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PIER Working Paper 03-033

"Do Vertical Mergers Facilitate Upstream Collusion?"

by

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http://ssrn.com/abstract=471782

Do Vertical Mergers Facilitate Upstream Collusion?*

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November 17, 2003

Abstract

In this paper we investigate the impact of *vertical* mergers on upstream firms' ability to sustain collusion. We show in a number of models that the net effect of vertical integration is to facilitate collusion. Several effects arise. When upstream offers are secret, vertical mergers facilitate collusion through the operation of an *outlets* effect: Cheating unintegrated firms can no longer profitably sell to the downstream affiliates of their integrated rivals. Vertical integration also facilitates collusion through a *reaction effect*: the vertically integrated firm's 'contract' with its downstream affiliate can be more flexible and thus allows a swifter reaction in punishing defectors. Offsetting these two effects is a possible *punishment* effect which arises if the integrated structure is able to make more profits in the punishment phase than a disintegrated structure.

Keywords: vertical mergers, collusion

JEL: L13, L42

1 Introduction

Many famous cases of collusion documented in the literature have involved intermediate goods industries. Further, a significant fraction of those cases have involved industries where one or more firms are vertically integrated.[1] Yet existing theories of collusion deal only with collusion

*We thank Glenn Ellison, George Mailath, Martin Peitz, Lars Stole, Dennis Yao, and especially Patrick Rey and Mike Riordan, as well as seminar participants at Helsinki University, the University of Michigan, the University of Pennsylvania, Washington University, the 2003 Duke-Northwestern-Texas I.O. Theory Conference, the 2003 I.O. Day at the N.Y.U. Stern School of Business, the Midwest Economic Theory Meeting (Pittsburgh), the North American Summer Meeting of the Econometric Society (Evanston), and the CEPR Workshop on *Competition Policy and Regulation in International Markets* (Madrid) for helpful comments.

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[1] See for example Tosdal's (1917) description of vertical mergers in the early twentieth century German Steel cartels, as well as Levinstein's (1997) description of the bromine cartel. Other examples of collusion involving some vertically integrated firms include railways (see, e.g., Porter 1983) and timber-cutting (Baldwin, Marshall and Richard, 1997). See also Hendricks, Porter and Tan (2000) on joint bidding for oil and gas tracts.

between firms selling to consumers (or atomistic buyers). In this paper, we provide the first examination of the often more relevant case where colluding firms sell to downstream firms which are strategic buyers with interdependent demands. Our particular focus is on the effect of vertical integration on the possibility of collusion in such markets. Why is vertical integration such a common feature of collusive industries? Does vertical integration facilitate upstream collusion, and if so, when should it be a concern for anti-trust regulators?

Following the Chicago School revolution of anti-trust policy in the early 1980s, vertical restraints were considered to be efficiency-enhancing. In the last decade, however, regulators and anti-trust authorities have shown an increased interest in prosecuting cases with vertical aspects (see, e.g., Kwoka and White (1999, part 3), Riordan and Salop (1995), Klass and Salinger, (1995)). At the same time, academics have been giving increased attention to the potential anti-competitive effects of vertical restraints and the nascent literature in this area has expanded considerably in recent years.[2] But this literature has – until now – taken a strictly static view of the interaction between firms. In contrast, we investigate the impact of vertical mergers in a *dynamic* game of repeated interaction between upstream and downstream firms.

In our model, upstream firms produce a homogeneous intermediate good that they sell to downstream firms. We allow downstream competition in the retail market to be in either prices or quantities. Contract offers to downstream firms may either be *public* and observed by all parties, or else *secret* and observed only by the contracting parties. In this set-up, we identify four ways in which a vertical merger affects upstream firms' incentives to collude.

First, and perhaps most importantly, vertical mergers facilitate collusion through the operation of an *outlets effect*. Cheating unintegrated firms can no longer profitably sell to the downstream affiliates of their integrated rivals, and foreclosure from these outlets makes defection from the collusive agreement less profitable. Second, vertical integration also facilitates collusion through a *reaction effect*: the vertically integrated firm's 'contract' with its downstream affiliate can be more flexible and thus allows a swifter reaction in punishing defectors. Third, and related to the reaction effect, is the *lack-of-commitment effect*. This again arises because of the flexibility of the integrated firm's relation with its downstream affiliate. The integrated firm cannot commit not to follow up deviating offers to other downstream firms with a best response through its own downstream division; and this can limit the profits it is able to extract in making deviant offers. Fourth, and working against the three previous effects is the *punishment effect*. This effect arises only if the integrated structure is able to make more profits in the punishment phase than a disintegrated structure. This increased rent in the punishment phase makes the integrated firm more tempted to deviate.

Table 1 illustrates which of these effects are at work in each of the four particular models that we study. We show that in each case, the net effect of vertical integration is to facilitate upstream collusion.

In the United States, policy makers' stance toward vertical restraints was laid out in the *Non-Horizontal Merger Guidelines* (1984). In contrast to the academic literature, the Guide-

[2] See for example, the recent contributions by Chen (2001), Choi and Yi (2001), Riordan (1998), Chen and Riordan (2003) and Rey and Tirole (2003) for a survey.

Table 1: The collusive effects of vertical integration.

	Public offers	Secret offers
Bertrand competition	Reaction effect	Outlets effect Lack-of-commitment effect
Cournot competition	Outlets effect Reaction effect Punishment effect	Outlets effect Lack-of-commitment effect Punishment effect

lines already anticipate the idea that vertical mergers may facilitate collusion,[3] envisaging two ways in which this may occur. Firstly, it may be that vertical merger facilitates upstream collusion by making it somehow easier to monitor prices. This is an old idea which has yet to be properly formalized,[4] but which has some relation to the theory proposed in this paper. Here, however, we do not impose any *ad hoc* changes in the ability to observe or punish deviations, but rather consider differences in contracts and incentives to cheat on a collusive agreement which arise *endogenously* as a result of vertical integration. Vertical integration clearly cannot improve the observability of prices when all contract offers are completely public, as in our base model. But the observability of defections from the collusive agreement is relevant only if firms can *react* to them, and vertical integration does enhance the integrated firm's ability to react to observed defections, so vertical integration facilitates collusion.[5]

The second way in which the Non-Horizontal Merger Guidelines envisage that vertical integration may facilitate collusion is through the acquisition of a "disruptive buyer". The Guidelines state that a disruptive buyer is one which is substantially different from the others, the idea being that price-cutting to this buyer is particularly attractive, so that the "removal" of this buyer from the downstream market may significantly reduce incentives to cheat on a collusive agreement. Again, this idea has some relation to our theory, but we derive, rather than impose, the result that the purchase of a downstream buyer by an upstream firm makes it less attractive for its rivals to cheat by removing an outlet for their cheating (our 'outlets'

[3] Interestingly, the Guidelines, like our paper, are concerned entirely with the effects on *upstream* collusion.

[4] But see Jullien and Rey (2001) for the related idea that resale price maintenance may facilitate collusion in an agency model where retailers face demand shocks which are not observed by wholesalers. In their model, resale price maintenance acts to smooth downstream prices which can make cheating easier to detect.

[5] Further, the reaction effect may in principle still arise even when contract offers are made secretly. We show that it is possible to design the collusive scheme so that some deviations will be detected and reacted to before they are revealed in the market price for the downstream output. Unintegrated firms will not make deviant offers to the integrated firm as this would allow the latter immediately to detect cheating. Deviant offers to unintegrated downstream firms also become more difficult if these firms are normally supplied by the integrated firm, because the integrated firm can detect such cheating if its downstream customers are led to reject contracts which they would normally accept.

effect). Our analysis suggests that the Guidelines may be too restrictive in focusing on buyers which 'differ substantially' from other firms in its market: even when downstream firms are symmetric, the removal of a downstream buyer can improve collusion possibilities. Moreover, it is not obvious that the acquisition of a particularly attractive buyer will facilitate upstream collusion. For the acquiring upstream firm it is a double-edged sword since it now owns an attractive outlet for cheating, so its own incentive to cheat may increase (in our model this shows up as the punishment effect). Nevertheless we show that integration with a low-cost or large downstream firm most facilitates collusion in our model of public offers.

The plan of the paper is as follows. In section 2 we set out a model of repeated competition between upstream firms to supply an input to downstream firms, where offers made to downstream firms are public information. Section 3 presents the equilibrium analysis for both Bertrand and Cournot competition downstream, demonstrating that vertical merger facilitates collusion in these models. Section 4 compares our results with the policy laid down in the U.S. Non-Horizontal Merger Guidelines and examines the idea they outline of a 'disruptive buyer' by introducing asymmetries downstream and investigating which vertical mergers are most damaging to competitive conduct. In section 5 we turn to an investigation of how the analysis is changed if upstream firms' offers are secret. In section 6 we investigate to what extent other vertical restraints can substitute for vertical merger in facilitating collusion. Section 7 concludes.

2 The Model with Public Offers

M identical upstream firms, $U1$, $U2, ..., UM$ produce a homogeneous intermediate good at constant and identical constant marginal cost c, which for simplicity we set equal to 0. They sell this good to N symmetric downstream firms (or retailers), $D1$, $D2, ..., DN$. Downstream firms transform intermediate inputs into a homogeneous final output on a one-to-one basis at zero marginal costs of production. For simplicity, we assume that $N \geq M \geq 2$. (If $N = 1$ or $M = 1$, the monopoly outcome could trivially be achieved.)

The M upstream firms make simultaneous and public take-it-or-leave-it two-part tariff offers to the downstream firms. Ui's offer to Dj takes the form $\phi_{ij} \equiv (w_{ij}, F_{ij})$, where w_{ij} is the marginal wholesale price and F_{ij} is the fixed fee. The fixed fee F_{ij} has to be paid when the offer is accepted, while the wholesale price w_{ij} has to be paid for each unit that is ordered and then sold in the retail market to consumers. If Ui does not make an offer to Dj, then $\phi_{ij} = \varnothing$. In the retail market, the N downstream firms compete either in prices or quantities. That is, Dj sets its retail price p_j (under price competition) or its quantity q_j (under quantity competition).

Time is discrete and indexed by t. Firms have an infinite horizon, and maximize the discounted sum of their future profits, using the common discount factor $\delta \in (0, 1)$. Vertically integrated firms are assumed to maximize their joint profits, independently of any "transfer prices" between the upstream and downstream affiliate. As pointed out by Bonanno and Vickers (1988), this implies that the vertically integrated downstream firm's true wholesale price is the marginal cost of its upstream affiliate, c.

Each period, an identical set of consumers come to the downstream market to buy the final good. We write $Q(\cdot)$ and $P(\cdot)$ for the market demand and inverse market demand functions, respectively.

The timing in each period is as follows:

1. *Offer stage:* Upstream firms $U1, ..., UM$ simultaneously make public offers to the downstream firms.

2. *Acceptance stage:* Downstream firms $D1, ..., DN$ simultaneously decide which contract(s) to accept.[6] If they decide to accept a contract, the relevant fixed fee is paid to the upstream firm. Dj's decision vis-à-vis Ui's offer is denoted by $\alpha_{ji} \in \{accept, reject\}$.

3. *Output stage:* Downstream firms $D1, ..., DN$ simultaneously set prices or quantities in the downstream (retail) market. Quantities demanded by consumers are then ordered from the upstream firms at the relevant wholesale prices.

The game is one of public information: all past actions become common knowledge at the end of each stage.

We seek the most collusive subgame perfect equilibrium that gives all monopoly rents to the upstream firms. For simplicity we assume that upstream firms sustain collusion through infinite Nash reversion (see the classic analysis of Friedman (1971) and the papers which followed it) and moreover that such Nash reversion will be triggered by *any* deviation by an upstream firm. In contrast, deviations by downstream firms (which get no rents in the collusive equilibrium) do not trigger any punishment. Unfortunately the analysis of optimal punishment schemes turns out to be a complex undertaking in our set-up, and is outside the scope of this paper. Optimal punishment strategies in oligopolies are explored in Abreu (1986) (see also the more general analysis in Abreu (1988)), but unfortunately, the results of these papers do not apply in our set-up for several reasons. In particular, the optimal punishment literature assumes that oligopolists sell directly to (atomistic, myopic) consumers. Whereas in our model, upstream oligopolists sell to downstream firms, which could potentially be involved in enforcing the collusive scheme. The optimal design of collusive schemes where effective deviation requires the agreement of two parties (a downstream firm must accept the upstream firm's deviant offer) is an important and interesting topic which we hope to address in future research.

In common with the majority of the non-cooperative collusion literature, we do not allow for direct side payments between upstream firms, on the grounds that these will invite scrutiny by anti-trust authorities. We will say that vertical integration *facilitates* upstream collusion if it *reduces* the critical discount factor above which the most collusive equilibrium can be sustained. Throughout the paper, we will denote by Q^M, p^M, and π^M the joint-profit maximizing industry output, retail price, and industry profit, respectively.

[6] Downstream firms are allowed to accept more than one offer, i.e., contracts are non-exclusive.

3 Equilibrium Analysis

We now turn to the equilibrium analysis of Bertrand and Cournot competition downstream, which will allow us to set out in more detail the outlets, reaction, and punishments effects of vertical integration. In the following, we compare two market structures: *non-integration (NI)*, where no upstream firm is vertically integrated, and *single integration (SI)*, where a single upstream firm and a single downstream firm are vertically integrated. Before doing so, let us consider the properties of the noncollusive equilibrium.

Noncollusive Equilibrium. In the following, we will refer to the static subgame perfect equilibrium of the stage game as the noncollusive equilibrium of the game. We assume that the punishment phase of the collusive equilibrium involves infinitely repeated play of this static equilibrium. Independently of market structure, and independently of whether downstream competition is in prices or quantities, any noncollusive (static) equilibrium must have the following two properties: (i) each upstream firm makes zero profit (on sales to unintegrated downstream firms), and (ii) each offer to an (unintegrated) downstream firm (that is accepted in equilibrium) maximizes the bilateral rents, given the set of other offers that are accepted in equilibrium. To see (i), suppose otherwise that Uj makes a positive profit on a particular contract in the noncollusive equilibrium. Then, its upstream rival Ui could profitably deviate by offering the same contract but with a slightly lower fixed fee. This would not affect downstream competition (as wholesale prices are unchanged), and so clearly the deviant contract would be accepted. Hence, no upstream firm can make a positive profit in any noncollusive equilibrium. To see (ii), suppose otherwise that Uj's offer to Di does not maximize the bilateral rents of the pair $Uj - Di$. Then, another (unintegrated) upstream firm, say Dk, can profitably deviate by offering a contract to Di that is more attractive to Di than Uj's offer and that leaves some rent to Dk. Since Dk does not receive any rents from any other contract (see (i)), any externalities that this new contract may exert will not affect the profitability of Uj's contracts with other firms.

3.1 Bertrand Competition

Suppose first that downstream firms compete in prices.

Noncollusive Equilibrium. Under both non-integration and single integration, upstream firms make two-part tariff offers of the form $(w_{ij}, F_{ij}) = (0, 0)$ to the (unintegrated) downstream firms. Consequently, all downstream firms set a retail price of 0, and industry profits are zero.

Non-Integration. Consider the market structure where no firm is vertically integrated. The collusive equilibrium offers are of the form $(w_{ij}, F_{ij}) = (p^M, 0)$, which allow the upstream firms to obtain monopoly rents. Since upstream firms are symmetric, the optimal collusive scheme involves sharing the market equally. That is, each upstream firm obtains a per-period profit of π^M/M along the (collusive) equilibrium path. If an upstream firm wants to deviate from the collusive scheme, it may offer the contract $(0, \pi^M - \varepsilon)$ to a single downstream firm, or contracts of the form $(p^M - \varepsilon, -\varepsilon)$ to any subset of downstream firms, for arbitrarily small ε. By optimally deviating, an upstream firm can thus obtain (almost) the monopoly profit π^M. Following any deviation, all firms revert forever to the noncollusive equilibrium, which yields zero industry profits.

Hence, under non-integration, no upstream firm has an incentive to deviate from the collusive agreement if

$$\frac{\pi^M}{M(1-\delta)} \geq \pi^M. \quad (1)$$

The critical discount factor above which collusion can be sustained is thus given by $\widehat{\delta}^{NI} = (M-1)/M$.

Single Integration. Consider now the market structure where one firm ($U1$, say) is vertically integrated (with $D1$, say). Along the collusive equilibrium path, each upstream firm makes contract offers of the form $(p^M, 0)$ to each (unintegrated) downstream firm, who then set a retail price of p^M. This is exactly as under non-integration. As will become clear, minimizing the critical discount factor requires giving a larger market share, which we denote by α, to the integrated firm, while the remaining share of the market is divided equally between the $M-1$ unintegrated upstream firms.

By simply undercutting the $M-1$ unintegrated downstream firms in the retail market, the integrated firm can obtain the monopoly profit if it decides to cheat. The ensuing punishment phase then entails zero profit even for the integrated firm: there is no punishment effect. Hence, $U1-D1$'s no-cheating-constraint can be written as

$$\frac{\alpha\pi^M}{1-\delta} \geq \pi^M. \quad (2)$$

Comparing (1) and (2), we see that, for a given market share of $\alpha = 1/M$, vertical integration does not affect the integrated firm's incentive to deviate. This is due to the absence of the punishment effect.

Let us now consider an unintegrated upstream firm's ($U2$'s, say) incentive to deviate. If the integrated firm $U1-D1$ were unable to react to a deviation (and were forced to continue to charge the monopoly price p^M), then $U2$ could simply offer contracts of the form $(w_{ij}, F_{ij}) = (p^M - \varepsilon, -\varepsilon)$ to the unintegrated downstream firms $D2$, ..., DN for arbitrarily small $\varepsilon > 0$. Clearly, all unintegrated downstream firms would accept these deviant offers, and $U2$ could obtain a profit arbitrarily close to the monopoly profit. Since $U2$ could thus still obtain the monopoly profit if $U1-D1$ were unable to react to $U2$'s deviation, there is no outlets effect. However, since offers are public, the integrated downstream firm $D1$ can react to $U2$'s deviation. In particular, since $D1$ has marginal cost of $c=0$, it can simply undercut any price $p_j > 0$ set by downstream firms Dj, $j = 2, ..., N$. It follows that Dj cannot make any profits from accepting a deviant offer, and so neither can the deviant unintegrated upstream firm $U2$. This constitutes an extremely strong reaction effect.

In our discussion, we have implicitly assumed that $U2$ will not find it profitable to sell through the integrated downstream firm $D1$. But this is indeed the case since if the integrated firm $U1-D1$ does not want to deviate on its own account (i.e., if (2) holds), it certainly will not wish to accept a (profitable to $U2$) deviating offer from its rival which splits the profits from deviation. If collusion is going to break down, the integrated firm is always at least weakly better off doing this by making a deviant move of its own at the output stage. This is the operation of the *outlets effect.* In the pathological case of homogeneous Bertrand competition, the outlets effect has no bite for the unintegrated firm since if it were not for the reaction effect it could obtain the whole monopoly profit without needing to sell through the integrated firm. This is obviously not the case in general, however, or indeed, as we will see, in the Cournot model we study below.

It follows from these observations that an unintegrated upstream firm in this environment cannot profit from cheating; it will be willing to collude as long as:

$$\frac{(1-\alpha)\pi^M}{(M-1)(1-\delta)} \geq 0. \tag{3}$$

Clearly, this inequality is satisfied for any market share allocation α. From the no-cheating constraint of the integrated firm, (2), the critical discount factor under single integration is thus given by $\widehat{\delta}^{SI} = 1-\alpha$. Hence, single integration facilitates upstream collusion for any market share allocation $\alpha > 1/M$. In fact, by choosing α arbitrarily close to 1, the critical discount factor above which collusion can be sustained under (single) vertical integration can be made arbitrarily small. We thus have the following result.

Proposition 1 *If downstream firms compete in prices, vertical integration facilitates upstream collusion.*

In the case of Bertrand competition, the collusive effect of vertical integration relies solely on the reaction effect. Under public offers, firms can immediately observe any deviation from the expected collusive contracts. However, upstream firms' ability to punish observed deviations is limited because they have already posted their take-it-or-leave-it contract offers, and are committed to these. Nevertheless, an integrated firm can react in this way because it does not need an inflexible contract to ensure that its downstream affiliate toes the collusive line. This is why vertical integration is helpful in maintaining collusion here - because the integrated firm has more flexibility to react. Effectively, through integration, $U1$ has recruited $D1$ as an 'enforcer' of the upstream collusive agreement.

Remark: Whilst the Bertrand homogeneous goods case is extreme, in that a single vertical merger allows perfect collusion, it illustrates the more general point that (provided the initial number of upstream firms is larger than two), a *vertical merger can easily be more collusive than a horizontal merger* between two upstream firms. This is in contrast to what seems to be widely believed and underlines the importance of studying the collusive impact of vertical mergers.

3.2 Cournot Competition

Noncollusive Equilibrium. In contrast to the case of Bertrand competition, downstream firms now obtain rents in the noncollusive equilibrium. Independently of the market structure, the noncollusive equilibrium involves two-part tariffs of the form $(w_{ij}, F_{ij}) = (0,0)$ to each (unintegrated) downstream firm. Each of the N downstream firms obtains a profit of $\pi^C(N)$, which is $1/N$th of the N-firm Cournot industry profit.

Non-Integration. Suppose no firm is vertically integrated. In the collusive equilibrium, the wholesale price is set such that the total Cournot output of all active downstream firms with this input cost would be exactly the monopoly output of a structure facing the true marginal cost. Specifically, suppose Dj produces a quantity of $q_j = \beta_j Q^M$, $0 \leq \beta_j \leq 1$, along the collusive equilibrium path, the input for which it purchases from Ui. Then, the marginal wholesale price is $w_{ij} = P(Q^M) + \beta_j Q^M P'(Q^M)$ and the fixed fee is $F_{ij} = [p^M - w_{ij}]q_j = -q_j^2 P'(Q^M) = \beta_j^2 \pi^M$. To equate the incentives to deviate for all upstream firms, each upstream firm should optimally receive, in each period, $1/M$th of the monopoly profit along the collusive equilibrium path. If an upstream firm wants to deviate, it can do so by slightly reducing the fixed fee, leaving unchanged the wholesale price. It is a dominant strategy for each downstream firm to accept such a deviant offer. Consequently, the deviant upstream can obtain the monopoly profit in the period of deviation. In the ensuing punishment phase, upstream firms do not obtain any rents. The no-cheating constraint under non-integration is thus given by:

$$\frac{\pi^M}{M(1-\delta)} \geq \pi^M. \tag{4}$$

Note that this constraint is the same as under Bertrand competition, and so the critical discount factor is again given by $\widehat{\delta}^{NI} = (M-1)/M$.

Single Integration. Suppose now that one firm ($U1$, say) is vertically integrated (with $D1$, say). The collusive equilibrium offers to unintegrated downstream firms take the same form as under non-integration. We again denote the market share of the integrated firm by α, while the remaining market share is divided equally between the $M-1$ unintegrated upstream firms.

Consider first the incentives to cheat for the integrated pair $U1 - D1$. We claim that the integrated pair can still make the monopoly profit in the period of deviation in the following way. $U1$ should offer contracts with a wholesale price of p^M or above to all of the unintegrated downstream firms, as well as a slightly negative (but arbitrarily small) fixed fee, $-\varepsilon$, to induce them to accept these offers. Because of our assumption that *any* deviation by an upstream firm will lead to Nash reversion, this action signals to the unintegrated downstream firms that there will be a punishment phase from next period onward. At the output stage (following $U1$'s deviant offers and the downstream firms' acceptance decisions) the downstream firms will therefore produce the static Nash equilibrium quantities, given the downstream firms' wholesale prices (since there will be punishment next period regardless of what firms do at the output stage). If one or more of the unintegrated downstream firms were to accept not only the deviant offer but also their equilibrium contracts , then in the induced subgame the industry equilibrium output would be larger than the monopoly quantity. Since the equilibrium offers are such that each unintegrated downstream firm makes zero profit when industry output is Q^M, it follows that each would now make a loss if it were to accept its equilibrium contract.[7] Hence, following

[7] Let Q' denote industry output in the subgame where (after observing $U1$'s deviation) one or more downstream firms accept their equilibrium contracts. Clearly, $Q' > Q^M$. We claim that if downstream firm Dj, $j \neq 1$, were to accept its equilibrium contract after $U1$'s deviation, its profit gross of the fixed fee would be lower than along the collusive equilibrium path. Since its equilibrium net profit is just zero, it would thus make an overall loss if it decided to accept its equilibrium contract following $U1$'s deviation.

From the first-order condition of profit maximization, the profit (gross of the fixed fee) of a downstream firm

$U1$'s deviation, all unintegrated downstream firms will reject their equilibrium contracts and produce nothing, while $D1$ will produce the monopoly output. Thus the integrated pair can achieve π^M in the period of deviation, the same profit as can be achieved by an unintegrated firm if no firm is integrated.[8]

In the ensuing punishment phase, which involves infinite reversion to the noncollusive equilibrium, $U1 - D1$ obtains a per-period profit of $\pi^C(N)$, the profit of a firm in Cournot competition when there are N competitors. Hence, the integrated firm has no incentive to cheat if

$$\frac{\alpha\pi^M}{1-\delta} \geq \pi^M + \frac{\delta}{1-\delta}\pi^C(N). \tag{5}$$

The last term on the right-hand side represents the punishment effect of vertical integration.

Consider now an unintegrated upstream firm's incentive to deviate ($U2$'s, say). Let π_{un}^{dev} denote the maximum profit $U2$ can get in the period of deviation; we discuss this term in detail below. Following any deviation, firms will revert forever to the noncollusive equilibrium, which gives zero profit to any (unintegrated) upstream firm. Hence, the no-cheating constraint for an unintegrated upstream firm can be written as

$$\frac{(1-\alpha)\pi^M}{(M-1)(1-\delta)} \geq \pi_{un}^{dev}. \tag{6}$$

The most 'aggressive' deviation for $U2$ would consist in offering to all $N-1$ unintegrated downstream firms contracts of the form $(w_{ij}, F_{ij}) = (0, \pi^C(N) - \varepsilon)$, where $\varepsilon > 0$ can be chosen to be arbitrarily small. However, $U2$ may do weakly better by increasing its wholesale prices and adjusting its fixed fees accordingly so as to extract (almost) all of the downstream rents (or by reducing the number of downstream firms to which it sells when deviating). This induces softer behavior from the unintegrated downstream firms (but more aggressive behavior from the integrated downstream firm $D1$). If the number of downstream firms N is sufficiently large, this will allow $U2$ to extract more downstream rents. (This intuition is closely related to the result

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with wholesale price w, conditional on industry output Q can be written as

$$\pi(w|Q) = -\frac{(P(Q)-w)^2}{P'(Q)}.$$

We want to show that $\pi(w|Q)$ is strictly decreasing in Q. The derivative of $\pi(w|Q)$ with respect to Q is negative if

$$2\left[P'(Q)\right]^2 - P''(Q)\left[P(Q)-w\right] > 0.$$

Using the first-order condition for profit maximization, this inequality can be rewritten as

$$P'(Q)\left\{2P'(Q) + qP''(Q)\right\} > 0,$$

where q denotes the firm's own output. The term in curly brackets must be negative for q to be profit-maximizing (this is just the second-order condition), and so the inequality holds. We thus have $\pi(w|Q') < \pi(w|Q^M)$.

[8] It might seem that $U1$ could also achieve a deviation profit of π^M in the same way as in the unintegrated case: by simply undercutting his rivals' offers to $D2, ...DN$. In fact this is not the case due to a *lack-of-commitment effect* - as explained in the text, the integrated $U1 - D1$ cannot commit not to best-respond to his own deviant offer. Since the lack-of-commitment effect does not affect profits here, we postpone discussion of it until section 5.

that a horizontal merger between a subset of firms that does not induce an efficiency gain leads to higher profits for the nonparticipating outsiders; see Salant, Switzer, and Reynolds, 1983.) This softer behavior imposes a positive externality on the integrated firm, and so $U1-D1$'s profit in the period of $U2$'s deviation must be greater than or equal to $\pi^C(N)$. Consequently, $\pi^C(N)+\pi_{un}^{dev}$ is less than the industry profit when $U2$ deviates, which in turn is clearly less than the monopoly profit. Hence, we must have $\pi^C(N)+\pi_{un}^{dev}<\pi^M$.

Assuming that the market share allocation α is chosen optimally to minimize the critical discount factor (see Compte et al. 2001), we can find the critical discount factor by adding up the incentive constraint of the integrated firm, (5), and the $M-1$ incentive constraints of the unintegrated upstream firms, (6). This yields

$$\widehat{\delta}^{SI}=\frac{M-1}{M-1+\frac{\pi^M-\pi^C(N)}{\pi_{un}^{dev}}}.$$

Comparing this critical discount factor with that under non-integration, we find that (single) integration facilitates upstream collusion if and only if $\pi^C(N)+\pi_{un}^{dev}<\pi^M$. As we have shown above, this inequality is satisfied, and so $\widehat{\delta}^{SI}<\widehat{\delta}^{NI}$. We thus have the following result.

Proposition 2 *If downstream firms compete in quantities, vertical integration facilitates upstream collusion.*

In contrast to the case of Bertrand competition downstream, not only the reaction effect but also the outlets and punishment effects of vertical integration are at work here. The integrated firm sells αQ^M along the equilibrium path and since a deviating unintegrated upstream firm cannot profitably sell through the integrated downstream firm, it cannot obtain the monopoly profit on deviating. This is the outlets effect. In addition, the integrated firm can react to the unintegrated firm's deviation by further increasing its output. This is the reaction effect. Both the outlets and reaction effects reduce the incentives to cheat for unintegrated upstream firms. On the other hand, for a given market share α, the punishment effect increases the integrated firm's incentives to cheat on the collusive agreement. However, as the proposition shows, the outlets and reaction effects jointly outweigh the punishment effect, so that (single) vertical integration makes upstream collusion easier to sustain.

Notice further that in proving the above result we have not made strong use of the fact that the downstream firms are in homogeneous goods Cournot competition. We have made use of the Cournot assumption only to write the integrated firm's profit in the punishment phase, π_{int}^{NC}, by π^C, but this was relatively incidental to the main thrust of the argument. This leads us to conjecture that the result that the punishment effect is outweighed by the other effects holds as long as $\pi_{int}^{NC}+\pi_{un}^{dev}<\pi^M$, which the above line of reasoning indicates may hold more generally than only in the particular models presented here. (The further results presented below are also suggestive of this.) Extending the result to differentiated goods downstream introduces some additional features which are somewhat tangential to the main issues we wish to address here, however, so we leave this topic for further research.[9]

[9] When downstream firms produce differentiated goods, the division of output between downstream firms

Remark 1: Punishments other than Nash Reversion. Allowing for punishments which are more severe than Nash reversion will only reduce the value of both π_{int}^{NC}, the profit of the integrated firm after it has deviated, and π_{un}^{dev}, the profit of the unintegrated firm in the period in which it deviates and the integrated firm immediately reacts in a Nash fashion. Therefore we conjecture that allowing for more severe punishment of deviating firms would only make our results hold more strongly. The analysis of optimal punishment in this environment is, however, a very complex undertaking which is outside the scope of this paper.

Remark 2: Incentives for Vertical Merger. We have shown that vertical merger makes upstream collusion easier. This in itself provides upstream firms with a reason to integrate downstream. However, it should be noted that the optimal collusive arrangement of market shares (where this is interpreted as the arrangement which minimizes the critical discount factor below which collusion cannot be sustained, see Compte et al., 2002) typically gives a larger market share to integrated firms than to unintegrated ones. The reason is that on integration, a firm's own incentive to deviate from the collusive agreement weakly increases, whereas its rivals have a reduced incentive to deviate. Consequently it is optimal to give more on the equilibrium path to the merged firm to equalize deviation incentives. Thus if the collusive agreement does indeed distribute market shares in order to minimize the critical discount factor, vertical mergers are not a pure public good for other upstream industry participants. While acquiring a downstream firm may be costly (in particular if, as in the Cournot model, downstream firms make positive rents in the noncollusive equilibrium), the acquirer would be rewarded by a larger market share. A positive analysis of the merger game would require us to develop a model of the bargaining process through which parties arrive at the collusive agreement to divide output, and of the merger process itself, and so would take us too far afield from the normative issues addressed paper, but it is quite conceivable that multiple integrations could arise.[10] It is to this topic that we now turn.

3.3 The Impact of Multiple Integration on Collusion

We have shown above that in a variety of models, vertical integration by a single firm facilitates collusion relative to the case where no firm is integrated. Do further integrations also facilitate collusion? As is by now clear, each integration removes an outlet for cheating, thus facilitating collusion. It also turns a firm with an inflexible wholesale contract into one with a flexible contract. In general, this latter change has both a cost and a benefit, though only the benefit shows up in the single integration case. The benefit comes in the form of the reaction effect that we have already seen: a firm with a 'flexible' wholesale contract can revert to non-cooperative

becomes more important, and this can complicate the design of a perfect collusive scheme. To take one example of the type of problems that arise, for symmetric differentiation, perfect collusion requires that downstream production is split equally between the two downstream firms, which conflicts with the need to give a larger market share to an integrated firm (since downstream firms will not accept more than one offer with a positive fixed fee). It may be possible to overcome this problem by having the unitegrated upstream firms supply subsidized output to the integrated downstream firm, but the analysis is clearly much messier without adding to the insight described in the text.

[10] Interestingly, this observation is consistent with the description of cartelized firms' strategic reasons for vertical mergers in Tosdal (1917), and Levinstein (1997). Both authors suggest that firms used vertical integration as a bargaining tool to acquire a larger share of the collusive output.

behavior immediately on observing a deviation, rather than having to wait for an opportunity to revise its contract. The cost is that a firm with a flexible contract is able to deviate secretly through its own downstream affiliate, whereas when offers are public an (unintegrated) firm with an inflexible contract must deviate publicly. That is, since an integrated upstream firm's posted internal transfer price does not provide any commitment or affect its downstream unit's behavior (which is always profit-maximizing with respect to the whole firm), there may be no public warning in the offer or acceptance stage that the downstream unit intends to deviate from the collusive path at the output stage. This cost of integration does not arise on the first integration because unintegrated firms are anyway unable to react even to public deviations within a period, so the fact that it becomes possible to conceal one's deviation until the next period is not relevant. However, once at least one firm is integrated, we have seen that the reaction effect makes 'within period' punishment of public deviations possible; whereas deviations which are made without any alteration in public contracts cannot be punished until the following period. Thus subsequent integrations may not facilitate collusion to as great an extent as the first integration, and, as we discuss in the following, it is difficult to prove general results.

3.3.1 Bertrand Competition

We have already seen that a single integration in this model can, with appropriate redistribution of market shares, allow collusion to be sustained even in the static case: $\widehat{\delta} = 0$. Therefore, further integration clearly cannot improve the prospects for collusion in this model. There is no further reaction effect since a single integrated firm is already able to reduce the price to marginal cost when cheating occurs; there is no outlets effect since it is not necessary to sell through more than one downstream firm to achieve monopoly profit; and there is no punishment effect since there are no rents in the punishment phase. Nevertheless, further vertical integration in the industry will actually harm the prospects for collusion for the following reason. Since a vertically integrated firm always has a transfer price of c whether or not its downstream unit is planning to collude or cheat in the output stage, vertical integration makes it impossible to detect and punish cheating within the same period. Whereas cheating by an unintegrated firm is immediately visible in the wholesale contracts it offers, it will only be known that a vertically integrated firm has cheated *after* the output stage. When at least one firm is integrated, and so could react to deviations within the same period, this difference is important. Essentially, the flip side of the flexibility of pricing associated with the reaction effect is that it reduces (delays) the observability of prices. This is interesting since it contrasts strongly with the standard view (mentioned in the introduction above) that vertical integration facilitates collusion by increasing the observability of prices.

3.3.2 Cournot Competition

When downstream firms compete in quantities, the optimal collusive scheme involves each integrated upstream firm selling all of its equilibrium output through its integrated downstream affiliate. This arrangement is designed to minimize the integrated firms' incentive to deviate secretly - the incentive to deviate publicly is unaffected by the market share arrangement, as will become clear shortly. If integrated firms were to make any profits on sales to unintegrated downstream firms, these profits could still be earned in a period of secret deviation, since the on-the-equilibrium path offers will already have been accepted and quantities ordered and paid for before the integrated firm's secret deviation becomes apparent after the output stage. So any profits from sales to unintegrated firms simply augment integrated firms' secret deviation profits, and therefore such sales should be set to zero.

Under this collusive scheme, the maximum profit that a secretly deviating integrated firm can obtain is the Cournot best-reply profit to $Q^M(1-\frac{\alpha}{K})$, the monopoly quantity less what it was itself supposed to sell through its downstream affiliate, where K is the number of integrated firms and α is their total market share. Alternatively, an integrated firm can decide to deviate publicly by altering its offers to the unintegrated downstream firms. Public deviation has both an advantage and a disadvantage compared to private deviation. The advantage is that when unintegrated downstream firms see that a firm plans to expand its output above the agreed collusive share, they will refuse their equilibrium contracts since they will be unable to cover the fixed fee. Thus it is actually valuable to 'announce' a deviation to unintegrated downstream firms making sales on the equilibrium path. The disadvantage comes in the form of the reaction effect from the other integrated firms: they can respond to the public deviation by increasing their output. An integrated firm's public deviation profit is bounded from above by the profit of a Stackelberg leader with $K-1$ followers. (This bound is tight if the number of downstream firms N is sufficiently large.) The deviation profit of an unintegrated firm (which can only deviate publicly) is given correspondingly by the profit of a Stackelberg leader with K followers. Thus the profit from a public deviation is clearly decreasing in K.

Evidently, when only one firm is integrated ($K=1$), public deviation is strictly preferred to secret deviation since there are no integrated rivals to react by increasing output and (since $N \geq M \geq 2$) there is at least one unintegrated downstream firm which will react by decreasing (to zero) its output. Hence, public deviations are preferable for sufficiently small K. Conversely, for $K=M$, there are no unintegrated rivals to react and when all firms are integrated each will prefer to make a private deviation. If integrated firms also prefer to make a private deviation when only $M-1$ firms are integrated, then the Mth and final integration will make collusion more difficult for the following reason. There is no outlets effect to this integration since all the other upstream firms, if they cheat, will do so only through their downstream affiliates; there is no reaction effect, since all deviations occur secretly; but, as long as N the number of downstream firms is finite, the integration will entail a punishment effect. Thus if the number of upstream firms M is large enough that private deviation would be optimal if all but one firm is integrated, then full integration of the industry is not optimal for sustaining collusion. If, however, the number of upstream firms M is sufficiently small that even when $M-1$ firms are integrated the preferred deviation is still a public one, then full integration of the industry can be optimal (especially if N is large so that the punishment effect is small).[11]

[11] Calculations for the Cournot model with linear demand show that this is indeed the case for $M=2$ as $N \to \infty$, for example.

3.3.3 Summary of Multiple Integration

In summary, our results would suggest that although the first integration always facilitates collusion, the same may not be said of all subsequent integrations and there is an intuitive sense in which each successive merger has less collusive benefit than the previous ones.[12] The resulting policy advice is thus in contrast to that laid out in the *Non-Horizontal Merger Guidelines* (1984, section 4.221), which suggest that vertical integration is unlikely to facilitate collusion unless a significant fraction of the industry is already vertically integrated. We analyze these Guidelines further in the next section.

4 Asymmetries Downstream: The 'Disruptive Buyer' and Anti-Trust Policy on Vertical Mergers

The U.S. 1984 Non-Horizontal Merger Guidelines (section 4.22) contain remarks pertaining to vertical mergers which may facilitate collusion. Although these Guidelines were formulated on the basis of theory which is now outmoded,[13] they nonetheless provide a useful point of comparison for our theory.[14] As in our analysis, these are entirely concerned with the idea that vertical integration by an upstream firm into the downstream industry may facilitate *upstream* collusion. The first set of remarks (4.221) pertains to the idea that vertical integration may facilitate the monitoring of price if downstream prices are more visible than upstream prices. We tackle this issue tangentially in section 5 where we consider the case when upstream offers are secret. In this section we continue to assume that upstream offers are public and we analyze the second set of remarks, (4.222) *Elimination of a Disruptive Buyer,* which state:

> The elimination by vertical merger of a particularly disruptive buyer in a downstream market may facilitate collusion in the upstream market. If upstream firms view sales to a particular buyer as sufficiently important, they may deviate from the terms of a collusive agreement in an effort to secure that business, thereby disrupting the operation of the agreement. The merger of such a buyer with an upstream firm may eliminate that rivalry, making it easier for the upstream firms to collude effectively. Adverse competitive consequences are unlikely unless the upstream market is generally conducive to collusion and the disruptive firm is significantly more attractive to sellers than the other firms in its market.
>
> The Department is unlikely to challenge a merger on this ground unless 1) overall concentration of the upstream market is 1800 HHI or above... and 2) the allegedly disruptive

[12] That is to say, each successive merger will increase the set of firms which can privately deviate and reduce the set of firms which suffer from a reaction effect. This is, of course, an extremely loose statement. Within the strict context of our model, the only vertical merger which is actually harmful is the one which reduces the critical discount factor below firms' actual discount factor, and this may or may not be the first integration.

[13] US government policy towards vertical mergers is now governed largely by consideration of what may loosely be termed theories of 'raising rivals costs' (see, e.g., Riordan and Salop, 1995) rather than the 'coordinated effects' model considered here.

[14] In particular, our outlets effect, whereby integration removes an outlet through which a deviating upstream firm would like to sell, resembles the type of exclusion envisioned by the informal pre-Chicago theories of foreclosure. Unlike these theories, however, our own is immune to Bork's stinging criticism that such problems could be solved by holding an "industry social mixer" (Bork, 1978). We thank Mike Riordan for this observation.

firm differs substantially in volume of purchases or other relevant characteristics from the other firms in its market...

We have already seen that the emphasis on the idea that integration is damaging only when the integrated downstream firm differs substantially from its rivals is misplaced: we showed that adverse competitive consequences arise in models with symmetric downstream firms. Nevertheless, if downstream firms are asymmetric, merger with one firm may facilitate collusion to a greater or lesser extent than merger with another. In the following subsections we investigate how one might identify a 'disruptive buyer'.

4.1 Bertrand Competition Downstream with Asymmetric Costs

We now reconsider Bertrand competition downstream but suppose that the downstream firms have asymmetric marginal costs. We simplify to the case of just two upstream and two downstream firms. As before, $U1$, $U2$, and $D1$ have zero marginal costs. However, we now assume that $D2$ is less efficient and has marginal cost $\overline{c} \in (0, p^M)$. Thus, in the non-collusive equilibrium, $D1$ makes profit $\overline{c}D(\overline{c})$, while $D2$ makes no profit. Note that for efficient collusion, along the equilibrium path no sales should be made through the inefficient downstream firm.

Non-Integration. Suppose first that no firm is vertically integrated. The optimal collusive scheme is for each upstream firm to offer a wholesale price of p^M to the efficient downstream firm $D1$ and of $p^M - \overline{c}$ to the inefficient downstream firm $D2$, with no fixed fees in either case. In the most collusive equilibrium, $D1$ must purchase $Q^M/2$ from each of the two upstream firms. The best deviation consists in offering $D1$ a wholesale price of $p^M - \varepsilon$ (or equivalently, a wholesale price of zero and a fixed fee equal to π^M). With symmetric market shares, the collusion incentive constraint is therefore:

$$\frac{\pi^M}{2(1-\delta)} \geq \pi^M$$

Efficient Firm (U1-D1) Integration. Suppose now that one upstream firm, $U1$ say, is vertically integrated with the efficient downstream firm, $D1$. Again, the efficient collusive scheme involves sales only through the efficient downstream firm, so that the integrated $D1$ must buy some output from the unintegrated $U2$. If $U1 - D1$ is allocated a collusive market share α, this can be achieved by having $U2$ offer $D1$ a wholesale price of zero with a fixed fee of $(1-\alpha)\pi^M$; a wholesale price of p^M with $D1$ purchasing a quantity $(1-\alpha)Q^M$ from $D2$, or any combination in between. When the integrated pair deviates, it can reject $U2$'s offer (indeed, it strictly prefers to do so if this offer involves a positive fixed fee) and make no purchases from $U2$. In this way, $U1 - D1$ can get π^M from deviating, and $\overline{c}D(\overline{c})$ in the punishment phase when both upstream firms supply both downstream firms at marginal cost. The efficient integrated pair will thus collude as long as:

$$\frac{\alpha\pi^M}{1-\delta} \geq \pi^M + \frac{\delta}{1-\delta}\overline{c}D(\overline{c})$$

Consider now the unintegrated upstream firm $U2$'s incentive to deviate. In contrast to the case with symmetric downstream firms, the outlets effect reduces the unintegrated upstream firm's incentives to deviate. Even if the integrated pair, $U1-D1$, were committed to charging the monopoly price in the downstream market, $U2$ could not get the monopoly profit by deviating through the unintegrated downstream firm, $D2$, but only $\left[p^M-\overline{c}\right] D(p^M) < \pi^M$. In addition to the outlets effect, there is a strong reaction effect (as in the case of symmetric downstream firms): any deviant offer to $D2$ can be immediately and severely punished by the integrated efficient pair, which will price just below whatever wholesale price $U2$ offers to $D2$. The combination of these two effects is such that $U2$ cannot profitably deviate at all, and its no-cheating constraint becomes:

$$\frac{(1-\alpha)\pi^M}{1-\delta} \geq 0.$$

Hence, by reorganizing market shares in favor of the efficient integrated pair, the critical discount factor (above which collusion can be sustained) can be made arbitrarily small. This turns out not to be the case when it is the inefficient downstream firm which is integrated.

Inefficient Firm (U1-D2) Integration. Suppose now that one of the upstream firms, $U1$ say, integrates with the inefficient downstream firm. As under non-integration, the optimal collusive scheme involves both upstream firms selling to $D1$ at wholesale price p^M (and zero fixed fee), with $D1$ purchasing a fraction α of its output from $U1$ and the remaining fraction $1-\alpha$ from $U2$. No sales are made through $D2$. The optimal deviation for the unintegrated $U2$ is to undercut the offer to $D1$, either by offering a wholesale price of 0 and a fixed fee of $\overline{c}D(\overline{c})$, or by offering a wholesale price of $\overline{c}$ and a fixed fee of 0 (or some equivalent combination in between).[15] There is no outlets effect here: if $D2$ were committed to charging the monopoly price, $U2$ could still obtain the monopoly profit by deviating through the efficient downstream firm, $D1$. However, $U1-D2$ can react to $U2$'s deviation, which reduces $U2$'s deviation profit. The reaction effect is smaller than in the case where it is the efficient firm that is integrated since the inefficient integrated pair cannot credibly threaten to price lower than their combined marginal cost in response to a deviation. The unintegrated firm's incentive constraint is thus:

$$\frac{(1-\alpha)\pi^M}{1-\delta} \geq \overline{c}D(\overline{c}).$$

Now consider the integrated firm, $U1-D2$. Because it is relatively inefficient, it will make no profits in the punishment phase, so there is no punishment effect. The integrated firm can deviate as in the unintegrated case by slightly undercutting $U2$ and offering a wholesale price of $p^M-\varepsilon$ (and no fixed fee), so that $D1$ makes all its purchases through $U1$ and the latter can earn arbitrarily close to the monopoly profit. The integrated firm will thus be willing to collude as long as:

$$\frac{\alpha\pi^M}{1-\delta} \geq \pi^M.$$

Reallocation of the market shares in favor of the integrated firm can improve collusive prospects relative to the unintegrated case, but cannot achieve as low a critical discount factor as in the case of integration with the efficient downstream firm. This result obtains because both the

[15] For the usual reasons, the unintegrated $U2$ cannot profitably deviate through the integrated $D2$ as $U1-D2$ would rather deviate on their own account if collusion were to break down anyway.

reaction and outlets effects are weaker than in the latter case. To see this, consider first the case of $U1 - D1$ integration. The outlets effect reduces $U2$'s deviation profit (relative to non-integration) by $p^M D(p^M) - \left[p^M - \bar{c}\right] D(p^M) = \bar{c}D(p^M)$, while the reaction effect further reduces the deviation profit by $\left[p^M - \bar{c}\right] D(p^M)$. Consider now the case of $U1 - D2$ integration. As explained above, there is no outlets effect. Moreover, the reaction effect reduces $U2$'s deviation profit (relative to non-integration) by $p^M D(p^M) - \bar{c}D(\bar{c})$, which is less than $\left[p^M - \bar{c}\right] D(p^M)$, the reaction effect under $U1 - D1$ integration.

Proposition 3 *In the Bertrand model with public offers and downstream cost differences, upstream collusion is most facilitated by integrating with the most efficient downstream firm.*

So, in the case of Bertrand competition with public offers, it is the efficient buyer which is 'disruptive' in the sense that integration with this buyer most facilitates collusion. While both the reaction and outlets effect are stronger under the efficient integration, the result is not a priori obvious since integrating with the efficient buyer also results in an offsetting punishment effect, while integrating with the inefficient buyer does not. As in section 3.2 above, however, the increased punishment effect is offset by the larger reaction and outlets effects.

4.2 Cournot Competition Downstream with Limited Capacities

We now turn to a different form of asymmetry downstream which seems to be important in practice: downstream firms may have different capacities to process upstream inputs. Specifically, we assume that downstream firms compete in quantities, simplify to the case of two upstream and downstream firms and suppose that: $D2$ faces a binding capacity constraint $Q^M/2 \leq k_2 < q^C(N = 2)$ (where $q^C(N = 2)$ denotes the downstream Cournot output when there are two competing firms), whereas $D1$ does not face a binding capacity constraint, since $k_1 \geq Q^M$. We continue to assume zero marginal costs both upstream and downstream.

In the noncollusive equilibrium, the smaller buyer, $D2$, faces a binding capacity constraint and produces k_2, while the larger $D1$ does not and produces the Cournot best-reply output to k_2, $r(k_2)$. This holds independently of whether or not some firms are vertically integrated.

Non-Integration. Suppose first that no firm is vertically integrated. As in the case without capacity constraints (section 3.2 above), on the equilibrium path offers are the two part tariffs which induce each downstream firm to produce half the monopoly quantity and extract all downstream firm rents. Deviating offers similarly just undercut rivals' fixed fee offers, leaving wholesale prices unchanged. In the absence of any integration in the industry, the collusive incentive constraint is therefore again given by (1) with $M = N = 2$:

$$\frac{\pi^M}{2(1-\delta)} \geq \pi^M,$$

and so the critical discount factor is $\widehat{\delta}^{NI} = 1/2$.

Single Integration with the Large Downstream Firm ($U1 - D1$). Suppose now that one upstream firm, $U1$ say, is vertically integrated with the larger downstream firm, $D1$. The integrated firm deviates as in the standard Cournot case by offering a contract with a prohibitive wholesale price $p \geq p^M$ to the small, unintegrated downstream firm $D2$, which then rejects its on-the-equilibrium-path offer and produces nothing; the integrated $D1$ then best-responds to this by producing Q^M. In the punishment phase, the integrated firm earns more than previously, namely $r(k_2)P(k_2 + r(k_2))$, since its downstream rival is capacity-constrained in the noncollusive equilibrium. Let the collusive market share of the integrated firm be α, which must exceed $\frac{1}{2}$ if vertical integration is to facilitate collusion.[16] The incentive constraint for the integrated firm $(U1 - D1)$ is thus:

$$\frac{\alpha\pi^M}{1-\delta} \geq \pi^M + \frac{\delta}{1-\delta} r(k_2)P(k_2 + r(k_2)). \tag{7}$$

Consider now the unintegrated upstream firm, $U2$. Along the collusive equilibrium path, $U2$ may make sales either through $D1$ or $D2$. But the presence of the outlets effect means that it can deviate only by selling to $D2$. The reaction of the integrated firm then further limits profits to $k_2 P(k_2 + r(k_2))$. The incentive constraint for the unintegrated $U2$ is therefore:

$$\frac{(1-\alpha)\pi^M}{1-\delta} \geq k_2 P(k_2 + r(k_2)). \tag{8}$$

Combining (7) and (8) yields the critical discount factor:

$$\widehat{\delta}^{SI}_{U1-D1} = \frac{k_2 P(k_2 + r(k_2))}{\pi^M - [r(k_2) - k_2]\, P(k_2 + r(k_2))}.$$

This is less than 1/2 if and only if $\pi^M > [k_2 + r(k_2)]\, P(k_2 + r(k_2))$, i.e., if and only if the monopoly profit is larger than the industry profit in the noncollusive equilibrium. This inequality clearly holds. Hence, single integration with the large downstream firm facilitates upstream collusion.

Single Integration with the Small Downstream Firm $(U1 - D2)$. Suppose now that one upstream firm, $U1$ say, is vertically integrated with the smaller downstream firm, $D2$. The optimal collusive scheme in this case depends on the exact level of $D2$'s capacity: if $U1$ were to sell through its own downstream affiliate, $D2$, and $U2$ through the unintegrated $D1$, $D2$'s equilibrium output would be αQ^M, where α is again the integrated firm's market share. Since optimal collusion requires giving a larger market share to the integrated firm, $\alpha > 1/2$, we may have $\alpha Q^M > k_2$, and so $D2$ may be unable to sell αQ^M. Hence, the optimal collusive scheme may require that the integrated $U1$ sells through the unintegrated $D1$. But $D1$ will accept only one offer involving a positive fixed fee, so it cannot be induced to accept any offer from the unintegrated $U2$. Therefore, $U2$ must sell $(1-\alpha)Q^M$ through the integrated $D2$ (at wholesale price 0 and fixed fee $(1-\alpha)\pi^M$). To minimize $U1$'s incentive to deviate secretly through $D2$, $U1$ should sell as much as feasible through $D2$, namely $k_2 - (1-\alpha)Q^M$, and sell the remaining $Q^M - k_2$ units through $D1$. When deviating, $U1 - D2$ can either deviate publicly or privately. By deviating publicly, it can obtain the monopoly profit, namely by rejecting $U2$'s offer to $D2$, and charging a wholesale price and fixed fee to $D1$ that induces $D1$ to produce $Q^M - k_2$, fully

[16] Because of the punishment effect, the integrated firm's incentive to deviate will rise on integration unless its market share is increased.

anticipating that $D2$ will produce its capacity output k_2. However, $U1 - D1$ might obtain a larger deviation profit by deviating secretly downstream: this will require that the fixed fee $(1-\alpha)\pi^M$ still be paid to $U2$, but allows the integrated firm to extract $p^M \left[Q^M - k_2\right]$ from $D1$ and, in addition, the Cournot best-reply profit to $D1$'s equilibrium output, $R(Q^M - k_2)$. In the ensuing punishment phase, the integrated firm obtains the noncollusive profit of its (capacity-constrained) downstream affiliate, $k_2 P(k_2 + r(k_2))$. The incentive constraint for the integrated firm $U1 - D2$ can thus be written as

$$\frac{\alpha\pi^M}{1-\delta} \geq \pi_{int}^{dev} + \frac{\delta}{1-\delta} k_2 P(k_2 + r(k_2)),$$

where $\pi_{int}^{dev} \geq \pi^M$ is the deviation profit of the integrated firm.

Since the integrated firm can react to any deviation by the unintegrated $U2$, the unintegrated upstream firm's deviation profit is $r(k_2)P(k_2 + r(k_2))$. The incentive constraint for the unintegrated $U1$ is thus:

$$\frac{(1-\alpha)\pi^M}{1-\delta} \geq r(k_2)P(k_2 + r(k_2)).$$

Consequently, the critical discount factor is

$$\begin{aligned} \widehat{\delta}_{U2-D2}^{SI} &= \frac{\pi_{int}^{dev} - \pi^M + r(k_2)P(k_2 + r(k_2))}{\pi_{int}^{dev} + \left[r(k_2) - k_2\right] P(k_2 + r(k_2))} \\ &\geq \frac{r(k_2)P(k_2 + r(k_2))}{\left[r(k_2) - k_2\right] P(k_2 + r(k_2))}. \end{aligned}$$

Comparing $\widehat{\delta}_{U2-D2}^{SI}$ with $\widehat{\delta}_{U1-D1}^{SI}$, it can easily be verified that the former is larger than the latter. We thus obtain the following result.

Proposition 4 *In the Cournot model with asymmetric capacities, upstream collusion is most facilitated by vertically integrating with the largest downstream firm.*

4.3 Implications for Policy

The preceding models try to formalize the idea of a "disruptive buyer" - which is to say, a buyer whose presence particularly disrupts collusion, and with whom vertical merger particularly enhances collusive possibilities. When upstream offers are public, then - intuitively - it is the more efficient or larger buyer who is 'disruptive'. The intuition behind this result is two-fold. First and most obviously, price-cutting to this buyer is particularly tempting, so the 'outlets effect' of removing a desirable recipient of deviating offers is greatest on integrating with this buyer. The outlets effect yields consequences similar to those envisioned in the *Guidelines* by removing an attractive outlet for deviant offers. But such integration is really a double-edged sword, as integration with a particularly efficient or large buyer also makes it *more* tempting for the integrated firm to cheat because it insulates him from punishment by his rivals. That is, the offsetting punishment is also larger. The second, less obvious reason why integration with the more efficient buyer is desirable is because it allows more flexible contracts with this particular buyer and hence a stronger 'reaction effect' to deviations by other firms. Integration with the efficient or larger buyer most facilitates collusion because, as above, the outlets and reaction effects together offset the punishment effect.

5 Secret Offers

In this section, we examine the robustness of our previous results to the assumption that upstream firms make public offers, since the reaction effect in particular appears prima facie to depend on this assumption. We show that our result that vertical integration facilitates collusion still goes through in two simple models of downstream competition where upstream offers are secret. The game examined is essentially the same as set out in section 2 above, except that the terms of each offer made in stage 1, and whether or not it was accepted in stage 2, are known only to the two contracting parties at stage 3 when downstream firms must choose their strategic variable (price or quantity). In order to avoid the complications associated with repeated games of imperfect public monitoring, we add a fourth stage, at the end of the period, after downstream profits are realized, when all actions (e.g., offers, *signed* contracts, etc.) are publicly revealed. For simplicity, we will continue to assume throughout this section that upstream firms' marginal cost $c = 0$.[17]

Public History vs. Private History. Let $\phi_{i\cdot} = (\phi_{i1}, \phi_{i2}, ..., \phi_{iN})$ denote the vector of Ui's offers to downstream firms, $\phi_{\cdot j} = (\phi_{1j}, \phi_{2j}, ..., \phi_{Nj})$ the vector of offers received by Dj, and $\phi = (\phi_{1\cdot}, \phi_{2\cdot}, ..., \phi_{M\cdot})$ the vector of all offers. Moreover, we denote by $h^{t-1} = (h^{t-2}, \phi^{t-1}, \alpha^{t-1}, p^{t-1})$ the public history at the end of period $t-1$. Since all actions are publicly revealed at the end of the period and since we do not allow for private randomization, h^{t-1} is also each firm's private history at the end of period $t-1$. At the beginning of each of the stages 1, 2, and 3 in period t, the public history is still h^{t-1}. At the beginning of stage 2 in period t, Ui's private history is (h^{t-1}, ϕ_i^t), while Dj's private history is $(h^{t-1}, \phi_{\cdot j}^t)$. At the beginning of stage 3 in period t, Ui's private history is $(h^{t-1}, \phi_{i\cdot}^t, \alpha_{i\cdot}^t)$, while Dj's private history is $(h^{t-1}, \phi_{\cdot j}^t, \alpha_{\cdot j}^t)$. Vertical integration between Ui and Dj means that the two affiliates share all information, and so have the same private history.

Because downstream firms must now choose their strategic variable in ignorance of the contracts accepted by their rivals, we now need to specify how they form beliefs about these contracts (and indeed about their rivals' strategic choices in response to these contracts). Along the equilibrium path, beliefs are pinned down by equilibrium play, since beliefs must be correct in equilibrium. Off-the-equilibrium-path, however, perfect Bayesian equilibrium does not restrict the downstream firms' beliefs. Clearly, if we allow an arbitrary choice of off-the-equilibrium path beliefs, then an extremely large set of outcomes can be sustained, even in the static game.[18] Therefore any interesting analysis of the effect of vertical integration on

[17] For $c > 0$, there are existence problems in the Bertrand game with passive beliefs, see Rey and Vergé (2002); although O'Brien and Schaffer (1992) show that there do exist 'contracts equilibria' (Cremer and Riordan, 1988).

[18] For example, in the Bertrand price-setting game, if downstream firms react to any deviant offer by anticipating that the same upstream firm will have priced at cost to its rivals, then on receiving a deviant offer with a wholesale price above cost each downstream firm expects to sell nothing and may as well choose to price arbitrarily highly. In this case, no profit can be made from deviation and the monopoly outcome can be sustained

collusion in this set up requires some form of restriction on off-the-equilibrium-path beliefs.

For the main body of this section we follow what is by now something of a convention in the literature on foreclosure and vertical restraints where this problem arises in making the assumption of so-called *passive beliefs*. If an upstream firm deviates from its equilibrium offer to a firm, we assume that the downstream firm obtaining the deviant offer continues to believe that its downstream rivals continue to be offered their equilibrium contracts (i.e., that it is the only firm receiving a deviant offer). This way of modeling beliefs certainly has the advantage of simplicity; for further description and motivation of passive beliefs, see Hart and Tirole (1990); Rey and Tirole (1997); Segal (1999); McAfee and Schwartz (1994). We briefly discuss robustness of our results to other forms of beliefs in the appendix.

With passive beliefs, as under public offers, the noncollusive equilibrium offers must be such that (i) each upstream firm makes zero profit (on sales to unintegrated downstream firms), and (ii) each offer to an (unintegrated) downstream firm (that is accepted in equilibrium) maximizes the bilateral rents, given the set of other offers that are accepted in equilibrium.

5.1 Bertrand Competition

Suppose downstream firms compete in prices. Independently of market structure, the noncollusive equilibrium then involves zero industry profits. In particular, the (secret) take-it-or-leave-it offers are of the form $(w_{ij}, F_{ij}) = (0, 0)$, and each downstream firm Dj sets price $p_j = 0$. In the collusive equilibrium, the upstream firms extract monopoly rents from the downstream firms by offering (secret) contracts of the form $(w_{ij}, F_{ij}) = (p^M, 0)$, and so downstream firms set prices equal to the monopoly price.

Non-Integration. Consider first the case where no firm is vertically integrated. On the collusive equilibrium path, each upstream firm will receive $1/M$th of the monopoly profit. (Since firms are symmetric, the symmetric market share allocation equates their incentives to cheat to minimize the critical discount factor.) An upstream firm optimally deviates by offering contracts of the form $(w_{ij}, F_{ij}) = (0, \pi^M - \varepsilon)$ to each of the N downstream firms, where $\varepsilon > 0$ is arbitrarily small. Given passive beliefs, a downstream firm receiving a deviant offer anticipates that its downstream rivals will still be receiving their equilibrium offers and pricing at p^M. The downstream firm thus believes that by accepting the deviant offer, it can slightly undercut its rivals and make some (arbitrarily small) rents. The deviant upstream firm can thus extract (almost) the monopoly profit from *each* of the N downstream firms. In the ensuing punishment phase, the deviant's profits are zero. Hence, each upstream firm's incentive constraint is given by

$$\frac{\pi^M}{M(1-\delta)} \geq N\pi^M.$$

The critical discount factor below which firms will be unable to collude is therefore

$$\widehat{\delta}^{NI} = \frac{NM-1}{NM}.$$

Single Integration. We now consider what happens to the ability to collude if one upstream-downstream pair - say $U1-D1$ - vertically integrates. On the equilibrium path, the integrated

even in the static game.

firm's market share is α, where α is optimally chosen so as to minimize the critical discount factor. Suppose an unintegrated downstream firm, say $D2$, receives a deviant offer from the integrated upstream firm $U1$. Having passive beliefs, $D2$ continues to believe that the other unintegrated downstream firms, $D3$ to DN, received their equilibrium offers. So, if $U1$'s deviant offer involves a wholesale price $0 < w_{12} < p^M$, $D2$ believes that it could undercut $D3$ to DN in the retail market if it decided to accept the deviant offer. However, since $U1$ and $D1$ act as one player, $D2$ correctly anticipates that if it were to accept $U1$'s offer at stage 2, then the integrated $D1$ (with effective marginal cost of 0) would charge a retail price of $w_{12} - \varepsilon$ at stage 3. This is an implication of subgame perfection and the extensive form of the game, which stipulates that $D1$ chooses its retail price only after learning whether an unintegrated downstream firm accepted $U1$'s offer. It follows that an unintegrated downstream firm will reject any deviant offer by $U1$ that involves a positive fixed fee, and so $U1$'s deviation profit is bounded from above by the monopoly profit. By slightly undercutting its rivals in the downstream market or by offering $(p^M - \varepsilon, -\varepsilon)$ to an unintegrated downstream firm, $U1 - D1$ can obtain (almost) the monopoly profit in the period of deviation. The integrated firm will thus maintain collusion as long as:

$$\frac{\alpha \pi^M}{1-\delta} \geq \pi^M. \tag{9}$$

Thus, unlike in the public offers cases studied above, vertical integration reduces the deviation profit of the *integrated* firm.[19] We call this the *lack-of-commitment effect*.[20] It arises because the integrated firm cannot commit not to choose the best reply in the downstream market to its own upstream affiliate's deviation vis-à-vis an unintegrated downstream firm.[21]

We now examine an unintegrated upstream firm's - say $U2$'s - incentive to cheat. Along the equilibrium path, each unintegrated upstream firm has a market share of $(1-\alpha)/(M-1)$. If it deviates, it can extract π^M from each of the $N-1$ unintegrated downstream firm by offering $(0, \pi^M - \varepsilon)$. However, the deviant upstream firm, $U2$, will not find it profitable to sell through the integrated downstream firm. To see this, note that the most rent that can (profitably) be offered by $U2$ in the deviating phase is π^M. By receiving the deviant offer, the integrated downstream firm knows that the industry will revert to the punishment in the next period. If collusion is expected to break down, then the integrated firm should optimally deviate on

[19] Vertical integration does not affect the integrated firm's incentive to deviate with public offers since the outlets and reaction effects affect only its rivals.

[20] The lack-of-commitment effect can in some ways be viewed as the flip side of the reaction effect. Both effects arise from the increased flexibility of contracting between an integrated pair. The reaction effect arises because such flexibility allows a swifter reaction to deviation; the lack of commitment effect because a deviating pair cannot commit not to best-respond to any deviating offers it makes to other firms. Both effects facilitate collusion.

[21] If $U1$'s offer and the choice of $D1$'s price were made simultaneously, then Perfect Bayesian Equilibrium would not pin down $D2$'s beliefs about $D1$'s price (in the event of $D2$ receiving a deviant offer from $U1$). Under our assumption of passive beliefs, $D2$ would be willing to accept the contract $(0, \pi^M - \varepsilon)$. Hence, $U1$ could extract $N-1$ times the monopoly profit from the $N-1$ unintegrated downstream firms, and by undercutting these in the retail market, obtain another π^M through its own downstream affiliate, $D1$. $U1 - D1$'s deviation profit would thus be $N\pi^M$, exactly as under non-integration. It can readily be seen however, that even in with this modified timing, vertical integration would still facilitate upstream collusion because it reduces the *unintegrated* upstream firms' incentive to cheat (see below).

its own account and (having passive beliefs) expect to make π^M. Hence, the unintegrated upstream firm has no incentive to cheat if

$$\frac{(1-\alpha)\pi^M}{(M-1)(1-\delta)} \geq (N-1)\pi^M.$$

Summing up the incentive constraints, we obtain that the critical discount factor under (single) integration is

$$\widehat{\delta}^{SI} = \frac{M(N-1)}{M(N-1)+1}.$$

It is easily verified that $\widehat{\delta}^{SI} < \widehat{\delta}^{NI}$, and so we have the following result.

Proposition 5 *In the Bertrand model with secret offers, (single) vertical integration facilitates upstream collusion.*

The intuition for this result is the following. By buying up one of the downstream firms, an upstream firm can foreclose a buyer from potential deviant offers from his rival, reducing his rival's incentive to deviate. This is the by now familiar *outlets effect.* Buying up a downstream firm reduces the number of outlets through which an upstream firm can sell the additional output when cheating by foreclosing access to the integrated downstream unit. One might think that when - as here - downstream firms' products are perfect substitutes, this effect would be irrelevant, as indeed it is in the Bertrand model with public offers. But with secret offers and passive beliefs it matters because upstream firms can *expropriate* downstream firms when they cheat on the collusive agreement, taking advantage of the downstream firms' passive belief that they are the only firm with whom the upstream firm has cheated. In addition to the outlets effect, the passive beliefs also give rise to a lack-of-commitment effect, whereby the integrated firm cannot commit to best respond to his own deviant offers. This reduces $U1$'s deviation profit, so that unlike in the public offers case, integration also reduces the integrated firm's incentive to deviate.[22]

Multiple Integration. The above intuition extends to the case of more than one integrated firm. If $U2$ were to integrate with $D2$, for instance, then $U2$'s incentive to deviate will be reduced (provided there is at least one other unintegrated downstream firm) due to the lack-of-commitment effect, and the remaining unintegrated upstream firms' incentives to deviate will be reduced due to the outlets effect. Consider the more general case when $1 \leq K \leq M$ firms are integrated. The optimal market share allocation is such that each integrated firm has a market share of α/K, and that each unintegrated firm has a market share of $(1-\alpha)/(M-K)$. An integrated upstream firm has no incentive to cheat if:

$$\frac{\alpha\pi^M}{K(1-\delta)} \geq \pi^M. \tag{10}$$

[22]Note that vertical integration would still facilitate collusion in this model even if the timing of the game were slightly changed, so that the upstream firm chooses his own downstream output simultaneously with his offers to the upstream firms, and the lack of commitment effect does not arise. We chose to study the more natural timing where all downstream firms choose output simultaneously, rather than $D1$ choosing beforehand in the offer stage.

Similarly, an unintegrated upstream firm will not deviate if

$$\frac{(1-\alpha)\pi^M}{(M-K)(1-\delta)} \geq (N-K)\pi^M. \tag{11}$$

Since an unintegrated firm has a higher deviation profit than an integrated firm (provided there is at least one unintegrated downstream firm), and since the deviation profit of all unintegrated firms is decreasing in the number of unintegrated downstream firms, the critical discount factor is decreasing in K, the number of integrated firms. Therefore each successive vertical merger facilitates collusion.[23] We summarize our result in the following proposition.

Proposition 6 *In the Bertrand model with secret offers, each successive vertical integration further facilitates upstream collusion. (The only exception arises if $M = N$, where the Mth vertical integration leaves the critical discount factor unchanged.)*

5.2 Cournot Competition

Suppose now that downstream firms compete in quantities. For simplicity, we restrict attention to the case of an equal number of upstream and downstream firms: $M = N$.

Noncollusive Equilibrium. Independently of market structure, the noncollusive equilibrium involves each upstream firms making (secret) take-it-or-leave-it offers of the form $(w_{ij}, F_{ij}) = (0,0)$ to all unintegrated downstream firms, and thus zero profits in the upstream market.[24] Facing such offers, each downstream firm Dj sells the Cournot equilibrium output of a symmetric M-firm Cournot oligopoly (with zero marginal cost). Each downstream firm's noncollusive profit is denoted by $\pi^C(M)$.

Non-Integration. Consider first the case where no firm is vertically integrated. In any perfectly collusive scheme, each upstream firm must sell to a different downstream firm. (Because of double marginalization, the optimal contract must involve a wholesale price below the monopoly price and a positive fixed fee. Hence, each downstream is willing to accept at most one contract in equilibrium.) Without loss of generality, suppose that along the equilibrium path Ui sells a fraction β_i of the monopoly quantity to Di, where $\beta_i \leq \beta_{i+1}$ and $\sum_i \beta_i = 1$. Ui's offer to Di then takes the form $w_{ii} = P(Q^M) + \beta_i Q^M P'(Q^M)$ and $F_{ii} = \beta_{ii}^2 \pi^M$. These contracts ensure that each downstream firm produces the desired fraction of the monopoly quantity and all rents are extracted. The optimal deviation for any upstream firm consists in offering the contract $(0, R\left((1-\beta_j)Q^M\right) - \varepsilon)$ to each downstream firm Dj, $j = 1, ..., M$.

[23] Instead of multiple vertical integration, the monopoly outcome could also be achieved by having $U1$ take over *all* the downstream firms. Most likely, however, this would be prevented by anti-trust authorities since it would make monopoly inevitable by completely foreclosing all other upstream firms from the downstream market.

[24] Obviously, an unintegrated upstream firm cannot profitably deviate by offering a different contract. One may think, however, that an integrated firm can profitably deviate by offering a positive wholesale price (perhaps in conjunction with a negative fixed fee) as its own downstream affiliate would benefit from softer competition downstream. This intuition is false, however. To see this, note that if the integrated upstream firm were to offer a contract with a positive wholesale price, the receiving unintegrated downstream firm would still accept the contract $(0,0)$ offered by the (unintegrated) upstream firm, and subsequently order all its inputs at this lower wholesale price.

Here, $R(Q)$ denotes the profit from the static Cournot best reply to a rival's output of Q, i.e., $R(Q) = \max_q qP(Q+q)$. Since the deviation profit is the same for all upstream firms but $U1$ has (by assumption) the (weakly) smallest equilibrium market share, in determining the critical discount factor we need to consider only $U1$'s no-cheating constraint. $U1$ has no incentive to deviate if

$$\frac{(1-\beta_1)\pi^M}{1-\delta} \geq \sum_{i=1}^{M} R\left((1-\beta_i)Q^M\right).$$

One can show that under very general conditions $R(Q)$ must be convex in Q, and so the optimal market share arrangement involves equal market shares, $\beta_i = 1/M$ for all $i = 1, ..., M$.[25] The no-cheating constraint thus simplifies to

$$\frac{\pi^M}{M(1-\delta)} \geq M \cdot R\left(\left(\frac{M-1}{M}\right)Q^M\right). \tag{12}$$

The resulting critical discount factor is thus

$$\widehat{\delta}^{NI} = 1 - \frac{\pi^M}{M^2 \cdot R\left(\left(\frac{M-1}{M}\right)Q^M\right)}.$$

Single Integration. We now examine the case where one upstream-downstream pair, $U1-D1$ say, is integrated. One can envisage various collusive schemes. One example would be a cross-selling arrangement where $U1$ sells its output through the unintegrated downstream firms, while the unintegrated downstream firms sell their output through the integrated downstream

[25] The best-reply profit $R(Q)$ is given by

$$R(Q) = r(Q)P(Q + r(Q)),$$

where $r(Q)$ is the best-reply to Q, i.e.,

$$P(Q + r(Q)) + r(Q)P'(Q + r(Q)) = 0.$$

From the envelope theorem,

$$R'(Q) = r(Q)P'(Q + r(Q)).$$

The second derivative of R is thus given by

$$\begin{aligned} R''(Q) &= r'(Q)P'(Q + r(Q)) \\ &\quad + r(Q)P''(Q + r(Q))\left[1 + r'(Q)\right]. \end{aligned}$$

From the implicit function theorem,

$$r'(Q) = -\frac{P'(Q + r(Q)) + r(Q)P''(Q + r(Q))}{2P'(Q + r(Q)) + r(Q)P''(Q + r(Q))},$$

and so $R''(Q)$ can be rewritten as

$$R''(Q) = -\frac{\left[P'(Q + r(Q))\right]^2}{2P'(Q + r(Q)) + r(Q)P''(Q + r(Q))}.$$

This expression is strictly positive since the denominator must be negative for $r(Q)$ to be the best-reply to Q.

firm $D1$. This scheme would generate a form of reaction effect even though offers are secret. $U1$'s offer to Dj, $j \neq 1$, must involve a fixed fee. This means that if an unintegrated Ui, $i \neq 1$, were to deviate by making an offer to Dj, Dj might be tempted to reject $U1$'s equilibrium offer to save the fixed fee, alerting $U1 - D1$ to the deviation and allowing them to react. To avoid such a rejection and have Dj instead accept both $U1$'s equilibrium offer and Ui's deviant offer, Ui's offer would need to leave some rents to Dj (so that Dj can pay the fixed fee to $U1$). This clearly reduces the attractiveness of deviation for Ui. It can easily be verified that, if demand is linear, this collusive scheme results in a critical discount factor that is lower than that under non-integration.

To show that vertical integration facilitates upstream collusion for general demand, it is sufficient to show that there exists one collusive scheme that implies a lower critical discount factor than without integration. We consider the simplest collusive scheme, where in equilibrium, the integrated $U1$ sells a fraction α of the monopoly output through its own downstream affiliate, while any unintegrated Ui sells a fraction $(1-\alpha)/(M-1)$ of the monopoly output through the unintegrated Di.

Consider first $U1$'s incentive to deviate. Suppose that if Di ($i \neq 1$) were to accept a deviant offer from $U1$, Di would in the ensuing subgame produce a quantity q. While Di has passive beliefs about $U1$'s offers to all other unintegrated downstream firms, subgame perfection requires that it rationally anticipates $D1$'s response to its acceptance at the output stage which follows. Di knows that $D1$ shares all information with its upstream affiliate $U1$, and so anticipates that $D1$ will produce the Cournot best-reply output to the output choice of all the unintegrated downstream firms, taking into account Di's acceptance of $U1$'s deviant offer. Because $U1$ is integrated with $D1$, $U1$ is unable to commit to maintaining D1's output at the collusive level. This lack of commitment hurts the integrated firm because its anticipated expansion of output will reduce Di's willingness to pay for a deviant contract compared to the case where $U1$ is not integrated (and where Di would have passive beliefs about $D1$'s output). For a contract which induces an output of q, Di is therefore willing to pay (through the combination of the wholesale price and the fixed fee) no more than a total amount[26] equal to

$$qP\left(q + \frac{(M-2)(1-\alpha)}{(M-1)}Q^M + r\left(q + \frac{(M-2)(1-\alpha)}{(M-1)}Q^M\right)\right),$$

where $(M-2)(1-\alpha)Q^M/(M-1)$ is the quantity that Di (having passive beliefs) expects the other $M-2$ unintegrated downstream firms to produce, and $r\left(q + \frac{(M-2)(1-\alpha)}{(M-1)}Q^M\right)$ is the quantity Di expects $D1$ to produce if Di accepts the deviant contract.

When deviating, $U1$ therefore chooses to sell q units through each of the unintegrated downstream firms (and to sell $r((M-1)q)$ through its own downstream affiliate, $D1$), where q

[26] With secret offers and Cournot competition, the precise division of Di's payment into fixed fee and quantity times wholesale price is, from $U1$'s point of view, less important than in the foregoing, since contracts are secret and quantities are ordered in ignorance of the market price. So unlike in the Bertrand or public offers cases, Di's actual payment will not vary with offers accepted by the other downstream firms.

is the solution to

$$\max_q (M-1)qP\left(q + \frac{(M-2)(1-\alpha)}{(M-1)}Q^M + r\left(q + \frac{(M-2)(1-\alpha)}{(M-1)}Q^M\right)\right)$$
$$+r((M-1)q)P\left((M-1)q + r((M-1)q)\right).$$

It is straightforward to show that the solution to this problem is to set $q = 0$. $U1$ will not sell through the unintegrated downstream firms but instead sell the monopoly quantity through its own downstream affiliate. By offering a wholesale price $w \geq p^M$ (and a slightly negative fixed fee), $U1$ can "signal" its intention to deviate and thus make sure that all the unintegrated downstream firms will accept $U1$'s offer, reject their other offers, and produce nothing (exactly as in the case of public offers). The integrated firm can thus obtain only the monopoly profit from deviating. The integrated firm's no-cheating constraint can thus be written as

$$\frac{\alpha\pi^M}{1-\delta} \geq \pi^M + \frac{\delta}{1-\delta}\pi^C(M), \tag{13}$$

where the last term represents the punishment effect. Comparing (13) with (12), we can see that integration reduces the integrating firm's deviation profit. This is due to the *lack-of-commitment effect* described above: an unintegrated firm does not trust the integrated firm not to best respond to its own deviant offer, so integration makes it more difficult for $U1$ to expropriate unintegrated downstream firms.[27]

Consider now the incentives to deviate for an unintegrated upstream firm, Ui, $i \neq 1$. Again, Ui cannot profitably cheat through the integrated $D1$ since the integrated firm (expecting collusion to break down) would then rather deviate on its own account (and, having passive beliefs, expand $D1$'s output from αQ^M to $R((1-\alpha)Q^M)$). Since Ui can profitably cheat only through the unintegrated downstream firms, each of which believes that all other downstream firms continue to produce their equilibrium output, its deviation profit is given by $(M-1)R\left(\left[1-\frac{1-\alpha}{M-1}\right]Q^M\right)$. Hence, Ui has no incentive to cheat if

$$\frac{(1-\alpha)\pi^M}{(M-1)(1-\delta)} \geq (M-1)R\left(\left[1-\frac{1-\alpha}{M-1}\right]Q^M\right). \tag{14}$$

Comparing (14) with (12), we observe that vertical integration reduces an unintegrated upstream firm's incentive to deviate, holding fixed the market share allocation $\alpha = 1/M$. This is due to the outlets effect. Ui cannot make a profitable offer expropriating $D1$, and at the output stage the latter, unaware of Ui's deviation, continues to produce its collusive output.

Thus in the Cournot model with secret offers, three effects are present: the outlets, lack-of-commitment, and punishment effects. As the following result indicates, it can be shown that as before, the outlets and lack-of-commitment effect jointly outweigh the punishment effect.

[27] To be more precise, one could also consider that there is another effect at play here: a *non-self expropriation effect*. Not only is it more difficult for the integrated firm to expropriate the remaining unintegrated downstream firms, it also no longer desires to expropriate the firm with which it has integrated.

Proposition 7 *In the Cournot model with secret offers, vertical integration facilitates upstream collusion:*

$$\widehat{\delta}^{SI} < \widehat{\delta}^{NI}.$$

Proof. See appendix. ■

6 Other Vertical Restraints

We now return to the model with public offers and consider to what extent other vertical restraints can substitute for vertical integration in facilitating collusion. Note first that there are several practical reasons why these other vertical restraints may be an imperfect substitute for vertical integration. Firstly, the vertical integration of two firms is presumably a long-term decision. In order to be effective in facilitating collusion, other vertical restraints must also have some form of long-term component. If they are simply offered together with the contract each period and expire at the end of each period, then a deviating firm can simply offer a contract without the restraint and the restraint will have no effect in preventing deviation in our model. Therefore in the following we consider vertical restraints which are long-term in nature and already in place when the game begins. Second, there is an issue as to the enforcement of these vertical restraints. The enforcement of some vertical restraints may require the use of third parties, which makes them somewhat more cumbersome to employ (Alexander and Reiffen, 1995). For example, if U1 and D1 sign an exclusive territories agreement, and then U1 sells more to D1 and D1 breaks the exclusive territories agreement by infringing on some other territory, it is not clear that U1 actually wishes to sue D1 for doing so - the damage has instead been done to some other downstream firm D2 (say), and it is up to D2 to sue. A similar issue arises with resale price maintenance contracts.

6.1 Exclusive Dealing

If upstream firms offer exclusive dealing clauses[28] together with their along-the-equilibrium-path collusive offers, then as noted above, this will not facilitate collusion. Therefore consider 'long-term' exclusive dealing contracts which have been signed before the game starts. We assume that signing an exclusive dealing contract commits a downstream firm to deal only with one particular upstream firm, although this downstream firm is still free to reject this upstream firm's offer in any given period. Upstream firms make take-it-or-leave-it two part tariff offers to their captive downstream firms.[29]

[28] See Bernheim and Whinston (1998), Rasmusen et al. (1991), and Segal and Whinston (2000) for an analysis of the anti-competitive effects of exclusive dealing in a static setting.

[29] This yields the closest parallel in the ex post division of rents to the vertical integration case. It may be necessary to compensate downstream firms with an ex ante lump sum payment for the loss of rents associated with signing an exclusive dealing contract, in the same way that it may be necessary to pay up front to acquire and vertically integrate with a downstream firm. For brevity we do not explicitly analyse the size of this payment, since it does not affect ex post incentives to collude. We simply assume that Coasian bargaining will occur so that it is possible to agree on such a payment where it is jointly efficient.

Suppose that there are M upstream and N downstream firms. Recall that without exclusive dealing it is possible simply to undercut all one's opponents' offers slightly and extract the monopoly profit on deviating, so that an upstream firm will collude only as long as $\delta \leq (M-1)/M$. We now consider the sustainability of collusion when only one upstream-downstream pair, $U1-D1$ say, have signed exclusive contracts.

Bertrand Competition. The exclusive dealing contract does not affect payoffs in the non-collusive equilibrium, which still involves zero profits, both upstream and downstream. The collusive equilibrium involves on-the-equilibrium-path wholesale prices of p^M and no fixed fees. Clearly, $U1$ can deviate exactly as without exclusive dealing and obtain a profit arbitrarily close to π^M, so exclusive dealing does not affect his incentive to deviate. Given the exclusive dealing agreement, however, no other upstream firm Uj ($j \neq 1$) can gain $D1$'s business on deviating. Nevertheless, in the Bertrand case there is no outlets effect since Uj is able to extract π^M by offering a wholesale price $p^M - \varepsilon$ to any other downstream firm Dj ($j \neq 1$). Hence we have the following proposition:

Proposition 8 *In the Bertrand model, exclusive dealing does not facilitate upstream collusion: the critical discount factor is the same as in the absence of exclusive dealing contracts.*

Evidently, in the Bertrand case, signing an exclusive dealing contract is not a substitute for vertically integrating as a means of facilitating collusion.

Cournot Competition. In the noncollusive equilibrium, exclusive dealing affects the distribution of rents since $U1$ can extract the Cournot profit of the 'captured' $D1$, $\pi^C(N)$. This implies that there is a punishment effect of exclusive dealing. As in the Cournot model of section 3.2, the collusive equilibrium involves strictly positive wholesale prices and fixed fees, with wholesale prices less than p^M. Upstream firm $U1$ can deviate by undercutting the fixed fees offered to $D2$ to DN, and thus obtain the monopoly profit. Hence, $U1$'s incentive constraint is exactly as if $U1$ and $D1$ were vertically integrated (equation (5)). Whether exclusive dealing is as effective as vertical integration in facilitating collusion thus hinges on Uj's incentive to deviate ($j \neq 1$). In fact, Uj has strictly more incentives to cheat, as we now show.

Proposition 9 *In the Cournot model, exclusive dealing is less effective than vertical integration in facilitating upstream collusion: the critical discount factor is higher if one upstream-downstream pair has an exclusive dealing contract than if the same firms are vertically integrated.*

Proof. We need to show that Uj, $j \neq 1$, has a larger deviation profit than when $U1$ and $D1$ are vertically integrated. Let (w'_{ji}, F'_{ji}) denote the optimal deviant offers of Uj to Di, $i, j \neq 1$, when $U1$ and $D1$ are vertically integrated. Upon acceptance of these offers, the unintegrated downstream firms have to compete with each other as well as with the integrated $D1$, which faces an effective wholesale price of 0.

Let $w^*_{11} > 0$ denote $D1$'s equilibrium wholesale price in the exclusive dealing case. Suppose a deviant Uj were to offer the same wholesale prices w'_{ji} as in the vertically integrated case to downstream firms $D2$ to DN. Upon acceptance of these deviant offers, these downstream firms would now compete with a $D1$ that faces the positive wholesale price w^*_{11}. They would thus make larger profits (gross of fixed costs) than in the case where $U1$ and $D1$ are vertically integrated. (In fact, $D1$ may decide to reject its equilibrium contract after observing Uj's deviation. This would further increase the profit of the remaining downstream firms.) Thus Uj can always obtain the same deviation profit as under vertical integration, and by raising the fixed fee to $F''_{ji} > F'_{ji}$ on at least one of these offers can extract strictly more while the downstream firms still accept his offers. ■

Loosely speaking, exclusive dealing does not do as well as vertical integration because the former entails the same outlets and punishment effects as the latter, but lacks a reaction effect. $U1$ cannot optimally adjust $D1$'s output to punish a deviant Uj within the period because he is committed to a given (high) wholesale price w_{11}^*. Moreover, to the extent that, given this wholesale price, $D1$ chooses to adjust his output in response to the public deviation by Uj, this will tend to make Uj's deviation even more profitable since $D1$ will choose to shrink his output or even reject his contract. That is, to the extent that there is any reaction effect at all with exclusive dealing, it will tend to be of the opposite sign to the vertical integration case, making collusion harder.

For this reason, it is not clear whether exclusive dealing *per se* actually facilitates collusion or not. In the limit case as the number of downstream firms becomes large, one can obtain the following neutrality result.

Proposition 10 *In the limit as the number of downstream firms becomes large, $N \to \infty$, exclusive dealing contracts do not affect upstream collusion: the critical discount factor is the same as in the absence of exclusive dealing contracts.*

Proof. Since $\pi^C(N) \to 0$ as $N \to \infty$, the RHS of $U1$'s incentive constraint becomes the same as in the absence of exclusive dealing. It remains to show that the RHS of Uj's incentive constraint ($j \neq 1$) is also unaffected by exclusive dealing in the limit as $N \to \infty$. Since offers are public, the deviation profit of any upstream firm is bounded from above by π^M. But Uj can obtain a deviation profit which is arbitrarily close to π^M by offering the contract $(p^M - \varepsilon, -\varepsilon/N^2)$ to downstream firms $D2$ to DN. For N large, the downstream price will converge to $p^M - \varepsilon$, and hence induce $D1$ to reject its equilibrium contract (as $D1$ just breaks even when the market price is p^M). $D2$ to DN will thus jointly produce (slightly more than) the monopoly quantity and their joint rents converge to zero. ■

For finite N, several issues arise. Firstly, the punishment effect of exclusive dealing will be non-negligible, making collusion harder. Secondly, this may or may not be offset by the combined outlets/negative reaction effect, which jointly imply that the deviation profit of Uj, $j \neq 1$, may be strictly less than the monopoly profit.

To see this, consider the polar case of two downstream firms. We claim that Uj's deviation profit is strictly less than π^M. To obtain the monopoly profit, Uj's deviant offer must induce $D1$ to reject its equilibrium contract and $D2$ to produce the monopoly quantity. To achieve the latter aim, the wholesale price offered to $D2$ must be 0, and to extract the monopoly profit, the fixed fee must approach π^M. In the ensuing subgame, it is clearly an equilibrium that $D1$ rejects its equilibrium contract and $D2$ accepts the deviant contract. However, it is also an equilibrium for $D1$ to accept its equilibrium contract and for $D2$ to reject the deviant contract. Since we seek the most collusive subgame perfect equilibrium of the game, in the subgame following Uj's deviation, we must select the equilibrium that gives the lowest profit to the deviant Uj, i.e., in this case, the equilibrium where it is the firm receiving the deviant offer that rejects its contract. It follows that it is impossible for Uj to obtain deviation profits of π^M in this case.[30]

The limit case as $N \to \infty$ is particularly simple since the punishment effect goes to zero and moreover the double-marginalization problem disappears. This makes it possible to extract all downstream rents using linear tariffs, so that the downstream firms accepting a deviant offer will never make losses independently of whether $D1$ rejects its contract. The multiplicity of equilibria at the acceptance stage, which limits rent extraction in the above example, can thus be costlessly avoided.

In summary, we have shown that the first vertical merger in an industry facilitates collusion to a greater extent than does the first exclusive dealing contract, essentially because of the beneficial reaction effect associated with the former. On the other hand, arms-length relationships may have the advantage that any deviations are publicly observed, which is advantageous for collusion if there is at least one integrated firm able to react. Thus it may be that the optimal collusive arrangement involves some combination of vertical merger and exclusive dealing arrangements. The complexities arising in the above example suggest, however, that the analysis of such a combination will not be straightforward.[31]

6.2 Long-term Resale Price Maintenance

We can interpret resale price maintenance as an industry-wide downstream agreement that no downstream firm may sell his product at a price less than some recommended price, which we will call p^{RPM}. This will limit the opportunity for price-cutting by downstream firms in response to upstream deviations. However, a simple argument shows that in the absence of other vertical restraints or vertical integration, resale price maintenance does nothing to facilitate *upstream* collusion (though it certainly limits downstream competition should upstream collusion break down). The reason is that if there is no vertical integration or exclusive dealing, upstream firms can capture the whole downstream market by simply slightly undercutting their rivals' offers and there is no need for downstream price reductions (indeed, these are undesirable since these limit the downstream rents to be captured from deviation). It is perhaps surprising that the only vertical restraint which is still treated as per se illegal in the United States is in fact innocuous.[32] We conjecture that resale price maintenance will have an impact on collusion in the presence of other vertical restraints such as integration or exclusive dealing (RPM limits the reaction effect and affects the size of both punishment and deviation profits),

[30] In fact, the most collusive equilibrium requires that Uj's deviant offer gives positive rents to $D2$ even if $D1$ were to accept its equilibrium contract, so that it is a dominant strategy for $D2$ to accept the deviant offer. It is possible to show that the optimal deviation gives Uj a rent of $\widehat{\pi}^C(0; w_{11}^*)$, the Cournot profit of a firm with zero marginal cost which faces a rival with marginal cost w_{11}^*.

[31] For an analysis of the static case where a vertically integrated firm can also offer exclusive dealing contracts to other downstream firms, see Chen and Riordan (2003).

[32] Jullien and Rey (2000) present a model where RPM does have an impact, because shocks to downstream firms are difficult to observe.

but we leave this analysis for future research.

6.3 Long-term Exclusive Territories

In our model of homogeneous upstream goods, we can interpret an exclusive territories agreement among downstream firms as an agreement to segment the market. Thus if total demand is $Q(p)$, then the N downstream firms agree to divide the market such that each faces demand curve $\frac{1}{N}Q(p)$, and they do not compete for customers, so the price charged by Di has no effect on demand in the market served by Dj. As with resale price maintenance, this restraint has no effect on upstream collusion as upstream firms are still perfectly able to slightly undercut one another and serve all downstream firms, although it is once again true that there may be complex effects arising from the interaction of exclusive territories and other vertical restraints.

7 Conclusion

Our analysis has highlighted four effects of vertical merger on upstream firms' ability to collude. The *outlets effect* facilitates collusion. It occurs when the vertical merger of a downstream firm removes an outlet through which a cheating upstream firm would otherwise wish to sell when it deviates from collusion. The outlets effect therefore arises when an unintegrated upstream firm needs to deviate with several downstream firms in order to obtain the maximum profits from defection, and yet cannot make a profitable offer to the integrated downstream firm. Thus, for example, it arises with Cournot competition and public offers, as well as with secret offers.

The *reaction effect* occurs when a vertical merger improves the ability of upstream firms to react to deviations by their rivals. Since a vertically integrated pair can collude with a lower wholesale price than otherwise, integration can expedite punishment and thence facilitate collusion. The reaction effect clearly arises when offers are public (or, presumably, have some chance of becoming public).

The flip side of this relatively "flexible contracting" between an integrated pair lies in the *lack-of-commitment effect*. Recipients of an integrated firm's deviant offers anticipate that its own downstream affiliate will be informed about such offers and will thus play best responses to them. This effect can limit the deviation profit available to an integrated firm when offers are secret, making collusion easier to sustain.

Acting against these three effects is the *punishment effect*, which arises in a Cournot setting because the Nash reversion profits of an integrated firm are larger than those of an unintegrated upstream firm. An integrated firm therefore suffers less from potential punishment and hence is more inclined to cheat.

In the Bertrand and Cournot models with public and secret offers we have studied, the punishment effect was always offset by either the outlets or reaction effect, so that vertical merger always facilitated collusion. On the basis of our analysis, we suggest the following tentative policy conclusion. Given a relatively concentrated *upstream* industry with barriers to entry, so that collusive behavior is a serious possibility, vertical merger is more likely to be harmful in facilitating collusion when: the *downstream* industry is *less* concentrated (higher N); and more competitive (e.g. less differentiated, price not quantity setting) because the punishment effect will be smaller. This prescription is in contrast to the current conventional wisdom which suggests that vertical merger is likely to be problematic only when the downstream industry is already concentrated.[33] We also considered the issue of identifying a *disruptive buyer* - one whose presence particularly disrupts collusion, and with whom a vertical merger would particularly facilitate collusion. Intuitively, in two models of downstream heterogeneity, we showed that the disruptive buyer is the larger or more efficient buyer. This result is less straightforward than it appears since it depends on a balance of the various effects described above, and in particular integration with a disruptive buyer will actually *increase* the integrated firm's incentives to cut prices, other things being equal. Finally we investigated the extent to which the use of other vertical restraints can substitute for vertical merger in facilitating collusion. The complex interaction between vertical integration and other vertical restraints we leave as a topic for future research.

8 Appendix

Proof of proposition 7. *First step.* We claim that the pooled incentive constraints under vertical integration, (13) and (14), are slack (hold with *strict* inequality) if evaluated at $\delta = \widehat{\delta}^{NI}$ and $\alpha = 1/M$. To see this, note that the sum of (13) and (14), can be written as

$$\frac{\pi^M}{1-\delta} \geq \pi^M + \frac{\delta}{1-\delta}\pi^C(M) + (M-1)^2 R\left(\left[1 - \frac{1-\alpha}{M-1}\right] Q^M\right),$$

while the sum of the corresponding incentive constraints under nonintegration, (12), is given by

$$\frac{\pi^M}{1-\delta} \geq M^2 \cdot R\left(\left(\frac{M-1}{M}\right) Q^M\right).$$

If evaluated at $\delta = \widehat{\delta}^{NI}$, the last inequality holds with equality. Hence, if evaluated at $\delta = \widehat{\delta}^{NI}$ and $\alpha = 1/M$, the first inequality is slack if

$$\begin{aligned} &\pi^M + \left(\frac{M^2 \cdot R\left(\left(\frac{M-1}{M}\right) Q^M\right) - \pi^M}{\pi^M}\right)\pi^C(M) + (M-1)^2 R\left(\left[1 - \frac{1-\alpha}{M-1}\right] Q^M\right) \\ < \;& M^2 \cdot R\left(\left(\frac{M-1}{M}\right) Q^M\right). \end{aligned}$$

This equation can be rewritten as

$$\left[M^2 \cdot R\left(\left(\frac{M-1}{M}\right) Q^M\right) - \pi^M\right]\pi^C(M) < \left[(2M-1)R\left(\left(\frac{M-1}{M}\right) Q^M\right) - \pi^M\right]\pi^M.$$

[33] Vertical merger by an essential upstream facility into a concentrated downstream industry might nevertheless be harmful for more standard, non-cooperative reasons relating to the potential foreclosure of other firms, see, e.g., Rey and Tirole (2003). These effects do not arise in our model, which allows us to better focus on the impact of merger on collusion in a repeated game context.

Since the industry profit is higher under monopoly than under oligopoly, $\pi^M > M\pi^C(M)$, the last equation holds if

$$M^2 \cdot R\left(\left(\frac{M-1}{M}\right)Q^M\right) - \pi^M < M\left[(2M-1)R\left(\left(\frac{M-1}{M}\right)Q^M\right) - \pi^M\right],$$

or

$$R\left(\left(\frac{M-1}{M}\right)Q^M\right) > \frac{\pi^M}{M},$$

which is clearly satisfied since $r((M-1)Q^M/M) \neq Q^M/M$.

Second step. The sum of the RHS of the pooled incentive constraints under vertical integration is decreasing in α (since $R(Q)$ is decreasing in Q), and so if the pooled incentive constraints are slack if evaluated at $\alpha = 1/M$, they continue to be slack for any $\alpha \geq 1/M$.

Third step. Comparing (12) and (14), we note that (14) is slack if evaluated at $\delta = \widehat{\delta}^{NI}$ and $\alpha = 1/M$. Hence, since the pooled incentive constraints under vertical integration are (i) slack if evaluated at $\delta = \widehat{\delta}^{NI}$ and any $\alpha \geq 1/M$, (ii) continuous in α, and (iii) since an unintegrated upstream firm's incentive constraint, (14), is clearly violated at $\alpha = 1$, there must exist a market share allocation $\alpha^* \in [1/M, 1)$ such that all incentive constraints under vertical integration are slack if evaluated at $\delta = \widehat{\delta}^{NI}$ and $\alpha = \alpha^*$. Consequently, under market share allocation α^*, the critical discount factor under vertical integration is strictly less than $\widehat{\delta}^{NI}$. ■

Secret Offers with Symmetric Beliefs. While passive beliefs are most commonly used in the literature on vertical restraints with secret contracts, other assumptions on beliefs have been proposed. One particularly simple class of beliefs are "symmetric beliefs", according to which a downstream firm receiving a deviant offer believes that each of its downstream rivals was offered the same contract; see McAfee and Schwartz (1994).[34]

In what follows, we re-examine the Bertrand and Cournot cases when unintegrated downstream firms have symmetric beliefs: whenever an unintegrated downstream firm receives an out-of-equilibrium offer, it believes that all other unintegrated downstream firms have received the same deviant offer. However, since the industry is asymmetric when one upstream-downstream pair is vertically integrated, we depart from the assumption of symmetric beliefs in two respects. (1) When an unintegrated downstream firm receives a deviant offer from an unintegrated upstream firm, it believes that no deviant offer has been made to the integrated downstream firm. (2) The integrated downstream firm has passive beliefs: when it receives a deviant offer from an unintegrated upstream firm, it believes that this was a "mistake", and no deviant offers have been made to the unintegrated downstream firms. We believe these two modifications of symmetric beliefs to be reasonable. Most importantly, as we will show below,

[34] We do not consider wary beliefs, the other type of beliefs introduced by McAfee and Schwartz (1994). To illustrate the problems that arise, consider the Cournot model. If we were to follow McAfee and Schwartz in imposing wary beliefs on firms' response to both on- and off-the-equilibrium path offers, then it would be impossible for upstream firms to extract the full monopoly rent in this model, no matter how large the discount factor. If by contrast we were to impose such beliefs only on off-the-equilibrium path offers, then for a sufficiently large number of downstream firms it would be possible to sustain the monopoly outcome for any value of the discount factor in the unintegrated case (even in the static game).

the most collusive equilibrium given these restrictions on out-of-equilibrium beliefs is such that, conditional on deviating, an upstream firm's optimal deviation can always be made in such a way that is consistent with these beliefs. In particular, whenever an upstream firm wants to deviate it can do so optimally by offering the same contract to all unintegrated downstream firms. Furthermore, whenever an unintegrated upstream firm wants to deviate, it should do so optimally by deviating only through the unintegrated downstream firms. Hence, no downstream firm will ex post regret accepting an upstream firm's optimal deviation contract, which implies that an upstream firm's deviation profit is bounded from above by the monopoly profit.

Bertrand Competition Downstream. We first analyze the case of price competition downstream. The collusive and noncollusive equilibrium contracts are as under passive beliefs, that is $(p^M, 0)$ and $(0, 0)$, respectively. Consider the case where no firm is vertically integrated. Since any downstream firm receiving an out-of-equilibrium offer believes (correctly or not) that each of its downstream rivals has received the same offer, we need to specify what equilibrium prescribes if an upstream firm were to offer (w, F) to all downstream firms. (i) Assume $F < 0$. Then, equilibrium is such that all downstream firms will accept this offer and price at w in the downstream market, provided $w < p^M$. (Having passive beliefs, a downstream firm that receives the offer (w, F) believes (correctly or not) that all other downstream firms have received the same offer, and that all other downstream firms will therefore price at w. Therefore, the downstream firm should accept the offer and price at w as well.) (ii) Assume $F \geq 0$. If $w < p^M$ and $0 < F < \pi^M$, there does not exist a pure strategy equilibrium. To see this, note that if two or more firms were to accept this offer with probability 1, then they would price at w in the retail market. But then, a downstream firm could profitably deviate by not accepting the offer. However, if only one firm were to accept this offer, then this firm would price just below p^M in the retail market. But then another downstream firm has a profitable deviation: accept the offer as well and undercut the other firm in the retail market (since acceptance decisions, like offers, are secret). Therefore, let us consider the symmetric mixed strategy equilibrium, where each downstream firm accepts the offer with probability λ. Not knowing how many other firms have accepted the offer (but believing – correctly or not – that all other downstream firms have received the same offer), each downstream firm will then randomize at the output stage. Since "miscoordination" (two downstream firms or more accepting this contract) reduces downstream firms' profits, a deviant upstream firm can obtain less than the monopoly profit in total by offering contracts with positive fixed fees. Hence, the optimal deviation for an upstream firm consists in offering $(p^M - \varepsilon, -\varepsilon)$ to each of the N downstream firms. Each downstream firm will accept this contract and then price at $p^M - \varepsilon$. In this way, the deviant upstream firm obtains (arbitrarily close to) the monopoly profit. The critical discount factor under non-integration is thus $(M - 1)/M$.

Suppose now that one upstream-downstream pair ($U1-D1$, say) is vertically integrated. By the same argument as under non-integration, no upstream firm can get more than the monopoly profit by deviating. The integrated firm can get the monopoly profit by simply undercutting its downstream rivals in the retail market. There is no punishment effect since noncollusive profits are zero. Consider now the incentives to deviate for an unintegrated upstream firm, say Dj. As under non-integration, the deviant Dj can receive (close to) the monopoly profit by offering $(p^M - \varepsilon, -\varepsilon)$ to all unintegrated downstream firms. Hence, the critical discount factor under vertical integration is $(M-1)/M$. Since there is no reaction or outlets effect, vertical integration has no effect upon upstream firms' ability to collude when beliefs are symmetric and downstream firms produce homogeneous goods and compete in prices.

Cournot Competition Downstream. We now turn to the case of quantity competition downstream. The collusive and noncollusive equilibrium contracts are as under passive beliefs. Consider first the case of non-integration. As discussed for the Bertrand case, we need to specify what equilibrium prescribes if a deviant upstream firm were to offer (w,F) to each of the N downstream firms. Let $\pi(w|n)$ denote the gross profit a downstream firm can make if it accepts the contract along with $n-1$ downstream rivals, and let $q(w|n)$ denote the associated output of the firm. Since $q(w|n) < q(w|n-1)$ and a firm's profit is decreasing in the joint output of its rivals, there always exists a pure strategy equilibrium at the acceptance stage. No firm accepts the deviant offer if $\pi(w|1) \leq F$, only DN accepts the offer if $\pi(w|2) \leq F < \pi(w|1)$, only DN and $D(N-1)$ accept the offer if $\pi(w|3) \leq F < \pi(w|2)$, and so on. Therefore, an upstream firm's deviation profit is bounded from above by the monopoly profit, even if it were to offer different contracts to different downstream firms. A deviant upstream firm can obtain the monopoly profit by slightly lowering the fixed fee, while keeping the wholesale prices at their collusive equilibrium level, i.e., by offering to each of the N downstream firms the contract $(P(Q^M) + \left(Q^M/N\right) P'(Q^M), \pi^M/N^2 - \varepsilon)$, where ε is arbitrarily small.

Consider now the case where one upstream-downstream pair ($U1 - D1$, say) is vertically integrated. As in the case of passive beliefs, the integrated upstream firm can get the monopoly profit in the period of deviation by "signaling" its intention to deviate to the unintegrated downstream firms (who will then reject their equilibrium contracts) and then selling the monopoly quantity through its own downstream affiliate, $D1$. In the punishment phase, the integrated firm gets $\pi^C(N)$ in each period. Because of the outlets effect, an unintegrated upstream firm can only get the Cournot best-reply profit to the integrated firm's equilibrium output, $R(\alpha Q^M)$, which can be accomplished by offering the same contract to all unintegrated downstream firms (but not to the integrated $D1$), vindicating their beliefs.[35]

Hence, under Cournot competition, vertical integration facilitates upstream collusion if $\pi^C(N) + R(\alpha Q^M) < \pi^M$, where α is the optimal market share arrangement. It follows immediately that if the number N of downstream firms is sufficiently large, then this inequality must hold. In fact, it is possible to show that vertical integration facilitates upstream collusion for any number of upstream and downstream firms, $M \leq N$, if demand is linear.

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$$(w,F) = \left(P(\alpha Q^M + r(\alpha Q^M)) + \left(\frac{r(\alpha Q^M)}{N-1}\right) P'(\alpha Q^M + r(\alpha Q^M)), \frac{R(\alpha Q^M)}{(N-1)^2} \right).$$

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