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"Do Vertical Mergers Facilitate Upstream Collusion?"

by

Volker Nocke and Lucy White

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Do Vertical Mergers Facilitate Upstream Collusion?[*]

Volker Nocke[†]
University of Pennsylvania

Lucy White[‡]
Harvard Business School

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Abstract

We investigate the impact of vertical mergers on upstream firms' ability to sustain tacit collusion in a repeated game. We identify several effects and show that the net effect of vertical integration is to facilitate collusion. Most importantly, vertical mergers facilitate collusion through the operation of an *outlets* effect: cheating unintegrated firms can no longer profitably sell to the downstream affiliates of their integrated rivals. However, vertical integration also gives rise to an opposing *punishment effect*: it is typically more difficult to punish an integrated structure, so that integrated firms are able to make more profits in the punishment phase than unintegrated upstream firms. When downstream firms can condition their prices or quantities on upstream firms' contract offers, two additional effects arise, both of which further facilitate upstream collusion. First, an unintegrated upstream firm's deviation profits are reduced by the *reaction effect* which arises since the downstream unit of the integrated firm will now react aggressively to upstream deviations. Second, an integrated firm's deviation profit is reduced by the *lack-of-commitment effect* as it cannot commit to its own downstream price when deviating upstream.

Keywords: vertical merger, collusion, vertical restraint, vertical integration, repeated game, penal code

JEL: L13, L42, D43

1 Introduction

Many famous cases of collusion documented in the literature have involved intermediate goods industries. Further, a significant fraction of those cases involved industries where one or more firms were vertically integrated.[1] Yet existing theories of collusion deal only with collusion between firms selling to consumers (or atomistic buyers). In this paper, we provide the first examination of the often more relevant case where colluding firms sell to downstream firms which are strategic buyers with interdependent demands. Our particular focus is on the effect of vertical integration on the possibility of collusion in such markets. Why is vertical integration such a common feature of collusive industries? Does vertical integration facilitate upstream collusion, and if so, when should it be a concern for anti-trust regulators?

Following the Chicago School revolution of anti-trust policy in the early 1980s, vertical restraints were considered to be efficiency-enhancing. In the last decade, however, regulators and anti-trust authorities have shown an increased interest in prosecuting cases with vertical aspects (see, e.g., Kwoka and White (1999, part 3), Riordan and Salop (1995), Klass and Salinger (1995)). At the same time, academics have been giving increased attention to the potential anti-competitive effects of vertical restraints and the nascent literature in this area has expanded considerably in recent years.[2] But this literature has – until now – taken a strictly static view of the interaction between firms. In contrast, we investigate the impact of vertical mergers in a *dynamic* game of repeated interaction between upstream and downstream firms.

In each period, M upstream firms produce a homogeneous intermediate good and make public two-part tariff offers to supply this good to N downstream firms. Downstream firms purchase the intermediate good and transform it into a homogeneous or differentiated final good, competing in either prices or quantities to supply consumers. This interaction is repeated over an infinite horizon. In the absence of collusion, vertical mergers do not affect the equilibrium allocation. This provides us with a particularly clean setting in which to evaluate the potential collusive effects of vertical mergers.

We focus our analysis on collusion between upstream firms. There are several reasons for such a focus. First, as mentioned above, collusion seems to be more prevalent in intermediate goods industries, perhaps because these industries are often more concentrated or sell goods which are more homogeneous. Second, policy towards vertical mergers also seems to reflect this fact. The *US Non-Horizontal Merger Guidelines* anticipate the idea that vertical merger may facilitate collusion but also focus exclusively on upstream collusion. Thirdly, from a theoretical point of view the analysis of collusion between downstream firms selling to consumers corresponds to the standard case of collusion which is by now well understood – although the acquisition of a supplier by one of these firms in a dynamic setting has yet to be considered. One would of course like to know how vertical integration might facilitate collusion between firms at each level of the vertical hierarchy; this is an open research question, but one can view our analysis as a first step along this road. We investigate the more limited question of how

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[1]See for example Tosdal's (1917) description of vertical mergers in the early twentieth century German Steel cartels, as well as Levenstein's (1997) description of the bromine cartel. Other examples of collusion involving some vertically integrated firms include railways (see, e.g., Porter (1983)) and timber-cutting (Baldwin, Marshall and Richard, (1997)). See also Hendricks, Porter and Tan (2000) on joint bidding for oil and gas tracts.

[2]See for example, the recent contributions by Chen (2001), Choi and Yi (2001), Riordan (1998), and Chen and Riordan (2003); see Rey and Tirole (2003) for a survey.

vertical integration can facilitate the maintenance of monopoly profit levels for the industry as a whole, where all of these profits are extracted by the upstream firms.

This focus is less restrictive than it might at first appear for the following reasons. First, we show that because of the Bertrand-like structure of competition in the upstream market, the ability to collude has an all-or-nothing feature when no firm is integrated: whenever it is possible to sustain an equilibrium with positive profits for upstream firms, it is possible to sustain an equilibrium in which all of the monopoly rents are extracted by upstream firms and shared between them. Thus we concentrate our analysis on the maintenance of such monopoly equilibria, although we briefly examine other equilibria and argue that in these too, vertical integration reduces total surplus. For the main part of the paper, we will follow the tradition of Friedman (1971) and consider whether such equilibria are supportable by the play of trigger strategies: infinite reversion to the repeated play of the static equilibrium of the stage game, following a deviation by one of the firms. Characterization of optimal punishment schemes is not straightforward in this setting because the stage game is not a normal-form game (acceptance decisions are made after contract offers) and so the results of Abreu (1986, 1988) do not apply. Nevertheless, we are able to determine the optimal punishment scheme when final goods are homogeneous. In each case, we will say that a vertical merger facilitates collusion if it reduces the critical discount factor above which the monopoly outcome is sustainable.

In our baseline model, we identify two important and counteracting effects of vertical mergers on upstream firms' ability to collude: the *outlets effect* and the *punishment effect*. In a variation of the baseline model we show that two other effects – the *reaction effect* and the *lack-of-commitment effect* – can also arise, depending on the timing of upstream and downstream moves.

Perhaps the most intuitive and important effect of vertical merger is the *outlets effect*. To understand this effect, consider first that the optimal way for an upstream firm to deviate is typically to undercut the fixed fees and wholesale prices of its rivals only marginally. This allows the deviant firm to steal all of its rivals' business whilst downstream output remains close to monopoly levels and hence the deviant firm's profits are close to monopoly profit. Note that such a strategy is no longer feasible when one or more downstream firms is integrated. Integrated downstream firms will always prefer to buy from their upstream affiliate at marginal cost than to buy from a deviant firm at any price which gives the latter positive profits (essentially, they would rather these profits went to their upstream affiliate than to another firm). Thus integrated downstream firms can be relied upon to reject any offer that would be profitable for a deviating upstream firm, which can help to enforce the collusive agreement. A deviating upstream firm cannot hope to attain the full monopoly profit from deviating if one or more downstream firms are integrated with its rivals. We call this the outlets effect of vertical integration since vertical integration by an upstream firm reduces the number of outlets through which its rivals can sell when deviating, generally reducing their profit from cheating and thus facilitating collusion.

Counteracting the outlets effect is the *punishment effect*. The punishment effect is also quite intuitive and arises in our set-up because downstream firms may earn rents in the non-cooperative equilibrium of the model. If an upstream firm integrates with a downstream firm, these rents now become part of the profit of the merged entity. Thus the merged entity can expect to make more profits in the non-cooperative punishment phase than the upstream firm could make alone. Conversely, absent any changes in market share, the merged entity will make the same profit as would the upstream firm alone when monopoly profits are sustained

upstream. So, for a given collusive market share, the merged entity suffers relatively less from a switch from collusive to punishment phases, and is correspondingly more tempted to cheat on any collusive agreement. We call this the punishment effect of vertical integration since it arises because the non-cooperative equilibrium is a less effective punishment of a vertically-merged firm than of a stand-alone upstream firm.

In order to counteract the punishment effect on an integrated firm's incentive to cheat, that firm's market share must be increased, leading to an asymmetric distribution of output in an otherwise symmetric industry. Given such a redistribution of output, we are able to show that when the first upstream-downstream pair in an industry integrates, the outlets effect always dominates the punishment effect, so vertical merger facilitates collusion. To gain some intuition for this result, consider that the outlets effect deprives all of the remaining $M - 1$ upstream firms of (at least) $\frac{1}{N}th$ of the monopoly profit because when deviating they cannot sell through the integrated downstream firm (which represents $\frac{1}{N}th$ of sales). The punishment effect, on the other hand, affects only the integrated upstream firm and increases its punishment profits by the amount of the non-cooperative profit in all future periods. Clearly, the integrated downstream firm's single-period non-cooperative profit is less than $\frac{1}{N}th$ of the monopoly profit, and so the key step is to show that at the critical discount factor in the absence of integration, the infinite sum of non-cooperative profits does not amount to more than $\frac{M-1}{M}$ times the monopoly profit. This follows by the argument given above that, in the absence of integration, upstream firms can obtain the whole monopoly profit when deviating, which determines that the critical discount factor in this case is $\frac{M-1}{M}$.

We examine several extensions to this basic set-up to show that our result is robust. First, we consider the optimal punishment scheme when final goods are homogeneous. Second, we consider what happens when upstream offers to downstream firms are private information to the contracting parties rather than being publicly observed.

Finally, we modify the timing of our original game to allow us to illustrate two further effects which can occur with vertical integration. Our baseline model has upstream firms making contract offers simultaneously with downstream firms setting prices (or choosing quantities). However, under some circumstances it may be more natural to think of downstream firms setting their strategic variable after they know what input costs they will face. This new timing introduces some further considerations to the dynamic game, however, because now downstream prices (or quantities) can potentially react to upstream deviations within the same period that they are made. The *reaction effect* arises from the fact that the integrated downstream firm can now react aggressively (reducing its price or increasing its quantity) *during the period of deviation,* reducing the profits of the deviator. Thus the reaction effect further facilitates collusion.

Whilst the flexibility of the integrated firm's downstream price is helpful in punishing its rivals' deviations, it becomes a liability for the integrated firm when it wants to deviate itself. The integrated firm can always do weakly better by posting its own downstream price simultaneously with its deviant upstream offers (as it does in our benchmark model). When downstream prices are set after upstream offers are made, downstream firms rationally anticipate that the deviating integrated firm will set its downstream price equal to the best response to their own anticipated prices. This means that the integrated firm's price will generally not be optimal from the point of view of maximizing the overall industry profit during the period of deviation (which the integrated firm could extract using fixed fees). Thus, when downstream

firms can condition their retail prices on upstream firms' contract offers, the integrated firm will suffer from a *lack-of-commitment effect*: its inability to commit to its own downstream price when making deviant contract offers reduces the integrated firm's deviation profit. The lack-of-commitment effect therefore makes upstream collusion easier to sustain.

Our analysis of the optimal punishment scheme employed by the upstream firms in our game is of independent interest to the particular application studied. The reason is that the literature on collusion has focused (almost) exclusively on repeated *normal*-form games, and very little is known about optimal punishment in repeated *extensive*-form games such as ours. In repeated normal-form games, it is well known that any subgame-perfect outcome can be supported by a *simple penal code* (Abreu (1988)), i.e., a profile with the property that any deviation by a player from the equilibrium path is punished by the same punishment path (penal code), so that the continuation play after a player's deviation is independent of the particular deviation chosen, and depends only on the identity of the deviator. Our game provides what is to our knowledge the first concrete example showing that the logic of simple penal codes breaks down in repeated extensive-form games. In our model, sustaining monopoly rents for upstream firms may require (for some discount factors) that the continuation play following an upstream firm's deviation depends not only on the identity of the deviator, but also on the details of the deviant contract offers, so "the punishment must fit the crime". Intuitively, in an extensive form game, inflicting the worst possible punishment on a cheating upstream firm not only involves punishing the deviator in all future periods but also, in so far as is possible, *within the same period*. A deviant (unintegrated) upstream firm can profitably deviate only if at least one downstream firm accepts the deviant offer. Hence, since downstream firms make acceptance/rejection decisions *after* observing upstream firms' contract offers, the optimal punishment scheme should provide incentives for downstream firms to reject deviant contracts by prescribing the play of an equilibrium in the continuation game that is favorable to those downstream firms that do indeed reject the deviant offers. The optimal punishment scheme differs from a simple penal code in two ways. First, depending on the details of the deviant offers, different downstream firms will be induced to reject these offers, and so different rewards should be offered in continuation play. Secondly, even given the actions of the downstream firms at the acceptance stage, the optimal distribution of rewards between those firms that reject the offers should optimally depend on the details of the offers that those firms received. In particular, downstream firms that received more profitable offers, and that were therefore more tempted to accept, should be offered larger rewards in continuation play. Again, this implies that the continuation play must depend not only on the identity of the deviator, but also on the details of the deviant contracts.

Plan of the Paper. The paper is organized as follows. In the next section, we describe the baseline model with public offers. In section 3, we investigate the collusive effects of a vertical merger in this model. The key result is that vertical merger facilitates upstream collusion relative to the unintegrated benchmark. We then investigate the robustness of this result to various extensions. In section 4.1 we set out the optimal punishment scheme for the case when downstream firms' goods are homogeneous, and show that the logic of simple penal codes breaks down in our repeated extensive-form game. Section 4.2 shows that our result still holds when upstream firms' contract offers are private information to the contracting parties. In section 4.3, we analyze a variation of the base model, where downstream firms can condition their retail prices or quantities on firms' contract offers, highlighting the additional effects which arise to

further facilitate collusion in this setting. Finally, we discuss our results and conclude in section 5. All proofs are deferred to the appendix.

2 The Baseline Model

We consider a vertically related industry with $M \geq 2$ identical upstream firms, $U1, U2, ..., UM$, and $N \geq 2$ symmetric downstream firms (or retailers), $D1, D2, ..., DN$. The upstream firms produce a homogeneous intermediate good at constant marginal cost c, which for simplicity we set equal to 0, and sell this good to the downstream firms. The downstream firms transform the intermediate good into a final good on a one-to-one basis at zero marginal costs of production, and sell it to consumers. Consumers view the final good as either homogeneous or else symmetrically differentiated (by downstream firm).

The M upstream firms make simultaneous and public take-it-or-leave-it two-part tariff offers to the downstream firms. Ui's offer to Dj takes the form $\omega_{ij} \equiv (w_{ij}, F_{ij})$, where w_{ij} is the marginal wholesale price and F_{ij} is the fixed fee. The fixed fee F_{ij} has to be paid when the offer is accepted, while the wholesale price w_{ij} has to be paid for each unit that is ordered and then sold in the retail market to consumers. If Ui does not make an offer to Dj, then $\omega_{ij} = \varnothing$. In the retail market, the N downstream firms compete either in prices or quantities. That is, Dj sets a retail price p_j (under price competition) or quantity q_j (under quantity competition).

Time is discrete and indexed by t. Each period, an identical set of consumers come to the downstream market to buy the final good. Demand for downstream firm Dj's final good is given by $Q(p_j; p_{-j})$, where p_j is the price of Dj's final good, and p_{-j} the vector of prices charged by Dj's downstream rivals. Symmetry between downstream firms means that the demand function $Q(\cdot; \cdot)$ is the same for all N downstream firms, and that $Q(p_j; p_{-j}) = Q(p_j; p'_{-j})$ whenever p'_{-j} results from p_{-j} by just changing the identities of the downstream firms that charge the various prices. Downstream firm Dj's inverse demand is denoted $P(q_j; q_{-j})$, where q_j is Dj's output, and q_{-j} a vector of outputs by the $N-1$ downstream rivals. Throughout the paper, we will denote by Q^M, p^M, and Π^M the joint-profit maximizing industry output, retail price, and industry profit, respectively.

We impose standard assumptions on the demand function. Holding fixed downstream rivals' prices, Dj's demand is positive if it charges a sufficiently low price, and weakly decreasing in its own price (and strictly decreasing if Dj's demand is positive). Holding fixed its own price, Dj's demand is weakly increasing in downstream rival Dk's price (and strictly increasing if both Dj and Dk face positive demand). Further, we assume that demand is such that in the associated one-shot simultaneous-move game in which N (downstream) firms compete in prices (respectively, quantities) with downstream firm Dj facing a constant marginal cost (wholesale price) w_j, there is a unique Nash equilibrium outcome.[3] See, for instance, Vives (1999) for conditions on demand that ensure uniqueness.

To fix ideas, consider the following linear demand system satisfying these assumptions, which is frequently used in oligopoly models. We will later illustrate some of our results using this example.

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[3]In the associated one-shot game, a firm with high marginal cost w_j may make zero profit in equilibrium. In this case, the firm's price/quantity choice will not be uniquely pinned down in equilibrium.

Example 1 (Linear Demand) *There is a unit mass of identical consumers with utility function*

$$U(\mathbf{x}; H) = \sum_{j=1}^{N} \left(x_j - x_j^2 \right) - 2\sigma \sum_{j=1}^{N} \sum_{k=1}^{j-1} x_j x_k + H, \tag{1}$$

where x_j is consumption of downstream firm's Dj's product, and H is consumption of the Hicksian composite commodity.[4] *The parameter $\sigma \in (0, 1]$, measures the degree of substitutability between products: products become perfect substitutes as $\sigma \to 1$, and independent as $\sigma \to 0$.*

The timing in each period is as follows:

1. *Pricing stage:* Upstream firms $U1, ..., UM$ simultaneously make public offers to the downstream firms. At the same time, downstream firms $D1, ..., DN$ simultaneously commit to prices (or quantities) in the retail market.

2. *Acceptance stage:* Downstream firms $D1, ..., DN$ simultaneously decide which contract(s) to accept.[5] If they decide to accept a contract, the relevant fixed fee is paid to the upstream firm.

3. *Output stage:* Consumers decide which final goods to purchase. Downstream firms then order the quantities demanded by consumers from the upstream firms at the relevant wholesale prices.

The game is one of perfect monitoring: all past actions become common knowledge at the end of each stage.

Observe that downstream firms set prices (or quantities) before deciding which contract(s) to accept, if any. Hence, we need to specify payoffs when downstream firm Dj sets a price p_j at which there is positive demand in equilibrium, $Q(p_j; p_{-j}) > 0$ (or, under quantity competition, when D_j sets a positive quantity $q_j > 0$), but Dj later decides to reject all of its offers, and so is unable to satisfy consumer demand. While this does not play any role for our analysis, we may assume that, in this case, the rationed consumers can adjust their demand for the rival final goods accordingly, and so demand for Dj's rivals is as if $p_j = \infty$ (or $q_j = 0$). To exclude equilibria of the stage game in which downstream firm Dj rations consumers on the equilibrium path we assume that Dj would have to pay a vanishingly small fine (which may be interpreted as a "reputation loss") $\phi > 0$ if it were to ration consumers, and consider the limit as $\phi \to 0$.[6]

Upstream and downstream firms have an infinite horizon. Each firm aims to maximize the discounted sum of its future profits, using the common discount factor $\delta \in (0, 1)$. Vertically integrated firms are assumed to maximize their joint profits, independently of any announced "transfer prices" between the upstream and downstream affiliates. As pointed out by Bonanno and Vickers (1988), this implies that the vertically integrated downstream firm's true wholesale

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[4] The linear-quadratic utility function (and the associated linear demand system) goes back to Bowley (1924).

[5] Downstream firms are allowed to accept more than one offer, i.e., contracts are non-exclusive.

[6] The need for this assumption arises because of the simultaneous timing of upstream offers and downstream price (or quantity) setting: we need to specify what happens if a downstream firm incorrectly anticipates upstream offers and sets its strategic variable in a way which would cause it to make losses if it were to accept the upstream contracts it is offered. With the alternative sequential timing which we consider in section 4.3, this problem does not arise as downstream firms set their strategic variable knowing the contracts which they have been offered.

price is the marginal cost of its upstream affiliate, $c = 0$. To focus on the potential collusive effects of vertical integration, we assume that a vertical merger does not affect costs or technology.

Since we are interested in tacit collusion between upstream firms, we will focus mostly on collusive equilibria that allow upstream firms to jointly extract all of the monopoly rents (in section 3.5 we will justify this focus and also discuss equilibria on upstream firms' Pareto frontier more generally). For simplicity we assume that upstream firms sustain collusion through infinite "Nash reversion" (see the classic analysis of Friedman (1971) and the papers which followed it): *any* deviation by an upstream firm is followed, in all subsequent periods, by the play of the "noncollusive equilibrium" (which is a subgame perfect equilibrium of the stage game).[7] In contrast, deviations by unintegrated downstream firms do not trigger any punishment.

We define the *critical discount factor* $\widehat{\delta}$ as the threshold value of δ such that there exists an equilibrium in which all of the monopoly rents are extracted by upstream firms if and only if the discount factor δ satisfies $\delta \geq \widehat{\delta}$. (Note that the way in which the monopoly profits are divided between upstream firms is part of the equilibrium description. For $\delta = \widehat{\delta}$, there will exist a unique market sharing arrangement that allows upstream firms to extract the monopoly profit.) We will say that a vertical merger *facilitates* upstream collusion if it *reduces* the critical discount factor $\widehat{\delta}$. (This notion of facilitating collusion is common in the industrial organization literature, see, e.g., Tirole (1988).) However, as we will discuss, the main prediction of our paper – that a vertical merger facilitates upstream collusion – is robust to a broader notion of facilitating collusion. Whenever some collusive upstream profits can be sustained under non-integration, the same level of upstream profits can be sustained under (single) vertical integration. In contrast, for some discount factors, collusive upstream profits are sustainable under vertical integration, but not under non-integration. Furthermore, those collusive equilibria that lie on upstream firms' Pareto frontier tend to be worse from society's point of view than those under non-integration.

Given any market structure, minimizing the critical discount factor above which upstream collusion is sustainable entails all of the upstream firms having the same net incentive to deviate. However, integer constraints will generally complicate this equalization of deviation incentives in a pure-strategy collusive equilibrium if downstream firms produce differentiated goods. (Upstream offers will generally involve positive fixed fees, and so each unintegrated downstream firm will typically accept at most one offer.) Since such integer constraints complicate the analysis while they are largely orthogonal to the issues of interest, we allow for public randomization so that firms can share the market in a continuous fashion. In particular, we will assume that there is a public random variable, θ, realized just after the pricing stage but before the acceptance stage. The random variable is independent of play, being uniformly distributed on $[0, 1]$; it serves only as a public correlating device.[8] Along the collusive equilibrium path, downstream firms accept different upstream firms' offers (between which they are indifferent), depending on the realization of θ. Since all upstream and downstream firms make their pricing decisions

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[7] The known results on *optimal* punishment in repeated normal-form games (e.g., Abreu 1988) generally no longer hold in repeated *extensive*-form games. As we will show in section 4.1, *simple penal codes* are no longer optimal in our repeated extensive-form game. While the analysis of optimal punishment schemes turns out to be a complex undertaking, we are able to derive the optimal punishment scheme for the case of homogeneous final goods; see section 4.1.

[8] We can dispense with the assumption of a public correlating device by using jointly controlled lotteries, introduced by Aumann, Maschler and Stearns (1968).

before θ is realized, the realization of θ does not affect firms' incentives to deviate.[9]

3 Equilibrium Analysis

In this section, we analyze the effect of vertical integration on upstream firms' ability to collude. The analysis proceeds in stages. We begin by examining the properties of the subgame-perfect equilibria of the stage game, since we assume that firms will revert to the infinite play of such equilibria after a deviation. We then analyze firms' abilities to collude under two market structures: *non-integration (NI)*, where no upstream firm is vertically integrated, and *single integration (SI)*, where a single upstream firm is vertically integrated with a single downstream firm. We then use the results from these analyses to show that vertical integration facilitates upstream collusion: the critical discount factor above which monopoly rents for the upstream firms can be sustained is lower under single integration than under non-integration.

3.1 Noncollusive Equilibrium

While the stage game has multiple equilibria, they all share the feature that upstream firms make zero profits on their contracts with unintegrated downstream firms, independently of the number of vertical mergers. This result obtains since upstream firms produce a homogeneous intermediate good and compete in (nonlinear) prices. The intuition behind the proof is thus similar to that in the classic Bertrand model: essentially, upstream firms can always slightly undercut any contract which allows their rivals make a positive profit.

Lemma 1 *Independently of market structure, each upstream firm makes zero profit on its contracts with unintegrated downstream firms in any (pure-strategy) equilibrium of the stage game.*

Define the *symmetric noncollusive equilibrium* as the play of the following set of strategies. Each upstream firm makes offers of the form $(w_{ij}, F_{ij}) = (0, 0)$ to each (unintegrated) downstream firm, and each downstream firm chooses the corresponding noncollusive price p^{NC} under price competition (or quantity q^{NC} under quantity competition), where $p^{NC} = \arg\max_p pQ\left(p; p^{NC}, ..., p^{NC}\right)$ and $q^{NC} = \arg\max_q qP(q; q^{NC}, ..., q^{NC})$. At the acceptance stage, each unintegrated downstream firm accepts at least one contract along the equilibrium path.[10]

Lemma 2 *Independently of market structure, the symmetric noncollusive equilibrium is a pure-strategy subgame-perfect equilibrium of the stage game.*

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[9]Instead of using a public correlating device, upstream firms may share collusive profits by making side payments at the end of each period. (Side payments are used only to resolve integer problems, and so no firm will need to make side payments larger than Π^M/N.) Failure to make side payments is assumed to trigger the (infinite) punishment phase; following a deviation at the pricing stage, however, side payments need not be made. It can be shown that if no firm has an incentive to deviate at the pricing stage, then no firm has an incentive to deviate by not making the required side payments.

[10]Following a deviation at the pricing or acceptance stage, we require the play of a subgame-perfect equilibrium in the ensuing subgame.

In the symmetric noncollusive equilibrium, each upstream firm makes zero profit. In contrast, downstream firms make positive profits, unless final goods are homogeneous and downstream competition is in prices. We denote by π^{NC} the profit that each downstream firm will make in the symmetric noncollusive equilibrium. Notice that the allocation in the symmetric noncollusive equilibrium is independent of the number of vertical mergers. This feature of our set-up is useful since it allows us to focus attention on the *collusive* effects of vertical integration.

While there may be a multiplicity of pure-strategy equilibria of the stage game, we believe that the symmetric noncollusive equilibrium is the most appealing one.[11] However, we do not require that firms coordinate on this equilibrium in the punishment phase. Since all noncollusive equilibria yield zero payoffs to the upstream firms, we allow firms to coordinate on *any* noncollusive equilibrium following the deviation of an *unintegrated* upstream firm. Following a deviation of an *integrated* firm, we assume that firms coordinate on the noncollusive equilibrium that yields the smallest profit to the deviant integrated firm. This is the symmetric noncollusive equilibrium.[12]

3.2   Collusive Equilibrium: Non-Integration

We now consider the collusive equilibrium when no firm is vertically integrated. As pointed out in section 2, we focus on the collusive equilibrium where the upstream firms jointly extract all of the monopoly rents Π^M.

For the monopoly rents to be extracted, each downstream firm Dj must, at the pricing stage, set the monopoly price $p_j = p^M$ (under price competition) or the monopoly quantity $q_j = Q^M/N = q^M$ (under quantity competition).[13] At the same time, each of the M upstream firms makes the same offer (w^M, F^M) to each of the N downstream firms. The collusive wholesale price $w^M \leq p^M$ is chosen such that it is a best response for each downstream firm Dj to charge the price p^M (or the quantity q^M), given the equilibrium behavior of its $N-1$ downstream rivals. The fixed fee $F^M \geq 0$ is chosen so as to extract all of the rents from a downstream firm. If final goods are homogeneous and downstream competition is in prices, $(w^M, F^M) = (p^M, 0)$;

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[11]We remark on some asymmetric equilibria below (section 4.1). The symmetric equilibrium is particularly appealing in the obviously symmetric case when no firm is integrated. The symmetric equilibrium is also immune to deviations by upstream-downstream pairs.

[12]In any Nash equilibrium at the output stage, a downstream firm's profit is weakly increasing in its downstream rivals' marginal costs, and in the symmetric noncollusive equilibrium, all downstream firms receive the intermediate good at a wholesale price of zero. Hence, if the integrated firm deviates, it makes a per-period profit of π^{NC} in the punishment phase (through its downstream affiliate).

[13]The following analysis assumes that each downstream firm produces the same quantity $q^M = Q^M/N$ in the collusive equilibrium. This is *necessary* to extract monopoly profits if final goods are symmetrically differentiated, but not if final goods are homogeneous. In the homogeneous case, *any* vector of downstream outputs that adds up to Q^M can be induced by appropriate contracts and be used to implement the monopoly outcome. The incentives to deviate set out below are unaffected by the way in which the production of Q^M is "shared" between downstream firms. A deviant upstream firm can always obtain the collusive industry profit in the period of deviation by slightly undercutting its rivals' offers.

otherwise, there is double marginalization, and so $w^M < p^M$ and $F^M > 0$.[14] At the acceptance stage, each downstream firm accepts one offer along the equilibrium path. Since a downstream firm is indifferent between each of the M identical offers, its choice of which contract to accept may depend on the outcome of the public randomization device. At the output stage, each downstream firm will then face a demand of $q^M = Q(p^M; p^M, ..., p^M)$, which it will order from its upstream supplier at wholesale price w^M.

Since all upstream firms are symmetric, the critical discount factor above which the monopoly outcome upstream can be sustained is minimized if the upstream firms share the market equally.[15] Hence, along the equilibrium path, each upstream firm receives an expected per-period profit of Π^M/M.

Consider now an upstream firm's incentive to deviate.[16] By offering the contract $(w^M - \varepsilon, F^M - \varepsilon)$ to each of the N downstream retailers, where ε is arbitrarily small, the deviator can obtain a deviation profit arbitrarily close to Π^M. Each downstream firm would find it profitable to accept such an offer, independently of the acceptance decisions of its downstream rivals, and to order all units demanded by consumers from the deviator. Production and pricing will be arbitrarily close to monopoly levels. This (or any other) out-of-equilibrium contract offer by an upstream firm triggers a switch to the punishment phase in which firms play a noncollusive equilibrium in all future periods and, as discussed above, upstream firms receive zero profit. Notice that the profits from deviation and punishment do not depend on whether downstream competition is in prices or quantities, or indeed whether goods are differentiated or homogeneous.[17] So, independently of the form that downstream competition takes, under

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[14]If downstream competition is in prices, we have

$$
\begin{aligned}
w^M &= p^M + \frac{Q(p^M; p^M, ..., p^M)}{Q_1(p^M; p^M, ..., p^M)}, \\
F^M &= \left[p^M - w^M\right] Q(p^M; p^M, ..., p^M) = -\frac{\left[Q(p^M; p^M, ..., p^M)\right]^2}{Q_1(p^M; p^M, ..., p^M)},
\end{aligned}
$$

where Q_1 denotes the derivative of Q with respect to its first argument. Similarly, under quantity competition,

$$
\begin{aligned}
w^M &= P(q^M; q^M, ..., q^M) + q^M P_1(q^M; q^M, ..., q^M), \\
F^M &= \left[P(q^M; q^M, ..., q^M) - w^M\right] q^M = -\left[q^M\right]^2 P_1(q^M; q^M, ..., q^M).
\end{aligned}
$$

[15]When N is not an integer multiple of M, downstream firms use the public randomization device to determine their acceptances accordingly.

[16]A deviation by an (unintegrated) downstream firm does not trigger punishment. However, if downstream firm Dj deviates by setting a price $p_j < p^M$ (or a quantity $q_j > q^M$), then the collusive strategy profile stipulates that the deviant Dj rejects all of its contract offers, while each of its downstream rivals accepts one contract (w^M, F^M). This ensures that no downstream firm has an incentive to deviate. To see that the prescribed acceptance decisions form a Nash equilibrium at the acceptance stage, note that if Dj rejects its offers, all other downstream firms make a nonnegative profit. (Recall that all downstream firms make zero profit when each one of them accepts one offer (w^M, F^M).) Given that Dj's rivals accept the contract (w^M, F^M), Dj's best response is to reject its offer, and incur a vanishingly small loss $\phi \to 0$: if it were to accept, it would incur a (non-vanishing) loss since the contract offer (w^M, F^M) is chosen such that each downstream firm's best-response is to set the monopoly price p^M (or quantity q^M), given that all downstream rivals charge p^M and accept their offers.

[17]Because optimal deviations will typically take the form of slightly undercutting of rivals' offers, most of the incentive constraints that we will deal with will have this property. We will continue to treat the general case throughout, except where otherwise noted.

non-integration, an upstream firm's "no-cheating" incentive constraint can thus be written as

$$\frac{\Pi^M}{M(1-\delta)} \geq \Pi^M. \tag{2}$$

The l.h.s. represents the discounted sum of profits along the collusive equilibrium path, and the r.h.s. represents the maximum profit a deviant upstream firm can obtain. Hence, the critical discount factor above which the monopoly outcome can be sustained is given by $(M-1)/M$.

By slightly undercutting its rivals' offers, a deviant upstream firm can always "sever the ties" between its upstream rivals and all of the unintegrated downstream firms. Since upstream offers and downstream prices (or quantities) are set simultaneously, this implies that a deviant upstream firm can obtain (arbitrarily close to) the collusive industry profit in the period of deviation. This holds not only for the monopoly profit Π^M, but more generally for any upstream industry profit level $\Pi \leq \Pi^M$. Hence, whenever upstream firms can sustain some positive profit level in the collusive equilibrium, they can also sustain the monopoly profit level Π^M. We summarize the main result for the case of non-integration in the following lemma.

Lemma 3 *Under non-integration, the critical discount factor above which it is possible to sustain any positive upstream industry profit $\Pi \in \left(0, \Pi^M\right]$ is given by $\widehat{\delta}^{NI} = (M-1)/M$.*

3.3 Collusive Equilibrium: Single Integration

We now turn to the collusive equilibrium when one upstream-downstream pair, say $U1$-$D1$, is vertically integrated. Since the market structure is no longer symmetric, it may be optimal for upstream collusion to allow the integrated $U1$-$D1$ to capture a share of the collusive equilibrium profit that is different from (in fact, larger than) that of the $M-1$ unintegrated upstream rivals. Let α denote the collusive market share of the integrated $U1$-$D1$. Symmetry of the $M-1$ unintegrated upstream firms implies that the critical discount factor (above which monopoly profits for the upstream firms can be sustained) is minimized if each one of them obtains the same share, $(1-\alpha)/(M-1)$, of the collusive equilibrium profit.[18] At the pricing stage, the collusive equilibrium behavior is as outlined above for the case of non-integration: each upstream firm offers the contract (w^M, F^M) to each of the (unintegrated) downstream firms, and each downstream firm charges a price of p^M (or sets a quantity of q^M). At the acceptance stage, the acceptance/rejection decisions of the downstream firms will, in expectation, reflect the market-sharing arrangement under vertical integration.

Consider first the incentives to deviate for the integrated firm $U1$-$D1$. Along the equilibrium path, the firm obtains an expected per-period profit of $\alpha\Pi^M$. By offering the deviant contract $(w^M - \varepsilon, F^M - \varepsilon)$ to all of the $N-1$ unintegrated downstream firms, where ε is arbitrarily small, the integrated firm can obtain their business and make a profit arbitrarily close to the monopoly profit Π^M in the period of deviation.[19] Following the integrated firm's deviation,

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[18]Recall from our discussion in section 2 that profits can be shared in an arbitrary fashion by using a public correlating device at the acceptance stage.

[19]The integrated $U1$-$D1$ cannot extract more than the monopoly profit Π^M in the period of deviation. To see this, note first that industry profits are bounded from above by Π^M. Hence, for $U1$-$D1$ to extract more than Π^M, some other firm would need to make a loss in the period of deviation. But each downstream firm can ensure itself a profit of approximately zero by rejecting all contracts, in which case it would need to pay only the vanishingly small fine ϕ. Finally, since all upstream firms make equilibrium offers involving nonnegative fixed fees, each upstream firm's profit must be larger than or equal to zero.

firms coordinate on the (symmetric) noncollusive equilibrium in all future periods. In this noncollusive equilibrium, the integrated firm makes a per-period profit of π^{NC} through its downstream affiliate $D1$. The integrated firm's incentive constraint is thus given by:

$$\frac{\alpha \Pi^M}{1 - \delta} \geq \Pi^M + \underbrace{\frac{\delta}{1 - \delta} \pi^{NC}}_{\text{punishment effect}} . \tag{3}$$

Comparing this equation with an upstream firm's incentive constraint (2) under non-integration, we see that there is an additional term on the r.h.s. of (3), $\delta \pi^{NC}/(1 - \delta)$. This term represents the *punishment effect* of vertical integration: it is more difficult to punish an integrated firm than an unintegrated upstream firm. Unless downstream products are homogeneous and retail competition is in prices, the integrated downstream affiliate makes positive profits in the punishment phase, $\pi^{NC} > 0$.

Consider now an unintegrated Ui's incentives to deviate, $i \geq 2$. There is no punishment effect for Ui since, in periods following a deviation, all unintegrated upstream firms make zero profits, as in the absence of integration. Along the equilibrium path, Ui obtains a per-period profit of $(1-\alpha)\Pi^M/(M-1)$. By offering instead the deviant contract $(w^M - \varepsilon, F^M - \varepsilon)$ to each of the $N-1$ unintegrated downstream firms, for arbitrarily small ε, Ui can gain the business of all the unintegrated downstream firms, and extract (arbitrarily close to) Π^M/N from each one of them, as above. Importantly, however, the deviant Ui will *not* be able to extract any profit from the integrated $D1$. Since $D1$ can obtain the intermediate input at zero marginal cost from its own upstream affiliate $U1$, $D1$ will not accept any (deviant) contract that does not leave $D1$ all of the rents. Vertical integration therefore reduces the deviation profit of an unintegrated upstream firm by Π^M/N: the amount that it would have made from selling to $D1$ if $D1$ were not integrated. This is what we call the *outlets effect* of vertical integration. Comparing with equation (2) above, we can clearly see how the outlets effect slackens the incentive constraint for an unintegrated upstream firm:

$$\frac{(1-\alpha)\Pi^M}{(M-1)(1-\delta)} \geq \Pi^M - \underbrace{\frac{1}{N} \Pi^M}_{\text{outlets effect}} . \tag{4}$$

Adding up incentive constraint (3) for the integrated firm and the $M-1$ incentive constraints (4) for the unintegrated firms, we obtain the following result.

Lemma 4 *Under single integration, upstream firms can collectively extract the monopoly profit if:*

$$\delta \geq \widehat{\delta}^{SI} = \frac{M - 1}{M + \frac{\Pi^M - N\pi^{NC}}{(N-1)\Pi^M}}.$$

In pooling the incentive constraints to obtain this lemma, we are making the implicit assumption that firms' market shares are set in such a way that the individual incentive constraints (3) and (4) are satisfied. The argument is that if there is sufficient total surplus from collusion to satisfy the pooled incentive constraints, then there is a way to arrange market shares to

divide that surplus such that no individual firm wants to deviate.[20] What does the market-sharing arrangement look like? Note that market shares are not completely pinned down for $\delta > \widehat{\delta}^{SI}$: there will be a range of α's which satisfy all the incentive constraints with some slack. But clearly, for the range of discount factors where vertical integration makes collusion feasible where it otherwise would not be $(\widehat{\delta}^{NI} > \delta > \widehat{\delta}^{SI})$ we must have $\alpha > \frac{1}{M}$. This is because vertical integration makes collusion feasible by *slackening* the unintegrated firms' incentive constraint (through the outlets effect), whilst *tightening* integrated firm's incentive constraint (through the punishment effect), so the integrated firm's share must rise relatively.

Sustaining collusion when one firm is integrated thus requires that the integrated firm receives a larger share of the collusive profits than it would if it were not integrated and collusion was nevertheless still sustainable. Clearly this affects a firm's incentive to perform a vertical merger: we remark upon this further in section 3.6. Before this, we ask whether our prediction that market shares will indeed be divided in this asymmetric way is reasonable. Firstly, and most obviously, one can argue from a theoretical point of view that whilst the unintegrated firms may apparently suffer from a reduction in market share when one firm integrates, if the integration facilitates collusion $(\widehat{\delta}^{NI} > \delta > \widehat{\delta}^{SI})$, they now have a smaller share of a much larger pie. Their profits rise from zero to $\frac{(1-\alpha)\Pi^M}{(M-1)}$, so the vertical integration benefits them as well as the integrated firm. Secondly, from an empirical point of view, the prediction that vertically integrated firms are on average larger does not seem empirically unreasonable. Acemoglu et al. (2003) find that vertical integration is positively correlated with firm size in the UK manufacturing sector. In fact, many case studies of cartels report instances of vertically integrated firms demanding and receiving larger shares of the collusive pie (e.g., Levenstein (1997), Tosdal (1917)).

Lemma 4 gives the critical discount factor for the case when final goods are (symmetrically) differentiated. If final goods are homogeneous, however, monopoly profits for upstream firms can be sustained for discount factors that are even lower than $\widehat{\delta}^{SI}$ – in fact, for *any* discount factor $\delta \geq 0$ when one firm is integrated. Along the equilibrium path, the integrated $D1$ sets the monopoly price p^M (or the monopoly quantity Q^M), while all unintegrated downstream firms charge a higher price (or set a zero quantity). At the same time, all upstream firms sell through the integrated $D1$, making unacceptable contract offers (involving high fixed fees and/or wholesale prices) to the unintegrated downstream firms. This asymmetric arrangement has the flavor of a "coordination failure" between unintegrated upstream and downstream firms. The deviation profit of an unintegrated firm is zero since any deviant offer to $D1$ would be rejected because of the outlets effect, while all unintegrated downstream firms have priced themselves "out of the market". The integrated firm's incentive constraint is the only one that can bind, so that as its collusive market share α becomes arbitrarily large, the critical discount factor becomes arbitrarily small. This yields the following result for the special case of homogeneous goods.

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[20]This is true since firms' deviation profits are independent of the collusive market sharing arrangement α. We are effectively assuming that if the discount factor is low enough to make it necessary, firms are able to organize market shares in a way which minimizes the collective incentive to deviate. One can argue that this method is implicit in most of the collusion literature dating back to Friedman's classic (1971) work. But in most previous analyses, the assumption has involved symmetric firms coordinating on a symmetric outcome, and so is uncontroversial. See Compte et al. (2002) for an example of the use of this method in an asymmetric setting.

Lemma 5 *If final goods are homogeneous, monopoly profits upstream can be sustained for any discount factor $\delta \geq 0$.*

3.4   The Collusive Effect of Vertical Integration

From lemmas 3 and 4, it follows immediately that $\widehat{\delta}^{SI} < \widehat{\delta}^{NI}$: under (single) vertical integration, perfect collusion upstream can be sustained for a larger set of discount factors than under non-integration. We summarize our key result in the following proposition.

Proposition 1 *In a vertically unintegrated industry, a vertical merger facilitates upstream collusion.*

A vertical merger between an upstream and a downstream firm has two opposing effects on upstream firms' incentives to collude. On the one hand, an unintegrated upstream firm cannot profitably deviate through a rival's integrated downstream affiliate, thereby reducing the firm's deviation profit (by at least Π^M/N). This *outlets effect* reduces an unintegrated upstream firm's incentive to deviate. On the other hand, an integrated firm captures the profit of its downstream affiliate (which is $\pi^{NC} < \Pi^M/N$) in the punishment phase, while all unintegrated upstream firms make zero profit in the punishment phase, independently of market structure. This *punishment effect* increases the integrated firm's incentive to deviate, holding fixed its market share.

Which effect is stronger? Following a single vertical merger, there are $M-1$ remaining unintegrated upstream firms, and so the combined outlets effect reduces overall deviation profits by $(M-1)\Pi^M/N$. The punishment effect increases the integrated firm's discounted sum of profits from deviating by $\delta \pi^{NC}/(1-\delta)$, holding fixed the integrated firm's market share. Under nonintegration, the critical discount factor is $\widehat{\delta}^{NI} = (M-1)/M$. Evaluated at $\delta = \widehat{\delta}^{NI}$, however, we have $\delta \pi^{NC}/(1-\delta) = (M-1)\pi^{NC} < (M-1)\Pi^M/N$. Hence, the outlets effect outweighs the punishment effect since $\pi^{NC} < \Pi^M/N$. This also shows that if final goods were completely independent, and so $\pi^{NC} = \Pi^M/N$, a vertical merger would have no collusive effect.

3.5   Other Collusive Equilibria under Vertical Integration

We have shown that a vertical merger facilitates upstream collusion in the sense that a vertical merger reduces the critical discount factor above which monopoly profits upstream can be sustained. One might wonder about the effect of vertical merger on upstream firms' ability to sustain profits above non-cooperative levels but still less than those associated with monopoly prices. In this subsection we argue that vertical integration merger facilitates upstream collusion in a broader sense, and that this should be of concern for anti-trust authorities.

Why might firms wish to sustain outcomes which are less collusive than monopoly? There are two reasons why such collusion schemes might be attractive. First, it may be that the monopoly outcome itself is not feasible, and second, it may be that whilst the monopoly outcome is feasible, the division of the pie required to sustain the monopoly outcome is not favorable for some particular firm(s). In either case, since this paper is concerned with upstream collusion, it seems appropriate to focus on equilibria that lie on upstream firms' Pareto frontier. We now deal with each of these concerns in turn.

First, would vertical merger harm firms' ability to sustain lower levels of collusive profit when monopoly itself is not feasible? The answer to this turns out to be negative. Recall from lemma 3 that, under non-integration, positive upstream profits are sustainable if and only if monopoly profits for upstream firms are sustainable. Proposition 1 then implies that, holding the discount factor fixed, any aggregate level of upstream profit that can be sustained under non-integration can also be sustained under vertical integration. Further, there is a range of discount factors for which collusive profits upstream can be sustained only if one upstream-downstream pair is vertically integrated. Note in addition that this range is larger than it would appear from a comparison of lemmas 3 and 4 since (unlike in the unintegrated case) when one upstream-downstream pair is vertically integrated, the feasibility of collusion does not have the same all-or-nothing feature that it has in the absence of vertical integration. It may be possible to sustain positive profit levels when the discount factor is too low for monopoly outcomes to be feasible. (We give an example of this below.) Thus, vertical integration also facilitates sustaining "imperfect collusion": it becomes possible to sustain positive profits below monopoly level even when the monopoly outcome itself is not sustainable, which was not feasible without integration.

Now let us turn to the second concern – the idea that, when monopoly outcomes are sustainable, firms might nevertheless prefer to implement some other collusive scheme. Clearly, for the reasons noted above, this is not a concern in the unintegrated case. For the case when one firm is integrated, one can also show that if it is sustainable, any equilibrium that involves monopoly profits upstream is better from the *unintegrated* upstream firms' point of view than one that does not. The *integrated* $U1$-$D1$, however, may be better off in an equilibrium in which upstream industry profits are less than Π^M. To see this, consider the thought experiment of increasing each unintegrated Dj's price from p^M to $p' > p^M$ (or reducing Dj's quantity from q^M to $q' < q^M$), which amounts to increasing the market share of the integrated $D1$. While this reduces the industry profit, it strengthens the outlets effect in the sense that the fraction of the industry profit that a deviant unintegrated upstream firm can capture is now reduced to less than $(N-1)/N$. Hence, the integrated $U1$-$D1$ may be better off since it may be able to obtain a larger share of the (albeit smaller) collusive pie.

This point can best be seen by considering the extreme case when final goods are almost perfect substitutes. If each unintegrated Dj sets $p_j = \infty$ (or $q_j = 0$), while the integrated $D1$ sets $p_1 = \arg\max_p pQ(p; \infty, ..., \infty)$ (or $q_1 = \arg\max_q qP(q; 0, ..., 0)$), the deviation profit of any unintegrated upstream firm is now zero (due to the outlets effect), and so it is possible to sustain a collusive equilibrium in which the integrated $U1$-$D1$ captures the entire industry profit. If final goods are close substitutes, $U1$-$D1$ will thus be able to obtain a collusive profit close to Π^M, which is more than what it would be able to capture if the monopoly outcome were sustained. Observe that this asymmetric market outcome – with profits for the integrated $U1$-$D1$ exceeding π^{NC} – can be sustained for any discount factor $\delta \geq 0$. This is a static equilibrium of the "coordination failure" type mentioned above.

To summarize the discussion so far, the collusive outcome we have focused on is the preferred one for unintegrated firms independently of market structure, but is not necessarily the best equilibrium for the integrated firm. Does this reduce the relevance of our analysis? Obviously not to the extent that it is the preferences of the unintegrated firms that prevail when the collusive "agreement" is made.[21] But we argue that even to the extent that the selected collu-

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[21]Non-cooperative game theory has very little to say about how the collusive surplus should be divided, as

sive equilibrium is an asymmetric one favored by the integrated firm, the anti-trust regulator should be very concerned about vertical merger in this context, because *welfare (as measured by total surplus) will tend to be lower in such an asymmetric collusive equilibrium than under monopoly.* Given that final goods are symmetrically differentiated, any asymmetry in retail pricing introduces an additional efficiency loss. So when both are feasible, collusion under a vertically integrated structure may be worse than collusion under an unintegrated structure, because, other things equal, the former is more likely to be asymmetric. We verify that this is indeed the case for our linear demand example.

Example 2 (Linear Demand) *Suppose there are two upstream and two downstream firms, $M = N = 2$, downstream competition is in prices, and one upstream-downstream pair, say U1-D1, is vertically integrated. It can be shown that if monopoly profits upstream are sustainable, then in any equilibrium on upstream firms' Pareto frontier, total surplus is less than or equal to total surplus under monopoly. For instance, consider the equilibrium that maximizes the integrated U1-D1's profit. If final goods are sufficiently good substitutes, $\sigma \geq \overline{\sigma} \equiv 2\delta/(1+\delta)$, then $p_1 = p^M$ and $p_2 = \infty$; obviously, welfare is lower than under monopoly. Otherwise, if $\sigma < \overline{\sigma}$, then $p_1 < p^M < p_2$, and demand is such that $q_1 + q_2 < 2q^M = Q^M$; again, total surplus is lower than under monopoly.*[22]

Thus vertical merger facilitates not only the maintenance of monopoly prices, but also the maintenance of lower levels of collusive profits. Moreover, when collusion is sustainable even in the absence of integration, vertical integration may result in the adoption of an asymmetric market structure which has lower total surplus than the monopoly outcome.

3.6 Incentives for Vertical Mergers

According to the Chicago School of anti-trust, firms have no (strict) incentive to vertically integrate if there are no efficiency gains from doing so. In our model, a vertical merger has no direct efficiency effects. In the symmetric noncollusive equilibrium, firms have therefore no incentive to vertically merge: the joint profit of any upstream-downstream pair in the symmetric noncollusive equilibrium is π^{NC}, independently of whether or not the pair is vertically integrated. Nevertheless, in our model, an upstream-downstream pair has two distinct (but related) motives to vertically merge: a *collusive motive* and a *market share motive*. To see this, let us assume that upstream firms collude whenever feasible.

The *collusive motive* for vertical merger arises since upstream collusion may be sustainable only if an upstream-downstream pair vertically integrates. Indeed, as we have shown above, if $\widehat{\delta}^{SI} \leq \delta < \widehat{\delta}^{NI}$, upstream collusion is not sustainable under nonintegration, while monopoly profits upstream can be sustained under (single) vertical integration. Consider an arbitrary upstream-downstream pair. Under nonintegration, the joint per-period profit of the pair is only π^{NC} if $\widehat{\delta}^{SI} \leq \delta < \widehat{\delta}^{NI}$. Under vertical integration, however, the integrated firm must obtain a larger per-period profit to permit upstream collusion, namely $\alpha\Pi^M \geq (1-\delta)\Pi^M + \delta\pi^{NC} > \pi^{NC}$, where the first inequality follows from the integrated firm's incentive constraint (3). The pair can merge and increase all upstream firms' profits by making collusion sustainable where it was not before.

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long as all incentive constraints are satisfied, and this large research question is outside the scope of this paper.

[22]Details are available from the authors upon request.

The *market share motive* for vertical merger arises since a vertically integrated firm may require a larger share of the collusive pie than an unintegrated upstream firm for upstream collusion to be sustainable. Suppose $\delta \geq \widehat{\delta}^{NI}$, and upstream firms jointly extract the monopoly profit. Then, under nonintegration, there is at least one upstream-downstream pair with a per-period profit not larger than Π^M/M. By vertically integrating, however, the upstream-downstream pair can ensure itself a collusive per-period profit of at least $\alpha\Pi^M \geq (1-\delta)\Pi^M + \delta\pi^{NC}$ since, otherwise, the integrated firm would have an incentive to cheat. Other firms must "concede" this increase in market share for collusion not to break down. Vertical integration thus increases the upstream-downstream pair's profit if $(1-\delta)\Pi^M + \delta\pi^{NC} > \Pi^M/M$, or $\delta < (M-1)/\left\{M\left[1-\pi^{NC}/\Pi^M\right]\right\} \equiv \overline{\delta}$. It can easily be verified that $\overline{\delta} > \widehat{\delta}^{NI}$ if $\pi^{NC} > 0$. Hence, if $\widehat{\delta}^{NI} \leq \delta < \overline{\delta}$, a vertically integrated upstream-downstream pair must obtain a larger share of the collusive profit for upstream collusion to be sustainable, and this pair has a market share motive for vertical merger.

3.7 Multiple Integrations

We have shown above that vertical integration by a single upstream-downstream pair facilitates upstream collusion relative to the case where no firm is vertically integrated. Do further integrations also facilitate collusion? To address this question, we take the same model as before but now suppose that $K \leq \min\{M, N\}$ upstream-downstream pairs (say, $U1$-$D1$ to UK-DK) are vertically integrated. Contract offers and downstream prices/quantities along the collusive equilibrium path are as outlined above for the case of single vertical integration.

Consider first the incentives to deviate for an unintegrated upstream firm Ui, $i \geq K+1$. Along the collusive equilibrium path, each unintegrated upstream firm obtains a share $(1-\alpha)/(M-K)$ of the monopoly profit, where α is again part of the equilibrium description (and will generally depend on the number K of vertically integrated firms). By slightly undercutting its upstream rivals' contract offers to unintegrated downstream firms, a deviant Ui can obtain the business of all unintegrated downstream firms, and thereby make a profit of Π^M/N on each of the $N-K$ unintegrated downstream firms. For the reasons discussed above, however, the unintegrated Ui cannot make a profit by offering a deviant contract to an integrated downstream firm: given that collusion will break down anyway, an integrated downstream firm would rather order the intermediate good from its own upstream affiliate. Hence, there is an *outlets effect* associated with each vertical integration, and this outlets effect reduces the incentives to cheat for unintegrated upstream firms. The incentive constraint for an unintegrated Ui can therefore be written as:

$$\frac{(1-\alpha)\Pi^M}{(M-K)(1-\delta)} \geq \left(\frac{N-K}{N}\right)\Pi^M. \tag{5}$$

Consider now the incentives to deviate for an integrated firm. To minimize the critical discount factor, each vertically integrated firm should obtain the same share, α/K, of monopoly profits. If $K > 1$, a deviant integrated firm Ui-Di, $1 \leq i \leq K$, can no longer obtain the monopoly profit by undercutting its rivals' contract offers since any deviant contract offer to an integrated rival's downstream affiliate would be rejected. The deviant Ui-Di can, however, obtain the business of all of the $N-K$ unintegrated downstream firms. In addition, if $K > 1$, it will be optimal for Ui-Di to deviate not only by changing the contract offers to unintegrated downstream firms but also by lowering its downstream price p_i (or increasing its output q_i).

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Formally, let $\pi_{int}^{dev}(K)$ denote the deviation profit of an integrated Ui-Di when K firms are vertically integrated. If downstream competition is in prices,[23]

$$\pi_{int}^{dev}(K) = \max_p pD(p; p^M, ..., p^M) + (N - K)p^M D(p^M; p, p^M, ..., p^M), \tag{6}$$

where the first term on the r.h.s. denotes the profit the integrated Ui-Di can make through its own downstream affiliate if it charges a price of p, and the second term denotes the rents that Ui-Di can extract from the $N - K$ unintegrated downstream firms by offering, say, the contract $(0, p^M D(p^M; p, p^M, ..., p^M) - \varepsilon)$, with ε being arbitrarily small, to each one of them. It can easily be verified that it is optimal for each one of the unintegrated downstream firms to accept this contract, independently of the acceptance decisions of the other downstream firms. Let $p_{int}^{dev}(K)$ denote the optimal downstream price that Ui-Di will charge in the period of deviation. Treating K as a continuous variable, and using the envelope theorem, we have

$$\frac{d}{dK}\pi_{int}^{dev}(K) = -p^M D(p^M; p_{int}^{dev}(K), p^M, ..., p^M) < 0.$$

Hence, any further vertical integration reduces the deviation profit of an already integrated firm. This is again due to the outlets effect. As before, there is a counteracting *punishment effect* associated with each vertical integration: when final goods are differentiated, an integrated firm makes a positive profit in the punishment phase. The integrated Ui-Di's incentive constraint can thus be written as

$$\frac{\alpha \Pi^M}{K(1 - \delta)} \geq \pi_{int}^{dev}(K) + \frac{\delta}{1 - \delta}\pi^{NC}. \tag{7}$$

Does the outlets effect outweigh the punishment effect for *each* vertical integration? The answer is, not necessarily. First, note that while the punishment effect is the same size as before, the outlets effect is smaller for an integrated firm than for an unintegrated upstream firm since the former can increase its deviation profit by changing its own downstream price. Second, while the vertical merger between UK and DK, $K > 1$, reduces the incentives to cheat for the first $K - 1$ integrated firms, $U1$-$D1$ to $U(K-1)$-$D(K-1)$, it *increases* UK's profit in the period of deviation. Effectively this is because by integrating UK can coordinate its upstream deviation with a downstream price reduction, which it could not before. For the first vertical merger, this coordination was not necessary and the effect did not occur: the deviation profit of an unintegrated upstream firm under non-integration is the same as that of the integrated $U1$-$D1$ under single integration, namely Π^M. But with $K - 1$ vertically integrated firms, the deviation profit of the unintegrated UK is $(N-K+1)\Pi^M/N$, which is less than UK's deviation profit after integration, $\pi_{int}^{dev}(K)$.[24]

Summing up the incentive constraints for the unintegrated and integrated upstream firms, (5) and (7), we obtain that monopoly profits can be sustained in equilibrium if

$$\delta \geq \widehat{\delta}(K) \equiv \frac{(M - K)(N - K)\Pi^M + N\left[K\pi_{int}^{dev}(K) - \Pi^M\right]}{(M - K)(N - K)\Pi^M + NK\left[\pi_{int}^{dev}(K) - \pi^{NC}\right]}.$$

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[23] If downstream competition is in quantities, the expression is analogous.

[24] To see this, note that the integrated UK-DK can always deviate by not changing its own downstream affiliate's price. This would result in the same deviation profit as UK's deviation profit prior to vertical integration, $(N - K + 1)\Pi^M/N$. However, if $K > 1$, it will always be optimal for the integrated UK-DK to set p_K below the monopoly price p^M, and so $\pi_{int}^{dev}(K) > (N - K + 1)\Pi^M/N$.

It is instructive to consider some limiting cases. First, suppose final goods are *almost* perfect substitutes and downstream competition is in prices, and so $\pi^{NC} \approx 0$ and $\pi^{dev}_{int}(K) \approx \Pi^M$. In this case, the critical discount factor becomes $\widehat{\delta}(K) \approx \left[M(N-K) + K^2 - N\right] / \left[M(N-K) + K^2\right]$, which is minimized at $K = M/2$, i.e., when half of the upstream firms are vertically integrated.[25]

Second, consider the thought experiment of increasing the number M of upstream firms, holding fixed the number N of downstream firms. Then, for M sufficiently large, the critical discount factor is minimized when each downstream firm is vertically integrated with an upstream firm, i.e., when $K = N$. To see this, note that $\widehat{\delta}(K) \to 1$ as $M \to \infty$ if $K < N$, but $\widehat{\delta}(N) = \left[N\pi^{dev}_{int}(N) - \Pi^M\right] / \left[N\pi^{dev}_{int}(N) - N\pi^{NC}\right] < 1$, independently of M.

Example 3 (Linear Demand) *Suppose there are two downstream firms, $N = 2$, and downstream competition is in prices. Then, if products are sufficiently differentiated, $\sigma < -1 + \sqrt{3}$, the second vertical integration further reduces the critical discount factor if there are at least three upstream firms, $M \geq 3$. If products are sufficiently close substitutes, $\sigma \geq -1 + \sqrt{3}$, the second vertical integration further reduces the critical discount factor if there are at least four upstream firms, $M \geq 4$.*

When considering the number K of vertical integrations that one would expect to see in an industry, one should keep in mind that, in addition to the *collusive motive*, there is a *market share motive* for vertical merger. One might expect to see more mergers than the number which minimizes the critical discount factor as firms may have an incentive to vertically integrate in order to obtain a larger share of the collusive pie, even when the vertical merger increases the critical discount factor. Nevertheless the results in this section are interesting in that they suggest a reason why we might see an intermediate degree of vertical integration with apparently symmetric firms making asymmetric integration choices. Vertical merger may appear to increase market share, but that does not imply that all firms in the industry should integrate as this could upset the collusive equilibrium.

4 Robustness and Extensions

In this section we investigate the robustness of our results to the use of optimal punishment schemes; the secrecy of upstream offers; and the timing of upstream offers and downstream pricing decisions. We show that our key result – that vertical merger facilitates collusion – is robust to these changes.

4.1 Optimal Punishment

In our analysis, we have assumed throughout that, in the collusive equilibrium, a deviation by an upstream firm triggers an infinite reversion to a noncollusive equilibrium in all subsequent periods. This may, however, not be the worst possible punishment that can be inflicted on the deviator. In the existing literature on collusion, which has focused on repeated *normal-*form games, inflicting the worst possible punishment on a deviator consists in playing, from

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[25] At the other extreme, if final goods were completely independent of one another (i.e., if each downstream firm sells in a separate market), $\pi^{NC} = \Pi^M/N$ and $\pi^{dev}_{int}(K) = (N - K + 1)\Pi^M/N$. In this case, the critical discount factor $\widehat{\delta}(K)$ would be independent of the number of vertical mergers.

the following period onward, the subgame-perfect equilibrium that yields the lowest payoff to the deviator. In the context of our model, a deviating unintegrated upstream firm cannot be punished any more harshly than by reversion to the noncollusive equilibrium where it receives zero profits. But it may be feasible to sustain a per-period payoff for an integrated upstream-downstream pair that is less than the noncollusive profit π^{NC}. If so, this would further reduce the critical discount factor under vertical integration, and only strengthen our main result (proposition 1) that (single) vertical integration facilitates upstream collusion by reducing the size of the punishment effect.

However, our model is a repeated *extensive*-form game, in which it may be possible to use *within-period* punishment. In repeated extensive-form games, inflicting the lowest feasible payoff in all *future* periods does not generally constitute the worst possible punishment for a deviator. For instance, in our model, an upstream firm can profitably deviate only if at least one downstream firm accepts the deviant offer. An optimal punishment scheme may therefore provide incentives for downstream firms to reject deviant offers. Studying optimal punishment schemes in general is beyond the scope of this paper, but we are able to do so for the case where final goods are homogeneous.

In this section, we consider the optimal punishment scheme for the case when downstream firms produce a homogeneous final good and compete in either prices or quantities. The section has two aims. First, we show that our conclusion – that a vertical merger between an upstream and a downstream firm facilitates upstream collusion – continues to hold under the optimal punishment scheme. Second, not much is known about optimal punishment in repeated *extensive*-form (rather than *normal*-form) games, and so the punishment scheme we derive is of independent interest. Indeed, we show that the logic of *simple penal codes* (Abreu (1988)) breaks down in our repeated-extensive form game: upstream collusion may be sustainable only under a strategy profile with the property that the continuation play after an upstream firm's deviation depends not only on the identity of the deviator, but also on the details of the deviation.

We assume that there is a public randomization device not only at the beginning of the acceptance stage (so as to allow an optimal sharing of collusive profits between upstream firms) as before, but also at the beginning of each period (so as to allow coordination on a particular equilibrium of the stage game). For simplicity, we will confine attention to the case of two downstream firms, $N = 2$. But it should be clear from our discussion below that the qualitative features of our results do not depend on this restriction.

Asymmetric Noncollusive Equilibria. In addition to the symmetric noncollusive equilibrium, there exist asymmetric subgame-perfect equilibria of the stage game. Indeed, as we have already shown above for the case of single vertical integration, there exists an equilibrium of the stage game, where the integrated firm obtains all of the monopoly rents (see lemma 5). Asymmetric equilibria of the stage game also exist when no firm is vertically integrated. In this case, for any downstream firm Dj, there exists a subgame-perfect equilibrium of the stage game, denoted $\widehat{\sigma}_j$, where Dj receives the monopoly profit Π^M, while all other firms make zero profits. Furthermore, when no firm is vertically integrated, there exists an equilibrium of the stage game, denoted $\widehat{\sigma}_0$, in which all upstream and downstream firms make zero profits: under price competition this is the symmetric noncollusive equilibrium; under quantity competition, this equilibrium is generated by a "coordination failure" between the quantity choices of and offers to the downstream firms.

Lemma 6 *Suppose final goods are homogeneous and no firm is vertically integrated. Then, there exists a subgame-perfect equilibrium of the stage game, denoted $\widehat{\sigma}_j$, in which an (arbitrary) downstream firm Dj captures all of the monopoly rents Π^M. Moreover, there exists a subgame-perfect equilibrium of the stage game, denoted $\widehat{\sigma}_0$, in which industry profits are zero.*

As we will now show, the asymmetric noncollusive equilibria are used in the optimal punishment scheme for rewarding those downstream firms that have rejected deviant offers, and the zero-profit equilibrium $\widehat{\sigma}_0$ for punishing downstream firms in the event where all of them have accepted deviant offers.

Collusive Equilibrium: Non-Integration. We now derive the optimal collusive scheme under non-integration, which differs from the collusive strategy profile described before only in the subgames following a deviation by an upstream firm. For simplicity, we restrict attention to the case where each of the two unintegrated downstream firms sells half of the monopoly output along the collusive equilibrium path (i.e., each downstream firm sets a quantity of $Q^M/2$, or charges a price of p^M and consumer demand is divided equally between the two retailers). Of course, the strategy profile will prescribe asymmetric downstream behavior in certain subgames off the equilibrium path.

The worst possible punishment that might be inflicted upon a deviant upstream firm is that– in addition to the play of one of the noncollusive equilibria in all future periods – all of its deviant contracts are rejected by the downstream firms in the period of deviation, leaving the unintegrated upstream firm with a deviation profit of zero. By playing one of the asymmetric noncollusive equilibria, $\widehat{\sigma}_1$ and $\widehat{\sigma}_2$, in all future periods, the maximum joint "reward" R that can feasibly be shared between downstream firms if they do reject deviant offers is:

$$R \equiv \frac{\delta}{1-\delta}\Pi^M. \tag{8}$$

For downstream firm Dj to accept upstream firm Ui's deviant offer (w'_{ij}, F'_{ij}), the deviator needs to leave rents to Dj, in the form of a reduced wholesale price and/or a reduced fixed fee. Let

$$b_{ij} \equiv [p^M - w'_{ij}]\frac{Q^M}{2} - F'_{ij}$$

denote Ui's "bribe" to downstream firm Dj.

An important feature of the optimal punishment scheme (discussed further below) is that downstream firm Dj's reward for rejecting upstream firm Ui's deviant offer (and accepting the equilibrium contract offered by a nondeviant upstream firm) depends on the bribes b_{i1} and b_{i2} offered by Ui, and on both downstream firms' acceptance decisions.[26] Specifically, the optimal punishment scheme is of the following form. *Case (i):* Suppose the bribes satisfy $\max\{b_{i1} b_{i2}\} \leq \Pi^M/2$. (a) If only one downstream firm, say Dj, rejects Ui's offer, then Dj receives the maximum reward R. That is, the asymmetric equilibrium $\widehat{\sigma}_j$ will be played in all future periods. (b) If both downstream firms reject the deviant offers, then $D1$ receives a fraction $\gamma_{i1} \in [0,1]$ of the maximum reward R, and $D2$ receives the remaining fraction $\gamma_{i2} = 1 - \gamma_{i1}$,

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[26] Formally, the optimal reward depends on the deviant contract offers (w'_{i1}, F'_{i1}) and (w'_{i2}, F'_{i2}) (in addition to downstream firms' acceptance decisions). But since downstream firms' prices/quantities are set at the same time as upstream firms' offers are made, what matters for downstream firms' incentives are not the offered wholesale prices and fixed fees *per se* but the induced bribes b_{i1} and b_{i2}.

where γ_{ij} will be chosen optimally as a function of b_{i1} and b_{i2}. That is, in any future period, the asymmetric equilibrium $\widehat{\sigma}_j$, $j = 1, 2$, will be played with probability γ_{ij} (using the public randomization device at the beginning of each period). (c) If both downstream firms accept the deviant offers, then the zero-profit equilibrium $\widehat{\sigma}_0$ will be played in all future periods. *Case (ii):* Suppose the bribes satisfy $b_{ij} < \Pi^M/2 < b_{ik}$. In this case, the deviant Ui would make a loss on its offer to Dk, and so Dk should not be rewarded for rejecting Ui's offer. Instead, downstream firm Dj will receive the maximum reward R for rejecting Ui's offer, independently of Dk's acceptance decision. *Case (iii):* Suppose the bribes satisfy $\min\{b_{i1}, b_{i2}\} > R$. In this case, the deviant Ui will make a loss on each contract if accepted, and so no downstream firm will be offered a reward for rejecting the offer. (The strategy profile may prescribe, for example, the play of the zero-profit equilibrium $\widehat{\sigma}_0$ in all future periods.)

Clearly, it will never be optimal for Ui to offer a bribe $b_{ij} > \Pi^M/2$, and so we will from now on confine attention to case (i) where $b_{ij} \leq \Pi^M/2$ for each $j = 1, 2$. Observe that the optimal punishment scheme is such that whenever a downstream firm accepts a deviant offer, it will receive zero profits in all future periods. Consequently, Dj will accept Ui's deviant offer if and only if the offered bribe b_{ij} exceeds Dj's expected reward for rejecting Ui's offer. We now derive the optimal "choice" of γ_{ij} as a function of the offered bribes b_{i1} and b_{i2}.

First, suppose that Ui deviates by offering bribes $b_{i1} > R$ and $b_{i2} > R$. In this case, even the maximum feasible reward R is insufficient to prevent any one downstream firm from accepting the deviant offer. Equilibrium thus prescribes that both downstream firms accept Ui's offers, independently of the value of γ_{ij}, and Ui's deviation profit is $\Pi^M - (b_{i1} + b_{i2})$. Hence, conditional on offering such bribes, Ui can obtain a maximum deviation profit of (approximately) $\Pi^M - 2R$ by setting $b_{i1} = b_{i2} = R + \varepsilon$ with ε being arbitrarily small.

Second, suppose that Ui deviates by offering bribes $0 < b_{ij} \leq R$ and $b_{ik} \geq R$. In this case, the available rewards are insufficient to induce both downstream firms to reject the Ui's offer; in fact, if $b_{ik} > R$, downstream firm Dk would accept Ui's offer even if offered the maximum reward R. The optimal punishment therefore involves setting $\gamma_{ij} = 1$, which induces downstream firm Dj to reject Ui's offer (and instead to accept the equilibrium contract (w^M, F^M) offered by a nondeviant upstream firm). The strategy profile then prescribes that Dj accepts and Dk rejects the deviant offer, and Ui's deviation profit is $\Pi^M/2 - b_{ik}$. Hence, conditional on offering such bribes, Ui can obtain a maximum deviation profit of $\Pi^M/2 - R$ by setting $b_{ij} \in (0, R]$ and $b_{ik} = R$.

Third, suppose that Ui deviates by offering bribes $b_{i1} \leq R$ and $b_{i2} \leq R$ with $b_{i1} + b_{i2} \leq R$. In this case, both downstream firms can be induced to reject Ui's deviant offer (and accept the equilibrium contract offered by a nondeviant upstream firm) by setting $\gamma_{ij} = b_{ij}/(b_{i1} + b_{i2})$. Hence, by offering such bribes, Ui's deviation profit is zero.

Fourth, suppose that Ui deviates by offering bribes $b_{i1} < R$ and $b_{i2} < R$ with $b_{i1} + b_{i2} > R$. (Given that $b_{i1}, b_{i2} \leq \Pi^M/2$, this case can arise only if $R < \Pi^M$, or $\delta < 1/2$.) In this case, the feasible rewards are insufficient to induce both downstream firms to reject Ui's offers for sure. But it is possible to offer rewards such that one downstream firm will reject for sure, or else that each downstream firm will reject with positive probability. By setting $\gamma_{ij} = 1$, downstream firm Dj can be induced to reject (and its rival Dk to accept) Ui's offer for sure. Alternatively, if $0 \leq \gamma_{i1} < b_{i1}$ and $0 \leq \gamma_{i2} < b_{i2}$ (with $\gamma_{i1} + \gamma_{i1} = 1$), there is a multiplicity of equilibria at the acceptance stage: (i) two pure-strategy equilibria where Dj accepts Ui's contract, while Dk rejects it, and Ui's deviation profit is $\Pi^M/2 - b_{ij}$, and (ii) a mixed-strategy equilibrium

where each downstream firm accepts with positive probability. The collusive strategy profile prescribes that the worst equilibrium from the deviant Ui's point of view will be played: as we will now show, this is the mixed strategy equilibrium.

Lemma 7 *Suppose upstream firm Ui deviates by offering a weakly larger bribe to Dj than to Dk, $b_{ik} \leq b_{ij} < \min\{R, \Pi^M/2\}$, and $b_{i1} + b_{i2} > R$. (The restrictions imply $R < \Pi^M$, i.e., $\delta < 1/2$.) Then, the optimal punishment scheme is such that $\gamma_{ij} = b_{ij}/R$ and $\gamma_{ik} = 1 - b_{ij}/R$. The strategy profile prescribes that Dk rejects Ui's offer with probability one and that Dj rejects the deviant offer with probability $(R - b_{ik})/b_{ij}$. Conditional on offering such bribes, Ui's optimal deviation consists in setting*

$$b_{i1} = b_{i2} = \frac{\sqrt{R\Pi^M}}{2} = \frac{\Pi^M}{2}\sqrt{\frac{\delta}{1 - \delta}},$$

which results in a deviation profit of

$$\left[1 - \sqrt{\frac{R}{\Pi^M}}\right]^2 \Pi^M = \left[1 - \sqrt{\frac{\delta}{1 - \delta}}\right]^2 \Pi^M.$$

When it is infeasible to induce both downstream firms to reject the deviant offer with probability one, the optimal punishment scheme provides rewards in such a way to permit "miscoordination" between downstream firms at the acceptance stage. To see why such mis-coordination is detrimental for the profits of the deviant upstream firm, suppose Ui offers bribes $b_{i1} = b_{i2} = (1 + \varepsilon)R/2 < \Pi^M/2$, where ε is small. The worst possible punishment involving a pure-strategy equilibrium at the acceptance stage is such that one downstream firm, say Dk, rejects the offer, while its downstream rival Dj accepts it. (This holds for any choice of $0 \leq \gamma_{i1} = 1 - \gamma_{i2} \leq 1$.) The resulting deviation profit is $\left[\Pi^M - (1 + \varepsilon)R\right]/2$. However, if $\gamma_{ij} = (1 + \varepsilon)/2$ and $\gamma_{ik} = (1 - \varepsilon)/2$, there exists a mixed-strategy equilibrium at the acceptance stage where downstream firm Dk rejects the deviant offer for sure, while Dj randomizes and accepts the deviant offer with probability $2\varepsilon/(1 + \varepsilon)$. Ui's deviation profit is only $\left[\Pi^M - (1 + \varepsilon)R\right]\varepsilon/(1 + \varepsilon)$, which is close to zero for ε small.

To summarize, by optimally choosing the bribes b_{i1} and b_{i2}, upstream firm Ui can get a maximum deviation profit of

$$
\begin{aligned}
\pi_{optimal}^{dev} &= \max\left\{\Pi^M - 2R, \Pi^M/2 - R, 0, \Pi^M\left[\max\left\{1 - \sqrt{\frac{R}{\Pi^M}}, 0\right\}\right]^2\right\} \\
&= \max\left\{\left(\frac{1 - 3\delta}{1 - \delta}\right)\Pi^M, \left(\frac{1 - 3\delta}{1 - \delta}\right)\frac{\Pi^M}{2}, 0, \Pi^M\left[\max\left\{1 - \sqrt{\frac{\delta}{1 - \delta}}, 0\right\}\right]^2\right\} \quad (9)
\end{aligned}
$$

where the arguments are listed in the order of our discussion above. (As regards the final argument, the maximum-term reflects that this fourth case can arise only if $R < \Pi^M$, or $\delta < 1/2$.) The incentive constraint under non-integration can then be written as

$$\frac{\Pi^M}{M(1 - \delta)} \geq \pi_{optimal}^{dev}. \quad (10)$$

We obtain the following result.

Lemma 8 *Suppose no firm is vertically integrated. Then, under the optimal punishment scheme, the critical discount factor above which monopoly profits upstream can be sustained is given by*

$$\widehat{\delta}_{optimal}^{NI} = \begin{cases} \frac{M-1}{3M} & \text{if } 2 \leq M \leq 13, \\ \frac{M-\sqrt{2M-1}}{2M} & \text{if } M \geq 13. \end{cases}$$

Comparing the critical discount factor under "Nash reversion" from lemma 3, $\widehat{\delta}^{NI}$, with that under the optimal punishment scheme, $\widehat{\delta}_{optimal}^{NI}$, we observe that the former goes to one as the number of upstream firms becomes large, $\widehat{\delta}^{NI} \to 1$ as $M \to \infty$, while the latter is bounded from above by $1/2$, $\widehat{\delta}_{optimal}^{NI} \to 1/2$ as $M \to \infty$. Under the optimal punishment scheme, the more patient are players (or the less frequent prices can be adjusted), the greater is the present value of future rewards. If $\delta \geq 1/2$, there exists a division of these future rewards and an equilibrium at the acceptance stage such that an upstream firm's maximum deviation profit is actually zero.

In the foregoing analysis, we have derived the critical discount factor assuming that each downstream firm produces one half of the monopoly output along the collusive equilibrium path. Intuitively, this arrangement facilitates upstream collusion: downstream asymmetries would make upstream collusion more difficult. To see this, suppose one downstream firm, say Dj, were to produce the whole monopoly output in the collusive equilibrium. Then, a deviant Ui would need to offer a bribe of only $b_{ij} = R + \varepsilon$ (with ε being arbitrarily small) to that downstream firm Dj, and obtain a deviation profit of (almost) $\Pi^M - R$. The resulting critical discount factor would be $(M-1)/(2M)$, which is clearly larger than the one when downstream firms share the market symmetrically.

The existing literature on collusion has focused almost exclusively on repeated *normal*-form games. As Abreu (1988) has shown, when deriving the optimal punishment scheme in such games, one can confine attention to *simple penal codes*. Any subgame-perfect outcome can be supported by a profile with the property that any deviation by a player from the current prescribed path is punished by the same punishment path (penal code). That is, the continuation play after a deviation by a player is independent of the details of the deviation, depending only on the identity of the deviator.

The optimal punishment scheme derived above is *not* a simple penal code. First, depending on the deviant upstream firm's contract offers, the strategy profile prescribes that different downstream firms reject the offers, and the associated "rewards" mean that different outcome paths are played in future periods. For example, suppose that the deviant Ui offers bribes $b_{i1} = b_{i2} = R/2 < \Pi^M/2$. Then, the strategy profile prescribes that both downstream firms reject the offer, and that in all future periods each one of the two asymmetric equilibria, $\widehat{\sigma}_1$ and $\widehat{\sigma}_2$, is played with probability $1/2$. In contrast, suppose that Ui offers bribes $b_{i1} = R < b_{i2}$. In this case, the strategy profile prescribes that $D1$ rejects Ui's offer (and that $D2$ accepts the offer), and that in all future periods, the asymmetric equilibrium $\widehat{\sigma}_1$ is played with probability one. Hence, the prescribed outcome path in all future periods depends on the details of Ui's deviant contract offers. Second, even in the event when both downstream firms reject Ui's deviant offers, the continuation play optimally depends on the details of these offers. Indeed, as lemma 7 shows, when the offered bribes satisfy $b_{i1} \leq b_{i2} < R$ and $b_{i1} + b_{i2} > R$, downstream firm $D2$ should obtain a fraction b_{i2}/R of the total reward R (and $D1$ the remaining fraction) when both $D1$ and $D2$ reject Ui's offer. Hence, the probability distribution over the two asymmetric

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equilibria, $\widehat{\sigma}_1$ and $\widehat{\sigma}_2$, in all future periods depends on the details of Ui's contract offers, even after both downstream firms have taken the same action at the acceptance stage.

Our results show that the logic of simple penal codes breaks down in repeated *extensive*-form games. In our model, upstream collusion may be sustainable only under a strategy profile with the property that the prescribed continuation play following a deviation not only depends on the identity of the deviant upstream firm, but also on the details of its contract offers.[27]

Proposition 2 *Under non-integration, the optimal punishment scheme is not a simple penal code.*

Collusive Equilibrium: Single Integration. Suppose now that one upstream-downstream pair is vertically integrated. As we have already shown in lemma 5, monopoly profits upstream are then sustainable for any discount factor $\delta \geq 0$. Hence, the conclusion of proposition 1 still holds under the optimal punishment scheme.

Proposition 3 *Suppose final goods are homogeneous. Under the optimal punishment scheme, (single) vertical integration facilitates upstream collusion.*

4.2 Secret Offers

While it is very natural to assume that downstream firms' retail prices are publicly observed (indeed, in most retail markets, prices are publicly posted), the details of a contract between an upstream and a downstream firm may not be easily verified by other firms. In this subsection, we will assume that only the downstream prices are publicly observed at the pricing stage, while upstream firm Ui's contract offer (w_{ij}, F_{ij}) to downstream firm Dj is private information to Ui and Dj. As we will show, the main conclusion of our paper is robust: even when contracts are secret, a (single) vertical merger facilitates upstream collusion.

To simplify the analysis, we will continue to assume that there is a publicly observed randomization device (so as to allow firms to share the market in an arbitrary way).[28] We will also assume that all contract offers (and acceptance decisions) are publicly revealed at the end of each period (after profits are realized).

At the acceptance stage, each downstream firm Dj must form beliefs about the secret contract offers to its downstream rivals. In perfect Bayesian equilibrium, these beliefs are pinned down by equilibrium play. However, if an integrated upstream-downstream pair, say $U1$-$D1$, deviates from equilibrium by charging a different retail price (or by setting a different quantity) – which is publicly observed – then, at the acceptance stage, each downstream firm Dj, $j > 1$, may hold arbitrary beliefs about the contracts that the deviant integrated firm has offered to any rival downstream firm Di, $i > 1$, $i \neq j$.[29] Similarly, if upstream firm Ui deviates by offering a different contract to downstream firm Dj, then perfect Bayesian equilibrium does not pin down Dj's beliefs about Ui's offer to Dk, $k \neq j$.

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[27]We provide some further discussion of the failure of simple penal codes in a separate note, Mailath, Nocke, and White (2004).

[28]Instead of assuming the existence of a public correlating device, we could alternatively allow firms to share the collusive profits by making secret side payments at the end of the period, after profits are realized.

[29]If an unintegrated downstream firm Dj were to deviate at the pricing stage, then perfect Bayesian equilibrium implies that, at the acceptance stage, all downstream firms would continue to believe that their downstream rivals were offered their equilibrium contracts.

In the literature on foreclosure and vertical restraints with secret contracts, where this problem arises, it is customary to impose restrictions on the set of out-of-equilibrium beliefs; see Hart and Tirole (1990), McAfee and Schwartz (1994), Segal (1999), and Rey and Tirole (2003). Our assumption that upstream and downstream firms set offers and prices (quantities) simultaneously implies that, unlike most of the literature, we do *not* need to restrict downstream firms' beliefs in the event that they receive an out-of-equilibrium contract offer from an unintegrated upstream firm, or from the integrated $U1$-$D1$ (unless $U1$-$D1$ deviates by also changing p_1 or q_1). Our model requires us to restrict downstream firms' beliefs only for the subgame in which the integrated firm, $U1$-$D1$, has deviated by changing the price (or quantity) of its downstream affiliate, $D1$. Having observed such a deviation, we assume that an (unintegrated) downstream firm Dj forms *wary beliefs* (see McAfee and Schwartz (1994)) at the acceptance stage: Dj believes that the cheating $U1$-$D1$'s secret contract offers to the other downstream firms maximize $U1$-$D1$'s profit, given $U1$-$D1$'s offer to Dj. An appealing implication of this assumption on beliefs is that no downstream firm will ex post regret accepting an upstream firm's optimal deviation contract, and so an upstream firm's deviation profit is bounded from above by the monopoly profit.[30]

We first consider the noncollusive equilibria, i.e., the perfect Bayesian equilibria of the stage game. It is straightforward to show the following two results. First, lemma 1 carries over to the case of secret offers: under any market structure, each upstream firm makes zero profit (on its contracts with unintegrated downstream firms) in *any* perfect Bayesian equilibrium of the stage game, independently of out-of-equilibrium beliefs. Second, for any set of out-of-equilibrium beliefs, it is possible to sustain the symmetric noncollusive outcome (see lemma 2), where all upstream firms offer $(0,0)$ to all (unintegrated) downstream firms, and each downstream firm makes profit $\pi^{NC} \geq 0$. Both claims are based on the observation that, at the acceptance stage, a downstream firm observes its downstream rivals' prices (or quantities), and given these prices, its profit does not directly depend on its rivals' contracts.

Consider now the collusive strategy profile where upstream firms extract all of the monopoly rents, and any deviation by an upstream firm or by an integrated upstream-downstream pair is followed, in all future periods, by the infinite play of the (symmetric) noncollusive equilibrium. Along the collusive equilibrium path, contract offers and downstream prices (or quantities) are as in the case of public offers.

We claim that upstream firms' deviation profits, and hence incentive constraints, are as in the case of public offers. The intuition is as follows. Recall that at the acceptance stage each downstream firm Dj observes its downstream rivals' prices/quantities. It then follows that downstream firm Dj's out-of-equilibrium beliefs about the secret offers made by the deviant Ui to its rivals affect Dj's profit only insofar as these secret offers may or may not lead to Dj's rivals rejecting all of their offers – since in this case, Dj would face additional demand

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[30] In the above-cited literature, which is concerned only with static models, the restriction of *passive beliefs* is often imposed: receiving an out-of-equilibrium offer, a downstream firm continues to believe that its rivals received their equilibrium offers. This restriction is typically justified by the argument that, from the upstream firm's point of view, downstream firms operate in separate markets. However, in our context of collusion in a repeated game, this argument no longer holds: conditional on deviating (i.e., conditional on triggering the punishment phase), it is optimal for the deviant upstream firm to deviate through *each* downstream firm. Under passive beliefs, an upstream firm's maximum deviation profit would be larger than the monopoly profit, and downstream firms would ex post regret accepting deviant offers. See also Rey and Tirole (2004) and Rey and Vergé (2004) for an alternative argument as to why wary beliefs may sometimes be preferred to passive or symmetric beliefs.

from the rationed consumers. Further, note that if Ui deviates but all retail prices/quantities remain unchanged (which is necessarily the case if Ui is an unintegrated upstream firm), then each downstream firm would make zero profit by accepting the contract (w^M, F^M) offered by Ui's nondeviant rivals. Hence, as long as downstream prices are unchanged, each Dj will correctly believe that its downstream rivals' will not reject all of their offers, and so Dj will not face additional demand from rationed consumers. When downstream prices remain unchanged, each Dj is thus willing to accept the same set of deviant contracts as in the case of public offers. Finally, out-of-equilibrium beliefs matter when the integrated $U1$-$D1$ deviates by changing its downstream price p_1 (or quantity q_1). In particular, if $p_1 < p^M$ (or $q_1 > q^M$), downstream firms would make a loss if all of them were to accept the nondeviant contract (w^M, F^M). What our assumption of wary beliefs ensures is that Dj cannot be exploited by falsely believing that its downstream rivals are rejecting all of their contracts (in which case Dj would face additional demand from rationed consumers). Since it is in the deviant Ui's best interest to deviate through all downstream firms (conditional on deviating), wary beliefs imply that Dj will believe that the deviant Ui has made "acceptable" contract offers to its rivals. Hence, Dj will accept the same set of deviant contract offers as under public offers.

Proposition 4 *Under secret offers, a vertical merger between an upstream and a downstream firm facilitates upstream collusion.*

As should be clear from our discussion above, we could dispense with *any* restriction on out-of-equilibrium beliefs – even in the event of a deviation involving the integrated firm's downstream affiliate – and still obtain the same result if we were to assume that downstream firm Dj's demand will not increase if its downstream rival Dk rejects all of its offers and is thus unable to serve its demand.

In our analysis, we have assumed that contract offers become common knowledge at the end of each period. Would our results still hold if contract offers were never publicly revealed? For the case of homogeneous final goods and price competition downstream, it is possible to construct a collusive equilibrium in which firms' incentive constraints (and the critical discount factor) are as under public offers. This is briefly described in the appendix. If final goods are differentiated (or downstream competition is in quantities), however, then it will in general not be possible for upstream firms to extract all of the monopoly rents along the collusive equilibrium path.

4.3   A Model with Sequential Moves

So far, we have assumed that upstream contract offers and downstream retail prices are chosen simultaneously. In many circumstances, however, it seems plausible that downstream firms have contracts with their input suppliers in place *before* deciding upon their own output prices. In this section, we analyze the robustness of our predictions to a change in the sequence of moves. In particular, we assume the following timing in each period:

1. *Contract offer stage:* Upstream firms $U1, ..., UM$ simultaneously make public two-part tariff contract offers to the downstream firms.

2. *Acceptance stage:* Downstream firms $D1, ..., DN$ simultaneously decide which contract(s) to accept.[31]

3. *Downstream pricing stage:* Downstream firms $D1, ..., DN$ simultaneously set prices (or quantities) in the retail market, and then order the quantities demanded by consumers from the upstream firms at the relevant wholesale prices.[32]

As before, we will analyze the impact of a single vertical merger on the critical discount factor above which monopoly profits upstream can be sustained, confining attention to a punishment scheme that involves infinite reversion to the (symmetric) noncollusive equilibrium.[33] Does vertical integration facilitate upstream collusion when downstream firms can condition their retail prices on upstream firms' contract offers in this way? It turns out that the outlets and punishment effects of vertical integration are still present with this modified timing. In addition, two new effects arise: the *reaction effect* and the *lack-of-commitment effect*. These new effects arise from the flexibility of the *integrated* downstream firm's price to changes in upstream offers which results from the new timing. Consider first a deviation by an unintegrated upstream firm, say, Ui. As before, vertical integration reduces Ui's deviation profits. The reduction in Ui's deviation profits can now be decomposed into two components. First, holding fixed all retail prices/quantities, the deviant Ui cannot obtain $D1$'s business. This is the by now familiar outlets effect. Second, the integrated $D1$ can *react* to Ui's deviation by reducing its retail price (or increasing its quantity), reducing the sales of the unintegrated downstream firms accepting Ui's offers. The deviant Ui may choose to anticipate this response by selecting a deviation which will also induce different (non-monopoly) choices from the unintegrated downstream firms, but the net effect of this downstream flexibility is still to reduce Ui's profit below what it would be without such flexibility. We will call the impact of this downstream price (quantity) flexibility on Ui's deviation profit the *reaction effect* of vertical integration.

Now consider a deviation by the integrated firm. The flip-side of the reaction effect in this context is the *lack-of-commitment effect*. This effect arises because the integrated firm's downstream affiliate can also react to *its own* upstream affiliate's deviation by changing its price (or quantity). The integrated firm's inability to commit to maintaining a high retail price (or a low output) when making deviant offers to unintegrated downstream firms reduces their willingness to pay and hence the integrated firm's deviation profit.

As we now show, both the reaction effect and lack of commitment effect reduce deviation profits and hence make upstream collusion easier to sustain.

Noncollusive Equilibrium. The symmetric noncollusive equilibrium of the stage game involves the same contract offers and downstream prices/quantities along the equilibrium path as in the base model. In this equilibrium, each upstream firm makes zero profit, while each downstream firm makes profit $\pi^{NC} \geq 0$ (where the inequality is strict unless final goods are homogeneous and downstream competition is in prices), and the equilibrium outcome is again independent of the number of vertical mergers.

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[31] As in the base model, we assume that either there exists a public randomization device at the beginning of the acceptance stage, or else upstream firms can make side payments at the end of each period.

[32] If downstream firm Dj has rejected all contract offers at the acceptance stage, it is forced to set $p_j = \infty$ (or quantity $q_j = 0$) at the downstream pricing stage, and thus make zero profit.

[33] Details of the optimal punishment schemes are available from the authors on request. Our results will be robust to the use of the optimal punishment scheme.

Lemma 9 *Independently of market structure, the symmetric noncollusive equilibrium is a pure-strategy subgame-perfect equilibrium of the stage game.*

Collusive Equilibrium: Non-Integration. Our analysis of the case of non-integration can be brief as, when no firm is vertically integrated, the sequential timing does not upset the collusive equilibrium outcome described in section 3.2. The same upstream contracts will implement the same monopoly prices (or quantities) downstream, and so each upstream firm's expected per-period profit along the collusive equilibrium path will be Π^M/M. The optimal deviation again entails slightly undercutting rival offers to obtain (approximately) the monopoly profit Π^M.[34] In the ensuing punishment phase, all upstream firms make zero profits, and so the incentive constraint under non-integration can be written just as in the base model:

$$\frac{\Pi^M}{1-\delta} \geq \Pi^M.$$

Lemma 10 *In the model with sequential moves, the critical discount factor (above which upstream firms can extract all of the monopoly profits) under non-integration is given by $\widehat{\delta}_{seq}^{NI} = (M-1)/M$.*

Collusive Equilibrium: Single Integration. Let us turn to the collusive equilibrium when one upstream-downstream pair, say $U1$-$D1$, is vertically integrated. Now the flexibility of downstream pricing (quantity setting) will affect deviation profits. But the collusive outcome itself will be implemented with the same set of contracts as in section 3.3, yielding monopoly profits Π^M to be divided among upstream firms. The market share of the integrated $U1$-$D1$ along the collusive equilibrium path is again denoted α, and minimizing the critical discount factor will again entail the $M-1$ unintegrated upstream firms sharing the remaining profits equally.[35]

Consider first the incentive to deviate of an unintegrated upstream firm, Ui, $i \geq 2$. As in the model with simultaneous moves, the unintegrated Ui will not be able to extract any rents from offering a deviant contract to the integrated downstream firm $D1$: the integrated $D1$ prefers to purchase its inputs from its own upstream affiliate $U1$ (at an effective cost of

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[34] When downstream prices are set after contracts are accepted, it is no longer a dominant strategy for each downstream firm to accept the deviant contract. However, it is straightforward to see that the unique subgame perfect equilibrium in this subgame involves each downstream firm accepting the deviant contract, and setting a price of (approximately) p^M.

[35] Suppose integer constraints on the sharing of collusive upstream profits are solved by using a public randomization device at the acceptance stage (rather than through side payments at the end of the period). Then, it may be optimal for upstream collusion that the integrated $U1$-$D1$'s profit be independent of the outcome of the randomization device since, otherwise, we may need to worry about $U1$-$D1$'s incentive constraint at the pricing stage. This can be achieved by prescribing that only a fraction of the integrated $D1$'s equilibrium output is ordered from its upstream affiliate $U1$, the remaining units being ordered from other upstream firms. For example, suppose $M = N = 2$ and $\alpha = 0.6$. Then, along the collusive equilibrium path, the unintegrated $D2$ accepts the integrated $U1$'s contract (w^M, F^M) with probability one, while the integrated $D1$ accepts a contract of the form $(p^M, 0)$, and orders eighty percent of its equilibrium output $q^M = Q^M/2$ from the unintegrated $U2$, and the remaining twenty percent of its output from its upstream division $U1$. It can easily be verified that this cross-selling arrangement does not affect upstream firms' incentives to offer deviant contracts. Alternatively, as before, the issue can be resolved by using side payments at the end of the period.

zero). If the integrated $U1$-$D1$'s price were *fixed* at the collusive price p^M (or its quantity were fixed at q^M), then the deviant Ui could obtain a deviation profit of $(N-1)\Pi^M/N$ in the same way as before, by offering each unintegrated downstream firm the contract $(w^M - \varepsilon, F^M - \varepsilon)$. This reduction in an upstream firm's deviation profit is the by now familiar *outlets effect* of vertical integration. However, since downstream prices are set *after* observing upstream firms' contract offers, the integrated $D1$ can change its retail price p_1 (or quantity q_1) in response to Ui's deviation. Anticipating this reaction, Ui may choose to make a different deviation which exploits the price flexibility of the unintegrated downstream firms and induces different behavior from them. The net effect of this price (quantity) adjustment by integrated and unintegrated downstream firms is to reduce the deviation profit of the unintegrated Ui beyond what is due to the outlets effect alone. We call this additional impact the *reaction effect* of vertical integration. As before, an unintegrated upstream firm makes zero profit in the punishment phase. Hence, the unintegrated Ui's incentive constraint is given by

$$\frac{(1-\alpha)\Pi^M}{(M-1)(1-\delta)} \geq \Pi^M - \underbrace{\frac{1}{N}\Pi^M}_{\text{outlets effect}} - \underbrace{\left[\left(\frac{N-1}{N}\right)\Pi^M - \pi_{unint}^{dev}\right]}_{\text{reaction effect}} = \pi_{unint}^{dev}, \qquad (11)$$

where π_{unint}^{dev} denotes the maximum deviation profit of an unintegrated upstream firm. The following lemma shows that the reaction effect of vertical integration does indeed reduce an unintegrated upstream firm's deviation profit.

Lemma 11 *An unintegrated upstream firm's maximum deviation profit, π_{unint}^{dev}, satisfies $\pi_{unint}^{dev} < (N-1)\Pi^M/N$.*

The lemma follows from two observations. First, since the integrated downstream firm $D1$ faces an effective wholesale price of zero (the marginal cost of its upstream division $U1$), $D1$ must receive a share of at least $1/N$ of industry profits when an unintegrated upstream firm Ui deviates, and the deviant Ui will not be able to extract $D1$'s profit. Second, the industry profit when Ui deviates will be less than the monopoly profit since the integrated $D1$ will choose its myopic best response to the other retail prices/quantities.

Let us now turn to the integrated $U1$-$D1$'s incentives to deviate. The integrated firm can no longer obtain the business of the unintegrated downstream firms simply by slightly undercutting its upstream rivals' contract offers (unless final goods are homogeneous and downstream competition is in prices). The reason is as follows. After it has made deviant offers to the unintegrated downstream firms, $U1$-$D1$ will at the downstream pricing stage set the price p_1 (or quantity q_1) that maximizes the integrated $U1$-$D1$'s deviation profit. To the extent that the wholesale price offered by $U1$-$D1$ is less than the monopoly price p^M (which it must be if goods are differentiated or competition is in quantities) the integrated firm will optimally set a retail price below p^M (or a quantity above q^M). Of course, this will be anticipated by the unintegrated downstream firms, which will therefore reject any offer with a "high" fixed fee. Indeed, as we will show below, the integrated firm's maximum deviation profit, π_{int}^{dev}, will be less than the monopoly profit Π^M if final goods are differentiated. We will refer to this reduction in $U1$'s deviation profit – as a result of its vertical merger with $D1$ – as the *lack-of-commitment effect* of vertical integration: it is the integrated firm's inability to commit to its own retail price that reduces its deviation profit. Of course, as in the model with simultaneous moves,

there is a countervailing *punishment effect* since the integrated firm will be able to capture its downstream affiliate's profit in the punishment phase, π^{NC}. Hence, the integrated $U1$-$D1$'s incentive constraint can be written as

$$\frac{\alpha \Pi^M}{1-\delta} \geq \Pi^M - \underbrace{\left[\Pi^M - \pi_{int}^{dev}\right]}_{\text{lack-of-commitment effect}} + \underbrace{\frac{\delta}{1-\delta}\pi^{NC}}_{\text{punishment effect}} . \tag{12}$$

Lemma 12 *The integrated firm's deviation profit π_{int}^{dev} satisfies $\pi_{int}^{dev} \leq \Pi^M$, where the inequality is strict if final goods are differentiated.*

Combining the upstream firms' incentive constraints (11) and (12), we obtain the critical discount factor under (single) vertical integration.

Lemma 13 *In the model with sequential moves, the critical discount factor (above which upstream firms can extract all of the monopoly profits) under vertical integration is given by*

$$\widehat{\delta}_{seq}^{SI} = \frac{(M-1)\pi_{unint}^{dev} + \pi_{int}^{dev} - \Pi^M}{(M-1)\pi_{unint}^{dev} + \pi_{int}^{dev} - \pi^{NC}}.$$

As in the baseline model, it is straightforward to show that upstream collusion can be sustained for *any* discount factor under vertical integration if final goods are homogeneous and downstream competition is in prices. (Details are available from the authors on request.) Intuitively, this is achieved by an asymmetric arrangement whereby all upstream firms sell through the integrated downstream firm, which stands ready to undercut the prices of any unintegrated downstream firm accepting a deviant upstream offer.

The Collusive Effect of Vertical Integration. In the baseline model with simultaneous moves, a vertical merger between an upstream and a downstream firm had two mutually counter-acting effects: an outlets effect and a punishment effect. We showed there that the outlets effect always dominates the punishment effect, and so a vertical merger in that model facilitates upstream collusion. When downstream firms can react to upstream firms' contract offers by changing their retail prices/quantities, two additional effects arise: the reaction effect and the lack-of-commitment effect. Since we have shown that the impact of the reaction effect on unintegrated firms' deviation profits is strictly negative; and that of the lack-of-commitment effect on the integrated firm's deviation profits is weakly negative (strictly when goods are differentiated), it should not be surprising that our key result – that vertical integration facilitates upstream collusion – is robust to allowing downstream firms to condition their retail prices/quantities on upstream firms' contract offers.

Proposition 5 *In the model with sequential moves, (single) vertical integration facilitates upstream collusion.*

We obtain as a corollary that the sequential-move structure facilitates collusion when one firm is vertically integrated, $\widehat{\delta}_{seq}^{SI} < \widehat{\delta}^{SI}$ (note that $\widehat{\delta}_{seq}^{NI} = \widehat{\delta}^{NI}$) . We can interpret this result as suggesting that if the industry contains one integrated firm, it will be helpful to collusion

if upstream prices are relatively sluggish in the sense that downstream firms can change their prices/quantities more quickly than upstream firms can.

We have also analyzed the optimal punishment for this sequential timing for the special case of homogeneous final goods and price competition in the downstream market (details are available from the authors on request). We show that our conclusion – that a vertical merger between an upstream and a downstream firm facilitates upstream collusion – continues to hold under the optimal punishment scheme. Our analysis again illustrates the observation that simple penal codes (Abreu (1988)) are not optimal in repeated extensive-form games.

5   Conclusion

In this paper we have shown how vertical mergers can facilitate upstream collusion. Two important effects are relevant to the analysis: the *outlets effect* and the *punishment effect*. The outlets effect arises because unintegrated upstream firms are unable to sell through the downstream affiliates of their integrated upstream rivals when they choose to deviate; this reduces the profitability of deviation and hence facilitates collusion. The punishment effect, on the other hand, arises because an integrated firm cannot be punished as severely as an unintegrated one when downstream firms make rents in the punishment phase; this makes sustaining collusion more difficult. We have shown that, under fairly general circumstances (downstream firms compete in prices or quantities to sell differentiated or homogeneous goods), the outlets effect dominates the punishment effect, so that vertical merger facilitates collusion. We have also examined the robustness of this conclusion to the secrecy of upstream offers, the use of the optimal punishment scheme (as opposed to Nash reversion) and to an alternative assumption about the timing of upstream and downstream pricing choices. The latter extension allowed us to identify two further competitive effects of vertical merger which may be of interest: the *reaction effect* and the *lack-of-commitment effect*. Both of these effects facilitate collusion and arise when downstream prices or quantities are set *after* upstream offers have been made. They result from the downstream firms' ability to adjust their downstream strategic variable in response to out-of-equilibrium upstream offers. The reaction effect arises because integrated firms can respond aggressively in downstream markets to the upstream deviations of their rivals, reducing the latters' profits from deviation. Integrated firms can also best-respond downstream to their own deviant upstream offers: this response is anticipated by the unintegrated downstream recipients of such offers and reduces their willingness to pay for deviant contracts. So this lack-of-commitment effect reduces the integrated firm's deviation profits and hence makes collusion easier to sustain.

Our paper contributes to our understanding of vertical mergers in three ways: by improving our theoretical understanding of games between vertically-related players; by helping us to understand the observed structure of vertically-related industries; and by helping us form policy on vertical mergers. Firstly, from a theoretical point of view, the operation of collusive equilibria where buyers are interdependent and behave strategically has received very little attention.[36] In this paper, we have modeled the case where upstream firms try to collude both to restrict supply and to extract all the rents from the downstream firms. One can view this analysis as

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[36]The only exception of which we are aware is Snyder (1996), who examines collusion in the presence of a single strategic buyer.

a step on the road towards the more ambitious goal of analyzing collusive agreements where the upstream and the downstream firms share the rents to cooperate in restricting supply to consumers. Our analysis of the design of optimal punishment schemes, where downstream firms are given rewards off-the-equilibrium path for participating in the punishment of upstream firms, gives us a flavor of just how complex such an analysis might become. It also serves to illustrate that our understanding of non-cooperative games where players need the "consent" of other players to deviate successfully is still in its infancy, even though this form is quite common in applications.[37] At a more technical level, our analysis of optimal punishment schemes has revealed that the logic of simple penal codes (Abreu (1988)) breaks down in repeated *extensive*-form games: for some discount factors, collusion may be sustainable only by a strategy profile with the property that the "punishment" depends not only on the identity of the deviator but is also "fine-tuned" to the details of the deviation made.

Secondly, from a more practical point of view, and despite the lack of theoretical attention, upstream collusion does seem to be a particular problem in intermediate goods industries, many of which exhibit substantial vertical integration (see, e.g., Scherer (1980), chapter 6). Our theory helps us understand why this may be the case, and also why one might see asymmetry in the degree of vertical integration in such industries. Leaving aside any efficiency gains or static foreclosure effects, our theory identifies two potential motives for vertical merger. Since the first vertical merger reduces the critical discount factor above which collusion is sustainable, vertical mergers could be driven by the desire to make collusion sustainable when it otherwise would not be ("the collusive motive"). Collusion becomes easier even though a merger will typically make deviation from a collusive agreement more tempting for the integrated firm itself. Indeed, this observation highlights a further motive for vertical merger even in cases where collusion is in any case feasible, the "market share motive". An integrated firm will typically need to be granted a larger market share to persuade it not to undercut the collusive price (see Levenstein (1997) for an example that seems to fit this case). Further, we have shown that while the first vertical merger always facilitates collusion, successive mergers after this may not do so, so that intermediate levels of integration may be optimal. This is interesting since many industries seem to have the feature that vertically integrated firms compete with separated ones, and it is not always clear why such differing arrangements should arise. (For examples, see Bindemann (1999) on the oil industry, Woodruff (2002) on the Mexican footwear industry, Slade (1998a, b) on the UK beer industry and the gasoline retail market in Vancouver, respectively, and Chipty (2001) and Waterman and Weiss (1996) on the US cable television industry. For an alternative theoretical rationale for asymmetric outcomes, see Ordover et al. (1990).)

Thirdly, our work is relevant to the formulation of policy towards vertical mergers. Interestingly, to the extent that anti-trust authorities have been concerned about the impact of vertical mergers on collusion, they too seem to have focused on intermediate goods industries as the relevant area for consideration. In contrast to the existing academic literature, the United States *Non-Horizontal Merger Guidelines* (1984) already anticipate the idea that vertical mergers may

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[37]Here, downstream firms must accept deviant upstream offers for such deviations to be profitable, but one can imagine many similar situations where in making a deviation, it "takes two to tango". Consider, for example, the case of labor unions trying to present a united front in negotiating with the firms in an industry – a union-firm pair must both defect from their respective collusive arrangements for the deviation to be successful. European soccer players' contracts typically prevent them from negotiating with other teams whilst under contract – to make a profitable deviation from this agreement (which does occur), players must find a team willing to negotiate with them. Similarly, bribed officials must accept and act on bribes if bribes are to be profitable.

facilitate collusion, envisaging two ways in which this may occur.[38] We relate these two ideas to our outlets and reaction effects. The first suggestion is that vertical merger facilitates upstream collusion by making it somehow easier to monitor prices. This is an old idea which has yet to be properly formalized, but which is loosely related to the reaction effect proposed in this paper. But rather than impose *ad hoc* changes in the ability to observe or punish deviations as an exogenous feature of vertically integrated firms, we have considered differences in contracts and incentives to cheat on a collusive agreement which arise *endogenously* as a result of vertical integration. In our base model, vertical integration clearly cannot improve the observability of prices *per se* since all contract offers are completely public; and indeed we have shown that incentives to deviate in our model will be the same even if upstream offers are private information, so that observability *per se* does not matter. But in the light of the results of our model with sequential timing, we can see that the observability of defections from the collusive agreement is very relevant if firms can *react* to them, and that vertical integration does enhance the integrated firm's ability to react to observed defections, so facilitating collusion. To put it another way, we show that when upstream prices are sluggish compared to downstream prices, vertical integration is particularly effective in sustaining collusion.

The second way in which the Non-Horizontal Merger Guidelines envisage that vertical integration may facilitate collusion is through the acquisition of a "disruptive buyer". The Guidelines state that a disruptive buyer is one which is substantially different from the others, the idea being that price-cutting to this buyer is particularly attractive, so that the "removal" of this buyer from the downstream market may significantly reduce incentives to cheat on a collusive agreement. Again, this idea has some relation to our theory, but we derive, rather than impose, the result that the purchase of a downstream buyer by an upstream firm makes it less attractive for its rivals to cheat by removing an outlet for their cheating (our 'outlets' effect). Our analysis suggests that the Guidelines may be too restrictive in focusing on buyers which "differ substantially" from other firms in the market: even when downstream firms are symmetric, the removal of a downstream buyer can improve collusion possibilities. Moreover, the theory in this paper suggests that it is not obvious that the acquisition of a buyer whose business is particularly attractive will especially facilitate upstream collusion. Such an acquisition is a double-edged sword since one upstream firm now owns an attractive outlet for cheating, so its own incentive to cheat increases (in our model this shows up as the punishment effect). We examine this issue more closely in a our earlier working paper (Nocke and White (2003)) where we allow for asymmetric downstream firms. We are able to show for a particular case that indeed integration with a low-cost or large downstream firm most facilitates collusion, but a general analysis identifying "disruptive buyers" awaits further research. Indeed, the dynamic effects of vertical restraints more generally remain largely unexplored.[39]

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[38] Though these guidelines are no longer generally used, they have not yet been either withdrawn or replaced.

[39] In our earlier working paper, Nocke and White (2003), we show that other vertical restraints (exclusive dealing or retail price maintenance) are only very imperfect substitutes for vertical merger in facilitating collusion. Jullien and Rey (2000) analyze the idea (related to the notion that vertical restraints may facilitate the monitoring of prices) that resale price maintenance may facilitate collusion. In an agency model where retailers face demand shocks which are not observed by wholesalers, resale price maintenance acts to smooth downstream prices, making cheating easier to detect.

6 Appendix

Proof of lemma 1. Suppose to the contrary that upstream firm Ui makes a positive profit on its contract with (unintegrated) downstream firm Dj. Hence, either $w_{ij} > 0$ or $F_{ij} > 0$, or both. Then we then claim that upstream firm Uk, $k \neq i$, has a profitable deviation. In particular, Uk can slightly undercut Ui's contract to Dj by offering $\left(w'_{kj}, F'_{kj}\right) = (w_{ij}, F_{ij} - \varepsilon)$ if $F_{ij} > 0$, or $\left(w'_{kj}, F'_{kj}\right) = (w_{ij} - \varepsilon, F_{ij})$ otherwise, where ε is arbitrarily small. Clearly, Dj prefers this contract to Ui's offer and will accept it, unless Uk's offer triggers a change in some other downstream firm's behavior which adversely affects Dj's profit were it to accept Uk's offer. Suppose for the moment that Uk's deviant offer leaves the acceptance/rejection decisions of the other downstream firms unchanged. (Recall that downstream prices are set at the same time as upstream firms' two-part tariff offers, and so cannot be changed in reaction to Uk's deviant offer.) In this case, Dj will accept Uk's deviant offer, and Uk makes a positive profit on this contract. Hence, Uk's deviation is profitable, unless it triggers a change in the acceptance/rejection decision of some other downstream firm.

Is it possible that, in equilibrium, some downstream firm Dl sets a positive output or some price at which it faces positive demand, and yet rejects all of its offers? The answer is no: Dj would have to pay a fine ϕ, and so its overall profit would be $-\phi$, while it could profitably deviate by setting a quantity of zero or a (very high) price at which demand is nil. Hence, Uk's deviation cannot trigger a change in the acceptance/rejection decision of some other downstream firm Dl (which, along the equilibrium path, rejects all offers) that is detrimental to Uk's profit.

The only possible change in the acceptance/rejection of some downstream firm Dl that may adversely affect Uk thus arises if in the candidate equilibrium Uk makes a positive profit on its contract with Dl and Uk's deviation induces Dl to switch suppliers. For this to hold, in the candidate equilibrium, Dl must be indifferent between accepting and rejecting Uk's offer. But then Uk can, in addition to offering a deviant contract to Dj, offer a slightly better contract to Dl so that Dl strictly prefers to accept it. The loss in profit on Uk's offer to Dl can always be chosen in such a way that it is outweighed by Uk's gain on its deviant offer to Dj. ■

Proof of lemma 2. It is immediate to see that there is no profitable deviation for a downstream firm, either at the pricing stage, or at the acceptance stage. Suppose now that upstream firm Ui deviates by offering contracts of the form $(w_{ij}, F_{ij}) \neq (0, 0)$ to one or more downstream firms. Clearly, a deviant contract can be profitable to Ui only if it involves a positive wholesale price, a positive fixed fee, or both. Suppose that Ui's deviant offer involves a positive fixed fee. Since each downstream firm is still offered the contract $(0, 0)$ by each of the other $M - 1$ upstream firms, each downstream firm would reject the deviant contract, and Ui would make zero profit. Suppose now that Ui's deviant offer involves a positive wholesale price. If it also involves a non-negative fixed fee, it would clearly be rejected in favor of the noncollusive contract offered by the other upstream firms. If the deviant offer involves a negative fixed fee, the downstream firm would accept it. However, the downstream firm would in addition accept the noncollusive contract $(0, 0)$ offered by, say Uk, and order all of the inputs from Uk at a wholesale price of zero, rather than from Ui at a positive wholesale price. Hence, the deviant Ui would make a loss. Finally, note that there is no incentive for an integrated firm to deviate at the pricing stage by changing both its upstream affiliate's contract offer to unintegrated downstream firms and its downstream affiliate's retail price. ■

Proof of lemma 5. At the pricing stage, an unintegrated downstream firm Dj, $j \geq 2$, sets a price $p_j > p^M$ (or a quantity $q_j = 0$). On the other hand, the integrated $D1$ sets the monopoly price p^M (or the monopoly quantity Q^M), being offered – for example – the contract $(0, F)$, $0 \leq F \leq \Pi^M$ by each of the $M - 1$ unintegrated upstream firms, where the fixed fee F (and the probabilities with which $D1$ accepts the various contracts) can be chosen so that each of the unintegrated upstream firms makes an expected per-period profit of $(1 - \alpha)\Pi^M/(M - 1)$. Consider now the incentives to deviate for an unintegrated upstream firm Ui, $i \geq 2$. Clearly, any deviant offer to $D1$ that does not leave all of the rents to $D1$ will be rejected. Moreover, Ui cannot profitably deviate through the unintegrated downstream firms since they face zero demand at their equilibrium prices (or, in the case of quantity competition, they produce zero output). The incentive constraint for an unintegrated Ui can thus be written as

$$\frac{(1 - \alpha)\Pi^M}{1 - \delta} \geq 0.$$

By rejecting all unintegrated upstream firms' contract offers, the integrated $U1$-$D1$ obtains a deviation profit of Π^M. In the punishment phase, it receives a profit of π^{NC} through its downstream affiliate, where $\pi^{NC} = 0$ in the case of price competition, and $\pi^{NC} > 0$ in the case of quantity competition. The integrated firm's incentive constraint is thus given by equation (3), and so monopoly profits upstream can be sustained if

$$\delta \geq \frac{(1 - \alpha)\Pi^M}{\Pi^M - \pi^{NC}}.$$

The assertion follows by setting $\alpha = 1$. ■

Proof of lemma 6. To see that there exists an equilibrium of the stage game where Dj obtains all of the monopoly profits, consider the following strategy profile. At the pricing stage, each upstream firm offers the contract $(0, 0)$ to Dj, and a contract with a sufficiently large wholesale price, say $(p^M + \varepsilon, F)$ with $\varepsilon > 0$, $F \geq 0$, to all other downstream firms Di, $i \neq j$. At the same time, downstream firm Dj sets price $p_j = p^M$ (or quantity $q_j = Q^M$), while Di, $i \neq j$, sets price $p_i > p^M$ (or quantity $q_i = 0$). At the acceptance stage, each downstream firm accepts any contract that leaves it with nonnegative rents, given the acceptance decisions of the other firms. Along the equilibrium path, Dj accepts at least one of the contracts offered to it, and makes the monopoly profit Π^M. It is straightforward to verify that no firm has an incentive to deviate unilaterally. To see that there exists an equilibrium of the stage game where all firms make zero profits, suppose all upstream firms make contract offers of the form (∞, ∞), and each downstream firm Dj sets a quantity $q_j = 0$ or a price $p_j = \infty$. Clearly, no firm has an incentive to deviate. ■

Proof of lemma 7. The proof proceeds in steps. (1) Assuming that Ui has deviated by offering bribes satisfying $b_{ik} \leq b_{ij} < \min\{R, \Pi^M/2\}$, and $b_{i1} + b_{i2} > R$, we derive the optimal choice of γ_{ij} such that the equilibrium at the acceptance is in *mixed* strategies. (2) We show that *any* choice of γ_{ij} (not necessarily the one derived in step (1)) that is followed by the play of a *pure*-strategy equilibrium at the acceptance stage, will lead to a higher deviation profit, and is hence not an optimal punishment. (3) We derive Ui's optimal deviation under the stated restrictions on the bribes.

(1) For at least one downstream firm to use a mixed strategy at the acceptance stage, the distribution of rewards must satisfy $1 - b_{ik}/R \leq 1 - \gamma_{ik} \equiv \gamma_{ij} \leq b_{ij}/R$ (where one inequality

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must be strict since $b_{ij} + b_{ik} > R$ by assumption). Suppose first that both inequalities are strict so that there exists an equilibrium in which both Dj and Dk use a mixed strategy. Let θ_k denote the probability that Dk accepts Ui's deviant offer. For Dj to use a mixed strategy, it has to be indifferent between accepting and rejecting, and so

$$b_{ij} = \theta_k R + (1 - \theta_k)\gamma_{ij}R,$$

where the l.h.s. gives Dj's payoff upon accepting Ui's offer, and the r.h.s. the expected payoff from rejecting. Solving for θ_k, we obtain

$$\theta_k = \frac{b_{ij} - \gamma_{ij}R}{(1 - \gamma_{ij})R}.$$

Deriving θ_j in the same fashion, we can write Ui's expected deviation profit as

$$\pi_i^{dev}(b_{ij}, b_{ik}; \gamma_{ij}) = \left(\frac{b_{ij} - \gamma_{ij}R}{(1 - \gamma_{ij})R}\right)\left[\frac{\Pi^M}{2} - b_{ik}\right] + \left(\frac{b_{ik} - (1 - \gamma_{ij})R}{\gamma_{ij}R}\right)\left[\frac{\Pi^M}{2} - b_{ij}\right] \quad (13)$$

The optimal division of rewards, γ_{ij}, minimizes this expression, subject to the constraint $1 - b_{ik}/R \leq \gamma_{ij} \leq b_{ij}/R$. The deviation profit is strictly concave in γ_{ij} over the relevant range, and so we obtain a corner solution $\gamma_{ij} \in \{b_{ij}/R, 1 - b_{ik}/R\}$. It can easily be verified that $\pi_i^{dev}(b_{ij}, b_{ik}; b_{ij}/R) \leq \pi_i^{dev}(b_{ij}, b_{ik}; 1 - b_{ik}/R)$ if and only if $b_{ij} \geq b_{ik}$, which holds by assumption. Hence, the deviation profit in (13) is minimized if $\gamma_{ij} = b_{ij}/R$. In the limit as $\gamma_{ij} \rightarrow b_{ij}/R$, downstream firm Dk rejects Ui's offer with probability one, while Dj accepts Ui's offer with probability $(b_{ij} + b_{ik} - R)/b_{ij} \in (0,1)$. (In fact, if $\gamma_{ij} = b_{ij}/R$, there exists a continuum of mixed-strategy equilibria. In all of these, Dk rejects Ui's offer with probability one, and Dj accepts the offer with probability larger than or equal to $(b_{ij} + b_{ik} - R)/b_{ij}$. Hence, the afore-mentioned equilibrium is the worst mixed-strategy equilibrium from the deviant Ui's point of view.) The minimized deviation profit is given by

$$\pi_i^{dev}(b_{ij}, b_{ik}) = \left(1 - \frac{R - b_{ik}}{b_{ij}}\right)\left[\frac{\Pi^M}{2} - b_{ij}\right]. \quad (14)$$

(2) In step (1), we have assumed that γ_{ij} is chosen such that a mixed-strategy equilibrium at the acceptance stage exists, and that downstream firms do indeed play this equilibrium. May the same or a different choice of γ_{ij}, followed by a pure-strategy equilibrium at the acceptance stage, lead to a lower deviation profit? The answer is, no. Since Ui offers a weakly larger bribe to Dj than to Dk (and, so if accepted, Ui's makes a weakly lower profit on its contract with Dj than with Dk), the worst possible (from Ui's point of view) pure-strategy equilibrium at the acceptance stage entails Dj accepting Ui's offer for sure, and Dk rejecting for sure. But in the mixed-strategy equilibrium derived in step (1), Dk also rejects Ui's offer for sure, but in addition Dj rejects the deviant offer with positive probability as well.

(3) In the previous steps, we have shown that $\gamma_{ij} = b_{ij}/R$, followed by the play of a mixed-strategy equilibrium (where Dk rejects for sure and Dj rejects with probability $(R - b_{ik})/b_{ij}$) is the most severe punishment. Given this punishment, we now derive the deviant Ui's optimal choice of bribes, under the conditions of the lemma, i.e., $b_{ik} \leq b_{ij} < \min\{R, \Pi^M/2\}$, and $b_{i1} + b_{i2} > R$. From equation (14), Ui's deviation profit $\pi_i^{dev}(b_{ij}, b_{ik})$ under the optimal

punishment scheme is strictly increasing in b_{ik}, given that $b_{ik} \leq b_{jk}$. Hence, Ui will optimally set $b_{ik} = b_{jk} = b$. Maximizing $\pi_i^{dev}(b, b)$ with respect to b, yields Ui's profit-maximizing bribe, $b = (1/2)\sqrt{R\Pi^M}$. The resulting deviation profit is $\left[1 - \sqrt{R/\Pi^M}\right]^2 \Pi^M$. $\blacksquare$

Proof of lemma 8. The assertion follows from equations (9) and (10). Observe that the second argument in (9) is one half times the first argument, and so this argument can be dropped. The first argument is positive only if $\delta < 1/3$, and the final argument is positive only if $\delta < 1/2$. That is, $\pi^{dev} = 0$ if $\delta \geq 1/2$. Inserting the first argument into the incentive constraint (10), we obtain

$$\frac{\Pi^M}{M(1-\delta)} \geq \left(\frac{1-3\delta}{1-\delta}\right)\Pi^M,$$

or

$$\delta \geq \frac{M-1}{3M}.$$

Inserting the final argument into the incentive constraint, yields

$$\frac{\Pi^M}{M(1-\delta)} \geq \left[\max\left\{1 - \sqrt{\frac{\delta}{1-\delta}}, 0\right\}\right]^2 \Pi^M,$$

or

$$\delta \geq \frac{M - \sqrt{2M-1}}{2M}.$$

Hence,

$$\widehat{\delta}_{optimal}^{NI} = \max\left\{\frac{M-1}{3M}, \frac{M-\sqrt{2M-1}}{2M}\right\}.$$

$\blacksquare$

Proof of proposition 4. The assertion obviously holds if we can show that upstream firms' incentive constraints are as under public offers. The argument proceeds in two steps.

First, we claim that each upstream firm can obtain at least the same deviation profit under secret offers than under public offers. To see this, observe that by making the same deviant contract offers as under public offers, a deviant upstream firm can make the same deviation profit as under public offers. Indeed, suppose an upstream firm offers the deviant contract $(w^M - \varepsilon, F^M - \varepsilon)$ to all unintegrated downstream firms, where (w^M, F^M) is the collusive equilibrium contract. Since each downstream firm Dj observes each rival's price (p^M) or quantity (Q^M/N), it is optimal for Dj to accept this deviant offer, independently of its downstream rivals' acceptance decisions.

Second, we claim that an upstream firm cannot obtain a larger deviation profit than under public offers. To see this, note that each firm has the option of rejecting all contracts, pay the (vanishingly small) fine $\phi \approx 0$, and make a profit of (approximately) zero. Hence, a deviant upstream firm can extract a higher deviation profit only if at least one downstream firm, say Dj, makes a loss ex post. Since downstream prices are publicly observed, this can occur only if Dj incorrectly believes that one or more of its downstream rivals reject all of their contracts – in which case Dj would face a larger demand (or, under quantity competition, would be able to fetch a higher price) than otherwise.

Suppose first that the deviant firm is an unintegrated upstream firm. In this case, all downstream prices are unchanged, and so each downstream firm can make a profit of exactly zero

by accepting the equilibrium offer of a nondeviant upstream firm (while rejecting all contracts would result in a vanishingly small loss). Hence, when a downstream firm receives a deviant offer from an unintegrated upstream firm, it must (correctly) believe that its downstream rivals will continue to be active in this period.

Suppose now that the deviant firm is the integrated upstream-downstream pair. For a deviant integrated firm, it is never optimal to make offers that would induce a downstream firm to reject all offers and be inactive as the deviant firm would make zero profit on such a contract. Instead, the deviant upstream firm can make a positive profit by offering the contract $(w', F'-\varepsilon)$, where ε is arbitrarily small and w' and F' are such that $\left[p^M - w'\right] Q(p^M; p', p^M, ..., p^M) = F'$, and p' is the integrated firm's retail price in the period of deviation. Clearly, it is optimal for each downstream firm to accept this offer, independently of its rivals' acceptance decisions. Given the assumption of wary beliefs, each downstream firm thus expects the deviant integrated firm to make offers to other downstream firms that will induce these to be active. Since downstream prices are publicly observed, this means that the integrated firm's deviation profit is bounded from above by the monopoly profit. ∎

Remarks on secret offers. In the main text, we briefly remarked on the case when secret offers are *never* publicly revealed. Here we examine that case in greater detail. Assume that final goods are homogeneous and retail competition is in prices, and that downstream firms have wary beliefs off the equilibrium path. To see that it is possible to construct a collusive equilibrium where the incentive constraints are as under public offers, suppose upstream firm Ui has deviated by undercutting Uj's offer to downstream firm Dk, and that Dk has accepted this deviant offer in preference to the equilibrium offer of Uj. The collusive strategy profile then prescribes that: (i) in all future periods, both Ui and Uj offer $(0,0)$ to all (unintegrated) downstream firms forever; (ii) in all future periods, downstream firm Dk charges a retail price of zero; (iii) whenever an off-equilibrium retail price is observed, all firms switch to the symmetric noncollusive equilibrium (involving contract offers of $(0,0)$ and retail prices of zero) in all future periods; here, this means that two periods after Ui has deviated, the whole industry has reverted to the symmetric noncollusive equilibrium.

Clearly, neither Ui nor Uj have an incentive to deviate from this collusive strategy profile: given that one upstream firm offers $(0,0)$ to all downstream firms, it is optimal for the other to do so as well. (For this, it is irrelevant that Uj does not observe the identity of the upstream firm that has undercut its offer to Dk.) Further, Dk has no incentive to deviate: having wary beliefs, Dk believes that Ui also has deviated through all other downstream firms (since this is in Ui's best interest) and will therefore expect all other downstream firms to charge a price of zero in the next period (and all periods thereafter). Therefore, it is optimal for Dk to charge a price of zero in the next period (and thereafter). Finally, when observing an off-equilibrium retail price, it is optimal for each upstream to forever offer the contract $(0,0)$ to all downstream firms since it (correctly) expects its upstream rivals to do the same.

However, if final goods are differentiated (or downstream competition is in quantities), then it will in general not be possible for upstream firms to extract all of the monopoly rents along the collusive equilibrium path if offers are never publicly revealed. This follows from two observations. First, in this case, downstream firms are better off in the punishment phase than along the collusive equilibrium path since $\pi^{NC} > 0$ in the symmetric noncollusive equilibrium. Second, since contract offers are never publicly revealed, firms face an "inference problem" when Dj rejects Ui's offer: Did Dj reject the offer because some other upstream firm deviated by

undercutting Ui's contract (which should trigger a punishment phase) or because Dj deviated on its own account (which should not trigger punishment) in the hope of triggering the punishment phase?

Proof of lemma 9. Off the equilibrium path, the strategy profile prescribes that strategies form a Nash equilibrium, and so no firm has an incentive to deviate. (The proof of existence of a Nash equilibrium in every subgame is standard, and is omitted here.) What remains to be shown is that no upstream firm has an incentive to deviate at the contract offer stage; the proof proceeds along the same lines as that of lemma 2. ∎

Proof of lemma 11. Consider a deviation by the unintegrated upstream firm Ui, $i \geq 2$. Suppose first that Ui deviates by offering contracts of the form $(0, F_{ij})$ to each of the $N-1$ unintegrated downstream firms, and that the fixed fee F_{ij} is low enough that each downstream firm will accept Ui's offer. Then, at the downstream pricing stage, each downstream firm will set a price of p^{NC} (or a quantity of q^{NC}), and so the integrated $D1$'s profit will be π^{NC}, while the deviant Ui can extract at most $(N-1)\pi^{NC} < (N-1)\Pi^M/N$ from the $N-1$ unintegrated downstream firms. Hence, in this case, Ui's deviation profit is indeed less than $(N-1)\Pi^M/N$.

Suppose now that Ui deviates by offering contracts that involve a positive wholesale price, $w_{ij} > 0$, for at least one downstream firm Dj (or a high fixed fee F_{ij} so that at least one unintegrated downstream Dj will reject the deviant offer). In this case, the integrated $D1$ will (in that period) obtain a share of the industry profit that strictly exceeds $1/N$ (since $D1$ faces an effective wholesale price of zero, while either at least one of its rivals faces a higher wholesale price or else at most $N-2$ unintegrated downstream firms are active), which the deviant Ui will not be able to extract. Since the industry profit is bounded from above by the monopoly profit, this means that the deviant Ui can extract strictly less than $(N-1)\Pi^M/N$ from the $N-1$ unintegrated downstream firms. ∎

Proof of lemma 12. The inequality $\pi_{int}^{dev} \leq \Pi^M$ holds trivially: if $\pi_{int}^{dev} > \Pi^M$, at least one downstream firm would make a loss and would thus have a profitable deviation. We now show that $\pi_{int}^{dev} < \Pi^M$ if final goods are differentiated. To see this, note that for the deviant upstream firm to extract all of the monopoly profits when final goods are differentiated (by downstream firm), it would need to sell though all of the N downstream firms, and each of the downstream firms would need to set the price p^M (or quantity $q^M = Q^M/N$). Suppose now that the integrated $U1$-$D1$ offers contracts of the form (w'_{1j}, F'_{1j}) to each unintegrated downstream firm Dj, $j \geq 2$. Suppose further that all unintegrated downstream firms accept and set the monopoly price p^M (or quantity q^M). Because of double marginalization (which must arise when products are differentiated), the wholesale price w'_{1j} must be less than the monopoly price, $w'_{1j} < p^M$. At the downstream pricing stage, the integrated $U1$-$D1$ would then face the following optimization problem:

$$\max_{p_1} p_1 Q(p_1; p^M, p^M, ..., p^M) + \sum_{j \geq 2} w'_{1j} Q(p^M; p_1, p^M, ..., p^M).$$

Since $w'_j < p^M$, the integrated $U1$-$D1$ would thus optimally set a retail price $p_1 < p^M$. But this means that the integrated firm cannot extract all of the monopoly profits. (The same argument applies when downstream competition is in quantities rather than prices.) ∎

Proof of proposition 5. >From lemmas 10 and 13, it follows that vertical integration

facilitates upstream collusion, $\widehat{\delta}_{seq}^{SI} < \widehat{\delta}_{seq}^{NI}$, if

$$(M-1)\left[\pi_{unint}^{dev} + \pi^{NC}\right] + \pi_{int}^{dev} < M \cdot \Pi^M. \tag{15}$$

Lemma 11 implies that $\pi_{unint}^{dev} + \pi^{NC} < \Pi^M$, while lemma 12 states that $\pi_{int}^{dev} \leq \Pi^M$. Hence, equation (15) must hold, and so $\widehat{\delta}_{seq}^{SI} < \widehat{\delta}_{seq}^{NI}$. ∎

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