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"A Gap for Me: Entrepreneurs and Entry"

by

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A Gap for Me: Entrepreneurs and Entry

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Abstract

We present a theory of entrepreneurial entry and exit decisions. Knowing their own managerial talent, entrepreneurs decide *which* market to enter, where markets differ in size. We obtain a striking sorting result: each entrant in a large market is more efficient than any entrepreneur in a smaller market. The result obtains since competition is endogenously more intense in larger markets. The sorting and price competition effects imply that the number of entrants (and hence product variety) may actually be smaller in larger markets. In the stochastic dynamic extension of the model, we show that the churning rate of entrepreneurs is higher in larger markets.

Keywords: entrepreneurship, entry, exit, firm turnover, industry dynamics.

JEL: L11, L13, M13.

If I can make it there, I'll make it anywhere
It's up to you - New York, New York
Frank Sinatra (New York, New York)

1   Introduction

This paper presents a simple theory of entrepreneurial entry (and exit) decisions. The two main questions addressed in this paper are the following. First, what is the relationship between the size of a market and the talent of its entrepreneurs - when entrepreneurs can decide which market to enter? Second, what is the relationship between the size of a market and the turnover (or "churning") rate of entrepreneurs?

Our model of entrepreneurship is based on the idea that (potential) entrepreneurs differ in their managerial talent. Moreover, we assume that (young) entrepreneurs are "mobile" in that they can freely choose the market they want to enter. For instance, the entrepreneurs may be chefs or restaurateurs who can decide in which city to open a new restaurant. Instead of deciding which geographical market to enter, the entrepreneurs may alternatively face the question which industry to enter. We posit that the entrepreneurial input at a given location is "essential" (or the entrepreneurial span of control exhibits strongly diminishing returns across markets), so that each entrepreneur enters at most one market.

To the best of our knowledge, this paper presents the first model in industrial organization, where a population of heterogeneous entrepreneurs decides *which* market to enter. The paper's main cross-industry predictions are not the results of exercises in comparative statics, but rather equilibrium outcomes of a multi-market model. However, our theory abstracts from some aspects of entrepreneurship that have been analyzed elsewhere in the literature. In particular, we do not consider the role of (attitudes toward) risk in entrepreneurial decision-making, which lies at the heart of the Knightian theory of entrepreneurship (see Kihlstrom and Laffont (1979)). In our model, entrepreneurs are risk-neutral profit maximizers. Moreover, we abstract from wealth constraints and imperfections in the capital market, which are explored in Evans and Jovanovic (1989). Also, in contrast to the Schultzian theory of entrepreneurship by Holmes and Schmitz (1990), we do not distinguish between entrepreneurial and managerial tasks. In our model, these tasks are inseparable, and so a business cannot be transferred from an entrepreneur to a manager. Our theory of entrepreneurship is more closely related to that of Lucas (1978), where different agents have different levels of entrepreneurial talent. However, in Lucas's model, entrepreneurs cannot choose between different markets.

The first main issue addressed in this paper - the relationship between market size and the talent of entrepreneurs - relates to a large empirical literature on productivity differences across firms and markets; see Bartelsman and Doms (2000) for a survey. Following Sveikauskas (1975) and Henderson (1986), many empirical studies have confirmed that total firms are more productive in larger cities or more densely populated regions. In the literature, these productivity differences have typically been interpreted as evidence for Marshallian externalities. Our theory provides an alternative explanation, which purely relies on self-selection of entrepreneurs.

The second main issue addressed in this paper - the relationship between market size and the turnover rate of entrepreneurs - is motivated by a large body of empirical literature (in

industrial organization and labor economics) analyzing the pattern of firm entry and exit, and gross job creation and destruction. Several interesting regularities have come out of this literature (see, for instance, Caves (1998), Cabral (1997), and Davis and Haltiwanger (1999)). First, the cross-industry differences in the level of firm turnover (or gross job reallocation) are large in magnitude and persistent over time. Second, the ranking of industries by the level of firm turnover is very similar from one country to another. Third, entry and exit rates are positively correlated across industries; that is, industries with high exit rates are likely to exhibit high entry rates as well. These regularities suggest that certain industry characteristics (such as the pattern of demand or technology) determine the turnover level. In this paper, we analyze the effect of market size on churning rates. Most previous theoretical models of dynamic industry equilibrium (e.g., Jovanovic (1982), Lambson (1991), Hopenhayn (1992), Ericson and Pakes (1995), and Asplund and Nocke (2003)) have assumed that firms are identical when they decide whether or not to enter a market, and so cross-industry predictions are typically the results of exercises in comparative statics. In contrast, in our model, we analyze the dynamic industry equilibrium in a multi-market model.

This paper considers entry (and exit) decisions of a pool of heterogeneous entrepreneurs: knowing her own talent, an entrepreneur decides which market to enter, where markets differ only in their size. What is the resulting relationship between the size of a market and the talent of its entrepreneurs? Some models of competition seem to suggest that (almost) no restriction can be placed on equilibrium outcomes. In particular, if firms behave as price takers producing a single homogeneous product, then any firm will enter the market with the highest market price. Free entry then implies that the equilibrium price is the same in all markets. Hence, all entrepreneurs are indifferent between all markets, and very little can be said about the relationship between market size and the efficiency levels of firms.[1]

In this paper, we therefore propose a model where each entrepreneur has a unique "idea", namely the knowledge to produce a distinct product. Consequently, post-entry competition is imperfect, and the intensity of competition in each market is the result of entrepreneurial entry decisions. We assume that the quality of an entrepreneur's idea varies with her talent. We then obtain a striking sorting result: in the unique equilibrium, the most capable entrepreneurs all enter the largest market, less capable entrepreneurs enter the next largest market, and so on. That is, the larger is the market, the more talented are its entrants. The sorting result follows from (little known) properties of standard models of imperfect competition with heterogeneous firms, and may be explained as follows. Free (but costly) entry implies that the toughness of price competition is positively related to the size of a market. Hence, comparing two markets of different size, an entrepreneur faces the following trade off: in a larger market, firms make greater sales (since there are more consumers) but price-cost margins are narrower (since competition is endogenously more intense). However, more efficient entrepreneurs face

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[1] The same result would obtain in a Dixit-Stiglitz type model of monopolistic competition. There, each entrant in a given market faces the same (residual) demand curve of the form $D(p) = \psi p^{-\sigma}$, where σ is some (exogenous) parameter of the utility function, whereas the demand level ψ is endogenous (and depends on market size, the product offerings of rival firms, and so on). Since each entrepreneur prefers to enter the market with the highest demand level ψ, free entry implies that, in equilibrium, each firm faces the same (residual) demand curve in all markets.

lower marginal costs, and benefit thus relatively more from the larger market. Consequently, in equilibrium, more efficient entrepreneurs will enter the larger market, while less efficient entrepreneurs will enter the smaller market. Our theory also makes testable predictions on cross-sectional differences in the size distribution of firms: we show that firms are larger in larger markets. Moreover, we show that the sorting result implies that the number of firms (and hence product variety) may actually be greater in a smaller market.

Assuming a unit transport cost (or tariff), the sorting result still obtains when firms are allowed to export their goods from one market to others. However, depending on the magnitude of transport costs, no entrepreneur may decide to enter the smallest market(s). In particular, if transport costs are sufficiently small, then all entrepreneurs will enter the largest market (or not enter any market). In an international trade context, this result may justify the fears of smaller countries that trade liberalization may lead to their de-industrialization (see Baldwin and Robert-Nicoud (2000)).

In the dynamic extension of the model, we analyze the relationship between turnover (or churning) of entrepreneurs and market size. To generate endogenous churning, we assume that the quality of an entrepreneur's idea changes stochastically over time. This may be explained by shocks to consumers' tastes. The stationary equilibrium again exhibits sorting of entrants (provided entrepreneurial efficiencies do not change at too fast a rate). Moreover, there is simultaneous entry and exit: entrepreneurs with good draws survive while entrepreneurs with sufficiently bad draws decide to leave the market and are replaced by new entrants. Most importantly, the equilibrium rate of firm turnover is higher in larger markets, and so the life span of firms is shorter. Consequently, entrepreneurial firms tend to be younger in larger markets. This is consistent with the empirical results presented in Asplund and Nocke (2000, 2003).

2   A Model of Entrepreneurial Entry

We consider a model of N imperfectly competitive markets which differ in their size, S. Markets are labelled in decreasing order of market size: $S_1 > S_2 > ... > S_N$. There is a population of (potential) entrepreneurs, each of whom may decide to enter one of the N markets. To avoid multiplicity of equilibria and integer problems, we assume that this population forms a continuum of mass M. Each (potential) entrepreneur has a unique "idea" (the know how to produce a unique product). However, the quality of her idea varies with her entrepreneurial talent. The entrepreneur's type is denoted by c, which may be the post-entry marginal cost of the entrepreneurial firm. Alternatively, an entrepreneur's type c may be inversely related to the perceived quality of her product. In any event, lower c's will be associated with better entrepreneurs. In the pool of potential entrants, the distribution of types is given by the cumulative distribution function $G(\cdot)$ with support $[0, 1]$.

If an entrepreneur decides to enter a market, she has to pay an irrecoverable entry fee $\varepsilon > 0$. In addition, an active entrepreneur pays a fixed production cost $\phi \geq 0$. Each entrepreneurial firm offers a unique differentiated product, and thus faces a downward-sloping demand curve. Any heterogeneity is assumed to be captured by the one-dimensional entrepreneurial type; firms are symmetric in all other respects. The profit (gross of entry and fixed costs) of a type-c

entrepreneur in market i is given by

$$\Pi(c; h(\mu_i), S_i) \geq 0.$$

The (Borel) measure μ_i summarizes the distribution of entrepreneurial types in market i. For any interval (or Borel set) A, the number $\mu_i(A)$ thus gives the mass of entrepreneurs active in market i whose types fall into the set A. The "intensity of competition" in market i depends on the (endogenous) distribution of entrepreneurial types and is summarized by $h(\mu_i) \in \mathbb{R}$.

We will impose a few assumptions on $\Pi(c; h(\mu), S)$. To motivate these, let us first consider an example.

Example: linear demand with a continuum of firms. There is a continuum of identical consumers (of mass S) with utility function

$$U = \int_0^n \left(x(k) - x^2(k) - 2\sigma \int_0^n x(k)x(l)dl \right) dk + H,$$

where $x(k)$ is the consumption of variety $k \in [0, n]$, and H the consumption of the Hicksian composite commodity (the price of which is normalized to one). The parameter $\sigma \in (0, 1)$ measures the substitutability between different varieties. The linear-quadratic utility function gives rise to the well-known linear demand system.[2] In particular, inverse demand for good k is given by $p(k) = 1 - x(k) - 2\sigma \int_0^n x(l)dl$. Firms have constant marginal costs of production. The distribution of the marginal costs of the entrepreneurs located in the market is summarized by the measure μ. Let $\overline{c}(\mu)$ denote the marginal type such that all firms with lower marginal costs make positive sales in equilibrium while less efficient firms make zero sales. Then, if firms compete in prices or quantities, each entrepreneur faces the same residual demand curve in equilibrium: $SD(p; \mu) = S(\overline{c}(\mu) - p)/2$, where

$$\overline{c}(\mu) = \frac{2 + \sigma \int_0^{\overline{c}(\mu)} z\mu(dz)}{2 + \sigma\mu([0, \overline{c}(\mu)])}.$$

The equilibrium gross profit of a type-c firm is given by

$$\Pi(c; h(\mu), S) = \begin{cases} S(\overline{c}(\mu) - c)^2/8 & \text{if } c \leq \overline{c}(\mu), \\ 0 & \text{otherwise,} \end{cases}$$

and so the intensity of price competition is negatively related to the marginal type $\overline{c}(\mu)$. The induced intensity of competition can thus be summarized by $h(\mu) \equiv f(\overline{c}(\mu))$, where f is an arbitrary decreasing function. The gross profit function $\Pi(c; h(\mu), S)$ has the following properties. First, it is decreasing in marginal cost c for $c \leq \overline{c}(\mu)$; however, sufficiently inefficient firms (those with $c \geq \overline{c}(\mu)$) make zero gross profit. Second, gross profits are proportional to market size S and can thus be written as $\Pi(c; h(\mu), S) = S\pi(c; h(\mu))$. More generally, an increase in market size raises the profit of more efficient entrepreneurs by at the least the same fraction as

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[2] The discrete version of the utility function goes back to Bowley (1924). The linear demand system is widely used in oligopoly models; see Vives (1999).

the profit of less efficient entrepreneurs, i.e., for $S' > S$, the ratio $\Pi(c; h(\mu), S')/\Pi(c; h(\mu), S)$ is non-increasing in c. Third, changes in the distribution of active firms affect the profit of all types in the same way (in this example, through $h(\cdot) = f(\overline{c}(\cdot))$); that is, we can completely order firm distributions by the implied intensity of competition. In particular, for $c < \overline{c}(\mu)$, $\Pi(c; h(\mu), S)$ is strictly decreasing in its second argument. Fourth, if firms are more efficient *and* there are more firms, then competition is more intense. More precisely, if $\mu([0, z]) \geq \mu'([0, z])$ for all $z \in (0, 1]$, then $h(\mu) > h(\mu')$. If, in addition, the inequality is strict for some $c < \overline{c}(\mu)$, then $h(\mu) > h(\mu')$. Fifth, $\Pi(c; h(\mu), S)$ is continuous. Finally, a change in the distribution of active firms (from μ to μ', say) which induces more intense competition (i.e., $\overline{c}(\mu') < \overline{c}(\mu)$) reduces the profit of more efficient firms by a lower fraction than that of less efficient firms: the ratio $\Pi(c; h(\mu'), S)/\Pi(c; h(\mu), S)$ is decreasing in c for all $c \in [0, \overline{c}(\mu))$. ◯

These properties of the gross profit function are not specific to the linear demand model but hold more generally in standard models of symmetric and non-localized competition. For instance, it is straightforward to show that the gross profit function in the Cournot model (with homogeneous products and a finite number of firms which differ in their (constant) marginal costs) has the same properties (under very mild restrictions on demand); see Appendix A. Let us now formally state our assumptions.

(MON) *There is a* $\overline{c}(\mu, S) \in (0, 1]$ *such that* $\Pi(c; h(\mu), S) = 0$ *for all* $c \in (\overline{c}(\mu, S), 1]$, whereas for $c < \overline{c}(\mu, S)$, $\Pi(c; h(\mu), S)$ is strictly decreasing in its first and second arguments, c and $h(\mu)$, and strictly increasing in its third argument, S.

That is, entrepreneurial firms with higher marginal cost c have lower gross profits. Moreover, a change in the distribution of active entrepreneurs that increases the toughness of competition (so that $h(\mu)$ increases) decreases profits, as does a decrease in market size S. We allow for the possibility that sufficiently inefficient entrepreneurs (those with marginal costs $c > \overline{c}(\mu, S)$) cannot make positive gross profits.

(DOM) *If* $\mu'([0, c]) \geq \mu([0, c])$ *for all* $c \in (0, 1]$, *then* $h(\mu') \geq h(\mu)$. *If, in addition, the inequality is strict for some* $c < \overline{c}(\mu, S)$, *then* $h(\mu') > h(\mu)$.

This assumption says that competition is more intense (in that $h(\mu)$ is larger) if the mass of active firms is larger, and the entrepreneurs are more efficient. It ensures that additional entry of entrepreneurs reduces profits, and hence the value of an entrant.

(CON) *The reduced-form profit function* $\Pi(c; h(\mu), S)$ *and* $h(\mu)$ *are continuous.*[3]

While (MON) ensures that an increase in the intensity of competition $h(\mu)$ or a decrease in market size S reduce the profits of all entrepreneurs, not all entrepreneurial types are likely to be affected to the same extent. This is formalized in the following two conditions.

C.1 *For* $h(\mu') > h(\mu)$, *the profit ratio* $\Pi(c; h(\mu'), S)/\Pi(c; h(\mu), S)$ *is strictly decreasing in* c *on* $[0, \overline{c}(\mu, S))$.

C.2 *For* $S' > S$, *the profit ratio* $\Pi(c; h(\mu), S')/\Pi(c; h(\mu), S)$ *is non-increasing in* c *on* $[0, \overline{c}(\mu, S))$.[4]

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[3] We endow the set of Borel measures on $[0, 1]$ with the topology of weak* convergence.

[4] Note that C.2 implies that $\overline{c}(\mu, S)$ is non-decreasing in S since $\Pi(c; h(\mu), S)$ is increasing in S. If $\overline{c}(\mu, S) < 1$, then assumption C.2 implies that $\overline{c}(\mu, S)$ is independent of market size S. To see this, note the marginal type cannot be increasing in S since this would imply that the profit ratio $\Pi(c; h(\mu), S')/\Pi(c; h(\mu), S) \to \infty$ as $c \uparrow$

Condition C.1 says that any change in the distribution of active types that makes competition more intense (and reduces the profits of all types), causes the gross profits of less efficient types to fall by a larger fraction than that of more efficient types.[5]

As is well known, if firms have constant marginal costs and an increase in market size means a replication of the population of consumers (leaving the distribution of consumers' tastes and incomes unchanged), gross profits are proportional to market size, holding everything else fixed; that is, $\Pi(c; h(\mu), S) = S\pi(c; h(\mu))$. In this case (which is standard in models of industrial organization), the elasticity of the profit function with respect to marginal cost is independent of market size, and so C.2 holds trivially.[6]

In models with a continuum of firms with constant marginal costs, C.1 is equivalent to assuming that an increase in the toughness of price competition (an increase in $h(\mu)$) results in lower equilibrium prices (for all types with positive sales). Moreover, in this case, C.2 is equivalent to assuming that the equilibrium price of a given type is non-increasing with market size (holding the distribution of active firms fixed). This is formally stated in the following proposition.

Proposition 1 *Suppose (entrepreneurial) firms have constant marginal costs $k(c)$, where $k'(c) > 0$, so that the gross profit of a type-c firm can be written as $\Pi(c; h(\mu), S) = [p(c; h(\mu), S) - k(c)]q(c; h(\mu), S)$, where $p(c; h(\mu), S)$ is equilibrium price, and $q(c; h(\mu), S)$ equilibrium output. Then,*

1. *Assumption C.1 holds if and only if, for $c \in [0, \overline{c}(\mu, S))$, the equilibrium price $p(c; h(\mu), S)$ is decreasing in the intensity of price competition $h(\mu)$.*

2. *Assumption C.2 holds if and only if, for $c \in [0, \overline{c}(\mu, S))$, the equilibrium price $p(c; h(\mu), S)$ is non-increasing in market size S.*

Proof. See Appendix B. ∎

Lower equilibrium prices when competition is more intense are a natural implication of models of imperfect competition. However, the Dixit-Stiglitz model (with a continuum of firms with constant marginal costs) does not have this property; there, firms' markups are completely independent of the state of competition, and so $\Pi(c; h(\mu'), S)/\Pi(c; h(\mu), S)$ is independent of c, violating our assumption C.1.

To make the model interesting (and simple), we want to ensure that, in equilibrium, there is a positive mass of entrants in each market and some entrepreneurs (obviously, the least

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$\overline{c}(\mu, S)$, contradicting assumption C.2. Observe that C.2 is equivalent to the assumption that the absolute value of the elasticity of the gross profit function with respect to c, $|c\Pi_1(c; h(\mu), S)/\Pi(c; h(\mu), S)|$, is nondecreasing in market size S.

[5]This is equivalent to assuming that the elasticity of the gross profit function with respect to c, $\Pi_1(c; h(\mu), S)/\Pi(c; h(\mu), S)$, decreases (in absolute value) as the market becomes more competitive (and $h(\mu)$ increases).

[6]Theoretical and empirical work in industrial organization that considers the effect of market size on industry structure includes Bresnahan and Reiss (1991), Nocke (2000), and Sutton (1991, 1998). Throughout this work, gross profits are assumed to be proportional to market size, holding the population of active firms fixed.

capable ones) do not enter any market. For this purpose, we assume that unbounded entry drives profits down to zero:

$$\lim_{\lambda \to \infty} \Pi(c; h(\lambda\mu), S) = 0 \text{ for all } c \in (0, 1].$$

Moreover, we posit that entry and fixed costs, ε and ϕ, are "sufficiently small" so that entering an "empty" market is preferred to not entering any market. Specifically,

$$\lim_{\lambda \to 0} \Pi(1; h(\lambda\mu), S_N) > \varepsilon + \phi.$$

Finally, we assume free entry: the total mass of potential entrepreneurs, M, is "sufficiently large" so that, in equilibrium, some entrepreneurs do indeed decide not to enter any market.

Formally, the model may be viewed as an *anonymous game* with a continuum of players. An entrepreneur's pure strategy s is a mapping $s : [0, 1] \to \{0, 1, ..., N\}$, where $s(c) = 0$ means "do not enter", and $s(c) = i$, $i = 1, ..., N$, stands for "enter market i". We seek the pure strategy Nash equilibrium of this game.

3   Endogenous Sorting of Entrepreneurs

In this section, we show that the (unique) equilibrium exhibits sorting of entrepreneurs: the most capable entrepreneurs all enter the largest market, less capable entrepreneurs enter the next largest market, and so on. This implies that the mass (number) of active entrepreneurs and, hence, product variety may actually be smaller in a larger market.

Since entry cost ε and fixed cost ϕ are assumed to be "small", and the size of the pool of entrepreneurs, M, "large", any equilibrium has the following features: there is a positive mass of entrepreneurs in each market and a positive mass of entrepreneurs who prefer not to enter any market. Denote by μ_i the measure of entrepreneurs who decide to enter market i in equilibrium. Then, we must have the following ordering:

$$h(\mu_1) > h(\mu_2) > ... > h(\mu_N).$$

That is, the larger is the market, the larger is $h(\mu)$, and so the more intense is competition. To see this, suppose otherwise; that is, there are markets i and $j > i$ such that $h(\mu_i) \leq h(\mu_j)$. This implies that *all* entrepreneurs (who are sufficiently efficient so as to make positive sales) strictly prefer to enter market i rather than market j: for all $c \in [0, \overline{c}(\mu_j, S_j))$,

$$\begin{aligned} \Pi(c; h(\mu_i), S_i) &\geq \Pi(c; h(\mu_j), S_i) \\ &> \Pi(c; h(\mu_j), S_j), \end{aligned}$$

where the first inequality follows from $h(\mu_i) \leq h(\mu_j)$ and the second inequality from $S_i > S_j$. However, if all entrepreneurs preferred market i over market j, no entrepreneur would decide to enter market j; in particular, we could not have $h(\mu_i) \leq h(\mu_j)$.

To simplify the exposition in the main text, suppose now that the gross profit takes the (general) form

$$\Pi(c; h(\mu), S) = p(c; h(\mu), S)q(c; h(\mu), S) - K(q(c; h(\mu), S); c),$$

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where $K(q;c)$ is the total variable production cost of a type-c firm producing q units of output. Suppose also that worse types (higher c's) have higher marginal costs for a given level of output, i.e., $K_{12}(q;c) > 0$. Observe that, for any given price p above marginal cost $K_1(q;c)$, gross profit $pq - K(q;c)$ is increasing in q. Suppose type c_{ij} is indifferent between entering market i and (the smaller) market $j > i$; i.e., $\Pi(c_{ij}; h(\mu_i), S_i) = \Pi(c_{ij}; h(\mu_j), S_j)$. Intuitively, since competition is endogenously more intense in the larger market, the price the firm can charge there is lower than in the smaller market:

$$p(c_{ij}; h(\mu_i), S_i) < p(c_{ij}; h(\mu_j), S_j).$$

In fact, proposition 1 says that this is an immediate consequence of C.1 and C.2 if firms have constant marginal costs. In order to be indifferent between the two markets, entrepreneurial type c_{ij} has thus to sell a larger quantity in the larger market:

$$q(c_{ij}; h(\mu_i), S_i) > q(c_{ij}; h(\mu_j), S_j). \tag{1}$$

The remaining question is now whether entrepreneurs who are marginally more capable than type c_{ij} prefer to enter the smaller or the larger market. From the envelope theorem, the additional profit from a marginal decrease in c is equal to $K_2(q;c)$, which is equal to output q if marginal costs are constant (i.e., $K(q;c) = cq$) but, more generally, is increasing in q since $K_{12}(q;c) > 0$. From (1), it therefore follows that an entrepreneur who is slightly more efficient than type c_{ij} strictly prefers to enter market i rather than the smaller market j. Similarly, a slightly less talented entrepreneur strictly prefers to enter the smaller market. Moreover, there can be at most one entrepreneur (with positive profit) who is indifferent between the two markets.

We therefore obtain the central sorting result of this paper.

Proposition 2 *There exists a unique equilibrium. In equilibrium, there are marginal types $0 \equiv c_0 < c_1 < ... < c_N$ such that (almost) all entrepreneurs of type $c \in [c_{i-1}, c_i)$ enter market i, while (almost) all entrepreneurs of type $c \in [c_N, 1]$ do not enter any market. Hence, each entrepreneur in a given market is more capable than any entrepreneur in a smaller market.*

Proof. See Appendix B. ∎

The proposition shows that the relationship between the characteristics of a market and the talents of its entrepreneurs takes a surprisingly extreme form: the larger is the market, the more talented are its entrepreneurs in that each entrepreneur in a large market is more efficient than any entrepreneur in a smaller market. Consequently, the total mass of entrepreneurs in market i whose types fall into the interval $[0, z]$ is given by

$$\mu_i\left([0, z]\right) = \begin{cases} 0 & \text{if } z < c_{i-1} \\ M\left[G(z) - G(c_{i-1})\right] & \text{if } z \in [c_{i-1}, c_i) \\ M\left[G(c_i) - G(c_{i-1})\right] & \text{if } z \geq c_i \end{cases}$$

The sorting result obtains since more capable entrepreneurs are better off in a larger and endogenously more competitive market whereas less capable entrepreneurs are better off in a smaller and hence less competitive market.

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As in Lucas (1978), the size distribution of firms in a given market is determined by the underlying distribution of entrepreneurial talent (namely, the distribution function $G(\cdot)$). While our theory thus does not impose testable predictions about the size distribution within a given market, it does allow us to make predictions across markets. Suppose we measure firm size by output $q(c; h(\mu), S)$, which is decreasing in the entrepreneur's type c. As discussed above, any entrepreneur, who is indifferent between entering two markets, would produce a greater output in the larger market. Since the more talented entrepreneurs enter the larger market, and the less talented ones the smaller market, our model predicts that firms located in larger markets are larger than those in smaller markets.

The Number of Entrants. Let us now reconsider the relationship between market size and the number of entrants. Following Bresnahan and Reiss (1991), a number of researchers have found that the ratio between the number of firms and market size is smaller in larger markets. This finding has been interpreted as evidence for the existence of the price competition effect: an increase in market size typically leads to more entry, and then the price competition effect implies a fall in price-cost margins. Hence, in larger markets, market size has to increase by a larger amount so as to sustain an additional firm in the market. The existing studies have implicitly assumed that the distribution of entrants' efficiency levels does not vary across markets. In particular, they have not allowed for self-selection of entrepreneurs at the entry stage.

In our model, the sorting effect may reinforce the price competition effect: since more efficient firms self-select into larger markets, entry causes price-cost margins to fall "at a much faster rate" with market size than without sorting. In fact, the sorting effect may be so strong that a larger market may have less entrepreneurs and, hence, less product variety to offer than a smaller market. This counterintuitive relationship may arise since competition in a market may be more intense for two reasons: (i) there is a larger population of active entrepreneurs, and (ii) the active entrepreneurs are more efficient. Moreover, the endogenous intensity of price competition changes continuously with market size, whereas the average efficiency of its entrepreneurs may change discontinuously. Suppose that the difference in size between markets i and $j > i$ is small. Then, competition in market i is not much more intense than in market j in the sense that $h(\mu_i) - h(\mu_j)$ is small. However, entrepreneurs in market i are much more capable than those in j. Hence, we may have $\mu_i([0,1]) = M\left(G(c_i) - G(c_{i-1})\right) < M\left(G(c_j) - G(c_{j-1})\right) = \mu_j([0,1])$, although $h(\mu_i) > h(\mu_j)$. This is illustrated in the following numerical example.

Numerical Example. Consider our leading example, the linear demand model. Suppose there are only two markets, i.e., $N = 2$, and entrepreneurial types are uniformly distributed on the unit interval, i.e., $G(c) = c$ for $c \in [0,1]$. In the unique equilibrium, the marginal types c_1 and $c_2 > c_1$ are determined by

$$S_1 \left(\frac{2 + \sigma M c_1^2/2}{2 + \sigma M c_1} - c_1 \right)^2 = S_2 \left(\frac{2 + \sigma M \left(c_2^2 - c_1^2\right)/2}{2 + \sigma M (c_2 - c_1)} - c_1 \right)^2,$$

and

$$\frac{S_2}{8} \left(\frac{2 + \sigma M \left(c_2^2 - c_1^2\right)/2}{2 + \sigma M (c_2 - c_1)} - c_2 \right)^2 = \varepsilon + \phi.$$

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Assume now that $\varepsilon+\phi = 1/8$, $\sigma M = 2$, $S_1 = 10$, and $S_2 = 8$. The marginal types are then given by $c_1 \approx 0.273$ and $c_2 \approx 0.525$. As one would expect, the total mass of entrepreneurial firms is larger in the larger market: $\mu_1([0,1])/M = c_1 \approx 0.273$, whereas $\mu_2([0,1])/M = (c_2-c_1) \approx 0.252$. However, assume now that the size of market 1 is equal to $S_1 = 9$. In this case, $c_1 \approx 0.239$ and $c_2 \approx 0.512$. Somewhat surprisingly, there are more (but less capable) entrepreneurs in the smaller market: $\mu_1([0,1])/M \approx 0.239 < 0.273 \approx \mu_2([0,1])/M$. More generally, whenever the difference in size between any two markets (as measured by $S_i - S_{i+1}$) is sufficiently large, then one will observe a positive correlation between market size and the number of entrepreneurs (or product variety) across markets. If, however, the differences are sufficiently small (in that $S_1 - S_N$ is small), then one will observe a *negative* cross-sectional correlation.$\bigcirc$

Perfect Competition and Dixit-Stiglitz. In the introduction of this paper, we have argued that the sorting result does not obtain if (i) firms are price takers or (ii) consumers have a CES utility function as in the Dixit-Stiglitz model of monopolistic competition. The reason is that, in these cases, the distribution of active firms, μ, and market size S enter the gross profit function in a very special way: μ and S are first aggregated into some one-dimensional variable φ, and affect gross profit only through this variable. In the case of perfect competition, this variable is the market price, $\varphi = p(h(\mu), S)$. Similarly, in the Dixit-Stiglitz model, where (residual) demand takes the form $D(p) = \psi p^{-\sigma}$, this variable is the demand parameter $\varphi = \psi(h(\mu), S)$. That is, the gross profit function can be written as $\Pi(c; h(\mu), S) = \widetilde{\Pi}(c; \varphi(h(\mu), S))$ in both models. Since the gross profit of price takers is increasing in market price p, decreasing in $h(\mu)$, and increasing in S, the market price $p(h(\mu), S)$ must be decreasing in the intensity of competition, $h(\mu)$, and increasing in market size S. But then, condition C.2 requires that $\widetilde{\Pi}_1(c; \varphi)/\widetilde{\Pi}(c; \varphi)$ is non-increasing in φ, whereas C.1 posits that $\widetilde{\Pi}_1(c; \varphi)/\widetilde{\Pi}(c; \varphi)$ is strictly increasing in φ. Obviously, C.1 and C.2 cannot both hold.[7] The same reasoning applies to the Dixit-Stiglitz model. For example, if firms have constant marginal costs (or, more generally, costs of the form $K(q; c) = cq^{\alpha}$) in this model, then C.2 holds but C.1 does not.

Empirical Evidence. Following Sveikauskas (1975) and Henderson (1986), there are a number of empirical papers analyzing productivity differences across cities and regions. A robust finding of this literature is that total factor productivity is higher in larger cities (or more densely populated regions). For example, in a recent paper using Japanese data, Davis and Weinstein (2001) find that, ceteris paribus, a doubling of region size raises productivity by 3.5 percent. Similarly, Syverson (2000) shows that cement plants are more efficient in more densely populated U.S. metropolitan areas. The urban and regional economics literature has traditionally attributed these productivity differences to *Marshallian externalities*. The present model suggests that a different force may be at work: more productive entrepreneurs or firms may endogenously select into larger markets. By failing to account for self-selection, however, the empirical literature may overestimate the role of externalities.

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[7]Suppose the cost function can be decomposed as $K(q; c) = k(c)l(q)$. Let $\kappa(q) \equiv ql'(q)/l(q)$ denote the elasticity of the cost function (with respect to output q). Then,

$$\frac{\Pi_1(c; h(\mu), S)}{\Pi(c; h(\mu), S)} = -\frac{k'(c)}{k(c)}\left(\frac{1}{\kappa(q)-1}\right).$$

Hence, C.2 holds if and only if $\kappa'(q) \leq 0$ for all q, whereas C.1 holds if and only if $\kappa'(q) > 0$ for all q.

4 Trade between Markets

Thus far, we have assumed that an entrepreneur can only sell her product in the market she chooses to be located in. In many real world markets, however, firms "export" their goods to other markets (countries or cities). In this section, we show that the central sorting result holds (under mild restrictions on the gross profit function) even when entrepreneurs are allowed to export their goods from one market to others. However, depending on the size of transport costs, no entrepreneur may decide to enter the smallest market(s). In particular, if transport costs are sufficiently small, then all entrepreneurs will enter the largest market.

As before, an entrepreneur is assumed to choose one market as her location. However, she can now sell her product in all other markets but has to incur a unit transport cost (or tariff) t. Assuming that firms have constant marginal costs, the unit cost (of production and transport) of a type-c entrepreneur in a "foreign" market is then equal to $c + t$. Since entrepreneurs can set different prices in the home and foreign markets, a type-c entrepreneur sets the same price (or quantity) as a foreign type-$(c + t)$ entrepreneur in the foreign entrepreneur's home market. There are no additional fixed costs associated with exports. We have to distinguish between the distribution of entrepreneurial types located in a given market and the distribution of types selling their products in this market. The distribution of types located in market i is summarized by the (Borel) measure $\widehat{\mu}_i$ on $[0, 1]$, the distribution of types selling in this market by the measure μ_i on $[0, 1 + t]$. Since a foreign type-$(c + t)$ entrepreneur behaves like a home type-c entrepreneur, the mass of entrepreneurs selling in market i whose types fall into the interval (or Borel set) A is given by

$$\mu_i(A) = \widehat{\mu}_i(A) + \sum_{j \neq i} \widehat{\mu}_j(A - t).$$

For simplicity of exposition, we assume that (i) an increase in market size means a replication of the population of consumers, and (ii) that firms have constant marginal costs of production. It then follows that the gross profit function is of the form $\Pi(c; h(\mu), S) = S\pi(c; h(\mu))$. Conditions (MON), (CON), and (DOM) are assumed to hold as before. Furthermore, again as in the basic model without trade, we assume the following: unbounded entry drives profit down to zero, entry and fixed costs are not too large so that some entrepreneurs decide to enter some market in equilibrium, and the population of potential entrants is sufficiently large so that, in equilibrium, some entrepreneurs decide not to enter any market. Assumption C.1, however, is now replaced by the following conditions on the reduced-form profit function.

C.3 *For any $t > 0$ and $c \in [0, \overline{c}(\mu))$, the profit difference $\pi(c; h(\mu)) - \pi(c + t; h(\mu))$ is strictly decreasing in $h(\mu)$.*

C.4 *For $h(\mu') > h(\mu)$, the ratio of profit differences,*

$$\frac{\pi(c; h(\mu')) - \pi(c + t; h(\mu'))}{\pi(c; h(\mu)) - \pi(c + t; h(\mu))},$$

is strictly decreasing in c on $[0, \overline{c}(\mu'))$.

Assumption C.3 says that the (absolute) profit increase from a reduction in marginal costs is smaller when competition is more intense. That is, an increase in the population of active

firms makes the profit function $\pi(\cdot; h(\mu))$ flatter. Assumption C.4 says that this fractional decrease in the slope of the profit function (due to an increase in $h(\mu)$) is smaller for more efficient types.

It is straightforward to check that the assumptions hold for our leading example, the linear demand model. In Appendix A, we show that the assumptions are satisfied in the Cournot model (with a finite number of firms) under quite general conditions on demand.

For sufficiently small transport costs (and assuming differentiability), the two assumptions can be rewritten as follows.

C.3' For $h(\mu') > h(\mu)$, the profit difference $|\pi(c; h(\mu)) - \pi(c; h(\mu'))|$ is strictly decreasing in c on $[0, \overline{c}(\mu))$.[8]

C.4' For $h(\mu') > h(\mu)$, the ratio of marginal profits $\pi_1(c; h(\mu'))/\pi_1(c; h(\mu))$ is strictly decreasing in c on $[0, \overline{c}(\mu'))$.[9]

In models of competition with a continuum of firms, C.3' and C.4' are equivalent to intuitive properties of the equilibrium output $q(c; h(\mu), S)$.[10]

Proposition 3 *For small transport costs, the following equivalence results hold.*

1. *C.3' holds if and only if, for all $c \in [0, \overline{c}(\mu))$, $q(c; h(\mu), S)$ is decreasing in $h(\mu)$; that is, if and only if an increase in the population of entrepreneurs leads to a lower equilibrium output of any given type.*

2. *C.4' holds if and only if, for all $c \in [0, \overline{c}(\mu))$, $|q_1(c; h(\mu), S)/q(c; h(\mu), S)|$ is increasing in $h(\mu)$; that is, if and only if the fractional decrease in equilibrium output due to an increase in marginal costs is greater when the distribution of entrepreneurs is larger (and hence competition more intense).*

Proof. See Appendix B. ∎

Thus, the larger is the distribution of entrepreneurs (and hence the more competitive is the market), the smaller is the equilibrium output for any given type and the larger is the fractional reduction in equilibrium output from a small increase in marginal costs.

We are now in the position to turn to the equilibrium analysis. The first observation to make is that since there are no fixed costs associated with exports, we can think of each entrepreneur selling her product not only in her own market but in all the other $N-1$ markets. (Of course, depending on the entrepreneur's type and the intensity of competition in a market, the entrepreneur may make zero sales in equilibrium in this market. In particular, a type-c entrepreneur makes zero sales in foreign market i if $c + t \geq \overline{c}(\mu_i)$.) The second observation is that the larger is the market, the tougher is the endogenous intensity of price competition of firms *selling* in the market:

$$h(\mu_1) \geq h(\mu_2) \geq ... \geq h(\mu_N),$$

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[8] This is equivalent to assuming that $\pi_{12}(c; h(\mu)) > 0$.

[9] This is equivalent to assuming that $\pi_{11}(c; h(\mu))/\pi_1(c; h(\mu))$ is decreasing in $h(\mu)$.

[10] Note that, in models (of monopolistic competition) with a continuum of firms, the equilibrium outcomes under price and quantity competition are identical.

and
$$h(\mu_1) > h(\mu_2) > ... > h(\mu_k),$$

where $k \in \{1, ..., N\}$ is the largest integer such that $\hat{\mu}_k([0,1]) > 0$ (i.e., k is the smallest market in which a positive mass of entrepreneurs locate). Suppose this assertion were not true. Then, there are markets i and j, where $k \geq j > i$ such that $h(\mu_i) \leq h(\mu_j)$ and $\hat{\mu}_j([0,1]) > 0$. We now show that, in this case, all entrepreneurs who choose to locate in market j would be strictly better off by locating in market i instead, contradicting $\hat{\mu}_j([0,1]) > 0$. Entrepreneurial type c strictly prefers to locate in market i if and only if

$$S_i\pi(c; h(\mu_i)) + S_j\pi(c+t; h(\mu_j)) > S_j\pi(c; h(\mu_j)) + S_i\pi(c+t; h(\mu_i)),$$

which is equivalent to
$$\frac{\pi(c; h(\mu_i)) - \pi(c+t; h(\mu_i))}{\pi(c; h(\mu_j)) - \pi(c+t; h(\mu_j))} > \frac{S_j}{S_i}. \tag{2}$$

The right-hand side of the last inequality is smaller than one. If $h(\mu_i) \leq h(\mu_j)$, as assumed, then C.3 implies that the left-hand side is larger than or equal to one. Hence, the inequality does indeed hold and all entrepreneurs are better off by locating in market i.

Suppose now that entrepreneurial type c_{ij} is indifferent between locating in markets i and j. That is, the l.h.s. of equation (2) is equal to S_j/S_i at $c = c_{ij}$. Assumption C.4 says that the left-hand side of the equation is strictly decreasing in c. It then follows that all types $c \in [0, c_{ij})$, strictly prefer to enter the larger (and endogenously more competitive) market i, while less capable entrepreneurial types, $c \in (c_{ij}, \overline{c}(\mu_j))$, are strictly better off by locating in the smaller (and less competitive) market j. We thus have the following result.

Proposition 4 *In the model with trade between markets, the equilibrium exhibits sorting of entrepreneurs by capabilities. In equilibrium, there exists a market $k \in \{1, ..., N\}$ and marginal types $0 \equiv \hat{c}_0 < \hat{c}_1 < ... < \hat{c}_k$ such that (almost) all entrepreneurs of type $c \in [\hat{c}_{i-1}, \hat{c}_i)$ enter market i, while (almost) all entrepreneurs of type $c \in [\hat{c}_k, 1]$ do not enter any market.*

The proposition shows the robustness of our earlier sorting result. As before, a larger market attracts entrepreneurs who are more capable than any entrepreneur in any smaller market. In contrast to the earlier result, however, what ultimately matters for the entrepreneur's location decision in the trade model is not in which home market she can obtain the highest profit. Rather, the entrepreneur will locate in the market in which her foregone profit from having to pay the transport cost (tariff) is the largest.

For small transport costs, this roughly means that she will enter the market where her equilibrium output in the home market is maximized (since the marginal loss from being slightly less efficient is equal to the equilibrium output). Now, suppose a particular entrepreneur is indifferent between entering any two markets. From the above argument, it follows that her equilibrium output in the two markets must (approximately) be the same. According to Proposition 3, assumption C.4 implies that more capable entrepreneurs have a higher equilibrium output in the larger and more competitive market, while less capable entrepreneurs would produce more in the smaller and less competitive market. That is, the better entrepreneurs

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prefer to locate in the larger market and export to the smaller market, whereas the less efficient entrepreneurs are better off doing the reverse. As in the basic model without trade, the sorting result would not obtain in a Dixit-Stiglitz type model, where consumers have a CES utility function.

Observe that consumers may enjoy a larger product variety in a smaller market, *even if the mass of entrepreneurs locating in this market is larger*. To see this, note that the more competitive is the market, the more talented entrepreneurs must be in order to make positive sales. Hence, entrepreneurs located in a large market (who are very efficient) will make positive sales in smaller markets (provided transport costs are not too large); in contrast, an entrepreneur who is located in a small market may not be efficient enough to make positive sales in a large market where competition is more intense.

Another empirical prediction of our model is that exporters are (on average) more efficient than non-exporters. This is for two reasons. First, consider firms located in the same market: in order to being able to profitably export to market i, a firm's marginal cost has to be less than $\overline{c}(\mu_i) - t$. Second, note that more efficient firms locate in larger markets and attempt to export to smaller and endogenously less competitive markets. For example, suppose there are only two markets, a large market 1 and a small market 2. In equilibrium, $\overline{c}(\mu_1) < \overline{c}(\mu_2)$. To profitably export to market 1, a firm's marginal cost has to be less than $\overline{c}(\mu_1) - t$, while the upper bound on marginal costs for exports to market 2 is $\overline{c}(\mu_2) - t > \overline{c}(\mu_1) - t$. Since the more efficient firms endogenously locate in market 1, they are more likely to export. Indeed, there is strong empirical evidence supporting this prediction; see Bernard and Jensen (1999).

In contrast to our basic model without trade, assuming that entry cost ε and fixed cost ϕ are sufficiently small, and the mass of potential entrepreneurs, M, sufficiently large, no longer ensures that, in each market, there is a positive mass of ("home") entrepreneurs locating in this market. The possibility of the existence of markets which solely rely on "imports" arises since entrepreneurs may find it optimal to locate in large markets and then export to other markets. The extent to which this may happen depends on the magnitude of transport costs. In particular, if transport costs are small, then all entrepreneurs either enter the largest market or do not enter any market.

Proposition 5 *Suppose that transport cost t is "sufficiently small". Then, in equilibrium, (almost) all entrepreneurs of type $c \in [0, \widehat{c}_1)$ enter market 1, while (almost) all entrepreneurs of type $[\widehat{c}_1, 1]$ do not enter any market.*

In the limit as transport costs go to zero, the most capable entrepreneurs enter the largest market, less capable entrepreneurs do not enter any market, and no entrepreneur enters any market other than the largest. This suggests that large markets profit more from free trade and free mobility of entrepreneurs than smaller markets. The intuition for the result is the following. The existence of a transport cost implies that, by entering a larger rather than a smaller market, an entrepreneur is more efficient in the larger (and endogenously more competitive) market, and less efficient in the smaller (and less competitive) market. Now, the marginal increase in profit from having slightly lower marginal costs is equal to a firm's output. If transport costs are small, the intensity of price competition is approximately the same in all markets. As a result, the equilibrium home market output for *any* type is greater in larger markets. It then

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follows that all entrepreneurs prefer to enter a larger rather than a smaller market (provided transport costs are sufficiently small). Put differently, in the limit as transport costs go to zero, firms locate so as to minimize transport costs. Moreover, in the case of small transport costs, *all* types will make the largest home sales in market 1.[11]

In fact, in the last twenty years, there have been numerous and well-documented fears by smaller countries that (symmetric) trade liberalization may lead to de-location of firms and even to de-industrialization; see Baldwin and Robert-Nicoud (2000). In our simple model, this fear seems to be well-grounded, even though it does not mean that trade liberalization has negative welfare implications.

5   Entry, Exit, and Market Size

Rates of firm turnover differ substantially across industries. These differences are similar from one country to another, and stable over time. While these cross-industry differences are not yet very well understood, there is a small number of models that attempt to relate firm turnover to observable industry characteristics. For example, Hopenhayn (1992) and Lambson (1991) consider the effect of sunk costs in a dynamic model with price-taking firms. In Asplund and Nocke (2003), we analyze the impact of market size and sunk costs in a dynamic model of imperfect competition. In these single-industry models, firms are ex-ante identical, and it is implicitly assumed that the distribution of entrants' characteristics (such as entrepreneurial "talent") are identical across industries.

In this section, we take a different approach. We consider a stochastic dynamic version of our basic model (without trade). This allows us to analyze the impact of market size on firm turnover in a multi-market model where the distribution of entrants' capabilities may vary endogenously across markets.

The Dynamic Model. We assume that time is discrete, firms have an infinite horizon and a common discount factor, denoted by $\delta \in [0, 1)$.[12] In each period, there is a mass M of "young" entrepreneurs whose (current) types are distributed according to $G(\cdot)$. Knowing her current type, a young entrepreneur decides whether to enter a market and if so, which one of the N markets. (As before, each entrepreneur can enter at most one market.) To generate turbulence, we assume that the quality of an entrepreneur's idea changes stochastically over time. This may be due to shocks to consumers' tastes for the entrepreneur's product. In particular, with probability $\alpha \in (0, 1)$, the entrepreneur will be of the same efficiency as in the last period, whereas with the remaining probability $1 - \alpha$, she gets a new draw of her type from $G(\cdot)$. The probability α thus measures the persistence in the evolution of an entrepreneur's type. The timing in each period is as follows. At the first stage, young entrepreneurs (potential entrants) and old entrepreneurs (incumbents) "learn" the realization of their current types. At the

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[11]Note that the proof of Proposition 5 does not use assumption C.4. Since the Dixit-Stiglitz model (with constant marginal costs) satisfies all of our other assumptions on $\Pi(\cdot)$, the prediction of proposition 5 also holds if consumers have a CES utility function.

[12]Suppose the probability of the entrepreneur's (physical) death in a period is equal to γ. Then, the discount factor δ is the product of the factor of time preference $\widetilde{\delta}$, and the probability of physical survival, $1 - \gamma$, and so $\delta = \widetilde{\delta}(1 - \gamma)$.

second stage, young entrepreneurs make their entry decisions and incumbents decide whether or not to exit the market. The (irrecoverable) entry cost is given by $\epsilon > 0$. Entrepreneurs who decide not to be active, take up an outside option, the value of which is normalized to zero. Re-entry after exit is not possible. Moreover, we assume that only young entrepreneurs are (geographically) mobile, which implies that old entrepreneurs cannot switch from one market to another. At the third and final stage, the active entrepreneurs in a given market compete on the product market, obtain a gross profit $\Pi(c; h(\mu), S)$, which depends on their current type c, the endogenous intensity of price competition $h(\mu)$, and the size of their market, S. Moreover, active entrepreneurs pay a fixed production cost $\phi > 0$. (The presence of this fixed cost provides sufficiently inefficient entrepreneurs with an incentive to leave the market.) We impose the same structure on the gross profit function as in the basic model without trade; in particular, (MON), (CON), (DOM) as well as C.1 and C.2 are assumed to hold.

Stationary Equilibrium. We confine attention to stationary equilibria in which the entrepreneurial entry and exit strategies, and hence the distribution of active types in each market, are time-independent.

Let $V(c; h(\mu), S)$ denote the value (at stage 2) of a type-c entrepreneur in a market of size S, where the distribution of types is given by μ. Since the entrepreneur has the option to leave the market, this value satisfies

$$V(c; h(\mu), S) = \max\{\overline{V}(c; h(\mu), S), 0\},$$

where

$$\overline{V}(c; h(\mu), S) = \Pi(c; h(\mu), S) - \phi + \delta \left\{ \alpha V(c; h(\mu), S) + (1 - \alpha) \int_0^1 V(u; h(\mu), S) G(du) \right\}$$

is the value conditional on staying in the market in the current period, and behaving optimally thereafter. It is straightforward to see that $\overline{V}(c; h(\mu), S)$ is decreasing in c on $[0, \overline{c}(\mu, S))$; moreover, it is continuous. Let $c^*(\mu, S)$ be such that $\overline{V}(c^*(\mu, S); h(\mu), S) = 0$. If $c^*(\mu, S) < \overline{c}(\mu, S)$ (as we assume for the moment), $c^*(\mu, S)$ is unique. It denotes the optimal exit policy: all more efficient types strictly prefer to stay in the market, all less efficient types strictly prefer to leave the market and take up the outside option.

The conditional value can be re-written as

$$\overline{V}(c; \mu, S) = \frac{1}{1 - \delta\alpha} \{ \Pi(c; h(\mu), S) - \phi + \frac{\delta(1 - \alpha)}{1 - \delta\alpha - \delta(1 - \alpha)G(c^*(\mu, S))} $$
$$\times \int_0^{c^*(\mu, S)} [\Pi(u; h(\mu), S) - \phi] \, G(du) \};$$

i.e., it is of the form $a\Pi(c; h(\mu), S) + b(\mu, S)$. Observe that $b(\mu, S) \to 0$ as $\delta(1 - \alpha)/(1 - \delta) \to 0$. That is, as the parameter of cost persistence, α, goes to one (or the discount factor δ goes to zero), a firm's value - conditional on staying in the market for another period - becomes proportional to its current gross profit.

Let us denote the value of a type-c entrant (before paying the entry cost) in market i by $V^e(c; h(\mu_i), S_i)$. This value can easily be related to the value of an incumbent (conditional on

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staying in the market for another period), and is given by

$$V^e(c; h(\mu_i), S_i) = \overline{V}(c; h(\mu_i), S_i) - \epsilon.$$

We are now in the position to state and prove the following result.

Proposition 6 *Suppose that costs are persistent over time (or the discount factor is small) such that $\delta(1-\alpha)/(1-\delta)$ is sufficiently small. Then, the unique stationary equilibrium of the dynamic model exhibits sorting of entrepreneurs by types. That is, there exist marginal types $0 \equiv c_0 < c_1 < ... < c_N$ such that (almost) all entrepreneurs of type $c \in [c_{i-1}, c_i)$ enter market i, while (almost) all entrepreneurs of type $c \in [c_N, 1]$ do not enter any market.*

Proof. See Appendix B. ∎

Note that, while the sorting result applies to new entrants, it is no longer true that any entrepreneur in a larger market is more talented than all entrepreneurs in smaller markets. However, as we will show below, a weaker result holds: the least efficient entrepreneur in a larger market is more talented than the least efficient entrepreneur in a smaller market.

From now on, let us assume that the (unique) stationary equilibrium exhibits sorting of types (as it indeed does under the condition of proposition 6). In particular, for any markets i and j, there exists a unique type c_{ij} such that $\overline{V}(c_{ij}; h(\mu_i), S_i) = \overline{V}(c_{ij}; h(\mu_j), S_j) \geq 0$. In the stationary equilibrium, the total mass of entrants per period is equal to the total mass of exiting firms:

$$[G(c_i) - G(c_{i-1})]M = (1-\alpha)[1 - G(c^*(\mu_i, S_i))]\mu_i([0,1]).$$

The total mass of entrepreneurs active in market i is then given by

$$\mu_i([0,1]) = \frac{[G(c_i) - G(c_{i-1})]M}{(1-\alpha)[1 - G(c^*(\mu_i, S_i))]}.$$

Since the value of the least efficient entrant is non-negative, $V^e(c_i; h(\mu_i), S_i) \geq 0$, it follows that $\overline{V}(c_i; h(\mu_i), S_i) > 0$, and hence that, in any market, the least efficient entrant is more efficient than the least efficient incumbent:

$$c_i < c^*(\mu_i, S_i). \tag{3}$$

The stationary equilibrium distribution of active types can now be characterized as follows. The mass of entrepreneurs active in market i whose types fall into the interval $[z, z']$ is given by

$$\mu_i\left([z, z']\right) = \begin{cases} \mu_i([0,1])\left[G(z') - G(z)\right] & \text{if } [z, z'] \subseteq [0, c_{i-1}] \\ & \text{or } [z, z'] \subseteq [c_i, c^*(\mu_i, S_i)], \\ \frac{M + (1-\alpha)\mu_i([0,1])}{1-\alpha}\left[G(z') - G(z)\right] & \text{if } [z, z'] \subseteq [c_{i-1}, c_i], \\ 0 & \text{if } [z, z'] \subseteq [c^*(\mu_i, S_i), 1]. \end{cases}$$

In each period, a share

$$\theta_i \equiv (1-\alpha)[1 - G(c^*(\mu_i, S_i))] \tag{4}$$

of entrepreneurs exit market i. Given our simple stochastic process, the probability of exit is independent of the entrepreneurial type (within the same market), and so θ_i is equal to each

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incumbent's probability of exit in market i. We will henceforth use θ as our measure of firm turnover.

As equation (4) shows, turnover rate θ varies across markets if different exit policies $c^*(\mu, S)$ are used in different markets. In particular, the tougher is the exit policy (the smaller is $c^*(\mu, S)$), the higher is the turnover rate θ. Since the marginal incumbent in market i is less efficient than the marginal entrant (see equation (3)), the conditional value of type $c^*(\mu_i, S_i)$ is strictly positive in the smaller (and hence less competitive) market $i + 1$:

$$\overline{V}(c^*(\mu_i, S_i); h(\mu_{i+1}), S_{i+1}) > \overline{V}(c^*(\mu_i, S_i); h(\mu_i), S_i) = 0.$$

It then follows that the marginal incumbent is the less efficient, the smaller is the market. That is, $c^*(\mu_i, S_i)$ is increasing in i. Since the equilibrium exit policy is tougher in larger markets, we then obtain that the turnover rate θ_i is decreasing in i. Our result on turnover can be summarized as follows.

Proposition 7 *Suppose the stationary equilibrium exhibits sorting by types. In particular, assume that for any two markets i and j, there exists a unique type c_{ij} such that $\overline{V}(c_{ij}; h(\mu_i), S_i) = \overline{V}(c_{ij}; h(\mu_j), S_j) \geq 0$. Then, the least efficient entrepreneur in a market is the less efficient, the smaller is the market, i.e., $c^*(\mu_i, S_i)$ is increasing in i. Consequently, the equilibrium turnover rate θ_i is larger in larger markets; i.e., θ_i is decreasing in i.*

Note that the proposition implies that the range of efficiency levels of firms within a market is smaller in larger markets.[13] Moreover, in smaller markets, the distribution of entrepreneurial types is shifted towards less efficient types in the sense of first-order stochastic dominance.

There is a close link between firm turnover and the age distribution of businesses. Intuition suggests that markets with higher turnover rates have on average younger firms. Let the period-t age of a firm that entered in period $t^e \leq t$ be given by $t - t^e + 1$. Then, in stationary equilibrium, the average firm age in market i is equal to $1/\theta_i$. Furthermore, the share of firms active in market i whose age is less than or equal to $y \geq 1$ is given by

$$\frac{\sum_{t=0}^{y-1}(1 - \theta_i)^t}{\sum_{t=0}^{\infty}(1 - \theta_i)^t} = 1 - (1 - \theta_i)^y.$$

For $y > 1$, this expression is increasing in θ_i. Since the turnover rate θ_i is decreasing in i, we obtain the following result.

Corollary 8 *Under the condition of proposition 7, in the stationary equilibrium, firms are on average older in smaller markets. More precisely, the age distribution of firms in smaller markets first-order stochastically dominates that in larger markets.*

From our earlier discussion, it is straightforward to see that the (conditional) value of *any* type is the same across markets if firms are price takers or competition is *à la* Dixit-Stiglitz. In this case, sorting of firms does not obtain, and the turnover rate and age distribution do not vary from one market to another.

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[13] This is consistent with the empirical evidence on cement plants; see Syverson (2000).

Empirical Evidence. How can our predictions be tested empirically? The magnitude of the underlying fluctuations in the pattern of demand (or technology) is likely to vary greatly across industries. As pointed out by Sutton (1997), this factor may be of primary importance, but it is very difficult to measure it or to control for its impact empirically. This causes a serious problem for any empirical test of cross-industry predictions on firm turnover. Fortunately, an attractive feature of our theory is that its predictions on turnover rates can be tested by comparing turnover rates of local service firms in geographically independent ("local") markets of different sizes (but within the same industry). This should control for many of those factors that would otherwise differ across industries. This is the route taken in Asplund and Nocke (2000), where we use data on the driving school industry in Sweden. Estimating the probability of exit in a Probit model, we find some supportive evidence for the prediction that turnover rates are higher in larger municipalities. In more recent work, Asplund and Nocke (2003), we analyze the age distribution of hairdressers in Sweden. Using non-parametric tests, we find that the age distribution of firms in smaller markets first-order stochastically dominates the age distribution in larger markets, as predicted by our theory. This is illustrated in Figure 1 (Appendix B): over virtually the entire range of firm ages, the cumulative frequency in the set of large markets is above that in the set of small markets.

6   Conclusion

The aim of this paper has been to present a simple theory of entrepreneurial entry and exit, where (young) entrepreneurs decide *which* market to enter. We have obtained a striking sorting result: in equilibrium, the most talented entrepreneurs all choose to enter the largest market, less talented entrepreneurs enter the next largest market, and so on. The larger the market, the more efficient are thus its entrants. This result follows naturally from properties of standard models of imperfect competition. It may provide an alternative explanation for the empirical finding that factor productivity is greater in larger cities or regions. Under some mild assumptions on the profit function, this sorting result still holds when entrepreneurs can export their goods or services from one market to another. However, in this case, no entrepreneur may decide to enter the smallest market(s). For sufficiently small transport costs, all active entrepreneurs locate in the largest market. Reconsidering the relationship between market size and the number of firms, we have shown that the sorting effect may reinforce the price competition effect. In fact, the sorting effect may be so strong that the number of active firms (and hence product variety) is not necessarily larger in larger markets.

In the dynamic extension of our model, we have shown that the churning rate of entrepreneurs is higher in larger markets (provided entrepreneurial efficiency levels do not change at too fast a rate), and so the life span of firms is shorter. Consequently, the age distribution of firms in larger markets is shifted towards younger firms. This is consistent with the empirical evidence on local service industries in Sweden, as shown in Asplund and Nocke (2000, 2003).

As discussed in the introduction, our theory abstracts from some important issues in the economics of entrepreneurship, such as the role of risk and liquidity constraints. Moreover, we have assumed that each entrepreneur cannot enter more than one market. Our theory therefore only applies to those industries where the entrepreneurial span of control has strongly

diminishing returns across different markets.[14] Also, we have assumed that all entrepreneurs are completely mobile and may decide to enter any one market. While this may be an extreme assumption, it allows us to analyze a benchmark case, without having to make assumptions on the initial (pre-entry) distribution of potential entrants over geographical locations. In any event, even if a fraction of entrepreneurs are not mobile, the intuition for the sorting result should still hold for all those entrepreneurs who are mobile, and thus imply that entrepreneurial firms in larger markets are more efficient.

Appendix A: The Cournot Model with Heterogeneous Firms

The aim of this appendix is to show that our assumptions on the reduced-form profit function $\Pi(c; h(\mu), S)$ are satisfied in a homogenous goods Cournot model, where firms differ in their (constant) marginal costs. We assume that an increase in market size means a replication of the population of consumers (leaving the distribution of consumers' tastes and incomes unchanged), so that inverse demand can be written as $P(Q/S)$, where Q denotes aggregate output, and S denotes market size. The demand function is assumed to be downward-sloping (and differentiable), i.e., $P'(\cdot) < 0$. The distribution of firm types is summarized by the vector $\mathbf{c}$ of firms' marginal costs. For a given population of firms, each firm's equilibrium output (and profit) is proportional to market size S. Consequently, equilibrium price is independent of market size. To see this, consider firm i's first-order condition for profit maximization:

$$P\left(\frac{\sum_j q_j}{S}\right) - c + \frac{q_i}{S} P'\left(\frac{\sum_j q_j}{S}\right) = 0, \tag{5}$$

where q_j is firm j's quantity. Since output and market size enter only through the ratio q_j/S, this ratio must be independent of market size. Hence, gross profit is of the form $\Pi(c; h(\mu), S) = S\pi(c; h(\mu))$. The monotonicity of gross profit with respect to market size, (MON), and condition C.2 are thus trivially satisfied. To simplify notation, we will henceforth set $S = 1$.

Changes in industry output reflect changes in the underlying distribution of firms' efficiencies. That is, equilibrium industry output Q is some (possibly complicated) function of the vector of firms' marginal costs: $Q = f(\mathbf{c})$. To study differences in output and profit levels across firms for a given market structure $\mathbf{c}$, we consider changes in firms' marginal costs conditional on industry output $Q = f(\mathbf{c})$. For this purpose, we denote by $q(c; Q)$ the equilibrium output of a type-c firm when industry output is Q. (The function $q(c; \cdot)$ is sometimes called the backward-reaction function.) From the first-order condition (5), the equilibrium output of a type-c firm is equal to

$$q(c; Q) = -\frac{P(Q) - c}{P'(Q)} \text{ if } c \leq \overline{c}(Q) \equiv P(Q), \tag{6}$$

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[14] In the extreme case, where a firm could enter any number of markets (and, on the cost side, these entry decisions are completely independent from one another), our results on the relationship between efficiency and market size would be reversed: the most efficient firms enter all markets, while less efficient firms only enter the larger markets.

and $q(c; Q) = 0$ otherwise. The associated second-order condition for profit maximization is given by

$$2P'(Q) + q(c; Q)P''(Q) < 0. \tag{7}$$

Conditional on industry output Q, the equilibrium profit of a type-c firm can be written as

$$\pi(c; Q) = -\frac{(P(Q) - c)^2}{P'(Q)} \text{ if } c \leq \overline{c}(Q) \equiv P(Q), \tag{8}$$

and $\pi(c; Q) = 0$ otherwise. Clearly, $\pi(c; Q)$ is decreasing in marginal cost c (provided $c < P(Q)$), and so one part of the assumption (MON) is satisfied.

To analyze how a firm's equilibrium output changes with the underlying distribution of firm efficiencies and hence industry output, we take the derivative of $\pi(c; Q)$ with respect to Q, and find

$$\frac{\partial \pi(c; Q)}{\partial Q} = -q(c; Q) \left\{ 2P'(Q) + q(c; Q)P''(Q) \right\},$$

where we have made use of (6). Hence, if the second-order condition (7) holds for all $c < \overline{c}(Q)$ and Q, then a firm's equilibrium equilibrium profit $\pi(c; Q)$ is strictly decreasing in industry output Q. Any change in the underlying distribution of efficiencies (summarized by $\mathbf{c}$) which reduces firms' profits must therefore induce an increase in industry output Q; an increase in Q is equivalent to an increase in the intensity of price competition $h(\mu)$ as defined in the main text. That is, the other part of assumption (MON) is satisfied. Moreover, $\pi(c; Q)$ is clearly continuous in c and Q, satisfying (CON).

We now consider an increase in the distribution of efficiencies, reflected in a rise in industry output from Q to $Q' > Q$. Assumption C.1 says that the profit ratio $\pi(c; Q')/\pi(c; Q)$ is decreasing in c for all $c < \overline{c}(Q)$. It is immediate to see that this condition holds if and only if $P(Q') < P(Q)$, which is clearly satisfied since the inverse demand function is downward-sloping.

Assumption C.3 requires that the profit difference $\pi(c; Q) - \pi(c + t; Q)$ is decreasing in Q for all $c < \overline{c}(Q)$. For $c \geq -t + \overline{c}(Q)$, $\pi(c + t; Q) = 0$, and hence C.3 is implied by (MON). For $c < -t + \overline{c}()Q$, on the other hand, the profit difference can be rewritten as

$$\pi(c; Q) - \pi(c + t; Q) = -2t \left(\frac{P(Q) - (c + t/2)}{P'(Q)} \right).$$

The derivative of this expression is negative if and only if

$$P'(Q) - \frac{P''(Q)}{P'(Q)}[P(Q) - (c + t/2)] < 0,$$

which (using the first-order condition for profit maximization) is equivalent to

$$P'(Q) + q(c + t/2; Q)P''(Q) < 0. \tag{9}$$

Hence, C.3 holds whenever quantities are strategic substitutes (firms' reaction curves are downward-sloping). This is a rather weak (and standard) restriction on demand in Cournot models.

Applied to the Cournot model, assumption C.4 says that, for $Q' > Q$,

$$\frac{\pi(c;Q') - \pi(c+t;Q')}{\pi(c;Q) - \pi(c+t;Q)} = \frac{P'(Q)\left[P(Q') - (c+t/2)\right]}{P'(Q')\left[P(Q) - (c+t/2)\right]}$$

is decreasing in c. It is straightforward to check that this is indeed the case if and only if $P(Q') < P(Q)$, i.e., if and only if demand is downward-sloping.

Summing up, the assumptions on the profit function made in the basic model are satisfied if demand is downward-sloping and the second-order condition for profit maximization, (7), holds (for all $c < \overline{c}(Q)$ and Q). For the assumptions in the extended model with trade to hold, we have to strengthen the conditions on demand slightly and require that quantities are strategic substitutes, i.e., equation (9) holds (for all $c + t/2 < \overline{c}(Q)$ and Q).

Appendix B: Proofs

Proof of Proposition 1. To see the first part of the proposition, note that C.1 holds if and only if

$$\frac{\Pi_1(c;h(\mu'),S)}{\Pi(c;h(\mu'),S)} < \frac{\Pi_1(c;h(\mu),S)}{\Pi(c;h(\mu),S)} \quad \forall c \in \left[0,\overline{c}(\mu',S)\right), \; h(\mu') > h(\mu).$$

Applying the envelope theorem, this inequality is equivalent to

$$-\frac{q(c;h(\mu'),S)}{[p(c;h(\mu'),S)-c]q(c;h(\mu'),S)} < -\frac{q(c;h(\mu),S)}{[p(c;h(\mu),S)-c]q(c;h(\mu),S)},$$

or simply

$$p(c;h(\mu'),S) < p(c;h(\mu),S).$$

To see the second part of the proposition, observe that the C.2 holds if and only if

$$\frac{\Pi_1(c;h(\mu),S')}{\Pi(c;h(\mu),S')} \leq \frac{\Pi_1(c;h(\mu),S)}{\Pi(c;h(\mu),S)} \quad \forall c \in \left[0,\overline{c}(\mu',S)\right), \; S' > S.$$

Applying the envelope theorem, this inequality is equivalent to

$$-\frac{q(c;h(\mu),S')}{[p(c;h(\mu'),S)-c]q(c;h(\mu),S')} \leq -\frac{q(c;h(\mu),S)}{[p(c;h(\mu),S)-c]q(c;h(\mu),S)},$$

or simply

$$p(c;h(\mu),S') \leq p(c;h(\mu),S).$$

∎

Proof of Proposition 2. Consider any two markets, i and j, where $j > i$. In the main text, we have shown that equilibrium requires that competition is endogenously more intense in the larger market, i.e., $h(\mu_i) > h(\mu_j)$. For any type-c entrepreneur, we can decompose the ratio of her gross profits between the two markets as follows:

$$\frac{\Pi(c;h(\mu_i),S_i)}{\Pi(c;h(\mu_j),S_j)} = \frac{\Pi(c;h(\mu_i),S_j)}{\Pi(c;h(\mu_j),S_j)} \times \frac{\Pi(c;h(\mu_i),S_i)}{\Pi(c;h(\mu_i),S_j)}. \tag{10}$$

Suppose now that entrepreneurial type c_{ij} is indifferent between entering market i and market $j > i$. Then, the profit ratio is equal to one at $c = c_{ij}$. Since $h(\mu_i) > h(\mu_j)$ (and $S_i > S_j$ by definition), conditions C.1 and C.2 then imply that the first term on the r.h.s. of equation (10) is strictly decreasing in c, whereas the second term is non-increasing in c. Hence, all entrepreneurs more capable than c_{ij} strictly prefer to enter (the larger but more competitive) market i, while less capable entrepreneurs strictly prefer to enter (the smaller but less competitive) market j. Since we assume that entry and fixed costs are sufficiently small, and the mass of potential entrepreneurs M sufficiently large, a positive mass of entrepreneurs must enter each market, while a positive mass of entrepreneurs does not enter any market. Hence, in equilibrium, there exist marginal types $\{c_i\}_{i=0}^{N}$, where $c_i < c_{i+1}$,

$$c_0 \equiv 0, \tag{E_0}$$

$$\Pi(c_i; h(\mu_i), S_i) = \Pi(c_i; h(\mu_{i+1}), S_{i+1}), \ i = 1, ..., N-1, \tag{E_i}$$

and

$$\Pi(c_N; h(\mu_N), S_N) = \varepsilon + \phi. \tag{E_N}$$

The existence of a pure strategy Nash equilibrium in games with a continuum of players and countable actions is well known; see Mas-Colell (1984), and Khan and Sun (1995). We now want to prove uniqueness of equilibrium. For this, suppose there exist marginal types $\{\widetilde{c}_i\}_{i=0}^{N} \neq \{c_i\}_{i=0}^{N}$ satisfying (E_0) to (E_N). Assume, for instance, that $\widetilde{c}_N < c_N$. For (E_N) to hold, we thus have $h(\widetilde{\mu}_N) > h(\mu_N)$. The last observation in turn implies that $\widetilde{c}_{N-1} < c_{N-1}$. To see this, suppose instead that $\widetilde{c}_{N-1} \geq c_{N-1}$; however, from $\widetilde{c}_N < c_N$ and (DOM), it would then follow that $h(\widetilde{\mu}_N) \leq h(\mu_N)$, contradicting our finding that $h(\widetilde{\mu}_N) > h(\mu_N)$. Observe now that, for given measures μ_i and μ_{i+1}, the marginal type c_i is *uniquely* defined by (E_i), where uniqueness follows from C.1; furthermore, c_i is decreasing in $h(\mu_i)$ and increasing in $h(\mu_{i+1})$. Hence, $\widetilde{c}_{N-1} < c_{N-1}$ and $h(\widetilde{\mu}_N) > h(\mu_N)$ imply that $h(\widetilde{\mu}_{N-1}) > h(\mu_{N-1})$. Following the same steps of argument, we obtain that $\widetilde{c}_i < c_i$ and $h(\widetilde{\mu}_i) > h(\mu_i)$ for all $i \in \{1, ..., N\}$. However, $\widetilde{c}_1 < c_1$ and $\widetilde{c}_0 = c_0 = 0$ imply that $h(\widetilde{\mu}_1) < h(\mu_1)$, contradicting $h(\widetilde{\mu}_1) > h(\mu_1)$. Hence, we cannot have $\widetilde{c}_N < c_N$. A similar reasoning yields that we cannot have $\widetilde{c}_N > c_N$. We therefore conclude that $\widetilde{c}_N = c_N$. Suppose now that $\widetilde{c}_N = c_N$ and $\widetilde{c}_{N-1} < c_{N-1}$. It then follows that $h(\widetilde{\mu}_{N-1}) > h(\mu_{N-1})$. As before, it is straightforward to show that this leads to a contradiction. Applying these arguments to all $i \in \{1, ..., N\}$, we find that $\widetilde{c}_i = c_i$ for all $i \in \{0, 1, ..., N\}$, proving uniqueness of equilibrium. ∎

Proof of Proposition 3. Suppose that $h(\mu') > h(\mu)$. To show the first part of the proposition, we note that

$$\frac{\partial}{\partial c} \left(\Pi(c; h(\mu), S) - \Pi(c; h(\mu'), S) \right) < 0$$

if and only if

$$\Pi_1(c; h(\mu'), S) > \Pi_1(c; h(\mu), S).$$

Applying the envelope theorem, the latter inequality can be shown to be equivalent to

$$q(c; h(\mu')) < q(c; h(\mu)).$$

To show the second part of the proposition, we note that

$$\frac{\partial}{\partial c}\left(\frac{\Pi_1(c; h(\mu'), S)}{\Pi(c; h(\mu), S)}\right) < 0$$

if and only if

$$\frac{\Pi_{11}(c; h(\mu'), S)}{\Pi_1(c; h(\mu'), S)} < \frac{\Pi_{11}(c; h(\mu), S)}{\Pi_1(c; h(\mu), S)}.$$

Applying the envelope theorem, the latter inequality can be shown to be equivalent to

$$\frac{q_1(c; h(\mu'), S)}{q(c; h(\mu'), S)} < \frac{q_1(c; h(\mu), S)}{q(c; h(\mu), S)}.$$

∎

Proof of Proposition 5. Let $\hat{c}_1$ be defined by

$$S_1 \pi(\widehat{c}_1; h(\mu_1)) + \sum_{j=2}^{N} S_j \pi(\widehat{c}_1; h(\mu_j)) = \varepsilon + \phi, \tag{11}$$

where μ_1 is such that $\mu_1([0, z]) = \mu_j([t, z + t]) = MG(\min\{z, c_1\})$, $j \neq 1$, for any interval $[0, z]$. (Our assumptions ensure that $\widehat{c}_1$ does indeed exist.) Then, we claim that

$$S_i \pi(\widehat{c}_1; h(\mu_i)) + \sum_{j \neq i} S_j \pi(\widehat{c}_1; h(\mu_j)) \leq \varepsilon + \phi, \ i = 2, ..., N. \tag{12}$$

Note that (11) and (12) imply that no type in $(\widehat{c}_1, 1]$ would find it profitable enter *any* market. Moreover, since type $\widehat{c}_1$ weakly prefers to enter market 1 than any other market, any (more capable) type in $[0, \widehat{c}_1)$ must strictly prefer to enter market 1; this follows from C.4. Hence, we prove the assertion by showing that (11) implies (12), provided transport cost t is sufficiently small. Now, observe that (11) implies (12) if

$$-\frac{S_1\left[\pi(\widehat{c}_1 + t; h(\mu_1)) - \pi(\widehat{c}_1; h(\mu_1))\right]}{t} \geq -\frac{S_i\left[\pi(\widehat{c}_1 + t; h(\mu_i)) - \pi(\widehat{c}_1; h(\mu_i))\right]}{t} \tag{13}$$

for $i = 2, ..., N$. Note also that $h(\mu_i) \to h(\mu_1)$ as $t \to 0$: in the limit as transport costs vanish, the distribution of entrepreneurial types selling in a market is the same for all markets. (For notational convenience, we do not explicitly state the dependence of μ_i, $i = 1, ..., N$, on t.) Thus, (13) holds for small enough t if

$$-S_1\pi_1(\widehat{c}_1; h(\mu_1)) > S_i\pi_1(\widehat{c}_1; h(\mu_1)), \ i = 2, ..., N,$$

or simply if

$$S_1 > \max_{i>1} S_i,$$

which holds by definition. ∎

Proof of Proposition 6. The proof is similar to that of proposition 2. The first step consists in showing that, in equilibrium, the distribution of active entrepreneurs is larger (in

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the sense of representing more intense competition) in larger markets: $h(\mu_i) > h(\mu_j)$ for any markets $i, j > j$. The proof of this assertion proceeds as before. The remaining steps are slightly more involved. Since we assume that each market is sufficiently large relative to entry and fixed costs (so that each market is non-empty in equilibrium) and since the conditional value is continuous in c, for any two markets $i, j > j$, there exists some type, say c_{ij}, who his indifferent between entering markets i and j: $\overline{V}(c_{ij}; h(\mu_i), S_i) = \overline{V}(c_{ij}; h(\mu_j), S_j)$. Similarly, there exists a unique type, say $\hat{c}_{ij}$, who would make the same (current) profit in both markets: $\Pi(\hat{c}_{ij}; h(\mu_i), S_i) = \Pi(\hat{c}_{ij}; h(\mu_j), S_j)$. Assumptions C.1 and C.2 ensure that the profit ratio $\Pi(c; h(\mu_i), S_i)/\Pi(c; h(\mu_j), S_j)$ is decreasing in c on $[0, \overline{c}(\mu_i, S_i))$. If $c_{ij} \leq \hat{c}_{ij}$, then it is straightforward to see that the ratio of conditional values, $\overline{V}(c; h(\mu_i), S_i)/\overline{V}(c; h(\mu_j), S_j)$, is decreasing in c at $c = c_{ij}$; this holds independently of the level of $\delta(1-\alpha)/(1-\delta)$. In this case, any type more efficient than c_{ij} strictly prefers to enter market i, whereas all less efficient types prefer to enter the smaller market j. Now, if c_{ij} is (much) larger than $\hat{c}_{ij}$, then the ratio of conditional values may not be monotonically decreasing in c. By assuming that $\delta(1-\alpha)/(1-\delta)$ is small, we ensure that c_{ij} is close to $\hat{c}_{ij}$, and hence that $\overline{V}(c; h(\mu_i), S_i)/\overline{V}(c; h(\mu_j), S_j)$ is decreasing in c at $c = c_{ij}$. The asserted sorting result follows then immediately. Uniqueness of equilibrium can be shown in a way similar to the proof of proposition 2. Note that the assumption that $\delta(1-\alpha)/(1-\delta)$ is small implies that the marginal incumbent $c^*(\mu, S)$ (who is just indifferent between exiting and staying in the market) makes a positive gross profit, $\Pi(c^*(\mu, S); h(\mu), S) > 0$, and hence $c^*(\mu, S) < \overline{c}(\mu, S)$ (as we posited before). $\blacksquare$
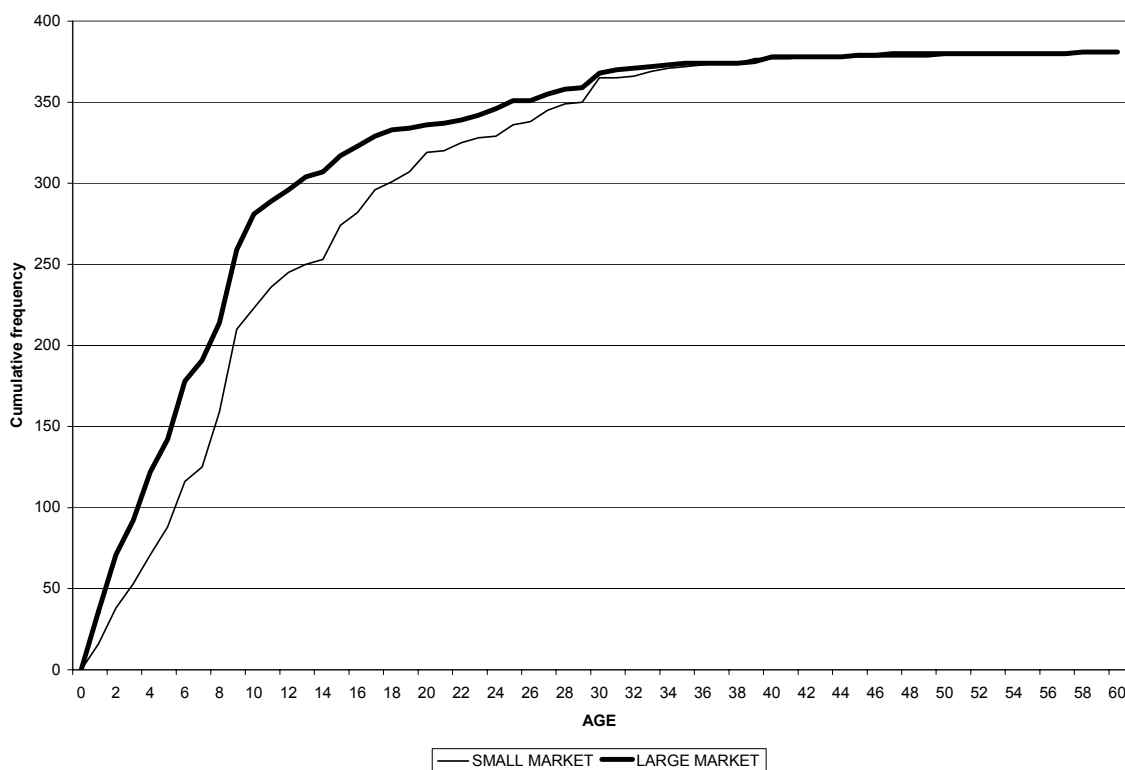
Figure 1: The age distribution of Swedish hairdressers in small and large markets. Data source: Asplund and Nocke (2003).

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