous section.

4.2 Worker-Consumers

Each worker-consumer lives for two periods. Let $\hat{y}(j)$ be the consumer's commitment in period j and $h_j(\hat{y}(j))$ be the corresponding cost, where

$$h_1(\hat{y}(1)) = \begin{cases} \beta + \kappa\hat{y}(1) & \text{if } \hat{y}(1) > 0 \\ 0 & \text{if } \hat{y}(1) = 0 \end{cases}$$

$$h_2(\hat{y}(2)) = \begin{cases} \beta + \kappa\hat{y}(2) & \text{if } \hat{y}(2) > 0 \\ 0 & \text{if } \hat{y}(2) = 0. \end{cases}$$

The consumer thus faces no constraints on the ability to adjust the level of the commitment good between periods. Given enough time, people can adjust their consumption of housing services not by incremental changes to their current house, but by moving to a new one.[8] As in the one-period model, the presence of the fixed cost β in the second period captures the fact that once again the commitment is drawn from a technology in which there is a premium on a sufficiently large scale of services.

If the consumer chooses a given level $\hat{y}$ in each period, then $\beta + \kappa\hat{y}$ is paid in each period. One might view the cost of purchased housing as more heavily weighted toward the beginning. We can readily interpret the model as one in which commitment $\hat{y}$ is made in the initial period at cost $(1+\delta)(\beta+\kappa\hat{y})$. Nothing further need be paid if $\hat{y}$ is maintained in the second period, while otherwise the value of the remaining service flow $(\delta(\beta + \kappa\hat{y}))$ must be sold and a new commitment made at cost $h(\hat{y})$ (again, with a transaction cost easily accommodated). Since we have incorporated no capital market imperfections into our model, this is equivalent to the current formulation.

The consumer's utility maximization problem is now

$$\max_{x_i(j),\hat{y}(j),y_i(j),z_i(j),i,j\in\{1,2\}} p(\gamma x_1(1)^\rho + (1-\gamma)(y_1(1) + z_1(1))^\rho)^{\frac{1}{\rho}} + (1-p)(\gamma x_2(1)^\rho + (1-\gamma)(y_2(1) + z_2(1))^\rho)^{\frac{1}{\rho}}$$

[8] There may be transaction costs associated with such a move. Adding such costs to the model will only reinforce the rigidities induced by consumption commitments and hence our results.

$$+\delta[p(\gamma x_1(2)^\rho + (1-\gamma)(y_1(2)+z_1(2))^\rho)^{\frac{1}{\rho}} + (1-p)(\gamma x_2(2)^\rho + (1-\gamma)(y_2(2)+z_2(2))^\rho)^{\frac{1}{\rho}}]$$

where $x_i(j)$, for example, is the quantity of good x consumed in period j in state i, subject to

$$z_i(j)\hat{y}(j) = 0,\ \ i,j = 1,2,$$

and, for each combination of state $i(1)$ and $i(2)$ in periods 1 and 2,

$$\begin{aligned}
& x_{i(1)}(1) + z_{i(1)}(1) + h_1(\hat{y}(1)) + \zeta[y_{i(1)}(1) - \hat{y}(1)]_+ \\
& \quad + \delta\left(x_{i(2)}(2) + z_{i(2)}(2) + h_2(\hat{y}(2)) + \zeta[y_{i(2)}(2) - \hat{y}(2)]_+\right) \\
\leq \quad & w_{i(1)}(1) + \delta w_{i(2)}(2) - \frac{1}{\psi}[y_{i(1)}(1) - \hat{y}(1)]_- - \delta\frac{1}{\psi}[y_{i(2)}(2) - \hat{y}(2)]_-,
\end{aligned}$$

where $w_i(j)$ is the wage paid in period state i and period j.

As before there are no restrictions on the consumer's ability to tailor x to the period and state. At the beginning of the contract, before learning the first-period state, the consumer has an opportunity to satisfy his service requirement via commitment, that is to choose a positive level $\hat{y}(1)$ at cost $h_1(\hat{y}_1)$. The consumer can buy additional units of y, but must do so at price ζ. The consumer can reduce consumption of good y below $\hat{y}(1)$, but in the course of doing so can recover only the fraction $\frac{1}{\psi}(\hat{y}(1) - y(1))$ of the cost. This sequence is repeated in the second period, beginning with a new commitment $\hat{y}_2$ made at cost $h_2(\hat{y}_2)$.

4.3 Equilibrium

Consider first a consumer facing an employment contract with no layoffs. Commitments will be optimal if β and κ are sufficiently small. Given our steady-state assumption, we will have $w_2(1) = w_2(2) = f'(n_2)$. If the optimal contract features no variation at all in the consumer's income, so that $w_1(j) = w_2(j)$, then the consumer would set $\hat{y}(1) = \hat{y}(2)$ and make no transfers between periods. In general, it will be optimal for the firm to smooth the consumer's income by setting $w_1(j) < \alpha f'(n_1 + n_2)$, but not to smooth income perfectly. In this case, a consumer who encounters the bad state in the first period will transfer income from the second period to the first, and a consumer encountering the good state will save some income for the second period. However, the consumer necessarily faces some income risk in the second period, and hence optimally stops short of equalizing first-period expenditures in the good state and the bad state, incurring some risk in the first period in order to smooth the extreme values of the risky second-period income. Borrowing and saving mitigate the risk faced by the consumer, but do not eliminate it.

A consumer facing a layoff risk will again prefer that the risk be concentrated. Given the lack of any capital market imperfection, the consumer will be indifferent between having the risk concentrated in the first or second period of employment. We make the realistic assumption that only young workers are laid off, a convention that we suspect reflects unmodelled capital market imperfections. Notice that consumers at risk of being laid off are no longer automatically precluded from making commitments. A consumer facing a layoff risk in (only) the first period can borrow from second-period income to cover a first-period commitment should the bad state occur in the first period.

Once again, layoffs allow the firm to relax the marginal product constraint on the relatively low state-2 wage. Section 6 proves:

Proposition 2 *For sufficiently small $\beta > 0$, there exist $\bar{\kappa}(\beta) > 0$, $\bar{\gamma}(\beta) > 0$, $\bar{\zeta}(\beta)$ and $\bar{\psi}(\beta)$ such that for all $\kappa < \bar{\kappa}(\beta)$, $\gamma < \bar{\gamma}(\beta)$, $\zeta > \bar{\zeta}(\beta)$ and $\psi > \bar{\psi}(\beta)$, the optimal contract features layoffs.*

The basic idea of the proof is that when it is costly to quickly adjust the level of the commitment in the face of income shocks, there will be a disproportionate adjustment in the quantity of the other good, x. Disproportionate adjustments of x will not be on an expansion ray out of the origin, and hence, the consumer will exhibit risk aversion over these adjustments. Borrowing and saving will mitigate the size of the optimal adjustment, but not eliminate them. The choice between contracts without employment and contracts with layoffs is then essentially between bearing an equal, moderate amount of risk in each period and bearing more risk in the first period in order to decrease the risk in the second period. When commitments are important, the latter wins.

Remark. It is clear that expanding the model beyond two periods only requires additional notation. Our assumptions that there are only two states and only two consumption technologies also make for rather stark equilibria. In the contracts of interest, those unaffected by layoff risks make commitments. Those at risk either make no commitments (in the single-period model) or possibly (in the multiperiod model) make a commitment involving the same fixed cost, though not necessarily the same level of service, as those not at risk. A richer model would allow for shocks of varying sizes and a variety of commitment technologies featuring different trade-offs between fixed and marginal costs, along with many periods. Workers at various stages of their tenure with the firm would face different layoff risks and make commitments of different types and sizes. Optimal employment contracts in such models are shaped by the same forces as in our simpler analysis, but with considerably more complicated details. ■

5 Discussion

Endogenous risk aversion. Consumers who make consumption commitments in our model behave is if they are risk averse over small variations in income, despite their linearly homogeneous utility functions. More generally, the utility functions we can hope to observe are inferred from behavior that is the product of an interaction between preferences and the technology for converting income into consumption. Different technologies may lead us to different and potentially misleading inferences concerning risk aversion. For example, we may observe that consumers are risk neutral, concluding that insurance has no value, while the opening of an insurance market may give rise to both risk-averse behavior and active demand for insurance.[9]

Concentrated risks. Conditional on facing a risk of being laid off, the worker would prefer to concentrate this risk in as few states as possible. In essence, there are economies of scale in bearing risk, inducing workers to lump risks together rather than disperse them.

Habit formation. Our model generates behavior that is similar to that of many habit formation models.[10] Attanasio [2] discusses a typical habit formation model which in essence decreases an individual's effective current consumption by a constant times the individual's depreciated aggregate previous consumption, making the individual averse to downward adjustments in consumption. If the force of habit formation is strong enough, it could lead to optimal employment contracts that include layoffs in a manner similar to that shown in this paper.[11]

[9] Chetty [5] also makes this point.

[10] See Deaton [7] and Attanasio [2] for surveys of the habit formation literature and Chetty and Szeidl [6] for an examination of the connection between consumption commitments and habit formation.

[11] The models exhibit some differences. For example, our model would suggest nontrivial heterogeneity, linked to observable characteristics, across individuals in their aversion to downward adjustments in consumption—an individual who has made consumption

Morale. Bewley [3, 4] discusses the tendency of employers to insure wage but not layoff risk, in order to avoid detrimental morale effects that especially accompany wage reductions. This differential effect on morale is in turn traced to a convention that wage reductions (but not layoffs) are a violation of fairness or social norms. We agree that adverse morale effects may pose significant barriers to wage reductions. But why are wage reductions devastating for morale, reductions in overtime for hourly employees less so, and appropriately conducted layoffs less so? One possibility is that morale effects reinforce employment practices that are customary, with these practices having become customary because they have economic advantageous linked to their interaction with consumption commitments.[12]

Permanent separations. Our model is based on the idea that workers may be laid off when young, in response to temporary shocks, in return for more secure subsequent employment. However, the basic logic of our results carries over to the case in which firms face permanent shocks that preclude recalling laid-off workers. In this case, workers who were employed under a no-layoff contract would face a permanent wage cut should the firm suffer a shock. If firms offered contracts incorporating layoffs, workers would be permanently laid off following a shock, and would seek employment at another firm. Consider a multiperiod environment in which workers are long-lived. A worker who accepts a contract with layoffs realizes that if a shock occurs soon after he is hired, he will be fired and will have to seek another job. If no shock occurs for some period of time, however, he will have sufficient seniority that he will no longer be at risk. The trade-offs in our model apply here: the benefits of making commitments gives the worker an incentive to bunch his risks, making layoff contracts attractive.

6 Appendix: Details of Proofs

Lemma 1.1. Suppose that the consumer sets $\hat{y} = 0$. Then the consumer is risk neutral and the consumer's indirect utility function can be written as $pw_1 + (1-p)\frac{n_2}{n_1+n_2}w_2$. Attaching multiplier λ to the consumer's participation constraint and multiplier μ to the constraint that $n_1 \geq 0$ (one

commitments will be more averse to income shocks than an individual who has avoided commitments.

[12] Bewley [3, Chapter 13] explains that layoffs have the advantage focussing adverse effects on those who are no longer with the firm, but also that they have morale effects for the entire workforce that are small compared to those of wage reductions (Chapter 11).

easily verifies that workers will not optimally be laid off in the good state), while ignoring the constraint that wages not exceed marginal products, the first-order conditions for the firm's profit maximization problem are:

$$n_1 : p(\alpha f'(n_1+n_2) - w_1) - \lambda(1-p)\frac{n_2 w_2}{(n_1+n_2)^2} + \mu = 0 \tag{2}$$

$$n_2 : p(\alpha f'(n_1+n_2) - w_1) + (1-p)(f'(n_2) - w_2) + \lambda(1-p)\frac{n_1 w_2}{(n_1+n_2)^2} = 0 \tag{3}$$

$$w_1 : -p(n_1+n_2) + \lambda p = 0 \tag{4}$$

$$w_2 : -(1-p)n_2 + \lambda(1-p)\frac{n_2}{n_1+n_2} = 0. \tag{5}$$

Assume that $n_1 > 0$, so that there are layoffs, and hence $\mu = 0$. Coupling this with the equality $\lambda = n_1 + n_2$, which we can derive from either of (4) or (5), we can rewrite (2)–(3) as

$$\begin{aligned} p(\alpha f'(n_1+n_2) - w_1) - (1-p)\frac{n_2}{n_1+n_2}w_2 &= 0 \\ p(\alpha f'(n_1+n_2) - w_1) + (1-p)(f'(n_2) - w_2) + (1-p)\frac{n_1}{n_1+n_2}w_2 &= 0. \end{aligned}$$

Substituting the first of these into the second, we have

$$(1-p)\frac{n_2}{n_1+n_2}w_2 + (1-p)(f'(n_2) - w_2) + (1-p)\frac{n_1}{n_1+n_2}w_2 = 0,$$

or $f'(n_2) = 0$, a contradiction. Hence, there must be no layoffs. We can also conclude, from (3) and the constraint that wages not exceed marginal products, that $w_1 = \alpha f'(n_2)$ and $w_2 = f'(n_2)$ which in turn implies that $w_1 > w_2$. ∎

Lemma 4. We proceed quickly through some obvious cases. If both wages fall short of the corresponding marginal products, then the firm could increase profits by hiring more labor at the existing wage rate while preserving the existing probability of a layoff (and hence preserving worker utility). If w_1 equals its marginal product and w_2 falls short of its marginal product, then either (1) $w_2 < w_1$, in which case the firm can increase w_2 and decrease w_1, preserving expected wage payments while preserving or increasing worker utility and leading to a state at which both wages fall short of their marginal products (at which point the firm can increase profits by hiring more labor); or (2) $w_2 \geq w_1$, in which case there must be layoffs in the bad state and the firm can increase profits and consumer utility by hiring more labor in the bad state (and hence reducing the layoff probability). Hence, we must have $w_2 = f'$. If $w_2 > w_1$, then either smoothing wages (if $w_1 < \alpha f'$) or reducing n_1 (if $w_1 = \alpha f'$) again increases consumer utility while preserving the firm's payoff, allowing the firm to exploit the resulting slack in the participation constraint to increase profits. Thus, we must have:

$$\alpha f' \geq w_1 \geq w_2 = f'. \tag{6}$$

The consumer's participation constraint can now be written as

$$\frac{n_2}{n_1+n_2}\Big(p\tilde{U}_1(w_1,w_2)+(1-p)\tilde{U}_2(w_1,w_2)\Big)+\frac{n_1}{n_1+n_2}pw_1 \geq \bar{U},$$

where $\tilde{U}_i(w_1, w_2)$ is the indirect utility function identifying the consumer's utility when state i is realized, when not vulnerable for layoffs and given wages w_1 and w_2. Notice that in the presence of commitments, both wages are relevant for determining state-i utility and $\tilde{U}_i$ is in general not linear.

Attaching multiplier λ to the participation constraint, η to the constraint $f' - w_2 \geq 0$, and μ to the constraint $n_1 \geq 0$, the first-order conditions for the firm's profit maximization problem are:

$$\begin{aligned} n_1 &: p(\alpha f'(n_1+n_2)-w_1)-\lambda\frac{n_2}{(n_1+n_2)^2}(p\tilde{U}_1+(1-p)\tilde{U}_2) \\ &+\lambda p\frac{n_2}{(n_1+n_2)^2}w_1+\mu=0 \end{aligned} \tag{7}$$

$$\begin{aligned} n_2 &: p(\alpha f'(n_1+n_2)-w_1)+(1-p)(f'(n_2)-w_2) \\ &+\lambda\frac{n_1}{(n_1+n_2)^2}(p\tilde{U}_1+(1-p)\tilde{U}_2)-\lambda p\frac{n_1}{(n_1+n_2)^2}w_1+\eta f''(n_2)=0 \end{aligned} \tag{8}$$

$$\begin{aligned} w_1 &: -p(n_1+n_2)+\lambda\frac{n_2}{(n_1+n_2)}\left(p\frac{d\tilde{U}_1}{dw_1}+(1-p)\frac{d\tilde{U}_2}{dw_1}\right) \\ &+\lambda p\frac{n_1}{n_1+n_2}=0 \end{aligned} \tag{9}$$

$$w_2 : -(1-p)n_2+\lambda\frac{n_2}{(n_1+n_2)}\left(p\frac{d\tilde{U}_1}{dw_2}+(1-p)\frac{d\tilde{U}_2}{dw_2}\right)-\eta=0. \tag{10}$$

Now suppose first that there are no layoffs, so that $n_1 = 0$. Then (8) becomes:

$$p(\alpha f'(n_2)-w_1)+(1-p)(f'(n_2)-w_2)+\eta f''(n_2)=0. \tag{11}$$

Now suppose that both wages equal marginal products. Then (11) can be satisfied only if $\eta = 0$. Using $\eta = 0$, we can write (9)–(10) as

$$\begin{aligned} -pn_2 + \lambda\left(p\frac{d\tilde{U}_1}{dw_1} + (1-p)\frac{d\tilde{U}_2}{dw_1}\right) &= 0 \\ -(1-p)n_2 + \lambda\left(p\frac{d\tilde{U}_1}{dw_2} + (1-p)\frac{d\tilde{U}_2}{dw_2}\right) &= 0, \end{aligned}$$

giving

$$\frac{p}{1-p} = \frac{p\frac{d\tilde{U}_1}{dw_1} + (1-p)\frac{d\tilde{U}_2}{dw_1}}{p\frac{d\tilde{U}_1}{dw_2} + (1-p)\frac{d\tilde{U}_2}{dw_2}} = \frac{\frac{d\tilde{U}(w_1,w_2)}{dw_1}}{\frac{d\tilde{U}(w_1,w_2)}{dw_2}} = \frac{p\frac{du(x_1^*,y_1^*)}{dx_1}}{(1-p)\frac{du(x_2^*,y_2^*)}{dx_2}}, \tag{12}$$

where $\tilde{U}(w_1, w_2) = p\tilde{U}_1(w_1, w_2) + (1-p)\tilde{U}_2(w_1, w_2)$ gives expected utility conditional on not being at risk of being laid off, u is the consumer's direct utility function (1), (x_i^*, y_i^*) is the optimal consumption bundle in state i, and the final equality follows from an envelope argument. The outer two terms of this equality, along with $w_1 > w_2$ and hence $\frac{x_1^*}{y_1^*} > \frac{x_2^*}{y_2^*}$, yield a contradiction.

Now suppose that the optimal employment contract features layoffs in state 2. (It is straightforward to exclude the optimality of layoffs in state 1.) Suppose the first weak inequality in (6) is an equality. Since $n_1 > 0$, we have $\mu = 0$. From (7), we then have

$$-\lambda\frac{n_2}{(n_1+n_2)^2}(p\tilde{U}_1 + (1-p)\tilde{U}_2) + \lambda p\frac{n_2}{(n_1+n_2)^2}w_1 = 0.$$

As a result, we have

$$pw_1 = p\tilde{U}_1 + (1-p)\tilde{U}_2.$$

This is a contradiction. The maximum utility achieved when making no consumption commitments and faced with wages w_1 in state 1 and 0 in state 2 is pw_1. A consumer who has income $w_2 > 0$ in state 2 and makes no commitments must then receive a higher utility, and a consumer with income w_2 who optimally makes commitments must receive a utility at least as high as the latter, giving the contradiction. Hence, we must have $\alpha f' > w_1 \geq w_2 = f'$. ■

Proposition 2. We assume that the optimal contract features no layoffs and seek a contradiction. The optimal no-layoff contract must then feature

$N = n_2$, $n_1 = 0$, and $\alpha f'(n_2) > w_1 \geq w_2 = f'(n_2)$. The firm's profits are in general given by,

$$p[\alpha f(n_1 + n_2) - w_1(n_1 + n_2)] + (1 - p)[f(n_2) - w_2 n_2],$$

where the assumption that there are no layoffs currently gives $n_1 = 0$. Beginning with the optimal no-layoff contract, we consider an adjustment that decreases n_2, adjusting w_2 so as to preserve equality with the marginal product of labor in the bad state (i.e., $dw_2/dn_2 = f''(n_2)$), increasing w_1 similarly, and adjusting n_1 so as to preserve expected profits. It is a contradiction to show that this adjustment increases consumer utility.

We differentiate with respect to n_2, giving:

$$p\left(\alpha f'(n_1 + n_2) - w_1\right)\left[\frac{dn_1}{dn_2} + 1\right] + (1-p)[f'(n_2) - w_2] - p\frac{dw_1}{dn_2}(n_1 + n_2) - (1-p)\frac{dw_2}{dn_2}n_2 = 0.$$

Because $f'(n_2) = w_2$, $dw_1/dn_2 = dw_2/dn_2 = f''(n_2)$, and $n_1 = 0$ by assumption, we can rearrange to obtain

$$\frac{dn_1}{dn_2} = -\frac{p[\alpha f'(n_2) - w_1] - f''(n_2)n_2}{p[\alpha f'(n_2) - w_1]}. \tag{13}$$

Let $\tilde{U}(w_1, w_2)$ be the indirect utility function, giving expected utility as a function of the wages w_1 and w_2, conditional on not being at risk of a layoff. This indirect utility is of the form

$$\begin{aligned}\tilde{U}(w_1, w_2) &= p(\tilde{U}_{11}(w_1, w_2) + \delta(p\tilde{U}_{21}(w_1, w_2, 1) + (1 - p)\tilde{U}_{22}(w_1, w_2, 1))) \\ &\quad + (1 - p)(\tilde{U}_{12}(w_1, w_2) + \delta(p\tilde{U}_{21}(w_1, w_2, 2) + (1 - p)\tilde{U}_{22}(w_1, w_2, 2))),\end{aligned}$$

where $\tilde{U}_{11}(w_1, w_2)$ is the first-period utility given the good state (with $\tilde{U}_{12}(w_1, w_2)$ in the case of the bad state) and $\tilde{U}_{21}(w_1, w_2, 1)$ (for example) is the second-period utility, given that the good state is realized in the second period (the second subscript) and given that the good state was also realized in the first period (the argument 1 in the function). The latter is relevant because the first-period state determines how much the consumer borrows or saves, and hence second-period (state-contingent) income. Let $\tilde{V}$ similarly be the indirect utility function for a consumer at risk of layoff in the first period (only). This function takes a similar form, but differs from $\tilde{U}$ in recognition of the zero income that is now attached to state 2. The consumer's utility is given by:

$$\frac{n_2}{n_1 + n_2}\tilde{U}(w_1, w_2) + \frac{n_1}{n_1 + n_2}\tilde{V}(w_1, w_2).$$

Differentiating gives (using $dw_1/dn_2 = dw_2/dn_2$)):

$$\frac{dw_2}{dn_2}\left[\frac{\left(\frac{d\tilde{U}(w_1,w_2)}{dw_1}+\frac{d\tilde{U}(w_1,w_2)}{dw_2}\right)n_2+n_1\left(\frac{d\tilde{V}(w_1,w_2)}{dw_1}+\frac{d\tilde{V}(w_1,w_2)}{dw_2}\right)}{n_1+n_2}\right]$$
$$+\tilde{U}(w_1,w_2)\left(\frac{n_1+n_2-n_2\left(\frac{dn_1}{dn_2}+1\right)}{(n_1+n_2)^2}\right)+\tilde{V}(w_1,w_2)\left(\frac{\frac{dn_1}{dn_2}(n_1+n_2)-n_1\left(\frac{dn_1}{dn_2}+1\right)}{(n_1+n_2)^2}\right).$$

Using the facts that $w_2 = f'(n_2)$, $n_1 = 0$, and $dw_2/dn_2 = f''(n_2)$, we have a contradiction if

$$\left(\frac{d\tilde{U}}{dw_1}+\frac{d\tilde{U}}{dw_2}\right)\frac{f''(n_2)n_2}{f'(n_2)}+\left(\frac{\tilde{U}(w_1,w_2)}{f'(n_2)}\right)\left(-\frac{dn_1}{dn_2}\right)+\left(\frac{\tilde{V}(w_1,w_2)}{f'(n_2)}\right)\frac{dn_1}{dn_2}<0,$$

or

$$-\left(\frac{d\tilde{U}}{dw_1}+\frac{d\tilde{U}}{dw_2}\right)\theta<\frac{dn_1}{dn_2}\left(\frac{\tilde{U}(w_1,w_2)}{f'(n_2)}-\frac{\tilde{V}(w_1,w_2)}{f'(n_2)}\right).$$

Using (13), this is

$$p[\alpha f'(n_2)-w_1]\left(\frac{d\tilde{U}}{dw_1}+\frac{d\tilde{U}}{dw_2}\right)\theta \;>\; \left(\frac{\tilde{U}(w_1,w_2)}{f'(n_2)}-\frac{\tilde{V}(w_1,w_2)}{f'(n_2)}\right)(p[\alpha f'(n_2)-w_1]-f''(n_2)n_2),$$

or, using the fact that the consumer's expected utility $\tilde{U}(w_1, w_2)$ must equal the reservation wage (given our working hypothesis of no layoffs),

$$p[\alpha-\frac{w_1}{f'(n_2)}]\left(\frac{d\tilde{U}}{dw_1}+\frac{d\tilde{U}}{dw_2}\right)\theta>\left(\frac{\overline{U}}{f'(n_2)}-\frac{\tilde{V}(w_1,w_2)}{f'(n_2)}\right)(p(\alpha-\frac{w_1}{f'(n_2)})+\theta).$$

Now fix β and κ sufficiently small that the consumer makes commitments, and hold β fixed while letting κ decrease. As κ and γ get small, $\overline{U}/f'(n_2)$ is bounded (because the firm optimally sets $f'(n_2) \geq \beta$, to ensure the feasibility of consumption commitments) while $\frac{d\tilde{U}}{dw_1}+\frac{d\tilde{U}}{dw_2}$ approaches infinity (because small κ allows increases in w_2 to yield ever larger increases in $\hat{y}$, the marginal utility of which remain large as γ gets small). The inequality (and contradiction) thus holds if $\alpha - w_1/f'(n_2)$ is positive and bounded away from zero. It is positive by (the counterpart for the two-period model of) Lemma 4. We then note that $w_1/f'(n_2)$ approaches one, for fixed β and κ, as γ gets small and ζ and ψ get large, since in the limit increments in the state-1 wage are worthless. We then need only set γ sufficiently small and ζ and ψ sufficiently large. ■

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