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PIER Working Paper 04-032

"Reputation and Turnover"

by

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http://ssrn.com/abstract=573421

Reputation and Turnover*

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August 4, 2004

Abstract

We consider a repeated duopoly game where each firm privately chooses its investment in quality, and realized quality is a noisy indicator of the firm's investment. We focus on dynamic reputation equilibria, whereby consumers 'discipline' a firm by switching to its rival in the case that the realized quality of its product is too low. This type of equilibrium is characterized by consumers' tolerance level - the level of product quality below which consumers switch to the rival firm - and firms' investment in quality. Given consumers' tolerance level, we determine when a dynamic equilibrium that gives higher welfare than the static equilibrium exists. We also derive comparative statics properties, and characterize a set of investment levels and, hence, payoffs that our equilibria sustain.

Keywords: Reputation, consumer switching, moral hazard, repeated games.

JEL classification numbers: C73, D82, L14, L15

*Part of this research was conducted while Sekiguchi was visiting the University of Pennsylvania. Sekiguchi thanks the University of Pennsylvania for its kind hospitality. Rob acknowledges NSF support under grant numbers 98-12681 and 01-36922, and Sekiguchi acknowledges financial supports from the Nomura Foundation for Academic Promotion and the Inamori Foundation. We are grateful to Michihiro Kandori, Hitoshi Matsushima, and Yossi Spiegel for helpful discussions. We are also grateful to Ray Deneckere and an anonymous referee for constructive suggestions. We presented the paper at the 2001 Decentralization conference, the 2001 Summer meetings of the Econometric Society, Kyusyu University, the University of Michigan, the University of Tokyo and Hitotsubashi University, where we received many useful comments. Remaining errors are, regrettably, our own.

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1 Introduction

This paper studies markets in which the quality of a product or a service varies over time, and in which consumers' expectations about future quality are determined by past quality. Firms in such markets are subjected to turnover; a firm that suffers a low quality realization loses favor in the eyes of consumers and, as a result, loses market share and profits. Our objective here is to set up and analyze a model focusing on this turnover phenomenon.

The point of view we take is that the presence of turnover disciplines firms in a world of moral hazard. Consider a world in which the quality of a firm's product is influenced by its investments, and in which these investments are private information. Assume also that consumers cannot ascertain quality at the point of purchase and that the legal system does not provide an effective and/or timely protection. Then, the equilibrium quality of products is prone to be minimal and, in particular, below the quality that is socially optimal. If consumers buy the product repeatedly, however, then the option to turn their business over to other firms has the potential to rectify this problem. Indeed, in such circumstances, a firm invests in quality because the delivery of high quality today generates high demand and high profits tomorrow, whereas the delivery of low quality results in low demand and low profits. Our purpose here is to make precise this intuition and study it systematically in the context of an oligopoly.

This line of research relates to two strands of literature. The first strand is the IO literature that originated with Klein and Leffler (1981), where the disciplinary role of markets was first recognized (see also Shapiro (1983)). Compared to this literature we make explicit the phenomenon of turnover. A firm's price, volume of sales, and stock market valuation all fluctuate over time because of the noise in quality, and these fluctuations are endogenous to the model. We also offer a unified and systematic analysis of a variety of duopoly games, including some of the most traditional games encountered in the IO literature. This allows us to link details of the competitive interaction between firms (market size, cost of investing in quality, degree of product differentiation, the noisy connection between investments and realized quality, etc.) and the nature of turnover equilibria.

A second strand of relevant literature is that on repeated games with imperfect monitoring, found in papers such as Fudenberg, Levine and Maskin (1994). This literature shows the full potential of repeated interaction to sustain outcomes that are unsustainable under one-shot interaction, and allows general (and arbitrarily complex) strategies to achieve that. Compared with this latter literature we concentrate on particular, simple strategies that are in the spirit of the IO literature. Then we characterize equilibria via standard marginal conditions, and use familiar fixed point arguments to prove their existence. Using this tractable approach we perform various comparative statics and quantitative exercises, and study welfare properties.

In greater detail, the turnover mechanism we study is characterized by a threshold realized quality level that we call *consumers' tolerance level*. If a firm delivers this, or higher, quality level, consumers believe that this firm is likely to deliver high quality in the future; otherwise, consumers believe that the firm is likely to deliver low quality. We embed this belief into a dynamic duopoly model in which firms and consumers are strategic players. Firms invest in quality and consumers form beliefs about future investments, based on last period's realized quality, and these beliefs determine consumers' purchasing behavior. Using this framework we derive three sets of the results.

The first set of results deals with existence and uniqueness of an equilibrium of the type described above. Given some tolerance level, we derive a closed form expression, written in terms of model primitives, which characterizes a set of investment levels that constitute equilibria. This allows us to pin down a condition under which positive investments are sustainable in equilibrium, so that some dynamic equilibrium is better than the static equilibrium that would occur without repeated purchase. The condition we state relates to the cost of quality provision (requiring it to be sufficiently low), although, as we comment, analogous condition can be stated relating to consumers' demand curve (requiring it to be sufficiently high). Hence, the question whether a dynamic equilibrium under repeated purchase improves upon the static equilibrium is settled by direct reference to model primitives. We also derive a condition under which the equilibrium (with positive investments) is unique for a given tolerance level. This condition, too, relates to model primitives.

The second set of results deals with comparative static properties of the equilibrium which is characterized by the largest investment level among all equilibria. Here we again take advantage of the simple characterization of the equilibrium set, and show that the 'largest' equilibrium is increasing with the discount factor. We also show that the largest equilibrium may increase or decrease in consumers' tolerance level, depending on properties of the noise distribution, and we elicit sufficient conditions under which one or the other occurs. Our analysis here is related to recent literature on monotone comparative statics (see Milgrom and Roberts (1990)) and, more specifically, to the property that a firm's best response curve shifts uniformly upwards or downwards when certain parameters change. As such our approach is easily extendable to other parametric changes, for instance, relating to cost or demand conditions.

The third set of results deals with welfare properties of the model. We delineate a set of investment levels and, thereby, a set of welfare levels that are sustainable via turnover equilibria if firms are sufficiently patient. Our analysis indicates that noisy observation of investments constrains the set of welfare levels that one can sustain via turnover equilibria. Finally, we show that a simple generalization of the turnover mechanism can do no better in terms of welfare.

Many of our results are illustrated numerically, using three parametric examples. Among other things, we illustrate the nature of fluctuations that turnover equilibria induce, which includes the duration of selling high and low quality products, period profits and stock market valuations. We also show how to numerically determine parameter values under which the best constrained equilibrium is achieved.

The rest of the paper is organized as follows. The next section is divided into three subsections. In the first subsection we set up a generally specified period game that is played in each period. In the second subsection we outline three IO-type examples that are special cases of the period game. In the third subsection we show how the period game is embedded into an infinitely repeated game. Section 3 introduces the concepts of tolerance level and turnover equilibria, and analyzes a firm's problem when consumers' belief is based on a tolerance level. Section 4 is divided into two parts. In the first part we prove existence and uniqueness of turnover equilibria, given consumers' tolerance level. In the second part the results are numerically illustrated. Section 5 studies comparative statics properties of the model, which includes sensitivity of equilibria with respect to the discount factor and consumers' tolerance level. Section 6 characterizes a set of welfare levels that are sustained via turnover equilibria with patient firms, and shows that one can do no better via what we call tournament equilibria, which are, essentially, a generalization of turnover equilibria. Section 7 concludes with further discussion of related literature and suggestions for future work.

2 The Model

We consider an infinite horizon model in discrete time, and index time periods by $t = 1, 2, \cdots$. There are two identical, risk-neutral firms who produce and sell to a continuum of price taking, risk-neutral consumers. Firms are indexed by $i, j = 1, 2$ and play a two-stage game within each period. The game is as follows.

2.1 The Period Game

In the first stage firms move simultaneously, choosing their *investments*, which determine the average quality of their products. We let $x_i \geq 0$ denote firm i's investment and x denote either firm's investment. The cost of choosing x is $c(x)$. We assume $c(0) = c'(0) = 0$, $c'(x) > 0$ and $c''(x) > 0$ for all $x > 0$, and $c'(\infty) = \infty$. x is a fixed cost: it affects the quality of *all* units a firm sells and is independent of the number of these units. We assume that x_i is privately chosen by firm i and is unobservable to firm j or to any consumer. We let y_i denote consumers' and firm j's *belief* about x_i. We discuss later how y_i's are determined.

In the second stage firms move simultaneously again, choosing either the quantity they sell or the price they charge (but not both). Since the formulation encompasses both possibilities we refer to quantity or price generically as an *action.* We let $z_i \geq 0$ denote firm i's action and z denote either firm's action.

Then, depending on the choice of actions, the market clears and firms' period profits are determined. The way the market clears depends on (z_1, z_2) and (y_1, y_2) but not on (x_1, x_2), which is not publicly observed, and has no effect on firms' period payoffs gross of $c(x_i)$ (which has already been sunk). We let $u_i(z_1, z_2; y_1, y_2)$ denote firm i's gross period payoff, when firms' action profile is (z_1, z_2) and when they are believed to have chosen (y_1, y_2). These u_i's are symmetric, i.e.,

$$u_1(z_1, z_2; y_1, y_2) = u_2(z_2, z_1; y_2, y_1).$$

Once the market clears and consumers consume the products, the quality of these products is realized. The realized quality of all units that a firm sells (if at all) is the same and is publicly observed.[1] We assume that if firm i's investment is x_i, then the realized quality of its products q_i is

$$q_i = x_i + \epsilon_i,$$

where ϵ_i is a noise term. We denote the joint c.d.f. of (ϵ_1, ϵ_2) by $G(\epsilon_1, \epsilon_2)$. G has a full support on $(-\infty, \infty) \times (-\infty, \infty)$, a $(0, 0)$ mean, and is symmetric, $G(\epsilon, \epsilon') = G(\epsilon', \epsilon)$. Hence the marginal c.d.f. of ϵ_1 is same as that of ϵ_2. We denote it by $F(\epsilon)$, $f(\epsilon)$ being its p.d.f. We assume that F is twice continuously differentiable, and that for any $\epsilon > 0$, $f'(-\epsilon) > 0 > f'(\epsilon)$, so that f is hump-shaped with a single peak at $\epsilon = 0$.[2] Since q_i has a full support $(-\infty, \infty)$ under any feasible investment profile, it cannot serve as a sure signal that firm i deviated from some (in fact any) investment level.

For a fixed (y_1, y_2), the profit functions, $u_i(z_1, z_2; y_1, y_2)$ for $i = 1, 2$, determine a static game in which firms compete in z_i's. Let us denote this game by $\Gamma(y_1, y_2)$, and let us make the following assumptions about its equilibria. We later exhibit three standard models of duopoly, where these assumptions are satisfied.

Assumption 1 *(i) Each $\Gamma(y_1, y_2)$ has a unique pure strategy equilibrium,*

$$(z_1^*(y_1, y_2), z_2^*(y_1, y_2)).$$

(ii) For $i = 1, 2$, let

$$\pi_i(y_1, y_2) \equiv u_i(z_1^*(y_1, y_2), z_2^*(y_1, y_2); y_1, y_2).$$

Then $\pi_i(y_1, y_2)$'s are continuous in (y_1, y_2).

(iii) Let

$$\Delta(x) \equiv \pi_1(x, 0) - c(x) - \pi_1(0, x).$$

[1] If a firm does not sell anything, we set the realized quality of its products to be zero. This specification is for the sake of concreteness, and our results are independent of how this quality is specified.

[2] One special case here is when ϵ_1 and ϵ_2 are independent of each other, and have the same density function, f.

Then $\Delta'(0) > 0$ *and there exists an* $\overline{x} < \infty$ *so that*

$$\{x|\,\Delta(x) > 0\} = (0, \overline{x}).$$

We rule out collusion between firms in terms of the actions (z_1, z_2). Therefore, Assumption 1(i) says that once a belief is fixed at (y_1, y_2), the only rational behavior for firms to follow is $(z_1^*(y_1, y_2), z_2^*(y_1, y_2))$. Assumption 1(ii) says that equilibrium payoffs are continuous in beliefs. Assumption 1(iii) concerns the profit differential Δ between one of the firms that we label the "high quality firm" (or HQ) and that chooses a positive investment, and the other firm that we label the "low quality firm" (or LQ) and that chooses zero investment. Then, Assumption 1(iii) says that the HQ firm gets a higher net period profit (than the LQ firm) as long as its investment is positive, but not too high. If the investment of HQ is too high, the cost of this investment overwhelms the quality advantage it gives rise to, so HQ ends up with lower profits. While 1(iii) is written from firm 1's viewpoint, by symmetry, it also holds for firm 2.

If firms choose (x_1, x_2) and if beliefs are correct, $(y_1, y_2) = (x_1, x_2)$, then the equilibrium play in the second stage determines social welfare for the period. Let us denote it by $W(x_1, x_2)$. If we let expected consumers' surplus under $(y_1, y_2) = (x_1, x_2)$ be $S(x_1, x_2)$, we have

$$W(x_1, x_2) = S(x_1, x_2) + \pi_1(x_1, x_2) + \pi_2(x_1, x_2) - c(x_1) - c(x_2). \qquad (1)$$

One establishes the existence of a maximizer (x_1^o, x_2^o) to (1) under the usual assumptions. We call this maximizer the second-best investment pair. The welfare level that corresponds to (x_1^o, x_2^o) results from product-market competition, which is imperfect and yields a price that is, in general, not equal to marginal cost; this is the reason we call (x_1^o, x_2^o) the 'second best'. We also define the third best investment level x^* as the one for which $(x^*, 0)$ maximizes (1) over all $(x, 0)$'s. Since the investment of one firm is constrained to equal zero under the third best, the welfare level associated with the third best is lower (in general) than the second best welfare level.

2.2 Examples

Now we present three parametric examples of period games that satisfy all our assumptions. The cost of investment in all three examples is quadratic, $c\,(x) = x^2$, and the variable manufacturing cost is 0.[3]

[3] These examples contain parameters, and the values of these parameters determine what type of equilibria arise in the dynamic game that we introduce later (see Section 4). The parameters can also be used to determine how equilibria vary with these parameters.

Example 1 (Cournot with heterogeneous consumers) There is a continuum of consumers, uniformly distributed on the interval $[0, a]$ with density 1. A consumer at $t \in [0, a]$ is referred to as a type t consumer. Each consumer buys zero or one unit, and the gross utility that a type t consumer derives from one unit of quality q product is $t + q$. If the consumer buys this product and pays p for it, her net utility is $t + q - p$. If a consumer (of any type) does not buy, her net utility is zero. Hence a type-t consumer buys a product of expected quality q only if $p \leq t + q$. If a consumer (of any type) has a choice between two products (q_1, p_1) and (q_2, p_2), she chooses the one with the higher $q_i - p_i$ as long as that product gives her a positive surplus; otherwise she does not buy anything.

Let us fix the investment vector (x_1, x_2) and analyze the Cournot game between firms. In this example, firm i's action z_i is quantity to sell and the cost of producing this quantity is 0 (it can be any constant independent of (x_1, x_2)). Given the quantity vector (z_1, z_2), together with a belief (y_1, y_2), the prices of the two firms are determined so that the market clears. Namely, all consumers who are willing to buy at these prices are served, and both firms get rid of the supplies they offer. If both firms sell positive quantities, this means consumers are indifferent between buying the product of firm 1 and firm 2. The market-clearing prices that have these properties are

$$p_i = a - Z + y_i \tag{2}$$

for $i = 1, 2$, where $Z = z_1 + z_2$. Since the marginal cost of production is zero, we obtain, from (2), the following second stage payoff functions

$$u_i(z_1, z_2; y_1, y_2) = z_i(a - Z + y_i), \;\; i = 1, 2.$$

Given these payoff functions one verifies that Assumptions 1(i)-(ii) are satisfied. Indeed the unique period equilibrium is such that

$$(z_1^*(y_1, y_2), z_2^*(y_1, y_2)) = \begin{cases} \left(\frac{a+y_1}{2}, 0\right) & \text{if } y_1 > a + 2y_2 \\ \left(0, \frac{a+y_2}{2}\right) & \text{if } y_2 > a + 2y_1 \\ \left(\frac{a+2y_1-y_2}{3}, \frac{a+2y_2-y_1}{3}\right) & \text{otherwise} \end{cases}$$

and

$$(\pi_1(y_1, y_2), \pi_2(y_1, y_2)) = \begin{cases} \left(\frac{(a+y_1)^2}{4}, 0\right) & \text{if } y_1 > a + 2y_2 \\ \left(0, \frac{(a+y_2)^2}{4}\right) & \text{if } y_2 > a + 2y_1 \\ \left(\frac{(a+2y_1-y_2)^2}{9}, \frac{(a+2y_2-y_1)^2}{9}\right) & \text{otherwise,} \end{cases} \tag{3}$$

which, as required, is continuous in (y_1, y_2). Moreover, using (3) one verifies that the profit differential is

$$\Delta(x) = \begin{cases} \frac{2}{3}x(a - x) & \text{if } \; x < a \\ \frac{a^2-3x^2+2ax}{4} & \text{if } \; x > a \end{cases}. \tag{4}$$

(4) shows that $\Delta(x)$ is concave and

$$\Delta'(0) = \frac{2a}{3} > 0 > \lim_{x \to \infty} \Delta'(x),$$

which imply that Assumption 1(iii) is also satisfied. Indeed, $\overline{x} = a$ in this example.

To compute welfare, let us first note that $S(x_1, x_2) + \pi_1(x_1, x_2) + \pi_2(x_1, x_2)$ is equal to

$$\begin{aligned} & \int_0^{z_1^* + z_2^*} (a - z)dz + z_1^* x_1 + z_2^* x_2 \\ = \quad & \frac{8a^2 + 8a(x_1 + x_2) + 11x_1^2 + 11x_2^2 - 14x_1x_2}{18}. \end{aligned}$$

Since $c(x) = x^2$, social welfare net of investments in quality is

$$\begin{aligned} W(x_1, x_2) \quad &= \quad \frac{8a^2 + 8a(x_1 + x_2) + 11x_1^2 + 11x_2^2 - 14x_1x_2}{18} - (x_1^2 + x_2^2) \\ &= \quad \frac{8a^2 + (x_1 + x_2)[8a - 7(x_1 + x_2)]}{18}. \end{aligned} \tag{5}$$

From (5) we see that any (x_1^o, x_2^o) such that

$$x_1^o + x_2^o = \frac{4a}{7}$$

is a second best investment pair. In particular, $(\frac{4a}{7}, 0)$ is a second best pair and, thus, $x^* = \frac{4a}{7}$ is the third best. In this example, therefore, the third best social welfare coincides with the second best level.

Example 2 (Bertrand with homogenous consumers) In this example, there is a continuum of identical consumers whose measure is a. Again, they have unit demands, and the gross benefit to a consumer who buys one unit of a quality-q product is q. If the consumer pays p for this product her net benefit is $q - p$. Thus, if a consumer buys from a firm which is believed to have chosen investment y and if the consumer pays price p, the consumer's expected net benefit is $y - p$. If a consumer buys nothing she gets zero net benefit.

In this game firms compete in prices at the second stage, so z_i denotes firm i's price. Variable costs continue to be zero. Given a price vector (z_1, z_2) and a belief (y_1, y_2), all consumers buy from the firm that gives them a higher consumer's surplus, i.e., they choose the i for which $y_i - z_i$ is maximal and non-negative (otherwise they don't buy). If $y_1 - z_1 = y_2 - z_2 \geq 0$ and $y_i > y_j$, consumers buy from firm i; if $y_1 = y_2$, $z_1 = z_2$, and $y_i - z_i \geq 0$, consumers divide equally between firms. Since variable costs are zero, the game $\Gamma(y_1, y_2)$ is similar to the Bertrand game with different quality levels and, in its equilibrium, a firm that is believed to provide the higher quality product makes sales, while its competitor makes no sales. As a result, if it is believed that $y_i > y_j$, then firm i serves the whole market at the price $y_i - y_j$, and we have $\pi_i(y_1, y_2) = a(y_i - y_j)$ and $\pi_j(y_1, y_2) = 0$. Clearly $\pi_i(y_1, y_2)$'s are continuous. Moreover, the profit differential is

$$\Delta(x) = ax - x^2 = x(a - x),$$

which is concave and

$$\Delta'(0) = a > 0 > \Delta'(\infty),$$

so Assumption 1 is again satisfied.

Concerning welfare, the social welfare function in this example is

$$W(x_1, x_2) = a \max\{x_1, x_2\} - c(x_1) - c(x_2).$$

This function is maximized if one of the x_i's is chosen to equal zero, while the other is chosen to satisfy $c'(x^*) = 1$. This gives $x^* = a/2$ for the quadratic cost case. Furthermore, in this example if firm i chooses $x_i > 0$ and firm j chooses $x_j = 0$ and if consumers' belief is correct, then firm i fully extracts consumers' surplus. But, then, firm i's objective coincides with the social objective, so there is no distortion in the second stage. Hence, the third best in this example coincides with the *first* best.

Example 3 (Bertrand with heterogeneous consumers) One curious feature of the previous example is that in its equilibrium one firm makes all the sales, while the other firm sells zero. This feature is due to the assumption of homogeneous consumers. The present example assumes heterogeneous consumers, so that the equilibrium has the more realistic feature that both firms make positive sales.

There is again a continuum of consumers whose measure is 1, but they are not identical. Instead, consumers are uniformly distributed on $[0, 1]$ and are indexed by t. A type t consumer derives gross utility tq from one unit of a quality-q product, and net utility of $tq - p$ if she pays p for this product. Consumers buy at most one unit and choose the product that gives them a higher consumer's surplus, provided this surplus is non-negative (otherwise they don't buy). The minimum quality that a firm supplies is $a \geq 0$, so that if $0 \leq x \leq a$, the resulting average quality is a and if $x > a$, the resulting average quality is x. The cost of investing x is $c(x) = (x - a)^2$. Therefore, it is wasteful for a firm to choose $x \in [0, a)$.

In this example there is a unique equilibrium in which the highest types buy the high quality product, lower types buy the low quality product and the lowest types don't buy anything. If $y_1 > y_2$, equilibrium prices are

$$z_1^*(y_1, y_2) = y_1 \frac{2(y_1 - y_2)}{4y_1 - y_2}, \quad z_2^*(y_1, y_2) = y_2 \frac{y_1 - y_2}{4y_1 - y_2}$$

and equilibrium payoffs are

$$\pi_1(y_1, y_2) = 4y_1 \frac{y_1(y_1 - y_2)}{(4y_1 - y_2)^2}, \quad \pi_2(y_1, y_2) = y_2 \frac{y_1(y_1 - y_2)}{(4y_1 - y_2)^2} \; . \tag{6}$$

As equation (6) shows, the $\pi_i(y_1, y_2)$'s are continuous in (y_1, y_2). Furthermore, (6) shows that equilibrium profits and quantities of both firms are positive as long as $y_2 > 0$, which holds if $a > 0$. Regarding the profit differential we have

$$\Delta(x) = (x - a)\left[\frac{x}{4x - a} - (x - a)\right].$$

Δ is therefore single peaked and positive over $x \geq a$ if and only if

$$a \leq x < \overline{x} \equiv \frac{5a + 1 + \sqrt{9a^2 + 10a + 1}}{8}.$$

So Assumption 1 is satisfied here too.

Concerning welfare we have

$$W(x, 0) = \frac{6x^3 - \frac{ax^2}{2} - a^2 x}{(4x - a)^2} - (x - a)^2.$$

This function is single peaked over $x \geq a$ and has a unique maximizer x^*.

Before concluding this subsection, let us note that if the period game described in subsection 2.1 is played just once, then firms are subject to the usual moral hazard problem, i.e., the unique equilibrium is for firms to invest 0. Indeed, suppose that (x_1, x_2) is an equilibrium investment pair of the static (two stage) game. Then $(y_1, y_2) = (x_1, x_2)$ is the only consistent belief given such an equilibrium. As a result, firm i earns the gross profit of $\pi_i(x_1, x_2)$, regardless of how much it invests in quality. Given that, firms optimally choose to invest 0. Thus $(0, 0)$ is the only equilibrium investment profile. Therefore, in a static play firms do not invest in quality. In the next section we show that this conclusion is no longer true in a dynamic play: we identify conditions under which firms do invest in equilibrium, and conditions under which the equilibrium implements, in fact, the third best efficient outcome (which, as we saw in the examples, is sometimes the second best or even the first best).

2.3 The Repeated Game

The period game specified in subsection 2.1 is played every period, $t = 1, 2, ...$ Firms and consumers observe all realized qualities and firms observe their own investments and actions in the past. They evaluate the stream of per-period payoffs according to the long run weighted average of these period payoffs, where the weight on profits received at t is $(1 - \delta)\delta^{t-1}$, $\delta \in (0, 1)$ being the common discount factor. That is, the long run payoff to a firm from the stream of (short run) period payoff $(\pi_t)_{t=1}^{\infty}$ is

$$v = (1 - \delta)\sum_{t=1}^{\infty} \delta^{t-1}\pi_t.$$

3 Turnover Equilibrium

We focus on certain sequential equilibria of the repeated game that we call *turnover equilibria*. A turnover equilibrium has the feature that one firm is believed to invest more than the other firm in quality. If the quality this firm delivers is above some threshold level, this belief is maintained into the next period. Otherwise, consumers are 'disappointed' and their belief is turned over, i.e., they start believing that the other firm is the one that invests more in quality.

More precisely, a turnover equilibrium is characterized by a pair $(x_H, \overline{q})$, the outcome of which is as follows. In each period, one firm is believed to invest x_H in product quality. We call this firm the *high quality firm*, or, in short, the HQ firm of that period. The other firm is called the *low quality firm*, or, in short, the LQ firm, and it is believed to invest 0. In each period, the HQ firm optimally chooses x_H, while the LQ firm optimally chooses 0 (the optimality of these choices is verified later). Given consumers' belief (which is either $(x_H, 0)$ or $(0, x_H)$, depending on which firm is HQ), the market clears in the way described in subsection 2.1. As discussed earlier, market clearing is independent of firms' actual investments in quality, so that if HQ deviates from x_H this has no effect on market clearing.

The rule that specifies who is the HQ firm in each period is as follows. The HQ firm of period 1 is firm 1.[4] Let a period $t > 1$ and the HQ firm of period $t-1$ be given. If the realized quality of the HQ firm in period $t-1$ was above $\overline{q}$, this firm continues to be the HQ firm of period t. Otherwise, turnover occurs, and it becomes the LQ firm of period t, and the firm that was the LQ firm of period $t-1$ becomes the HQ firm of period t. We refer to $\overline{q}$ as *consumers' tolerance level*. Note that the realized quality of the LQ firm has no influence on the occurrence of turnover.

Next we determine the conditions required to make the strategy profile corresponding to the above description a sequential equilibrium of the repeated game. Let v_H and v_L be the average long run payoffs of HQ and LQ, respectively, given $(x_H, \overline{q})$ and given the discount factor δ. Since turnover occurs with probability $F(\overline{q} - x_H)$, these payoffs are determined by the following equations

$$v_H = (1-\delta)\pi_H + \delta[F(\overline{q} - x_H)v_L + (1 - F(\overline{q} - x_H))v_H] \tag{7}$$

$$v_L = (1-\delta)\pi_L + \delta[F(\overline{q} - x_H)v_H + (1 - F(\overline{q} - x_H))v_L], \tag{8}$$

where $\pi_H \equiv \pi_1(x_H, 0) - c(x_H)$ and $\pi_L \equiv \pi_1(0, x_H)$.

From (7) and (8) we get

$$v_H - v_L = \frac{(1-\delta)\Delta(x_H)}{1 - \delta[1 - 2F(\overline{q} - x_H)]}. \tag{9}$$

[4] That, obviously, is an arbitrary specification; a 'twin' equilibrium exists whereby firm 2 is the HQ firm of period 1.

Since turnover is independent of the realized quality of LQ, there is no incentive constraint for LQ: it is just optimal for LQ to choose its static best response; namely, 0. The only incentive constraint is for HQ: it must be in HQ's best interest to choose x_H. So let us study the best response problem facing HQ. Assume HQ faces the belief y. Its choice of x has no effect on y and, hence, has no effect on HQ's gross period payoff, which is $\pi_1(y,0)$. The only effect that x has is on the cost $c(x)$ and on the turnover probability $F(\overline{q}-x)$. Consequently, the objective of HQ is

$$(1-\delta)[\pi_1(y,0)-c(x)]+\delta[F(\overline{q}-x)v_L+(1-F(\overline{q}-x))v_H].$$

After some rearrangement and substitution from (9), HQ's objective is to maximize

$$-(1-\delta)c(x)-\delta F(\overline{q}-x)\frac{(1-\delta)\Delta(y)}{1-\delta[1-2F(\overline{q}-y)]}+(1-\delta)\pi_1(y,0)+\delta v_H \quad (10)$$

with respect to x. Considering an interior maximum, the first-order condition is:

$$-c'(x)+\delta f(\overline{q}-x)\frac{\Delta(y)}{1-\delta[1-2F(\overline{q}-y)]}=0. \quad (11)$$

Let $R(y)$ be the set of solutions (possibly empty or multi valued) to (11). Our first result states a condition under which there is a unique solution to (11), which is in fact the maximizer to (10). Furthermore, the condition guarantees that this solution is continuous in y.

Lemma: Assume $\overline{q}<0$ and $y<\overline{x}$, where $\overline{x}$ is defined in Assumption 1. Then $R(y)$ is a singleton, and is the maximizer of (10). Furthermore, $R(y)$ is continuous in y.

Proof. Since $f'(\overline{q}-x)>0$ for $\overline{q}<0$ and $\Delta(y)\geq 0$ for $y<\overline{x}$, $\overline{q}<0$ and $y<\overline{x}$ guarantee that HQ's objective is strictly concave over $x\geq 0$. Hence $R(y)$ characterizes a unique maximizer of (10). Furthermore, since $c'(\infty)=\infty$, the optimal x can be no larger than some $\widetilde{x}<\infty$. Therefore, the theorem of the maximum applies, and the result follows. ■

From this point onward we limit attention to $\overline{q}<0$ and $y<\overline{x}$, in order to take advantage of the first-order approach and the continuity of R.

A turnover equilibrium is defined now by the fixed point requirement that $R(x_H)=x_H$ or, equivalently, $(x_H,\overline{q})$ is a *turnover equilibrium* if x_H is a solution to

$$h(x)\equiv -c'(x)+\delta f(\overline{q}-x)\frac{\Delta(x)}{1-\delta[1-2F(\overline{q}-x)]}=0. \quad (12)$$

We also say that x_H is a *turnover equilibrium under* $\overline{q}$ if $(x_H,\overline{q})$ is a turnover equilibrium. In subsequent sections we study the existence of a turnover equilibrium, and then establish its comparative statics properties.

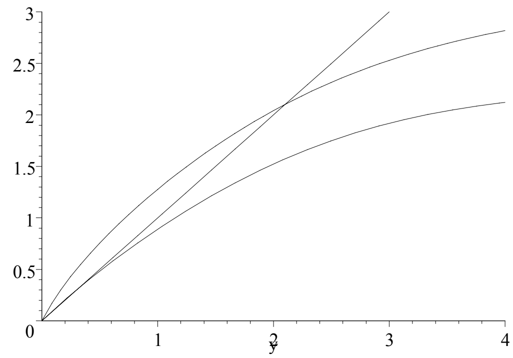

Figure 1: Two best response curves for Example 2 when $f(\epsilon) = \frac{1}{2}e^{\epsilon}$ for $\epsilon < 0$, $\delta = 0.9$, $\overline{q} = -1$ and $a = 10$, 20

4 Existence

In this section we prove the existence of a turnover equilibrium, which is equivalent to the existence of a solution to (12) or a fixed point to $R(x) = x$. To start with, let us note that a trivial turnover equilibrium exists for *any* fixed tolerance level $\overline{q}$; namely, $(0, \overline{q})$. Consumers' belief in this equilibrium is so low (i.e., $y = 0$) that firms invest $x = 0$ in quality - irrespective of consumers' tolerance level. The outcome of this equilibrium is an indefinite repetition of the static equilibrium of the period game.

4.1 General Results

Obviously, we are interested in ensuring the existence of more efficient equilibria, i.e., equilibria with $x_H > 0$. To illustrate what it takes to ensure that such equilibria exist, we depict two best response curves R, for Example 2 under two different parameterizations. This is shown in Figure 1, where both curves are drawn for example 2 under $f(\epsilon) = \frac{1}{2}e^{\epsilon}$ for $\epsilon < 0$, $\delta = 0.9$, $\overline{q} = -1$. One curve - the low one - is drawn for low demand, i.e., it specifies $a = 10$ and, consequently, it lies everywhere below the 45^0 line, except at $y = 0$. The other curve is drawn for high demand, i.e., it specifies $a = 20$ and, consequently, it rises above the 45^0 line for small y's. Both curves are hump shaped, and lie below the 45^0 line for large y's.

Comparing the two curves, we see that the low demand curve intersects the 45^0 line only once at $y = 0$ and yields only the trivial equilibrium, whereas the high demand curve intersects the 45^0 line also at a positive y, so it yields a nontrivial equilibrium. The conclusion we draw from this is that to ensure the existence of a nontrivial equilibrium it suffices to ensure that R is steep enough at $y = 0$, and one way to ensure that is to require that demand is sufficiently high.

An analogous way to ensure the existence of a nontrivial equilibrium, which is the approach we take in Proposition 1, is to require that the investment cost function is sufficiently low (at least for small investments). Then the incentive to invest is strong, which produces the same effect, i.e., it makes the best response curve steep at $y = 0$. More precisely, we invoke the following assumption:

Assumption 2 *$c''(0) = 0$.*

A natural family of cost functions that satisfy Assumption 2 is $c(x) = kx^{\alpha}$ for $\alpha > 2$ and any $k > 0$. If $\alpha \leq 2$, this assumption is obviously not satisfied, but we can still ensure the existence of an equilibrium by requiring k to be small enough. This again will have the effect of making R have a slope greater than 1 at $y = 0$, which is all that one needs to get a nontrivial equilibrium.

Proposition 1 *Suppose Assumption 2 holds. Then, for any $\overline{q} < 0$ and $\delta \in (0, 1)$,*

(i) an $x_H > 0$ exists so that $(x_H, \overline{q})$ is a turnover equilibrium under δ, and

(ii) the set of all x_H's such that $(x_H, \overline{q})$ is a turnover equilibrium is compact.

Proof. As stated above, we want to show that there is a positive x for which $R(x) = x$. Let us first observe that, since $\Delta(\overline{x}) = 0$, we have $0 = R(\overline{x}) < \overline{x}$.

Invoking Assumptions 1 and 2 together with $\Delta(0) = 0$, we also have that

$$h'(0) = \delta f(\overline{q}) \frac{\Delta'(0)}{1 - \delta[1 - 2F(\overline{q})]} > 0.$$

Hence, there exists an $x_1 > 0$ such that $h(x_1) > 0$. (12) shows that $h(x_1) > 0$ implies $R(x_1) > x_1$. Therefore, since $R(x_1) > x_1$ and $R(\overline{x}) < \overline{x}$, and since R is continuous, there exists an x_H, $0 < x_1 < x_H < \overline{x}$, so that $R(x_H) = x_H$. Hence (i) is proven.

Since R is continuous, the set of its fixed points is closed. The above argument shows that any fixed point x_H must satisfy $x_H < \overline{x} < \infty$, i.e., the set of fixed points is bounded. This proves (ii). ■

Note that Proposition 1 places no restrictions on δ. As long as $c''(0) = 0$ and $\overline{q} < 0$ the existence of a nontrivial turnover equilibrium is guaranteed no matter how small δ is. This is not to say that the equilibrium x_H does not depend on δ; it certainly does. However, the *mere existence* of a nontrivial equilibrium is independent of the value of δ.

Since the equilibrium x_H in the above proof is generated as a fixed point of a continuous function and since a continuous function may have more than one fixed point, the nontrivial equilibrium under a fixed $\overline{q}$ need not be unique. Figure

1 shows, however, that the nontrivial equilibrium *is* unique in some examples, and the next result generalizes this observation. To prove this result we need an extra assumption, which is

Assumption 3 $\frac{\Delta(x)}{c'(x)}$ *is decreasing in* x.

Suppose the investment cost in Examples 1, 2 and 3, which we specified in subsection 2.2, is cubic rather that quadratic, $c(x) = x^3$ (with $c(x) = (x-a)^3$ in Example 3). Then, all three examples satisfy Assumptions 2 and 3 under this cost specification. This shows that the next result holds non-vacuously.

Proposition 2 *Suppose Assumptions 2 and 3 hold. Then, for any* $\overline{q} < 0$, *there exists a* $\overline{\delta}$ *such that for any* $\delta \leq \overline{\delta}$, *the nontrivial turnover equilibrium under* $\overline{q}$ *is unique.*

Proof. Fix some $\overline{q} < 0$ and $\delta \in (0,1)$. Then, by Assumption 2 and Proposition 1, we know that a nontrivial turnover equilibrium exists. So let $x > 0$ be such an equilibrium. Then it must satisfy the fixed point requirement (12), which is rewritten as

$$1 = \delta \frac{f(\overline{q}-x)}{1-\delta[1-2F(\overline{q}-x)]} \frac{\Delta(x)}{c'(x)}. \tag{13}$$

Let us evaluate the derivative of the RHS of (13) at this x. Differentiating the RHS and using the equality to simplify, we obtain

$$-\frac{f'(\overline{q}-x)}{f(\overline{q}-x)} + \left(\frac{\Delta(x)}{c'(x)}\right)' \frac{c'(x)}{\Delta(x)} + \frac{2\delta f(\overline{q}-x)}{1-\delta[1-2F(\overline{q}-x)]}. \tag{14}$$

Since $\overline{q} < 0$ is fixed, both $f(\overline{q}-x)$ and $f'(\overline{q}-x)$ are continuous and bounded away from 0 for $x \in [0, \overline{x}]$, which implies that $\gamma \equiv \min_{x\in[0,\overline{x}]} \frac{f'(\overline{q}-x)}{f(\overline{q}-x)} > 0$. Also, $x > 0$ together with assumption 3 imply that $\left(\frac{\Delta(x)}{c'(x)}\right)' \frac{c'(x)}{\Delta(x)}$ is well defined and non-positive. Furthermore, $x \in [0, \overline{x}]$ and the single-peakedness of f imply that $\frac{2\delta f(\overline{q}-x)}{1-\delta[1-2F(\overline{q}-x)]} \leq \frac{2\delta f(0)}{1-\delta}$. Putting these three facts together we conclude that (14) is no larger than

$$-\gamma + \frac{2\delta f(0)}{1-\delta}. \tag{15}$$

Let $\overline{\delta}$ be such that (15) is negative for all $\delta \leq \overline{\delta}$. Then, the function on the RHS of (13) is negatively sloped at any point at which it crosses the horizontal line with height equal to 1. This implies there can be at most one such crossing point, i.e., the equilibrium is unique. ■

The analysis, therefore, shows that the multiplicity of turnover equilibria for a fixed $\overline{q}$ is possible, under Assumptions 2 and 3, only for a large δ. This is analogous to Folk-Theorem-type results, where the multiplicity of equilibria is shown for large δ's, but not (necessarily) for small δ's. One major difference, however, is that for *any* δ, whether it is small or not, we have a nontrivial turnover equilibrium with a positive investment level.

4.2 Numerical Illustration of the Dynamic

As described earlier, turnover equilibria are characterized by the turnover of consumers' belief. In addition, they exhibit the turnover of more tangible variables such as prices and quantities. Consider, in particular, the three examples introduced in subsection 2.2. Then, when a firm loses its HQ status and becomes LQ, the average quality of its products deteriorates, it loses market share, it receives a lower price, it earns lower period profit, and its stock market valuation goes down. Therefore, the well being of firms is subject to fluctuations; a firm gets large period profits for a while, then it gets small profits, only to eventually return to its large profit state. In this subsection we numerically illustrate the fluctuations that a turnover equilibrium induces. We do this by selecting one particular example and assigning numerical values to its parameters. Then, we solve for the nontrivial equilibrium of this example, and compute the values of various endogenous variables that are associated with this equilibrium.

To be specific we adopt the following procedure. Given some $\overline{q}$ and δ we determine the equilibrium investment level x_H from equation (12). Then, we compute the turnover probability, $F = F(\overline{q} - x_H)$, the duration of being in the HQ (and the LQ) state, $T = \frac{1}{F}$, the period profit of HQ and LQ, π_H and π_L, and the corresponding average long run profit, v_H and v_L. Also, since there are multiple equilibria, depending on which $\overline{q}$ is the equilibrium tolerance level, we illustrate our results for 2 different $\overline{q}$ values. Relatedly, we illustrate our results for 2 different discount factors. All these computations are done for Example 1 with cubic cost, $a = 20$, and $f(\epsilon) = \frac{1}{2}e^{\epsilon}$ for $\epsilon < 0$. We report the results in the following table.

	$(0.9, -1)$	$(0.95, -1)$	$(0.9, -2)$	$(0.95, -2)$
x	1.101	1.340	0.770	1.016
F	0.061	0.048	0.031	0.024
T	16.342	20.773	31.927	40.843
π_H	53.433	54.750	51.098	52.890
π_L	39.687	38.686	41.087	40.041
v_H	49.831	49.557	49.293	49.794
v_L	43.289	43.879	42.892	43.137

Table 1: The values of various endogenous variables for 4 parameter configurations.

As this table shows, equilibrium investments increase in δ and $\overline{q}$. The next section provides an analytical counterpart to these numerical results, giving conditions under which these results hold in general.

5 Comparative Statics

This section studies the comparative static properties of turnover equilibria. Our approach here bears some resemblance to the literature on monotone comparative statics; see Milgrom and Roberts (1990). In particular, we show that changes in certain parameters shift the best response curve R upwards at all points of its domain, which makes the largest equilibrium shift upwards as well. Note, however, that R itself is not upward sloping and the underlying game is not supermodular. Nonetheless, it is still possible to prove and take advantage of the property that the best response curve shifts uniformly in our setting.

To pursue this approach we assume that a nontrivial turnover equilibrium exists, and that the set of equilibria is compact. Proposition 1 provides a set of sufficient conditions to ensure that. Given this assumption, we know that a largest turnover equilibrium exists for any pair $(\delta, \overline{q})$. We denote this turnover equilibrium by $\overline{x}(\delta, \overline{q})$, and prove comparative statics results with respect to this equilibrium. Since our approach is to show that changes in parameters shift R, we make the dependence of R on $(\delta, \overline{q})$ explicit in this section and write $R(y; \delta, \overline{q})$.

5.1 Patience

We start by studying the effect of firms' patience parameter δ on the equilibrium investment in quality. The following result conforms with the usual intuition that more patient firms try harder to keep a good reputation, which they do by investing more in quality.

Proposition 3 *Fix a $\overline{q} < 0$. Then $\overline{x}(\delta, \overline{q})$ is increasing in δ.*

Proof. We first show that $R(y; \delta, \overline{q})$ is increasing in δ for (arbitrarily) fixed y and $\overline{q} < 0$. In other words, we show that for any $\delta \in (0, 1)$ and $\delta' > \delta$,

$$R(y; \delta', \overline{q}) \geq R(y; \delta, \overline{q}). \tag{16}$$

Let $x = R(y; \delta, \overline{q})$. Then (16) immediately follows if $x = 0$, so let us assume otherwise. Then (11) implies that

$$-c'(x) + \delta f(\overline{q} - x) \frac{\Delta(y)}{1 - \delta[1 - 2F(\overline{q} - y)]} = 0. \tag{17}$$

One readily verifies that the LHS of (17) is increasing in δ (note that (17) requires that $\Delta(y) > 0$). Therefore (17) implies

$$-c'(x) + \delta' f(\overline{q} - x) \frac{\Delta(y)}{1 - \delta'[1 - 2F(\overline{q} - y)]} \geq 0, \tag{18}$$

since $\delta' > \delta$. Since HQ's objective is concave, (18) implies that $R(y; \delta', \overline{q}) \geq x = R(y; \delta, \overline{q})$.

Specializing this to $y = \overline{x}(\delta, \overline{q})$, we get

$$R(\overline{x}(\delta, \overline{q}); \delta', \overline{q}) \geq R(\overline{x}(\delta, \overline{q}); \delta, \overline{q}) = \overline{x}(\delta, \overline{q}),$$

where the last equality follows from the definition of $\overline{x}(\delta, \overline{q})$. Since $R(x; \delta', \overline{q}) = 0$ for all sufficiently large x's, there exists an $x_H \geq \overline{x}(\delta, \overline{q})$ so that $R(x_H; \delta', \overline{q}) = x_H$. Hence, $\overline{x}(\delta', \overline{q}) \geq \overline{x}(\delta, \overline{q})$. ■

5.2 Tolerance Level

Next, we determine how $\overline{x}(\delta, \overline{q})$ varies with $\overline{q}$. The question we ask is whether less tolerant consumers, a larger $\overline{q}$, encourage or discourage the HQ firm to invest more in quality.

The effect of an increase in $\overline{q}$ is not as clear cut as the effect of an increase in δ. An increase in $\overline{q}$ either raises or lowers the marginal return to investment, $\delta f(\overline{q}-x)\frac{\Delta(y)}{1-\delta[1-2F(\overline{q}-y)]}$, and, therefore, either raises or lowers the best response curve, R. This depends on which of two opposing effects dominates. The first effect is that an increase in $\overline{q}$ increases the marginal probability $f(\overline{q}-x)$ that a turnover is averted, and this increases the marginal return to investment. The second effect is that an increase in $\overline{q}$ increases the rate of turnover in the industry, $F(\overline{q}-x)$. This decreases the payoff from averting turnover and, thus, decreases the marginal return to investment. In general either effect maybe stronger than the other, depending on model primitives. Given this, our approach here is to elicit sufficient conditions under which one effect dominates the other, so that the overall effect of a change in $\overline{q}$ has a definitive sign.

These sufficient conditions involve the hazard rate function

$$H(\epsilon) \equiv \frac{f(\epsilon)}{F(\epsilon)},$$

which partially summarizes the two effects discussed above.[5]

Our first result here is that if H is increasing, then investments increase in $\overline{q}$. We state the condition and the result as follows.

Assumption 4 *For any $\epsilon < 0$,*

$$H'(\epsilon) \geq 0.$$

Since H is the derivative of $\log F$, Assumption 4 is equivalent to logconvexity of F.

Proposition 4 *Suppose Assumption 4 holds. Then for any δ, $\overline{x}(\delta, \overline{q})$ is increasing in $\overline{q}$ on $(-\infty, 0)$.*

Proof. Fix some $\delta \in (0,1)$ and $q^1 < q^2 < 0$, and let $\overline{x}^1 = \overline{x}(q^1, \delta)$, $\overline{x}^2 = \overline{x}(q^2, \delta)$. Since the assertion immediately holds if $\overline{x}^1 = 0$, let us assume $\overline{x}^1 > 0$, so we have:

$$-c'(\overline{x}^1) + \delta f(q^1 - \overline{x}^1)\frac{\Delta\left(\overline{x}^1\right)}{1-\delta\left[1-2F\left(q^1-\overline{x}^1\right)\right]} = 0. \tag{19}$$

[5] Although they are not formally used in our proofs, two interpretations of H may help explain its role. The first interpretation is that H is the long run return to an increase in investments, assuming consumers follow cutoff rules. The second interpretation is that H is the probability that a small increase in investment averts turnover - conditional on the event that turnover is about to occur.

Let's rewrite (19) as

$$-c'(\overline{x}^1) + \frac{\delta\Delta\left(\overline{x}^1\right)}{\frac{1-\delta}{f(q^1-\overline{x}^1)} + \frac{2\delta}{H(q^1-\overline{x}^1)}} = 0. \tag{20}$$

Now, since $\frac{1}{f(q^1-\overline{x}^1)}$ and $\frac{1}{H(q^1-\overline{x}^1)}$ are both decreasing in q^1, we infer that

$$-c'(\overline{x}^1) + \frac{\delta\Delta\left(\overline{x}^1\right)}{\frac{1-\delta}{f(q^2-\overline{x}^1)} + \frac{2\delta}{H(q^2-\overline{x}^1)}} \geq 0$$

or that

$$-c'(\overline{x}^1) + \delta f(q^2 - \overline{x}^1)\frac{\Delta\left(\overline{x}^1\right)}{1-\delta\left[1-2F\left(q^2-\overline{x}^1\right)\right]} \geq 0.$$

Consequently, $R(\overline{x}^1; \delta, q^2) \geq \overline{x}^1$. Therefore, as in the proof of Proposition 3, there exists an $x_H \geq \overline{x}^1$ so that $R(x_H; \delta, q^2) = x_H$, which implies that $\overline{x}^2 \geq \overline{x}^1$. $\blacksquare$

Examples of logconvex distributions include $f(\epsilon) = \frac{1}{2}e^{\epsilon}$ for $\epsilon < 0$, which is the distribution underlying Figure 1 and the numerical illustration in subsection 4.2. Then, $\log F(\epsilon) = \epsilon$ (up to an additive constant), which is indeed (weakly) convex. More generally, consider the family of functions

$$F(\epsilon) = \begin{cases} \frac{1}{2}\exp\left[-g(-\epsilon)\right] & \text{if } \epsilon < 0 \\ 1 - \frac{1}{2}\exp\left[-g(\epsilon)\right] & \text{if } \epsilon \geq 0, \end{cases} \tag{21}$$

where $g : [0, \infty) \to R$ is a twice continuously differentiable function for which $g(0) = 0$, $g(\infty) = \infty$, and $g'(\epsilon) > 0 > g''(\epsilon)$. Any member from this family satisfies

$$H'(\epsilon) = -g''(-\epsilon) > 0.$$

for any $\epsilon < 0$. Another example of a member from this family is constructed by letting $g(\epsilon) = \log(\epsilon + 1)$.

We next consider the case where H is decreasing, i.e., the case of a logconcave F. For this case we prove a partial converse to Proposition 4. It is a converse because we prove that investments *decrease* rather than increase in $\overline{q}$. It is partial because we have to impose one extra restriction, namely, that firms are sufficiently patient.

The proof of Proposition 4 makes clear why an extra restriction is needed. Indeed, if we look at equation (20) we see - under logconcavity - that an increase in q^1 affects $\frac{1-\delta}{f(q^1-\overline{x}^1)}$ and $\frac{2\delta}{H(q^1-\overline{x}^1)}$ in opposite directions (which is not the case under logconvexity). However, if δ is sufficiently large, the effect on $\frac{2\delta}{H(q^1-\overline{x}^1)}$ dominates, and investments decrease in q^1. In proving our next result one follows the same logic as in the proof of Proposition 4, except for the qualification we just mentioned. Consequently, we state the result and provide only a sketch of the proof.

Assumption 5 *For any $\epsilon < 0$,*

$$H'(\epsilon) < 0. \tag{22}$$

Proposition 5 *Suppose Assumption 5 holds. Then, for any $M > 0$, there exists a $\underline{\delta}$ so that for any $\delta \geq \underline{\delta}$, $\overline{x}(\delta, \overline{q})$ is decreasing in q on $(-M, 0)$.*

Sketch of the Proof. Since the interval $[-M, 0]$ is compact for any fixed $M > 0$, there exists $\underline{\delta}$ such that

$$\frac{1-\delta}{f(\epsilon)} + \frac{2\delta}{H(\epsilon)}$$

is increasing in ϵ on $[-M, 0]$ for any $\delta \geq \underline{\delta}$. Now the result follows, if we fix $\delta \geq \underline{\delta}$ and $\overline{q}$'s on $(-M, 0)$ and apply the same argument employed in the proof of Proposition 4. ■

In analogy with logconvexity, Assumption 5 is satisfied for distributions of the form given in (21), except that g is required to be convex rather than concave, and that it satisfies

$$(g'(\epsilon))^2 - g''(\epsilon) > 0. \tag{23}$$

(23) is necessary to ensure that $f'(\epsilon) > 0$ for $\epsilon < 0$. Concrete examples include $g(\epsilon) = \exp(\epsilon) - 1$. We also note that many textbook examples have a logconcave c.d.f; this includes the normal and the extreme value distribution, $F(\epsilon) = \exp[-\exp(-\beta\epsilon)]$, where $\beta > 0$.

6 Welfare

We conclude our analysis by characterizing a set of investment levels that can be sustained by turnover equilibria for some tolerance level and for sufficiently patient firms.

Proposition 6 *Let*

$$X^* = \left\{ x > 0 : \sup_{\overline{q}<0} H(\overline{q} - x)\,\Delta(x) > 2c'(x) > \inf_{\overline{q}<0} H(\overline{q} - x)\,\Delta(x) \right\}. \tag{24}$$

Then for any $x \in X^*$, there exists a $\underline{\delta}$ such that for any $\delta \geq \underline{\delta}$, there exists a $\overline{q}$ so that $(x, \overline{q})$ is a turnover equilibrium.

Proof. Fix some $x \in X^*$. Then, by (24), there exist $q', q'' < 0$ so that

$$H(q' - x)\,\Delta(x) > 2c'(x) > H(q'' - x)\,\Delta(x).$$

Therefore, there exists a $\underline{\delta}$, so that for any $\delta \geq \underline{\delta}$,

$$\frac{\delta f(q' - x)}{1 - \delta[1 - 2F(q' - x)]}\Delta(x) > c'(x) > \frac{\delta f(q'' - x)}{1 - \delta[1 - 2F(q'' - x)]}\Delta(x). \tag{25}$$

(25) implies that for any $\delta \geq \underline{\delta}$, there exists a $\overline{q} < 0$ such that

$$c'(x) = \frac{\delta f(\overline{q} - x)\,\Delta(x)}{1 - \delta\left[1 - 2F(\overline{q} - x)\right]},$$

which is (12). Hence, x is a turnover equilibrium under $\overline{q}$ and δ. ■

Proposition 6 delineates a set of investment levels and, therefore, a set of welfare levels that are asymptotically (as $\delta \to 1$) sustainable via turnover equilibria. In particular, if $x^* \in X^*$, then some turnover equilibrium sustains the third best investment level x^*, the socially optimal investment level of one firm given that the other firm chooses zero. Remember that the LQ firm invests zero in any turnover equilibrium, so this is the best outcome we can hope for from the turnover mechanism. One implication of Proposition 6 is that if $x^* \notin X^*$, then efficiency or even constrained efficiency is not attainable - even if we let $\delta \to 1$. This implication is analogous to results reported in Radner *et al.* (1986), except that they analyze a partnership game.

Let us numerically illustrate the possibility of implementing x^* by calculating a restriction on parameter values that ensures it. Let us consider Example 1. Then $\Delta(x) = \frac{2x(a-x)}{3}$, $c(x) = x^2$ and $x^* = \frac{4a}{7}$. Let us also assume logconcavity (Assumption 5 or (22)). Therefore, if x^* is implementable, the RHS of (24) says we must have

$$4x^* > H(-x^*)\,\frac{2x^*(a - x^*)}{3}, \tag{26}$$

which, after some manipulations, is equivalent to

$$\frac{14}{a} > H\left(-\frac{4a}{7}\right). \tag{27}$$

Now, if we specify F, we can pin down a closed-form restriction on a. For example, let $F(\epsilon) = \exp\left[-\exp(-\epsilon)\right]$. Then $H(\epsilon) = e^{-\epsilon}$, so (27) reads

$$\frac{14}{a} > \exp\left(\frac{4a}{7}\right),$$

which holds if and only if $a < 2.81$. Furthermore, since $\lim_{x \to -\infty} H(x) = \infty$, the LHS of (24) is automatically satisfied, so $a < 2.81$ is sufficient to ensure that x^* is implementable. As another example, if we specify $F(\epsilon) = \exp\left[-\epsilon^2\right]$, we get $H(\epsilon) = -2\epsilon$. Then (26) is equivalent to

$$\frac{14}{a} > 2\frac{4a}{7},$$

which holds if and only if $a < 3.5$. Again, since $\lim_{x \to -\infty} H(x) = \infty$, this condition is also sufficient.

Equation (24) helps us better understand the idea that noise in the observation of behavior acts as an obstacle to achieving efficiency. For example, suppose

F is normally distributed: $f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{2\sigma^2}}$, where σ is its variance. Since F is logconcave, it is easily seen that

$$\sup_{\overline{q}<0} H\left(\overline{q} - x\right) = H(-x) = \frac{f(-x)}{F(-x)}, \tag{28}$$
$$\inf_{\overline{q}<0} H\left(\overline{q} - x\right) = 0.$$

In general, $H(-x)$ depends on σ in a complicated way, but (28) implies that

$$\lim_{x\to 0} H(-x) = \sqrt{\frac{2}{\pi}}\frac{1}{\sigma},$$

which is decreasing in σ. Hence, to the extent that x^* is close to zero, a smaller σ makes $x^* \in X^*$ more likely to happen. In this way, our analysis suggests that the smaller the noise is, the more likely it is that x^* is implementable.

As stated earlier, Proposition 6 delineates a set of investment levels that are implementable via turnover equilibria. A bigger set is potentially implementable if one allows for arbitrary strategies, including those that depend in complex ways on history. Such strategies might be too complicated for consumers to carry out, however, so it is interesting to determine whether a simple generalization of turnover implements a bigger set of investments. Specifically, let us consider the strategy whereby the realized quality *difference* between the two firms determines turnover. This generalized mechanism 'pitches' one firm against the other, so we refer to it as a 'tournament' mechanism. To be concrete, let x_L and x_H be the investments of LQ and HQ, and let q_L and q_H be their realized qualities. Define a tournament mechanism by the property that turnover occurs if and only if $q_H - q_L < \overline{q}$ for some $\overline{q}$.[6] The question we address now is whether equilibria under the tournament mechanism implement a positive investment by LQ and, if so, whether these equilibria improve social welfare - compared with turnover equilibria. Our last result provides a negative answer to these questions and, thereby, provides a partial justification for focusing on the turnover mechanism.

Proposition 7 *Assume $x_L > 0$ and $x_H > 0$. Then there exists no $\overline{q}$ so that $(x_L, x_H, \overline{q})$ is a tournament equilibrium.*

Proof. Suppose to the contrary that there exists a $\overline{q}$ under which (x_H, x_L) is a tournament equilibrium. Let $\widehat{G}$ be the c.d.f. of $\epsilon_1 - \epsilon_2$ and let $\widehat{g}$ be the corresponding p.d.f. By symmetry, $\widehat{G}$ is also the c.d.f. of $\epsilon_2 - \epsilon_1$. Thus, given (x_H, x_L), turnover occurs with probability $\widehat{G}(\overline{q} - x_H + x_L)$ in each period on the equilibrium path.

For this equilibrium, we have the following value equations:

$$v_H = (1-\delta)\pi_H + \delta[\widehat{G}(\overline{q} - x_H + x_L)v_L + (1 - \widehat{G}(\overline{q} - x_H + x_L))v_H] \tag{29}$$

[6]Note that if $\overline{q} < 0$, then the LQ firm is 'handicapped', i.e., it has to deliver a quality which is sufficiently higher, not simply marginally higher, to earn the HQ status.

$$v_L = (1-\delta)\pi_L + \delta[\widehat{G}(\overline{q} - x_H + x_L)v_H + (1 - \widehat{G}(\overline{q} - x_H + x_L))v_L], \qquad (30)$$

where $\pi_H = \pi_1(x_H, x_L) - c(x_H)$ and $\pi_L = \pi_2(x_H, x_L) - c(x_L)$. Since $x_H > 0$ and $x_L > 0$, the following first-order conditions must hold in equilibrium:

$$(1-\delta)c'(x_H) = \delta\widehat{g}(\overline{q} - x_H + x_L)(v_H - v_L) \qquad (31)$$

$$(1-\delta)c'(x_L) = \delta\widehat{g}(\overline{q} - x_H + x_L)(v_H - v_L) \qquad (32)$$

However, (31) and (32) imply $x_H = x_L$, which in turn implies $v_H = v_L$ by (29) and (30). Hence (31) and (32) reduce to

$$c'(x_H) = c'(x_L) = 0.$$

But this contradicts $x_H > 0$ and $x_L > 0$. ■

7 Conclusion and Related Literature

This paper explores the hypothesis that consumers condition their beliefs and, hence, their purchasing behavior on the past performance of firms they buy from. This hypothesis implies that market shares, profits and stock market valuations fluctuate over time. The hypothesis and its implications apply to a broad set of markets, ranging from markets where the downside risk of experiencing low quality is relatively small, such as restaurants or books, to markets where the downside risk is catastrophic, such as airplane crashes or cars that tip over or equipment that can be dangerous to operate. In closing let us mention some empirical studies and cases that are consistent with our working hypothesis.

One such study is Mitchell (1989), who documents and analyses the poisoning of Tylenol capsules at the retail level, which took place some 20 years ago. As he shows, Johnson and Johnson reacted by cutting the price of Tylenol by 2.50 dollars (by distributing coupons) and, notwithstanding, lost significant market share and suffered significant loss of stock market valuation. Similarly, Chalk (1987) shows that the loss of stock market valuation of airlines in the wake of fatal crashes comes from 'market discipline,' as opposed to regulatory and legal penalties. A recent case exhibiting similar features is the Ford/Bridgestone/Firestone debacle, which occupied the popular press during the summer of 2000. As reported in various publications, Bridgestone/Firestone lost market share, which is evidenced by the fact that it fired a significant portion of its labor force. While these catastrophic events receive a lot of publicity, this phenomenon is certainly not limited to catastrophic losses. For instance, Ippolito (1992) documents the reaction of investors to performance in the mutual-fund industry, finding that poor relative performance results in investors shifting their assets to other funds. Recent events in the same industry provide another illustration. Mutual funds that did well for their customers during the 90's, were later hit by low returns and corruption charges. Customers lost faith in these companies, withdrew their funds, and moved them to alternative investment channels.

We also suggest several extensions and further relationship to the theoretical literature on reputations. An important approach to reputation, that originated in Kreps and Wilson (1982) and Milgrom and Roberts (1982), stresses the role of adverse selection. A firm invests in quality because high realized quality is construed as (statistical) evidence that the firm is a good type. We have deemphasized this approach, focusing instead on a pure moral hazard formulation. One obvious extension, then, is to view the incentive to invest in quality and the consequent turnover through these (adverse selection) lens and explore their implications.

There are many other extensions that are possible. Extensions that relate to the way the model is set up, and that constitute robustness checks, include oligopoly with more than two firms, entry and exit, more general noise structures, and more general turnover strategies. Extensions that relate to practical aspects include the derivation of further predictions of the theory that are, in principle, testable. For example, do profits correlate with the frequency of turnover or with prices and quantities (and how does that relate to 'fundamentals')? Another possibility is to consider parametric examples in which high quality does not necessarily mean high profit. A high quality product may be geared towards a niche market (e.g., an elite group of customers) and, as such, may generate a high markup, but not high volume of sales and, therefore, not high profits. The interaction between quality, profits and the incentive to build reputation is then an open question. Finally there is the possibility of exploring the disciplinary role of the legal system and how it interacts with market discipline. Therefore, while our work provides an analytical framework and answers some questions about the dynamics of reputation and turnover, many other questions await further analysis.

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Reputation and Turnover

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August 4, 2004

PIER Working Paper No. 04-032
Penn Institute for Economic Research (PIER), Research Paper Series

Abstract:
We consider a repeated duopoly game where each firm privately chooses its investment in quality, and realized quality is a noisy indicator of the firm's investment. We focus on dynamic reputation equilibria, whereby consumers 'discipline' a firm by switching to its rival in the case that the realized quality of its product is too low. This type of equilibrium is characterized by consumers' tolerance level - the level of product quality below which consumers switch to the rival firm - and firms' investment in quality. Given consumers' tolerance level, we determine when a dynamic equilibrium that gives higher welfare than the static equilibrium exists. We also derive comparative statics properties, and characterize a set of investment levels and, hence, payoffs that our equilibria sustain.

Keywords: Reputation, consumer switching, moral hazard, repeated games

JEL Classifications: C73, D82, L14, L15

Working Paper Series

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