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"Monopoly Pricing under Demand Uncertainty: Final Sales versus Introductory Offers"

by

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Monopoly Pricing under Demand Uncertainty: Final Sales versus Introductory Offers[1]

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Abstract

We study rationing as a tool of the monopolist's selling policy when demand is uncertain. Three selling policies are potentially optimal in our environment: uniform pricing, final sales, and introductory offers. Final sales consist in charging a high price initially, but then lowering the price while committing to a total capacity. Consumers with a high valuation may decide to buy at the high price since the endogenous probability of rationing is higher at the lower price. Introductory offers consist in selling a limited quantity at a low price initially, and then raising price. Those consumers with high valuations who were rationed initially at the lower price may find it optimal to buy the good at the higher price.

We show that the optimal selling policy involves either uniform pricing or final sales. Introductory offers may dominate uniform pricing, but can never be optimal if the monopolist can also use final sales.

Keywords: rationing, priority pricing, sales, demand uncertainty, introductory offer, price dispersion, advance purchase discount

JEL-Classification: L12, M31

1 Introduction

Firms frequently charge different prices for the same good at different points in time (and sometimes even contemporaneously). Such (intertemporal) price dispersion is often generated by *priority pricing* or *final sales*: a firm initially charges a high price and subsequently lowers the price for any remaining items. Some consumers (namely those with a high willingness to pay or those who believe that demand will be high) may prefer to purchase the good at the high price (so as to obtain the good with certainty) rather than buying the good at a lower price and risking that the good may be sold out. Other consumers (namely those with a low willingness to pay or those who believe that demand will be low) may resolve the trade off by opting for a lower price and a higher probability of being rationed. Under demand certainty, a final sales strategy may thus allow a firm to screen between different consumers with high and low valuations. In the presence of demand uncertainty, setting different prices may in addition allow a firm to discriminate between demand states.

The selling policies of many opera houses, theaters, and concert venues involve priority pricing: advance ticket sales are complemented by lower priced "community rush tickets", "day seats", or standby tickets. Similarly, holiday tour operators and airlines typically offer both regular and last-minute deals. (In the case of last-minute holiday packages, the consumer may not obtain the destination or hotel of choice, while in the case of stand-by airline tickets, the consumer buys the ticket in advance but risks to be forced to take a later flight, which he views as an inferior substitute.) Winter or summer sales for fashion goods may also partially be explained by the idea of priority pricing. A less obvious example of priority pricing concerns season tickets for sporting events (such as baseball or soccer) or cultural events (such as concerts or operas); see Ferguson (1994). Some consumers may decide to buy a season ticket knowing that they will miss many events so that the season ticket is likely to turn out to be more expensive than buying single tickets only for those events the consumer actually attends. However, if consumers choose to buy tickets only shortly prior to the event, they risk not obtaining the desired ticket.[1]

In this paper, we consider the selling policy of a monopolist who faces uncertain demand. Before the state of demand is realized, the monopolist has to commit to prices and capacities for each period. Then, consumers (who want to buy only one unit of the good) learn their own willingness to pay, update their beliefs about the underlying state of the world, and decide when to buy the good. Consumers rationally anticipate the behavior of other consumers and thus the endogenous probabilities of rationing in each period. Three selling policies are potentially optimal for the monopolist: uniform pricing, final sales, and introductory offers. *Uniform pricing* means that the monopolist commits to charging the same price in each period. As explained above, *final sales* consist in charging a high price initially, but then lowering the price while committing to a total capacity. High valuation consumers may decide to buy at the high price since the endogenous probability of rationing is higher at the lower price. *Introductory offers* (or "advance-purchase discounts") consist in selling a limited quantity at a low price initially, and then raising price. Those consumers with high valuations who were rationed at the lower price may find it optimal to buy the good later at the higher price.

We consider a simple environment with two types of consumers (with high and low valuations, respectively) and two demand states (a good and a bad state). We show that introductory offers are never optimal in our model: the monopolist's optimal selling policy either involves uniform pricing or final sales.

[1] This means that product bundling can implement priority pricing for those spectators who are only interested in certain events for which rationing may occur. While living in England, the first author of this article often bought a season ticket for the famous BBC Promenade Concert Series at the Royal Albert Hall, London, knowing that he would only attend a small fraction of the more than seventy concerts. His rationale was to make sure that he could attend some of the more popular concerts.

An example. Suppose there are two states of the world, a good demand state and a bad demand state, which are equally likely. Consumers have unit demand and may either have a high or a low valuation for the good. High types have a willingness to pay of 7, and low types a valuation of 1, independently of the demand state. In the good demand state, there is a mass 1 of high types, and a mass 5 of low types. In the bad demand state, there are no high types, and a mass 4 of low types. A consumer who learns that he has a high valuation can thus infer that the demand state must be good. (In this example, it is therefore immaterial whether consumers directly learn the demand state or only learn their own valuation before making their purchasing decision.) The monopolist produces at zero cost and may set prices and capacities for two periods. In the case of excess demand in one period, consumers are rationed randomly. There is no discounting.

First, consider uniform pricing and introductory offers. Conditional on charging a single price, the monopolist will optimally set a price of 1. This yields expected profits of $0.5 \times 4 + 0.5 \times 6 = 5$. Alternatively, the monopolist can make a limited introductory offer and charge a higher price to consumers who are rationed in period 1. The monopolist's optimal introductory offer strategy is to offer 4 units at a price of 1 in the first period, and to serve any unserved consumers at a price of 7 in the second period. Consumers arrive at random, and so are rationed with probability 1/3 in period 1 if the state of demand is good. Hence, consumers of mass 1/3 buy the good at the high price in the good demand state. In expected terms, the monopolist makes a profit of $0.5 \times 4 + 0.5 \times (4 + (1/3) \times 7) = 5.167$. This strategy dominates uniform pricing.

Second, consider final sales. The optimal strategy with final sales consists in setting total capacity equal to 4, charging a first-period price p_1 with $7 > p_1 > 1$, and offering all remaining units in the second period at a price of $p_2 = 1$. In the bad demand state, there are no high valuation consumers, and all low valuation consumers purchase the good in the second period. In the high demand state, all high type consumers buy the good in the first period, while the low types demand the good in the second period (and hence are rationed with probability 0.4). Indeed, high type consumers weakly prefer to demand the good in the first period rather than in the second period if $7 - p_1 \geq 0.6 \times (7 - 1)$, where 0.6 is the probability of not being rationed in the second period. Hence, the monopolist will optimally set $p_1 = 3.4$, which results in an expected profit of $0.5 \times 4 \times 1 + 0.5 \times (1 \times 3.4 + 3 \times 1) = 5.2$.

Comparing profits, we observe that final sales perform better than introductory offers and uniform pricing.

Intuitively, the attraction of a uniform price is that it allows the monopolist to extract all of the surplus from a given consumer type and to flexibly serve all demand at this price. On the other hand, however, if the monopolist charges a high uniform price, she effectively excludes low type consumers, while if she charges a low uniform price, she cannot discriminate between consumers with high and low valuation. Moreover, under demand uncertainty, the monopolist cannot discriminate between demand states by charging different prices (but only by selling different quantities at the same price).

Both introductory offer and final sales strategies allow the monopolist to effectively charge different prices and thus to discriminate between consumers. Under both selling strategies, high type consumers always obtain the good, while low type consumers obtain the good in each demand state with a different probability. This implies that the monopolist can (partially) discriminate between demand states as the number of consumers of each type varies with the demand state. By charging a second-period price equal to the valuation of the high type consumers, the introductory offer strategy allows the monopolist to fully extract the surplus of those consumers who are forced to purchase the good in the second period. However, a fraction of the high type consumers will be able to buy the good at the low first-period price. In contrast, under the final sales strategy, all high type consumers pay a higher price than the low type consumers, but the monopolist has to leave them some rent (as they have the option to purchase the good at the lower second-period price and be rationed with positive probability).

The profit comparison between the different selling policies seems to be rather complex as they differ not only in the expected prices the two consumer types have to pay, but also in the quantities sold. Using insights from mechanism design theory, however, it can be reduced to a comparison of the implied probabilities of rationing high and low type consumers in the two demand states. In the above example, if the monopolist could condition her prices on the demand state (but not on the consumer type), she would optimally charge a (uniform) price equal to the valuation of the high type when demand is in the good state, and a (uniform) price equal to the valuation of the low type when demand is in the bad state. That is, the monopolist would not like to ration the high type consumers in any demand state, nor the low type consumers in the bad demand state, but the low type consumers with probability 1 in the good state. In the example, when demand is in the bad state, the optimal uniform price, optimal final sales, and optimal introductory offer strategies all induce the same rationing probabilities as the optimal state-contingent pricing policy. Hence, the profit differences between these selling policies are due to different induced probabilities of rationing in the good demand state. When demand is in the good state, low type consumers are rationed with probability 0 under the optimal uniform pricing policy, with probability 1/3 under introductory offers, and 2/5 under final sales. The optimal final sales strategy thus comes closest to the optimal state-contingent pricing policy. As we will show in this paper, this insight holds more generally.

Related Literature. Our paper complements the existing literature on pricing strategies with commitment and provides a stronger theoretical underpinning for the use of final sales strategies. Earlier work has considered the use of introductory offer and final sales strategies under demand certainty. Wilson (1988) analyzes the problem of a monopolist who wants to sell a given quantity q of a good. He shows that an introductory offer strategy may be more profitable than a uniform pricing strategy (namely if and only if there exists a neighborhood around q where the single-price revenue function is non-concave in quantity). However, if the monopolist can choose the quantity q she wants to sell, and the marginal cost of production is non-increasing, then uniform pricing is always optimal.[2] Ferguson (1994) shows that whenever uniform pricing is not optimal, the best final sales strategy is revenue equivalent to the best introductory offer strategy, for any given quantity q. Hence, the existing results under demand certainty do not allow us to predict when a monopolist should prefer a final sales strategy over an introductory offer strategy, or *vice versa*. Moreover, they indicate that non-uniform pricing should be observed only if the single-price revenue function is non-concave in quantity *and* there are decreasing returns to scale in production.

In a model similar to ours, Dana (2001) considers a monopolist who faces uncertain demand and can serve any demand at constant marginal cost of production. He shows that introductory offers may dominate uniform pricing. However, Dana does not allow for final sales strategies.

Harris and Raviv (1981) also analyze monopoly pricing under demand uncertainty. They show that priority pricing is an optimal strategy for a monopolist who produces at constant marginal costs but faces a (binding) capacity constraint. However, in their model, demand uncertainty is of a very special kind: there are a finite number of (large) buyers with i.i.d. valuations. Hence, as the number of buyers increases, demand uncertainty vanishes in the limit. Furthermore, Harris and Raviv show that if the monopolist can costlessly choose capacity (and thus serve any demand at constant marginal cost), then uniform pricing dominates other pricing schemes (see also Riley and Zeckhauser, 1983).

To summarize, the above-mentioned papers – with the exception of Dana (2001) – rely on sunk costs (or decreasing returns) to generate price dispersion. The same is true for the literature on price dispersion in competitive markets (Prescott, 1975; Eden, 1990; Dana, 1998).[3] Following Dana (2001),

[2] As Denicolo and Garella (1999) have shown, introductory offers are also a useful strategy if the monopolist lacks commitment power beyond the first period. Clearly, final sales strategies can only be used if commitment for two periods is possible.

[3] Dana (1999b) extends the Prescott model of price dispersion to monopoly and imperfect competition. Gale and Holmes (1993) consider a capacity-constrained monopolist selling two substitute products, a “peak” and an “off-peak”

we do not assume that costs are sunk before sales occur. We show that the revenue equivalence between introductory offers and final sales does not hold when demand is uncertain. Introductory offers are never optimal in our model: whenever they perform better than uniform pricing, they are dominated by final sales.

Our plan of the paper is as follows. In section 2, we present our model. In section 3, we analyze the monopolist's optimal selling strategy. First, we consider the case of demand certainty. Then, we turn to the main concern of this paper, the optimal selling policy under demand uncertainty. Finally, in section 4, we discuss our key assumptions and model extensions.

2 The Model

We consider a monopolist who sells a homogeneous product over $T = 2$ periods.[4] As in the example of concert tickets and the like, consumption takes place (simultaneously) after period 2. Hence, the time of purchase does not directly affect consumers' utility, and so there is no discounting. More generally, consumers view one unit of the good in period 1 as a perfect substitute for one unit of the good in period 2. That is, we may also think of consumers consuming the good immediately after purchase, provided their discount rate is zero.

Consumers. There is a mass M of potential consumers with unit demand. Demand is random and can be in either one of two states $\sigma \in \{G, B\}$: a good demand state, G, and a bad demand state, B. In each demand state, each consumer gets a random draw of his willingness to pay. Consumers can have a high valuation or a low valuation for the good. Moreover, some consumers may not value the good at all; we call such consumers "null types". High types are denoted by H, low types by L, null types by $\varnothing$, and the generic consumer type by $\theta \in \{H, L, \varnothing\}$. In demand state σ, the probability that a consumer's type is θ is given by $m(\theta|\sigma)/M$. Here, $m(\theta|\sigma)$ denotes the mass of consumers of type θ in state σ. We assume that there are (weakly) more high types in the good demand state than in the bad demand state; that is,

$$m(H|G) \geq m(H|B).$$

Moreover, the total mass of consumers with positive valuation is at least as large in the good demand state as in the bad state:

$$m(H|G) + m(L|G) \geq m(H|B) + m(L|B).$$

Also, we require that $m(L|\sigma) > 0$ for $\sigma \in \{G, B\}$.

Conditional on buying one unit of the product at price p, a consumer of type θ has (indirect) utility

$$v(\theta) - p,$$

where $v(\theta)$ is the consumer's willingness to pay.[5] High types have a higher willingness to pay than low types, $v(H) > v(L)$. The valuation of a null type is equal to zero, $v(\varnothing) = 0$.[6] We normalize the willingness to pay of the high type (in both demand states) to 1, and so the willingness to pay of the low type satisfies $v(L) \in (0, 1)$.

Before making his purchasing decision, each consumer observes a signal s about the underlying aggregate demand state. In this paper, we focus on the information structure, where each consumer

flight. They show that introductory offers (or advance-purchase discounts) are optimal. See also Dana (1999a) and Gale and Holmes (1992).

[4] We will also briefly discuss the more general case, where the good can be sold over $T \geq 2$ periods.

[5] In Nocke and Peitz (2003), we also analyze a variant of the model, where valuations depend on the demand state; see our discussion.

[6] Clearly, null types will never purchase the good. We introduce the construct of a null type for technical reasons so as to be able to use Bayesian updating; see footnote 9.

only observes his own valuation , i.e., $s = v(\theta)$, and updates his beliefs about the demand state σ using Bayes' rule. We only briefly discuss the alternative information structure, where each consumer directly observes the true state of demand, i.e., $s = \sigma \in \{G, B\}$. This alternative information structure is fully analyzed in our discussion paper, Nocke and Peitz (2003).

The Monopolist's Strategies. The monopolist can produce any amount of the homogeneous good at constant marginal cost c. Without loss of generality, we set $c = 0$. The monopolist can sell the product over two periods, $t = 1, 2$. Before the demand state is realized, she sets prices p_1 and p_2 for periods 1 and 2, respectively. In addition, she can commit to a capacity for period 1, k_1, and to a total cumulative capacity (for both periods), $k \geq k_1$. Ex ante, the monopolist thus sets an overall capacity k, and may commit not to sell more than a certain fraction of this capacity (namely, k_1 units) in $t = 1$. Any capacity unsold in the first period is then available in $t = 2$. (One extreme interpretation, consistent with our assumptions, is that the monopolist has to produce all k units before the state of the world is realized. In fact, this interpretation reflects quite well the motivating examples in the introduction.) By setting a price p_t and a capacity, the monopolist commits to serving all demand up to capacity at price p_t. There are no capacity costs. (This assumption is for expositional simplicity. As we discuss in the conclusion, our main results hold for any positive and constant costs per unit of capacity.) Following Dana (2001), we assume that prices and capacities are set before demand uncertainty is resolved. Therefore, the monopolist cannot condition them on the state of demand σ.[7] Moreover, the identity of consumers is unknown to the monopolist ahead of time, and so forward contracts with consumers cannot be written.

Depending on the intertemporal profile of prices, we can distinguish between three different types of selling policies:

Uniform pricing. The monopolist sets prices such that, with probability 1, all items are sold at the same price. In particular, setting the same price in both periods, $p_1 = p_2$, is a uniform pricing strategy.

Introductory Offers. The monopolist sets a lower price in the first period, $p_1 < p_2$, and some units are sold in each period with positive probability (that is, in at least one demand state).

Final Sales. The monopolist sets a lower price in the second period, $p_1 > p_2$, and some units are sold in each period with positive probability (that is, in at least one demand state).

Since consumers clearly prefer to purchase the good at the lowest possible price, an introductory offer strategy must have the property that first-period capacity k_1 is binding in at least one demand state (otherwise all units would be sold at the low price in the first period). Similarly, a final sales strategy must have the property that total capacity k is binding with positive probability (otherwise, all consumers would always prefer to buy the good at the low price in the second period).

Consumer Rationing. Since the monopolist can commit to capacities, consumers may be rationed in period 1, period 2, or both periods. Again following Dana (2001), we assume that rationing is *proportional* (or *random*). Under the proportional rationing rule (see, for example, Beckmann (1965), and Davidson and Deneckere (1986)), each consumer who is willing to purchase the good has the same probability of obtaining the good. That is, if a mass m of consumers demand the good, but only a quantity $k < m$ is available, then each consumer – independently of his type – is served with probability k/m. Note that this rationing rule is consistent with a queuing model, where consumers arrive in random order, and consumers who arrive first are served first.

Consumer Equilibrium. Observing the monopolist's strategy (p_1, p_2, k_1, k) and their (private) signals about the demand state σ, consumers make their purchasing decisions. For any (p_1, p_2, k_1, k) and distribution of signals, consumers thus play an anonymous game with discrete actions. A consumer

[7] There are a number of other papers on price discrimination where firms have to commit to prices before the demand uncertainty is resolved; see Stole (2001) for a survey.

equilibrium is a (Bayesian) Nash equilibrium of this (sub-)game. For $p_1 \leq p_2$, consumers have a (weakly) dominant strategy: "demand the good in the first period if and only if your willingness to pay is equal to or higher than p_1; if you are rationed in the first period, demand the good in the second period, provided your valuation is at least p_2." Moreover, any consumer equilibrium is revenue equivalent for the monopolist. In contrast, if the monopolist chooses a final sales strategy (and so $p_1 > p_2$), a consumer may not have a dominant strategy. The only reason why consumers may be willing to buy the good at the higher price in the first period is that they expect to be rationed with a higher probability at the lower price in the second period. However, if consumers expect that more consumers postpone their purchase until $t = 2$, they expect a lower probability of rationing in the second period (as the monopolist will sell all unsold units in the second period), and hence buying in the second period becomes more attractive. This may give rise to the existence of multiple consumer equilibria with different revenues. The best consumer equilibrium from the monopolist's point of view (and the worst from the consumers' point of view) is the one that maximizes sales at the high price (in $t = 1$).

The Monopolist's Maximization Problem. The monopolist optimally chooses her strategy (p_1, p_2, k_1, k) assuming that, in each subgame, consumers' purchasing decisions form a (Bayesian) Nash equilibrium. To obtain a unique solution, we select, for each (final sales) strategy of the monopolist, the best consumer equilibrium (from the monopolist's point of view).[8]

3 The Optimal Selling Policy

3.1 Pricing under Demand Certainty

To understand the role of demand uncertainty for non-uniform pricing, we first consider the case of demand certainty. Suppose there are no demand shocks, i.e., $m(\theta|\sigma)$ is independent of the state of demand σ. To simplify notation, we can then write the mass of consumer type θ as $m(\theta)$, and set the total mass of consumers with positive valuation to 1, so that $m(L) = 1 - m(H)$. We proceed in two steps. First, we analyze the problem where the monopolist faces an exogenous quantity ceiling q, where w.l.o.g. $q \leq 1$. Then, we allow the monopolist to choose the quantity without any restrictions, which amounts to setting the quantity ceiling $q = 1$.

A mechanism design perspective. It will prove helpful to first analyze the following mechanism design problem since the monopolist's selling policies are equivalent to some mechanisms. We restrict attention to mechanisms which consist, for each consumer type θ, of a probability $R(\theta)$ at which the consumer obtains the good and a price $P(\theta)$ that the consumer has to pay if and only if he receives the good. By the revelation principle we can restrict attention to direct-revelation mechanisms. Clearly, any selling policy (p_1, k_1, p_2, k) considered in this paper, including uniform pricing, introductory offers, and final sales, induces probabilities $R(\theta)$ and prices $P(\theta)$, and can thus be represented by an element of this class of mechanisms. We will show that, under demand certainty, the solution to the mechanism design problem can be implemented by at least one of the three selling policies. That is, any optimal selling policy implements the solution to the mechanism design problem.

Formally, the monopolist's design problem can be written as

$$\max_{R(H),R(L),P(H),P(L)} m(H)R(H)P(H) + [1 - m(H)]R(L)P(L)$$

subject to

- the quantity constraint $m(H)R(H) + [1 - m(H)]R(L) \leq q$, and

[8] In section 4, we discuss a perturbation of the model, which introduces heterogeneity in consumers' valuations, and can lead to a unique consumer equilibrium.

- the incentive (IC_θ) and individual rationality (IR_θ) constraints for each type θ.

Since the standard single-crossing property is satisfied, it is well known that the solution to such a problem satisfies the (IC_H) and (IR_L) constraints with equality, while the (IC_L) and (IR_H) constraints are nonbinding. Using the binding (IC_H) and (IR_L) constraints, we can rewrite the monopolist's design problem purely in terms of the two probabilities $R(H)$ and $R(L)$:

$$\begin{aligned} &\max_{R(H),R(L)} \; m(H)R(H) + [v(L) - m(H)]\, R(L) \\ &\text{s.t. } m(H)R(H) + [1 - m(H)]R(L) \le q. \end{aligned}$$

The solution to this problem is

$$R(H) = \begin{cases} q/m(H) & \text{if } q \le m(H) \\ 1 & \text{otherwise,} \end{cases}$$

and

$$R(L) = \begin{cases} \frac{q-m(H)}{1-m(H)} & \text{if } q > m(H) \text{ and } v(L) > m(H) \\ 0 & \text{otherwise.} \end{cases}$$

The optimal selling policy. If $q \le m(H)$ or $m(H) \ge v(L)$, the solution to the mechanism design problem can be implemented by the uniform price $p = 1$: the low type consumers do not purchase the good, while all high types attempt to purchase the good (and are rationed with positive probability if $q < m(H)$). If $q = 1$ and $m(H) \le v(L)$, the uniform price $p = v(L)$ implements the optimum: all consumers obtain the good with probability 1. Hence, if the monopolist can freely choose how much to sell, i.e., $q = 1$, the optimal selling policy is a uniform price, namely $p = 1$ if $m(H) \ge v(L)$ and $p = v(L)$ if $m(H) \le v(L)$.

We now claim that if $m(H) < q < 1$ and $v(L) > m(H)$, then the solution to the mechanism design problem can be implemented by both an introductory offer and a final sales strategy, but *not* by a uniform price. In fact, the optimal final sales and introductory offer strategies are revenue equivalent and implement the same allocation. As should be clear, this result is closely related to the well-known revenue equivalence result in auction theory. In the case of final sales, the monopolist sets a total capacity of $k = q$, charges $p_1 = (1 - r^*(q))1 + r^*(q)v(L)$ in period 1, and $p_2 = v(L)$ in period 2, where $r^*(q) = R(L) = [q - m(H)]/[1 - m(H)]$ is the probability of obtaining the good in period 2. This policy makes the high type consumers just willing to purchase the good at the high first-period price, and so $R(H) = 1$, as they face a probability $1 - r^*(q)$ of being rationed at the low second-period price. In the case of introductory offers, the monopolist charges a first-period price of $p_1 = v(L)$ and commits to the first-period capacity $k_1 = r^*(q)$. In the second period, she charges the price $p_2 = 1$, and sells $(1 - r^*)m(H)$ units.

Another way of illustrating the different selling policies is the following. Consider the single-price revenue or profit function

$$\pi(x) = \begin{cases} x & \text{if } x \le m(H), \\ v(L)x & \text{if } x \in (m(H), 1], \end{cases}$$

where $x \le q$ is quantity sold. By charging a uniform price, the monopolist can obtain the maximum (over $x \le q$) of this single-price profit function: $\max_{x \le q} \pi(x) = \max\{\pi(m(H)), \pi(q)\}$. Observe now that $\pi(x)$ has a downward-jump at $x = m(H)$ and is thus not concave. Let $\overline{\pi}(x)$ denote the "concavified" single-price revenue or profit function, defined by

$$\overline{\pi}(x) = \begin{cases} x & \text{if } x \le m(H), \\ \left(1 - \frac{x-m(H)}{1-m(H)}\right) m(H) + \left(\frac{x-m(H)}{1-m(H)}\right) v(L) & \text{if } x \in (m(H), 1]. \end{cases}$$

This concavified profit lies everywhere weakly above the single-price profit function, $\overline{\pi}(x) \geq \pi(x)$ for all x. Moreover, if $m(H) < q < 1$ and $v(L) > m(H)$, we have $\overline{\pi}(q) > \max_{x \leq q} \pi(x) = \max\{\pi(m(H)), \pi(q)\}$. Importantly, the monopolist can obtain the profit $\overline{\pi}(q) = (1-r^*(q))m(H)+r^*(q)v(L)$ by either using the final sales or the introductory offer strategies described above. However, in this case, $\overline{\pi}(q)$ is increasing in q, and so the monopolist optimally chooses the uniform price $v(L)$ if she does not face the exogenous quantity ceiling. We summarize our results in the following proposition.

Proposition 1 *Under demand certainty, the optimal final sales and introductory offer strategies for a fixed quantity ceiling q are revenue equivalent. However, if the monopolist does not face an exogenous quantity ceiling, then the optimal selling policy is a uniform pricing strategy.*

3.2 Pricing under Demand Uncertainty

Proposition 1 shows that uniform pricing is always optimal in our model when demand is certain. We now turn to the main concern of this paper, the monopolist's optimal selling policy when demand is uncertain. We focus on the information structure where consumers do not directly observe the demand state, but only their own valuation. Since the distribution of consumer types varies with the state of demand, a consumer's best reply to the purchasing strategies of other consumers may depend on his beliefs about the state of demand. Learning his own type, a Bayesian consumer uses this information to update his beliefs about the underlying demand state. However, a consumer's private signal s is typically not perfectly revealing (the example in the introduction is an exception in this respect).[9] A consumer who learns that he has a high valuation, will (using Bayes' rule) compute the probability of the state of demand being good as[10]

$$Q(G|H) = \frac{\rho m(H|G)}{\rho m(H|G) + (1-\rho)m(H|B)}.$$

The optimal state-contingent selling policy. From proposition 1 we know that if the monopolist could condition her selling policy on the state of demand σ (which, of course, she cannot in our model), she would optimally set a uniform price $p(\sigma)$. The optimal state-contingent (uniform) prices are as follows. In the good demand state, $p(G) = 1$ if

$$m(H|G) > [m(H|G) + m(L|G)]v(L), \tag{1}$$

and $p(G) = v(L)$ if the inequality is reversed. In the bad demand state, $p(B) = v(L)$ if

$$m(H|B) < [m(H|B) + m(L|B)]v(L), \tag{2}$$

[9] To be able to use Bayes' rule in our setting, we must specify the size of the population of consumers, which must be independent of the realization of the state of the world. Since the total mass of high and low type consumers may be larger in the good demand state than in the bad demand state, $m(H|G)+m(L|G) \geq m(H|B)+m(L|B)$, we introduce the construct of a "null type". Recall that a null type has a valuation of zero, and is thus not willing to buy at any (positive) price. The mass of null types in demand state σ is the difference between the total mass of consumers and the mass of high and low type consumers, $m(\varnothing|\sigma) = M - [m(H|\sigma) + m(L|\sigma)] \geq 0$. That is, while the total mass of high, low, and null types is independent of the demand state, the shares of the different types are state-dependent.

[10] The probability of being a consumer of type $\theta \in \{L, H, \varnothing\}$, given that the demand state is $\sigma \in \{G, B\}$, can be written as $Q(\theta|\sigma) = m(\theta|\sigma)/M$. Recall that the unconditional probability of a good and bad demand state is given by $Q(G) = \rho$ and $Q(B) = 1 - \rho$, respectively. Hence, the unconditional probability of being a high type is equal to

$$\begin{aligned} Q(H) &= Q(G)Q(H|G) + Q(B)Q(H|B) \\ &= \rho \frac{m(H|G)}{M} + (1-\rho)\frac{m(H|B)}{M}. \end{aligned}$$

Using Bayes' rule, we then obtain

$$Q(G|H) = \frac{Q(H|G)Q(G)}{Q(H)}.$$

and $p(B) = 1$ if the inequality is reversed. In our analysis of the optimal selling policy, these optimal state-contingent prices will provide a useful benchmark.

A mechanism design perspective. As in the case of demand certainty, it will prove helpful to consider the related problem of a mechanism designer. We restrict attention to mechanisms which consist, for each consumer type θ and demand state σ, of a probability $R(\theta|\sigma)$ at which the consumer obtains the good and a price $P(\theta|\sigma)$, which is paid by the consumer if and only if he obtains the good. (Formally, the probabilities and prices do not directly depend on the state of demand but rather on the consumer's report of his own type and the reports of all other consumers. However, if almost all consumers report their types truthfully, the monopolist knows the demand state for sure. Abusing notation, we therefore write the probabilities and prices directly as a function of the consumer's own reported type and the state of demand.) In addition to the (interim) incentive constraints (IC_θ) for the high and low types, we require that the mechanism has to satisfy the ex post individual rationality constraints ($\mathrm{XIR}_{\theta,\sigma}$): in each demand state, the price a consumer has to pay cannot be larger than his willingness to pay, $P(\theta|\sigma) \leq v(\theta)$. (These ex post individual rationality constraints appear reasonable whenever consumers cannot be forced (by contracts) to purchase a good.) Clearly, any selling policy (p_1, k_1, p_2, k) considered in this paper, induces probabilities $R(\theta|\sigma)$ and prices $P(\theta|\sigma)$ and satisfies the ex post individual rationality constraints, and can thus be represented by an element of this class of mechanisms. Below, we will show that the solution to this mechanism design problem is revenue-equivalent to the optimal state-contingent prices.

Formally, the monopolist's design problem can be written as

$$\begin{aligned} \max_{R(\theta|\sigma),P(\theta|\sigma)} \quad & \rho\left\{m(H|G)R(H|G)P(H|G) + m(L|G)R(L|G)P(L|G)\right\} \\ & +(1-\rho)\left\{m(H|B)R(H|B)P(H|B) + m(L|B)R(L|B)P(L|B)\right\} \end{aligned}$$

subject to

- the (interim) incentive constraints (IC_θ) for each type θ:

$$\begin{aligned} & Q(G|\theta)\left\{R(\theta|G)\left[v(\theta) - P(\theta|G)\right] - R(\overline{\theta}|G)\left[v(\theta) - P(\overline{\theta}|G)\right]\right\} \\ & +\left[1 - Q(G|\theta)\right]\left\{R(\theta|B)\left[v(\theta) - P(\theta|B)\right] - R(\overline{\theta}|B)\left[v(\theta) - P(\overline{\theta}|B)\right]\right\} \\ \geq \; & 0, \end{aligned}$$

  where $\overline{\theta} \in \{H, L\}$, $\overline{\theta} \neq \theta$, and

- the ex post individual rationality constraints ($\mathrm{XIR}_{\theta,\sigma}$) for each type θ and demand state σ:

$$P(\theta|\sigma) \leq v(\theta).$$

It is straightforward to show that the solution to this problem satisfies the high valuation consumer's incentive constraint (IC_H) with equality. Also, the low type's (ex post) individual rationality constraints ($\mathrm{XIR}_{L,\sigma}$) are binding for both demand states whenever $R(L|\sigma) > 0$.[11] As will become clear later, any selling policy (final sales, introductory offers, and uniform pricing) that is optimal within its own class (i.e., within the class of final sales, introductory offers, or uniform pricing strategies) also satisfies the same constraints with equality.[12] Replacing $P(L|G) = P(L|B) = v(L)$ in (IC_H), we obtain that

$$\begin{aligned} P(H|G) \;=\;& 1 - \frac{R(L|G)}{R(H|G)}\left[1 - v(L)\right] \\ & + \frac{1 - Q(G|H)}{Q(G|H)}\left\{\frac{R(H|B)}{R(H|G)}\left[1 - P(H|B)\right] + \frac{R(L|B)}{R(H|G)}\left[1 - v(L)\right]\right\}. \end{aligned}$$

[11] If $R(L|\sigma) = 0$, the ($\mathrm{XIR}_{L,\sigma}$) constraint becomes irrelevant and $P(L|\sigma)$ cancels out in the monopolist's expected revenue.

[12] The constraint $P(\theta|\sigma) \leq v(\theta)$ holds with equality whenever $R(\theta|\sigma) > 0$. Hence, if the monopolist charges a uniform price $p = 1$, she does not sell to the low types, i.e., $R(L|\sigma) = 0$, and so the constraint becomes irrelevant.

This allows us to write the monopolist's expected revenue purely in terms of probabilities:[13]

$$\begin{aligned}
& \rho \left\{ m(H|G) \left[R(H|G) - R(L|G)\,(1 - v(L)) + \frac{1 - Q(G|H)}{Q(G|H)} \left[R(H|B) - R(L|B)\,(1 - v(L)) \right] \right] \right. \\
& \left. + m(L|G) R(L|G) v(L) \right\} + (1-\rho) m(L|B) R(L|B) v(L) \\
= \;& \rho m(H|G) R(H|G) + \rho \left\{ \left[m(H|G) + m(L|G) \right] v(L) - m(H|G) \right\} R(L|G) \\
& + (1-\rho) m(H|B) R(H|B) + (1-\rho) \left\{ \left[m(H|B) + m(L|B) \right] v(L) - m(H|B) \right\} R(L|B) \qquad (3)
\end{aligned}$$

Maximizing this expression, we obtain that the monopolist should serve high type consumers with probability 1 and low type consumers with probability 0 whenever the optimal state-contingent price is $p(\sigma) = 1$. Similarly, she should serve all consumers with probability 1 whenever the optimal state-contingent price is $p(\sigma) = v(L)$. That is,

$$\begin{aligned}
R(H|\sigma) &= 1 \text{ for } \sigma = G, B, \\
R(L|G) &= \begin{cases} 0 & \text{if (1) holds} \\ 1 & \text{otherwise,} \end{cases} \\
R(L|B) &= \begin{cases} 1 & \text{if (2) holds} \\ 0 & \text{otherwise.} \end{cases} \qquad \text{(ProbOPT)}
\end{aligned}$$

When comparing the revenues of our different selling policies, we can thus focus on comparing the implied probabilities of serving the different consumers in the two demand states and analyze how "close" these are to the probabilities in the optimal benchmark.

Comparison of different selling policies. We now turn to the main concern of this paper, the analysis of the optimal selling policy (p_1, k_1, p_2, k) under demand uncertainty. We first follow Dana (2001) in restricting attention to selling strategies with non-decreasing price paths, and compare introductory offers with uniform pricing. Then, we also allow for decreasing price paths and compare final sales strategies with introductory offers and uniform pricing.

Uniform pricing. Let us first consider uniform pricing, where the monopolist sets the same price p in both periods. Under uniform pricing, the monopolist has no incentive to ration consumers, and will thus set capacities $k = k_1 = m(H|G) + m(L|G)$ so that demand can always be met. Independently of his beliefs about the demand state (and the behavior of other consumers), a consumer will optimally purchase the good (in either period 1 or 2) if and only if the price is lower than his willingness to pay. Clearly, the monopolist will optimally extract all of the surplus from one of the two consumer types. Hence, we can confine attention to two uniform prices, $p = 1$ and $p = v(L)$. The induced probabilities of serving consumers are given by

$$\begin{aligned}
R(H|\sigma) &= 1, \\
R(L|\sigma) &= \begin{cases} 0 & \text{if } p = 1, \\ 1 & \text{if } p = v(L). \end{cases} \qquad \text{(ProbU)}
\end{aligned}$$

for $\sigma = G, B$. These induced probabilities in conjunction with the prices $P(\theta|\sigma) = p$ constitute a mechanism of the class defined above. In particular, the (interim) incentive constraint of the high consumer type is binding, as are the (ex post) individual rationality constraints of the low type whenever he obtains the good with positive probability (i.e., $p = v(L)$). Note that the high uniform price, $p = 1$, implements the optimal mechanism if (1) holds, but (2) does not. Similarly, the low uniform price, $p = v(L)$, implements the optimum if (2) holds, but (1) does not. Expected profits, which can be obtained by inserting the probabilities (ProbU) into equation (3), are

[13] Note that $P(H|B)$ cancels out so that $P(H|B)$ and $P(H|G)$ are not uniquely determined.

$$\pi^U(p) = \begin{cases} \rho m(H|G) + (1-\rho)m(H|B) & \text{if } p = 1 > v(L), \\ v(L)[\rho(m(H|G) + m(L|G)) \\ \quad +(1-\rho)(m(H|B) + m(L|B))] & \text{if } p = v(L). \end{cases}$$

Hence, the profit-maximizing uniform price is $p = v(L)$ if

$$\begin{aligned} & \rho\left\{[m(H|G) + m(L|G)]v(L) - m(H|G)\right\} \\ +&(1-\rho)\left\{[m(H|B) + m(L|B)]v(L) - m(H|B)\right\} \quad > \quad 0, \end{aligned} \tag{4}$$

and $p = 1$ if the reverse inequality holds. Observe that the first term on the l.h.s. is negative if (1) holds, while the second term is positive if (2) holds.

Introductory offers. Next, let us consider introductory offers, where $p_1 < p_2$. Independently of his beliefs, each consumer has a dominant strategy when facing an increasing price path, namely to demand the good at the low price in period 1, provided the price is not higher than his willingness to pay. If the consumer is rationed at the low price, his dominant strategy is to demand the good at the high price in period 2, provided again this price is less than his valuation. Clearly, the monopolist has no incentive to ration consumers at the high price. Without loss of generality, she may thus set total capacity $k = m(H|G) + m(L|G)$ so as to always meet demand in the second period. In each period, the monopolist optimally extracts all of the surplus of some consumer type. Under introductory offers, the monopolist will therefore set prices $p_1 = v(L)$ and $p_2 = 1$. We can thus restrict attention to the following family of introductory offer strategies, $(v(L), 1, k_1, m(H|G) + m(L|G))$, which is parametrized by k_1. We denote these strategies by $IO(k_1)$. Without loss of generality, we can assume that first-period capacity $k_1 \leq m(H|G) + m(L|G)$. Expected profits are then given by

$$\begin{aligned} & \pi^{IO}(k_1) \\ = \; & (1-\rho)\left[v(L)\min\{k_1, m(H|B) + m(L|B)\}\right. \\ & \left. +1\left(\frac{\max\{0, m(H|B) + m(L|B) - k_1\}}{m(H|B) + m(L|B)}\right) m(H|B)\right] \\ & +\rho\left[v(L)k_1 + 1\left(\frac{m(H|G) + m(L|G) - k_1}{m(H|G) + m(L|G)}\right) m(H|G)\right]. \end{aligned}$$

Since this expression is piecewise linear in k_1, the unique candidate for an optimal introductory offer strategy is $IO(m(H|B) + m(L|B))$. That is, the monopolist optimally sets the first-period capacity k_1 so as to just serve all demand at the low price $p_1 = v(L)$ when demand is in the bad state. In the good demand state, on the other hand, the monopolist sells $m(H|B) + m(L|B)$ units at the low first-period price, and serves all rationed high valuation consumers at the high second-period price. The induced probabilities of serving consumers are thus given by

$$\begin{aligned} R(H|\sigma) &= 1 \text{ for } \sigma = G, B, \\ R(L|G) &= \frac{m(H|B) + m(L|B)}{m(H|G) + m(L|G)}, \\ R(L|B) &= 1. \end{aligned} \tag{ProbIO}$$

These induced probabilities in conjunction with the induced prices $P(L|\sigma) = v(L)$ for $\sigma = G, B$, $P(H|B) = v(L)$, and

$$P(H|G) = \left(\frac{m(H|B) + m(L|B)}{m(H|G) + m(L|G)}\right) v(L) + \left(1 - \frac{m(H|B) + m(L|B)}{m(H|G) + m(L|G)}\right) 1$$

constitute a mechanism of the class defined above. In particular, the (interim) incentive constraint of the high consumer type is binding, as are the (ex post) individual rationality constraints of the low type.

The expected profits can be obtained by inserting the probabilities (ProbIO) into equation (3). As can be seen from (ProbIO), $IO(m(H|B)+m(L|B))$ never implements the optimal mechanism.

Uniform pricing versus introductory offers. We first compare the optimal introductory offer strategy with the low uniform price $v(L)$. The induced probabilities $R(\theta|\sigma)$ only differ for the low type in the good demand state: under the low uniform price, $R(L|G)=1$, while under the optimal introductory offer strategy, $R(L|G)<1$.[14] Hence, the introductory offer strategy is more profitable than the low uniform price if and only if $R(L|G)=0$ in the optimal mechanism. That is, $\pi^{IO}(m(H|B)+m(L|B))>\pi^{U}(v(L))$ if and only if equation (1) holds, i.e.,

$$m(H|G)>[m(H|G)+m(L|G)]v(L).$$

Second, we compare introductory offers with the high uniform price of 1. The induced probabilities only differ for the low type: while the low type consumers never purchase the good under the high uniform price, they obtain the good with positive probability in both demand states when the monopolist uses the optimal introductory offer strategy. Inserting the difference in the expected probabilities into the expression for the monopolist's expected revenue, equation (3), we obtain that $\pi^{IO}(m(H|B)+m(L|B))>\pi^{U}(1)$ if and only if

$$\rho\left\{v(L)-\frac{m(H|G)}{m(H|G)+m(L|G)}\right\}+(1-\rho)\left\{v(L)-\frac{m(H|B)}{m(H|B)+m(L|B)}\right\}>0. \tag{5}$$

Observe that the first term on the l.h.s. is negative if (1) holds, whereas the second term is positive if (2) holds. From the profit comparison with the low uniform price, introductory offers can only dominate uniform pricing if $R(L|G)=0$ in the optimal mechanism, i.e., the first term on the l.h.s. of (5) is negative. Given that the optimal $R(L|G)$ is equal to 0, condition (5) can only be satisfied if $R(L|B)=1$ in the optimal mechanism, i.e., only if the second term on the l.h.s. of (5) is positive. In this case, there exists a trade off between introductory offers and the high uniform price. In the good demand state, too many units are sold to low type consumers under introductory offers, while in the bad demand, too few units are sold under the high uniform price. Relative to the high uniform price, the loss of introductory offers in the good demand state is

$$\{m(H|G)-[m(H|G)+m(L|G)]\,v(L)\}\,\frac{m(H|B)+m(L|B)}{m(H|G)+m(L|G)},$$

while the gain in the bad demand state is

$$\{m(H|B)-[m(H|B)+m(L|B)]\,v(L)\}\,.$$

The expected gain outweighs the expected loss if and only if (5) holds. This shows that if $R(L|G)=0$ and $R(L|B)=1$ in the optimal mechanism, introductory offers may improve upon uniform pricing, but fail to implement the optimal mechanism.

Put differently, for conditions (5) and (1) to hold simultaneously, it is necessary that equation (2) holds, i.e., $m(H|B)<[m(H|B)+m(L|B)]v(L)$. This equation says that, in the bad demand state, the optimal state-contingent price is $p=v(L)$. Conditions (2) and (1) are thus necessary (but not sufficient) for introductory offers to dominate uniform pricing. To see this, suppose otherwise. There are three cases to consider. (i) Condition (1) holds, but (2) does not, and so the ex post profit-maximizing price in both demand states is $p=1$. Clearly, in this case, the uniform price $p=1$ is (ex ante) more profitable

[14] If total demand does not expand in the good demand state, i.e., $m(H|G)+m(L|G)=m(H|B)+m(L|B)$, we have $R(L|G)=1$ in (ProbIO) and the optimal introductory offer strategy is degenerate in that all units are always sold at the low price $v(L)$.

than the introductory offer strategy. (ii) Condition (1) does not hold, but (2) does, and so the ex post profit-maximizing price in both demand states is $p = v(L)$. Clearly, in this case, the uniform price $p = v(L)$ is (ex ante) more profitable than the introductory offer strategy. (iii) Both conditions (1) and (2) do not hold, and so the ex post profit-maximizing price is $p = 1$ ($p = v(L)$) in the bad (good) demand state. In this case, the introductory offer strategy $IO(m(H|B) + m(L|B))$ performs worse than the uniform price $p = v(L)$: in the bad demand state, both strategies lead to the same profits, while in the good demand state, the uniform price $p = v(L)$ is ex post optimal, and thus yields higher profits.

We summarize our results in the following proposition.

Proposition 2 *The introductory offer strategy $IO(m(H|B) + m(L|B))$ maximizes profits among the set of selling policies with $p_1 \leq p_2$ if and only if conditions (5) and (1) are satisfied.*

Final sales. Let us now consider final sales strategies, where $p_1 > p_2$. Clearly, the monopolist has no incentive to ration demand at the high price. Without loss of generality we can therefore set first-period capacity $k_1 = k$. For consumers to be willing to purchase the good in the first period, there must exist a positive probability that consumers are rationed in the second period. Since consumers cannot condition their purchasing decision on the state of the world, but only on their own valuation, any optimal final sales strategy must have the property that all high type consumers demand the good in the first period, while all low types demand the good in the second period (and are rationed with positive probability). Hence, it is sufficient to consider the family of final sales strategies (parameterized by capacity k), $(\widehat{p}_1(k), v(L), k, k)$, where $\widehat{p}_1(k)$ is set so as to make high type consumers just indifferent between demanding the good in the first period at price $\widehat{p}_1(k)$, and postponing the purchase (so as to demand the good in the second period at price $v(L)$). We denote these strategies by $FS(k)$. For $k \geq m(H|G)$, the indifference condition can be written as

$$\begin{aligned} 1 - \widehat{p}_1(k) &= [1 - v(L)] \left[Q(G|H) \left(\frac{k - m(H|G)}{m(L|G)} \right) \right. \\ &\quad \left. + (1 - Q(G|H)) \min \left\{ 1, \frac{k - m(H|B)}{m(L|B)} \right\} \right], \qquad (6) \end{aligned}$$

where $\min\{[k - m(H|\sigma)/m(L|\sigma)], 1\}$ is the probability of obtaining the good at the low price in demand state σ. (For $k < m(H|G)$, rationing occurs even at the high price. This case is considered in the proof of proposition 3.) Since $\widehat{p}_1(k)$ is piecewise linear in k, there are (at least) two potentially optimal capacity levels: $k = m(H|G)$ and $k = m(H|B) + m(L|B)$.

It will prove useful to distinguish between two demand regimes; see figure 1 for a graphic illustration.

- *Weak demand shifts.* In this case, the rightward shift of the demand curve is sufficiently small in the sense that the number of high type consumers in the good state is less than the total number of high and low types in the bad state, i.e., $m(H|G) < m(L|B) + m(H|B)$.
- *Strong demand shifts.* In this case, the rightward shift of the demand curve is sufficiently large in the sense that the number of high type consumers in the good state is greater than the total number of high and low types in the bad state, i.e., $m(H|G) \geq m(L|B) + m(H|B)$.

The set of potentially optimal final sales strategies depends on the demand regime. As the following proposition shows, we only have to consider a single final sales strategy under strong demand shifts and two final sales strategies under weak demand shifts.

Proposition 3 *Under strong demand shifts, the only potentially optimal final sales strategy is $FS(m(H|G))$. Under weak demand shifts, the only potentially optimal final sales strategies are $FS(m(H|G))$ and $FS(m(H|B) + m(L|B))$.*

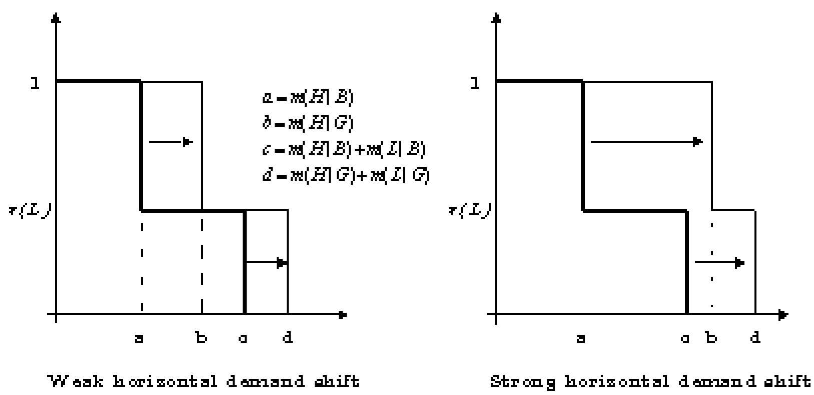

Figure 1: Weak and strong demand shifts.

Proof. See Appendix. ■

Strong demand shifts. Suppose first that demand shifts are strong, and so $m(H|G) \geq m(H|B) + m(L|B)$. We first consider the final sales strategy $FS(m(H|G))$, where, from equation (6), the first-period price $\widehat{p}_1(m(H|G))$ is given by

$$\widehat{p}_1 = \frac{\rho m(H|G) + (1-\rho)m(H|B)v(L)}{\rho m(H|G) + (1-\rho)m(H|B)}.$$

That is, the optimal first-period price is a weighted average of $p = 1$ and $p = v(L)$, where the weight on the higher price is $Q(G|H)$, the probability that demand is in the good state, conditional on drawing a high valuation. Capacity $k = m(H|G)$ is such that, in both demand states, high type consumers are not rationed in the first period. In contrast, low type consumers, who demand the good in the second period, are rationed with probability 1 in the good demand state, while they are not rationed in the bad demand state. The induced probabilities of serving consumers are thus given by

$$\begin{aligned} R(H|\sigma) &= 1 \text{ for } \sigma = G, B, \\ R(L|G) &= 0, \\ R(L|B) &= 1. \end{aligned} \tag{ProbFSstrong}$$

The expected profits can be obtained by inserting the probabilities (ProbFSstrong) into equation (3). Observe that the optimal final sales strategy implies the same probabilities as the optimal mechanism if and only if (1) and (2) hold. It follows that the final sales strategy is revenue equivalent to the (nonfeasible) optimal state-contingent selling policy, where the monopolist charges the high (uniform) price $p = 1$ in the good demand state and the low price $p = v(L)$ in the bad demand state. Note that the revenue equivalence does not necessarily imply the same induced prices for the high type consumers: in the optimal mechanism, $P(H|G)$ and $P(H|B)$ are not uniquely determined. Indeed, $P(H|G) = P(H|B) = \widehat{p}_1(m(H|G))$ under final sales, while $P(H|G) = 1$ and $P(H|B) = v(L)$ under the optimal state-contingent pricing policy.[15]

If conditions (1) and (2) hold, the final sales strategy $FS(m(H|G))$ is thus revenue equivalent to the optimal state-contingent selling policy, while uniform pricing and introductory offers lead to lower profits. Moreover, recall that introductory offers are more profitable than uniform pricing *only if* these two conditions are satisfied. Hence, introductory offers are never optimal: they are either dominated by uniform pricing or by final sales.

Above, we have shown that the final sales strategy $FS(m(H|G))$ is optimal amongst all feasible selling strategies when conditions (1) and (2) hold. Which strategy is optimal when one of the two conditions is not satisfied? If equation (1) does not hold, then the optimal state-contingent price in the good demand state is $p = v(L)$. In this case, the uniform price $p = v(L)$ dominates the state-contingent strategy ($p(G) = 1$, $p(B) = v(L)$): the uniform price yields the same revenues in the bad demand state and higher revenues in the good state. Hence, if equation (1) does not hold, the final sales strategy is dominated by the uniform price $p = v(L)$. Similarly, if condition (2) is not satisfied, then the optimal state-contingent price in the bad demand state is $p = 1$. In this case, the uniform price $p = 1$ dominates the state-contingent strategy ($p(G) = 1$, $p(B) = v(L)$) and, by revenue equivalence, the final sales strategy: the uniform price yields the same revenues as the state-contingent strategy in the good demand state and higher revenues in the bad state. We can summarize our results as follows.

[15] The final sales strategy leads to lower profits than the optimal state-contingent pricing policy when demand is in the good state since the high type consumers obtain the good at price $\widehat{p}_1(m(H|G)) < 1$. The loss in profit is thus equal to $[1 - \widehat{p}_1(m(H|G))]m(H|G)$. In expected terms, this loss is exactly offset by the gain in the bad demand state, which amounts to $[\widehat{p}_1(m(H|G)) - v(L)]m(H|B)$.

Lemma 1 *Under strong demand shifts, introductory offers are never optimal, and hence the profit-maximizing selling policy involves a non-increasing price path, $p_1 \geq p_2$. If conditions (1) and (2) hold, the final sales strategy $FS(m(H|G))$ is optimal, and hence $p_1 > p_2$. Otherwise, uniform pricing is optimal.*

As pointed out in the introduction, price dispersion is a widely observed phenomenon. In our framework, introductory offers and final sales trivially exhibit dispersion of *posted* prices. Moreover, these strategies induce dispersion of prices at which trade occurs within at least one demand state. The final sales strategy $FS(m(H|G))$ induces dispersion of prices at which trade occurs only when demand is in the bad state: all high type consumers buy the good in the first period, and all low type consumers demand the good in the second period. (The same happens in the good demand state, but there is no residual supply at the low price.) In contrast, the introductory offer strategy $IO(m(H|B)+m(L|B))$ induces price dispersion in the good state.

Weak demand shifts. Suppose now that demand shifts are weak, and so $m(H|G) < m(H|B)+m(L|B)$. First, let us consider the final sales strategy $(\widehat{p}_1(m(H|B)+m(L|B)), v(L), m(H|B)+m(L|B))$, where the first-period price is given by

$$\begin{aligned} & \widehat{p}_1(m(H|B)+m(L|B)) \\ = \quad & v(L)+[1-v(L)]Q(G|H)\left(\frac{m(H|G)+m(L|G)-m(H|B)-m(L|B)}{m(L|G)}\right). \end{aligned}$$

If the monopolist employs this final sales strategy, then all high types will purchase the good in the first period, and all low types try to purchase in the second period. Total capacity k is chosen such that there is no rationing in either period when demand is in the bad state, but when demand is in the good state, (low type) consumers are rationed with probability $[m(H|G)+m(L|G)-m(H|B)-m(L|B)]\,/m(L|G)$ in the second period. That is, the induced probabilities of serving consumers are given by

$$\begin{aligned} R(H|\sigma) &= 1 \text{ for } \sigma = G, B, \\ R(L|G) &= \frac{m(H|B)+m(L|B)-m(H|G)}{m(L|G)}, \\ R(L|B) &= 1. \end{aligned} \tag{ProbFS1weak}$$

The expected profits can be obtained by inserting the probabilities (ProbFS1weak) into equation (3). Note that this final sales strategy never implements the optimal mechanism. Nevertheless, as we will show below, it can be the optimal selling policy.

Observe that the final sales strategy $FS(m(H|B)+m(L|B))$ differs from the introductory offer strategy $IO(m(H|B)+m(L|B))$ only in the induced probability of serving low type consumers in the good demand state, $R(L|G)$. Comparing (ProbFS1weak) and (ProbIO), it is easily checked that this probability is lower under the final sales strategy. This should not be surprising: total capacity is $k = m(H|B)+m(L|B)$ under the final sales strategy, while the introductory offer strategy uses the same capacity as first-period capacity k_1 only and induces additional trade in the second period when demand is in the good state. Hence, the final sales strategy dominates the introductory offer strategy if and only if $R(L|G) = 0$ in the optimal mechanism, i.e., if and only if condition (1) is satisfied (which says that the optimal state-contingent price in the good demand state is 1). However, recall that condition (1) is necessary for introductory offers to dominate the low uniform price. It follows that introductory offers are never optimal: they are either dominated by the final sales strategy $FS(m(H|B)+m(L|B))$ or the low uniform price. By the same argument, the final sales strategy dominates the low uniform pricing strategy if and only if condition (1) holds.

Second, let us consider the final sales strategy $FS(m(H|G))$, where the first-period price is given by

$$\widehat{p}_1(m(H|G)) = 1 - \frac{[1-v(L)]\,(1-\rho)m(H|B)}{\rho m(H|G) + (1-\rho)m(H|B)} \left(\frac{m(H|G) - m(H|B)}{m(L|B)} \right).$$

Note that the price $\widehat{p}_1(m(H|G))$ (and hence the profit of $FS(m(H|G))$) under weak demand shifts differs from the price $\widehat{p}_1(m(H|G))$ (and hence the resulting profit) under strong demand shifts since, under weak demand shifts, the low type consumers are rationed with positive probability in the bad demand state. Under weak demand shifts, the induced probabilities of serving consumers are given by

$$\begin{aligned} R(H|\sigma) &= 1 \text{ for } \sigma = G, B, \\ R(L|G) &= 0, \\ R(L|B) &= \frac{m(H|G) - m(H|B)}{m(L|B)}. \end{aligned} \tag{ProbFS2weak}$$

The expected profits can be obtained by inserting the probabilities (ProbFS2weak) into equation (3).

Observe that the induced probabilities differ from those of the high uniform price $p = 1$ only in the higher value of $R(L|B)$. Hence, the final sales strategy $FS(m(H|G))$ dominates the high uniform price if and only if $R(L|B) = 1$ in the optimal mechanism, i.e., if and only if condition (2) is satisfied (and so the optimal state-contingent price in the bad demand state is $v(L)$).

We now claim that the optimal selling policy is a final sales strategy if and only if conditions (1) and (2) hold. The "if part" of this claim follows directly from above: the low uniform price is dominated by $FS(m(H|B) + m(L|B))$ if (1) is satisfied, while the high uniform price is dominated by $FS(m(H|G))$ if (2) holds What about the "only if part"? Clearly, if neither condition (1) nor (2) hold, both final sales strategies are dominated by a uniform pricing strategy. On the other hand, if (1) holds, but (2) does not, then the high uniform price is the only selling strategy (of the class considered in this paper) that implements the optimal mechanism. If the reverse holds, then the low uniform price is the only selling policy that implements the optimum. We thus have the following result.

Lemma 2 *Under weak demand shifts, introductory offers are never optimal, and hence the profit-maximizing selling policy involves a non-increasing price path, $p_1 \geq p_2$. If conditions (1) and (2) hold, either the final sales strategy $FS(m(H|B) + m(L|B))$ or $FS(m(H|G))$ is optimal, and hence $p_1 > p_2$. Otherwise, uniform pricing is optimal.*

As under strong demand shifts, the final sales strategy $FS(m(H|G))$ induces intertemporal dispersion of prices (at which trade occurs) only when demand is in the bad state: in the good demand state, all $m(H|G)$ units are sold at the high first-period price. In contrast, the final sales strategy $FS(m(H|B) + m(L|B))$, which employs a larger capacity, induces intertemporal price dispersion in *both* demand states: even in the good demand state, some units are sold at the low second-period price.

To complete the analysis of the optimal selling policy, we have to compare the two potentially optimal final sales strategies. Final sales strategy $FS(m(H|B) + m(L|B))$ involves a larger capacity than $FS(m(H|G))$, and thus a larger quantity sold in both demand states. In the good state, the difference in the probability of serving the low type is

$$\frac{m(H|B) + m(L|B) - m(H|G)}{m(L|G)},$$

while it is

$$\frac{m(H|B) + m(L|B) - m(H|G)}{m(L|B)}$$

in the bad state. If conditions (1) and (2) hold (and uniform pricing dominated by one of the final sales strategies), any optimal mechanism satisfies $R(L|G) = 0$ and $R(L|B) = 1$. Under these conditions, $FS(m(H|B)+m(L|B))$ performs thus better than $FS(m(H|G))$ in the bad demand state and worse in the good demand state. Inserting the difference in the probabilities into the expression for the monopolist's expected revenue, equation (3), we obtain that $FS(m(H|B)+m(L|B))$ dominates $FS(m(H|G))$ if and only if

$$\frac{\rho}{m(L|G)}\{[m(H|G)+m(L|G)]v(L)-m(H|G)\}+\frac{1-\rho}{m(L|B)}\{[m(H|B)+m(L|B)]v(L)-m(H|B)\}>0.$$

The first term on the l.h.s. is negative if (1) holds, whereas the second terms is positive if (2) holds.

Main results. In this paper, we have shown that the revenue equivalence between the best introductory offers and final sales strategies breaks down under demand uncertainty. Perhaps more surprisingly, introductory offers are never optimal in our model. For an introductory offer strategy to be more profitable than uniform pricing, there has to exist a tension between the good and the bad demand state in that the ex post optimal (uniform) price in the good state has to be higher than the one in the bad demand state, i.e., $m(H|G) > [m(H|G)+m(L|G)]v(L)$ and $m(H|B) < [m(H|B)+m(L|B)]v(L)$, which are conditions (1) and (2), respectively. However, it is exactly this tension that makes also the final sales strategy more attractive than uniform pricing. Moreover, as shown above, in this case, final sales dominate introductory offers as they involve a smaller quantity in the good demand state (and the same quantity in the bad demand state). Combining lemmas 1 and 2, yields our main result:

Proposition 4 *Introductory offers are never optimal, and hence the profit-maximizing selling policy involves a non-increasing price path, $p_1 \geq p_2$. If conditions (1) and (2) hold, final sales are optimal, and hence $p_1 > p_2$. Otherwise, uniform pricing is optimal.*

Strategy space. It is possible to show that the monopolist cannot do better by charging more than two prices. Essentially, the argument is that a consumer of a given type cannot condition his purchasing decision on the state of demand. Moreover, all consumers of the same type have the same willingness to pay and the same beliefs about the state of demand. Since there are two consumer types with positive valuation, the monopolist does not need to charge more than two prices.

Alternative information structure. We have derived our results under the assumption that consumers do not directly observe the state of demand, but only their own valuation. We now briefly discuss how our analysis would change under the alternative information structure, where consumers learn the demand state before making their purchasing decision. For more details, the interested reader is referred to our working paper, Nocke and Peitz (2003).

Under the alternative information structure, the mechanism design problem described at the beginning of this (sub-)section has to be modified in only one point: each consumer type now has a separate (interim) incentive constraint for each state. This implies that there is no "interaction" between the demand states in the design problem. For each demand state $\sigma \in \{G, B\}$, the solution to the design problem satisfies ($\mathrm{IC}_{H,\sigma}$) and ($\mathrm{XIR}_{L,\sigma}$) with equality. Rewriting the monopolist's expected revenue purely in terms of probabilities, we obtain the same expression as before, equation (3). In particular, the optimal state-contingent (uniform) prices $p(\sigma)$ implement the optimal mechanism.

Turning to the optimal selling policy, recall first that uniform pricing and introductory offers are unaffected by changes in consumers' information structure (as consumers have a (weakly) dominant strategy). This is not true for final sales strategies, however. Indeed, under the alternative information structure, each optimal final sales strategy has the property that high types purchase the good at the high first-period price only when demand is in the good state. Accordingly, for any capacity k, the monopolist now optimally charges an even higher price in the first period.

Under strong demand shifts, the unique candidate for an optimal final sales strategy is $p_1 = 1$, $p_2 = v(L)$, and $k = m(H|G)$. Facing this selling policy, high valuation consumers purchase the good in the first period only when demand is in the good state. The final sales strategy thus implements the state-contingent (uniform) pricing strategy $(p(G) = 1, p(B) = v(L))$, and is thus an optimal mechanism whenever conditions (1) and (2) are satisfied. In fact, the strategy is revenue equivalent to the final sales strategy $FS(m(H|G))$ in the original information setting as it induces the same probabilities. Consequently, introductory offers are never optimal, while uniform pricing is optimal whenever (1) or (2) do not hold.

Under weak demand shifts, there are two potentially optimal final sales strategies. One consists of

$$p_1 = \frac{1}{m(L|G)}\{m(H|G) + m(L|G) - [m(H|B) + m(L|B)] + [m(H|B) + m(L|B) - m(H|G)]v(L)\},$$

$p_2 = v(L)$, and $k = m(H|B) + m(L|B)$. Again, high valuation consumers only purchase the good in the first period when demand is in the good state. However, the induced probabilities are the same as those of final sales strategy $FS(m(H|B) + m(L|B))$ in the original information setting. Consequently, the two strategies are revenue equivalent, and so introductory offers can never be optimal. We thus have the following result.

Proposition 5 *Suppose that consumers learn the demand state before making their purchasing decision. Then, introductory offers are never optimal, and the optimal strategy of the monopolist involves a non-increasing price path,* $p_1 \geq p_2$.

Under weak demand shifts, the second potentially optimal final sales strategy involves $p_1 = 1$, $p_2 = v(L)$, and $k = m(H|G)$. The induced probabilities of serving consumers are

$$\begin{aligned} R(H|G) &= 1 \\ R(H|B) &= R(L|B) = \frac{m(H|G)}{m(H|B) + m(L|B)} \\ R(L|G) &= 0. \end{aligned}$$

High valuation consumers purchase the good in the first period only when demand is in the good state. In the bad state, all consumers demand the good at the low second-period price and are rationed with the same positive probability. In other words, in the bad demand state, some high valuation consumers do not obtain the good, while some low valuation consumers do obtain the good, and so this final sales strategy leads to an inefficient allocation for a given quantity sold. Nevertheless, it can be the optimal selling policy. Not surprisingly, then, this strategy leads to lower payoffs than the final sales strategy $FS(m(H|G))$ in the original information setting. Hence, under weak demand shifts, conditions (1) and (2) are necessary, but no longer sufficient, for final sales to be the optimal selling policy.[16]

4 Discussion and Conclusion

In this paper, we have analyzed the optimal intertemporal price-capacity decision of a monopolist who faces uncertain demand. In particular, we have investigated the optimality of uniform pricing, final sales, and introductory offer strategies. To this end, we have analyzed a model where the number of high and low valuation consumers depends on the demand state, which can be either good or bad. Our main results can be summarized as follows.

[16] Under weak demand shifts, it is optimal for some parameter constellations to use a three-price strategy, which is a hybrid pricing strategy containing introductory offer and final sales. Interestingly, allowing for such a hybrid three-price strategy, we restore our previous result that uniform pricing is suboptimal if and only if conditions (1) and (2) hold. For details, see Nocke and Peitz (2003).

- Introductory offers can dominate uniform pricing only if the optimal state-contingent price is high in the good state and low in the bad state. However, in this case, they are dominated by final sales.
- Whenever the optimal state-contingent price in the good state is high and in the bad state is low, final sales are the optimal selling policy. (If, in addition, demand shifts are strong, final sales implement the optimal mechanism.)
- Otherwise, uniform pricing is the optimal selling policy.

We have shown that for a profit comparison of the different selling policies, we only need to consider the induced probabilities of serving the two consumer types in the two demand states. Clearly, uniform pricing can only be suboptimal if the optimal state-contingent prices differ across demand states. In particular, suppose that the optimal state-contingent price is high in the good demand state and low in the bad state. In this case, the monopolist would optimally like to always serve high valuation consumers, but to serve low valuation consumers only in the bad demand state. A high uniform price thus leads to too little trade because, in the bad demand state, low valuation consumers are never served. A low uniform price leads to too much trade because, in the good demand state, low valuation consumers are always served. Certain final sales and introductory offers perform better than the low uniform price because they imply that only a share (which may be zero) of low valuation consumers are served in the good demand state, while all consumers are served in the bad demand state. Furthermore, certain final sales perform better than the high uniform price because they imply that a positive share (which may be one) of low valuation consumers are served in the bad demand state, while no low valuation consumers are served in the good demand state. We have shown that the best final sales strategy dominates uniform pricing and introductory offers as its induced probabilities come closest to those of the optimal state-contingent pricing policy. If demand shifts are strong (so that the number of high and low valuation consumers in the bad demand state is less than the number of high valuation consumers in the good demand state), the best final sales strategy is revenue-equivalent to the optimal state-contingent pricing policy. Otherwise, it leads to lower revenues since either some low valuation consumers obtain the good in the good demand state or some consumers do not obtain the good in the bad demand state.

Below, we discuss our key assumptions and comment on modifications and extensions. We distinguish between assumptions concerning demand side and supply side characteristics.

Demand side: Demand shifts. We have derived our results under the assumption that demand shifts are horizontal in that they only affect the mass of consumers of each type, but not a given type's valuation. Alternatively, we may assume that demand shifts are vertical in that the mass of each consumer type is independent of the demand state, but each type's willingness to pay is greater in the good demand state than in the bad state. Formally, $m(\theta|\sigma)$ is independent of σ, while the valuation of high and low type consumers is increasing in the demand state: $v(\theta|G) > v(\theta|B)$ for $\theta \in \{H, L\}$. This implies that a consumer's valuation fully reveals the underlying demand state σ. In our discussion paper (Nocke and Peitz, 2003), we show that the optimal selling strategy under vertical demand shifts always entails final sales, while uniform pricing and introductory offer strategies are dominated. Thus, the monopolist optimally sets a high price in period 1 and a low price in period 2. (It can be shown that ex ante the monopolist has no incentive to offer the good in later periods.) However, this selling policy induces intertemporal dispersion only of posted prices but not of prices at which trade occurs since all units are either sold in period 1 (when $\sigma = G$) or in period 2 (when $\sigma = B$).

Demand side: Rationing scheme. In our analysis, we have assumed random or proportional rationing (where high type consumers are rationed with the same probability as low types). In general, the optimal strategy of the monopolist depends on the particular rationing rule. Clearly, any change in the rationing rule toward efficient (or parallel) rationing (so that high type consumers are rationed with a lower probability than low types) makes it more difficult for the monopolist to use rationing as a tool of her optimal strategy. This will reduce the profitability of final sales and introductory offer strategies. However, if the optimal strategy involves rationing with probability 1 (as it does under vertical demand shifts), then the rationing rule becomes irrelevant.

Demand side: Multiplicity of consumer equilibrium. As pointed out before, for any given final sales strategy, there may be a multiplicity of consumer equilibria. In our analysis, we have selected the consumer equilibrium that is most favorable for the monopolist. We believe that it is reasonable to select this equilibrium. In particular, it is often the unique consumer equilibrium if one introduces some heterogeneity in the willingness-to-pay amongst consumers of a particular type. This observation is illustrated in our discussion paper (Nocke and Peitz, 2003).

Demand side: Consumer decision making. For a final sales strategy to work, consumers have to form beliefs about the likelihood of being rationed in the future. Since the probability of rationing depends on the behavior of other consumers, high type consumers do not have a dominant strategy. This requires consumers to be quite sophisticated in their decision-making. In contrast, when facing an introductory offer strategy (or a uniform price), consumers have a dominant strategy (and can follow a simple decision rule): "demand the good at the low price; if it is sold out at this price, buy it at the higher price (provided the price is less than your valuation)." In a world where consumers are not sophisticated decision makers, a firm may thus favor an introductory offer strategy (or a uniform price) over a final sales strategy.

Supply side: Capacity costs. We have assumed that the monopolist faces zero costs of capacity. Would introductory offers still be dominated if we allowed for positive capacity costs? In our analysis, we have shown that whenever an introductory offer strategy performs better than uniform pricing, it is dominated by some final sales strategy. Consider the generalization to positive and constant marginal costs of capacity, c_k. As the monopolist changes her strategy from a high uniform price to final sales to introductory offers to a low uniform price, she has to increase total capacity with each change. This means that with an increase of the capacity cost c_k the condition which ensures that introductory offers dominate the low uniform price becomes less strict. However, the same happens to the condition that final sales dominate introductory offers. It can be shown that introductory offers dominate the low uniform price if

$$\rho\{m(H|G) - v(L)[m(L|G) + m(H|G)]\} + c_k m(L|G) > 0.$$

Exactly the same condition implies, however, that final sales dominate introductory offers. Hence, the optimal selling policy never involves introductory offers.

Supply side: Fixed capacity. In our analysis, we have assumed that the monopolist chooses total capacity k. Suppose now instead that the monopolist faces the restriction $k \leq \overline{k}$, where $\overline{k}$ is an exogenous capacity limit. This hypothesis may apply well to the case of ticket sales for concerts and the like, where the maximum number of seats in the concert hall is fixed. How does this assumption affect the profitability of the different selling policies? In the absence of an exogenous capacity limit, the (maximum) output sold in the good demand state is lower under the best final sales strategy than under the best introductory offer strategy. We can then easily show that whenever there exists a final sales strategy which dominates introductory offers in the absence of exogenous capacity limits, there exists some final sales strategy which dominates introductory offers in the presence of exogenous capacity limits.[17] Hence, final sales strategies perform even better - relative to introductory offer strategies - when the monopolist faces an (exogenous) capacity constraint.

However, this does not mean that introductory offer strategies cannot be optimal in this case since parameters may be such that introductory offers dominate final sales. In section 3, we have shown that, in this case, introductory offer strategies are dominated by the uniform price $p = v(L)$. This may no longer

[17] Note that for sufficiently small exogenous capacities, namely $\overline{k} \leq m(H|B)$, optimal final sales and introductory offer strategies degenerate to uniform pricing.

hold when the monopolist faces an exogenous capacity limit. To be precise, without exogenous capacity limits, introductory offers are dominated by final sales strategies if $m(H|G) > [m(H|G)+m(L|G)]v(L)$, and by the uniform price $p = v(L)$ if the reverse inequality holds. The best introductory offer strategy involves, in the good demand state, a total quantity of

$$m(H|B) + m(L|B) + \rho \left(1 - \frac{m(H|B) + m(L|B)}{m(H|G) + m(L|G)}\right) m(H|G) \equiv \widehat{k},$$

which is always less than $m(H|G) + m(L|G)$. Hence, for $\overline{k} \in [\widehat{k}, m(H|G) + m(L|G))$, the exogenous capacity limit is not binding when the monopolist uses this strategy. On the other hand, for $\overline{k} < m(H|G)+m(L|G)$, the capacity constraint would be binding if the monopolist charged the uniform price $p = v(L)$. Therefore, there exist parameter constellations under which introductory offer strategies are optimal in the presence of exogenous capacity limits. In particular, at $k = \widehat{k}$, the best introductory offer strategy and the uniform price $p = v(L)$ induce the same quantity sold in both demand states, but in the high demand state some units are sold at the high price when the monopolist uses the introductory offer.

To summarize, exogenous capacity limits make final sales more attractive relative to introductory offers. However, there exist parameter constellations under which introductory offer strategies are optimal since a (low) uniform price becomes less attractive when the monopolist faces an exogenous capacity constraint.

Supply side: The monopolist's decision making. In our model, we have assumed that the monopolist ex ante commits to a price for each period, a first-period capacity, and a total capacity. Not all of the available selling strategies require such commitment, however.

- *Price commitment.* Introductory offer strategies may require less (intertemporal) commitment power than final sales strategies. When using an introductory offer strategy, the monopolist has no incentive to change her price (or capacity) ex post in period 2. (In fact, it can be shown that an introductory offer strategy does not require commitment to the second-period price.) In contrast, when using a final sales strategy, the monopolist has ex post an incentive to raise her capacity or price in period 2.

- *Capacity commitment.* A final sales strategy requires a commitment to total capacity. Such commitment can be implemented if total production is determined ex ante and if the production of additional units is sufficiently costly (high marginal costs or high fixed costs for an additional run). An introductory offer strategy requires a commitment to first-period capacity. This can be implemented if the monopolist produces in each period and has limited production capacity per period.

Hence, while our (simple) model predicts that we should not observe introductory offers (but rather final sales and uniform prices), the demanding commitment requirements for final sales strategies may give a rationale, within our framework, for the use of introductory offer strategies.[18]

Another reason for the use of introductory offers strategies is that, in contrast to final sales, goods can be offered *concurrently* at different prices, and consumers can freely choose at which price they want to buy the good. Clearly, consumers will select the cheaper units first, and once these items are stocked out, high valuation consumers purchase the high priced units.

Our paper is also connected to several general themes in the industrial organization and microeconomics literature.

[18] In particular, introductory offer strategies can be used in durable goods monopoly with a Coasian commitment problem (see Denicolo and Garella, 1999).

The economics of rationing. In this paper, we provide a justification for the use of rationing as part of the optimal selling strategy of a monopolist. However, ours is not the first paper to point out that consumer rationing may be an equilibrium phenomenon. Apart from consumer segmentation, reasons for rationing include sunk investments by consumers (Gilbert and Klemperer, 2000), buying frenzies (DeGraba, 1995), bundling (DeGraba and Mohammed, 1999), and direct demand externalities (e.g., Becker, 1991, Karni and Levin, 1994). In our model, we do not need any of these demand-side considerations to generate rationing as an equilibrium outcome.

Price dispersion. Our paper contributes to the literature on price dispersion, initiated by Salop (1977). In the model presented in this paper, the optimal final sales strategy induces price dispersion *within states*. That is, in at least one demand state, the good is traded at different prices.[19] In our model, intertemporal price dispersion (within states) may thus arise in the absence of discounting (see, for instance, Stokey, 1979, for an analysis of price dispersion with discounting). We may interpret the probability of rationing in our model as a "discount rate", which is endogenously determined.

Endogenous quality. Our paper is loosely connected to the literature on product differentiation. In the context of final sales, we may interpret the probability of obtaining the good as the quality of the good. If the price of the good is adjusted by this probability, the model corresponds to a model of quality differentiation such as Mussa and Rosen (1978). In contrast to Mussa and Rosen (and the literature on vertical product differentiation in general), the good's quality under a priority pricing scheme is endogenously determined by demand (and thus ultimately by prices and capacity). In the literature on product differentiation, on the other hand, the firm directly controls quality. The use of different qualities or classes of service is common in the pricing of tickets (see Rosen and Rosenfield, 1997, for an economic analysis).

Final sales as a marketing strategy. More generally, our paper contributes to the literature on selling strategies of a firm with market power. We have shown that a final sales strategy, which involves restricting total capacity and thus leads to consumer rationing, may effectively separate between demand states and consumers types. Sophisticated price-capacity strategies can thus be used by a monopolist to segment the market for a homogeneous good. Price-capacity strategies may be superior to other non-price strategies such as product differentiation, which may be more costly to implement or may reduce consumers' reservation values (as in the case of damaged goods). Introductory offer strategies are less "sophisticated" than final sales strategies in that they do not fully segment the market by consumer types. In our world with commitment and rational consumers, we have shown that introductory offer strategies are never the optimal marketing strategy. Nevertheless, as argued above, since they require less sophisticated consumer behavior (and perhaps less commitment power by the seller), introductory offers may sometimes be the preferred selling strategy.

[19] As discussed in our discussion paper (Nocke and Peitz, 2003), under vertical demand shifts, final sales induce price dispersion only *across states*. That is, while the monopolist posts different prices for different periods, in each demand state, trade occurs in only one period. In the alternative information setting of our model, both types of price dispersion are induced by (different) final sales strategies.

Appendix

Proof of Proposition 3

As pointed out in the main text, any optimal final sales strategy must be such that all high types are just willing to demand the good at the high price. Hence, any optimal final sales strategy is of the form $(\widehat{p}_1(k), v(L), k)$, where $\widehat{p}_1(k)$ is chosen so as to make high type consumers just indifferent between purchasing at $\widehat{p}_1(k)$ and delaying the purchase.

Step 1. Suppose $k \in [m(H|B), m(H|G)]$. In this case, rationing occurs even at the high first-period price (when demand is in the good state). If $k \leq \min\{m(H|G), m(H|B) + m(L|B)\}$, a high type consumer is indifferent between purchasing at $\widehat{p}_1(k)$ and postponing his purchase if

$$\begin{aligned} & \left\{ Q(G|H) \frac{k}{m(H|G)} + 1 - Q(G|H) \right\} [1 - \widehat{p}_1(k)] \\ = \quad & [1 - Q(G|H)] \left(\frac{k - m(H|B)}{m(L|B)} \right) [1 - v(L)] \, , \end{aligned}$$

and so

$$\widehat{p}_1(k) = 1 - \frac{[1 - Q(G|H)] \left(\frac{k - m(H|B)}{m(L|B)} \right)}{1 - \left(\frac{m(H|G) - k}{m(H|G)} \right) Q(G|H)} [1 - v(L)] \, .$$

The monopolist's expected profit is then

$$\pi^{FS}(k) = \rho \widehat{p}_1(k) k + (1 - \rho) \{ \widehat{p}_1(k) m(H|B) + v(L)[k - m(H|B)] \},$$

which is non-linear in k.

Similarly, if $k \in [\min\{m(H|G), m(H|B) + m(L|B)\}, m(H|G)]$, the first-period price is equal to

$$\widehat{p}_1(k) = 1 - \frac{[1 - Q(G|H)]}{1 - \left(\frac{m(H|G) - k}{m(H|G)} \right) Q(G|H)} [1 - v(L)] \, ,$$

and the expected profit is given by

$$\pi^{FS}(k) = \rho \widehat{p}_1(k) k + (1 - \rho) [\widehat{p}_1(k) m(H|B) + v(L) m(L|B)],$$

which again is non-linear in k.

Let

$$\overline{p}_1(k) \equiv 1 - [1 - Q(G|H)] \max \left\{ \frac{k - m(H|B)}{m(L|B)}, 1 \right\} [1 - v(L)] \, ,$$

and note that $\overline{p}_1(k) > \widehat{p}_1(k)$ for all $k < m(H|G)$, and $\overline{p}_1(m(H|G)) = \widehat{p}_1(m(H|G))$. Next, let

$$\overline{\pi}^{FS}(k) \equiv \rho \overline{p}_1(k) m(H|G) + (1 - \rho) \left\{ \overline{p}_1(k) m(H|B) + v(L) \min\{k - m(H|B)\}, m(L|B) \right\} ,$$

and note that $\overline{\pi}^{FS}(k) > \pi^{FS}(k)$ for all $k < m(H|G)$, and $\overline{\pi}^{FS}(m(H|G)) = \pi^{FS}(m(H|G))$. Moreover, observe that $\overline{\pi}^{FS}(k)$ is linear in k for $k \leq \min\{m(H|G), m(H|B) + m(L|B)\}$, and independent of k on $[m(H|B) + m(L|B), m(H|G)]$.

We now claim that $\overline{\pi}^{FS}(m(H|B))$ is equal to $\pi^U(1)$, the profit from the uniform price $p = 1$. To see this, note that $\overline{p}_1(m(H|B)) = 1$, and

$$\overline{\pi}^{FS}(m(H|B)) = \rho m(H|G) + (1 - \rho) m(H|B) = \pi^U(1).$$

Since $\overline{\pi}^{FS}(k)$ is linear for $k \leq \min\{m(H|G), m(H|B) + m(L|B)\}$, it follows that an optimal final sales strategy must have $k \geq \min\{m(H|G), m(H|B) + m(L|B)\}$. Moreover, since $\overline{\pi}^{FS}(k)$ is constant on $[m(H|B) + m(L|B), m(H|G)]$, strictly larger than $\pi^{FS}(k)$ for all $k < m(H|G)$, and $\overline{\pi}^{FS}(m(H|G)) = \pi^{FS}(m(H|G))$, an optimal final sales strategy must have $k \geq m(H|G)$. Hence, there cannot be rationing at the high price.

Step 2. Suppose $k \in [m(H|G), m(H|G) + m(L|G)]$. In this case, rationing can only occur at the low price. The indifference condition for high type consumers can now be written as

$$\begin{aligned} 1 - \widehat{p}_1(k) &= \left\{ Q(G|H) \left(\frac{k - m(H|G)}{m(L|G)} \right) \right. \\ &\left. + [1 - Q(G|H)] \min \left[1, \frac{k - m(H|B)}{m(L|B)} \right] \right\} [1 - v(L)] \, . \end{aligned}$$

The expected profit is then

$$\begin{aligned} \pi^{FS}(k) &= \rho \left\{ \widehat{p}_1(k) m(H|G) + v(L) \left[k - m(H|G) \right] \right\} \\ &+ (1 - \rho) [\{ \widehat{p}_1(k) m(H|B) + v(L) \min \left[m(L|B), k - m(H|B) \right] \} \, , \end{aligned}$$

which is linear in k on $[m(H|G), m(H|B) + m(L|B)]$, provided this interval is non-empty (i.e., when horizontal demand shifts are weak), and on $[\max\{m(H|G), m(H|B) + m(L|B)\}, m(H|G) + m(L|G)]$.

Hence, under strong horizontal demand shifts (where $m(H|G) \geq m(H|B) + m(L|B)$), the unique candidate for an interior optimum is at capacity $k = m(H|G)$. Under weak horizontal demand shifts (where $m(H|G) < m(H|B) + m(L|B)$), there are two candidates: $k = m(H|G)$ and $k = m(H|B) + m(L|B)$. ■

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