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"Moral Hazard and Capital Structure Dynamics"

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Moral Hazard and Capital Structure Dynamics[*]

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Abstract

We base a contracting theory for a start-up firm on an agency model with observable but nonverifiable effort, and renegotiable contracts. Two essential restrictions on simple contracts are imposed: the entrepreneur must be given limited liability, and the investor's earnings must not decrease in the realized profit of the firm. All message game contracts with pure strategy equilibria (and no third parties) are considered. Within this class of contracts/equilibria, and regardless of who has the renegotiating bargaining power, debt and convertible debt maximize the entrepreneur's incentives to exert effort. These contracts are optimal if the entrepreneur has the bargaining power in renegotiation. If the investor has the bargaining power, the same is true unless debt induces excessive effort. In the latter case, a non-debt simple contract achieves efficiency – the non-contractibility of effort does not lower welfare. Thus, when the non-contractibility of effort matters, our results mirror typical capital structure dynamics: an early use of debt claims, followed by a switch to equity-like claims.

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1. Introduction

As is well known, the classical agency model of, e.g., Mirrlees (1999) and Holmström (1979), fails to yield optimal schemes that resemble standard instruments like debt or equity. The "security design" literature has therefore looked elsewhere to show that debt (or equity) are optimal. For example, Townsend (1979) and Gale and Hellwig (1985) consider "costly state verification" models in which output can be observed only at a cost. Bolton and Scharfstein (1990), Berglof and von Thadden (1994), and Hart and Moore (1994, 1998) consider "stealing models" in which output is entirely unverifiable, but debt holders can seize assets in some contingencies. In Aghion and Bolton (1992), and Dewatripont and Tirole (1994), output is costlessly verifiable, but actions that affect continuation values are not contractible.

In this paper we derive a simple theory of capital structure dynamics for a start-up firm. It is based on a model that, relative to those in the above papers, is closer to the classical moral hazard paradigm. It departs from the classical paradigm in three ways. First, contracts can be renegotiated after effort is chosen, but before output is realized. This is an appropriate assumption when the input of the entrepreneur (agent) is crucial to the initial business stage, before its fruits are realized. Our paper thus joins the literature on renegotiating moral hazard contracts, as discussed below.

Second, although the entrepreneur's effort remains noncontractable, it is observed by the investor (principal). This abstraction from issues of imperfect observability is a reasonable approximation when investors have expertise and engage in monitoring, as venture capitalists frequently do (Kaplan and Stromberg, 2002). Our observability and renegotiation assumptions resemble those of Hermalin and Katz (1991).

Third, feasible contracts must take account of the entrepreneur's limited resources, and give the investor a payoff that does not decrease in the firm's output. The former "limited liability" restriction holds naturally for an entrepreneur with little wealth. The latter "monotonicity" restriction can be derived as an equilibrium outcome from ex post moral hazard considerations. It arises, for example, if the investor can "burn output"

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in order to make the firm's performance appear lower than it really was[1]. Alternatively, it arises if the entrepreneur can secretly borrow from an outside lender in order to make the firm's performance appear greater than it really was. Assuming such ex post moral hazards weakens the assumption that output is verifiable, but less so than in the costly state verification models, and much less so than in the stealing models.

Under these liability and monotonicity restrictions, Innes (1990) shows that debt is optimal if the parties are risk neutral. Debt gives the entrepreneur a return of zero – the minimal possible return when he has limited liability – if the firm's realized earnings are lower than the face value of the debt. This property of debt is useful for giving the entrepreneur incentives to choose an effort that lowers the probability of this low return. But it also makes debt a poor risk-sharing scheme if the entrepreneur is risk averse, in which case debt is not optimal in Innes' no-renegotiation model.

On the other hand, if the debt can be renegotiated after the effort is chosen, possibly it can be renegotiated to a better risk-sharing contract without destroying incentives. A result like this is due to Hermalin and Katz (1991). They examine a model like ours, with renegotiation, effort that is observable but not verifiable, and a risk averse entrepreneur (but risk neutral investor). They show that if the entrepreneur has the renegotiation bargaining power, then a riskless debt contract, i.e., a contract that pays the investor a fixed amount regardless of the realized output, achieves a first-best outcome.[2] Riskless debt provides appropriate incentives, and it is renegotiated to an efficient risk-sharing contract after the effort is chosen.

Riskless debt, however, will generally give the investor too low a return when the limited liability of the entrepreneur prevents him from paying back more than the firm earns. In this case, if the smallest possible output of the firm is less than the required start-up investment, and if the investor has no bargaining power in the renegotiation,

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[1] The investor could for example engage in sabotage activities, or play a negative role in the certification process of the firm's performance.

[2] This describes both the proof and statement of Proposition 3 in Hermalin and Katz (1991). Matthews (1995) obtains a similiar result for unobservable effort. These results do not rely on the monotone likelihood ratio propety, unlike unlike those of Innes (1990), and ours, regarding risky debt.

any feasible riskless debt contract gives her a negative return on her investment. Our task in this paper, therefore, is to determine the nature of an optimal contract that gives the investor a higher payoff than would any feasible riskless debt contract.

1.1. Preview of Results

To investigate this problem, we first restrict attention to "simple contracts", which are contracts that specify a fixed rule for sharing the firm's output. To ease the exposition we start with the most tractable case of interest, that in which (i) the investor is risk neutral, (ii) the entrepreneur is risk averse, and (iii) the entrepreneur has all the bargaining power in the renegotiation stage. The main result is that, within the class of simple contracts satisfying the liability and monotonicity restrictions, debt contracts are optimal. Thus, risky, rather than riskless, debt emerges when the latter gives the investor too low a payoff. The reason, roughly, is that within the set of simple contracts that give the investor some payoff, a debt contract elicits the greatest effort. Unless the debt is riskless, this effort is not high enough to be efficient, i.e., if effort were to be contractible, prescribing a higher effort would make both parties better off.

We next consider a general setting in which (a) both parties may be risk averse, (b) bargaining powers may be shared, and (c) contracts may require the parties to send messages to the contract enforcer after the effort is chosen. These messages determine a (possibly random) simple contract for sharing output; the prescribed (random) simple contract can then be renegotiated. This is along the lines of the literature on mechanism design with renegotiation, especially Maskin and Moore (1999) and Segal and Whinston (2002). Within this broad class of contracts, an "investor-option contract" is one in which only the investor sends a message; it is equivalent to a set of (random) simple contracts from which the investor will select after the effort is chosen. Our first result in this setting is that investor-option contracts are optimal, given a restriction to pure strategy equilibria of the message game. There is thus, subject to the pure strategy proviso, no need to consider contracts that require the entrepreneur to send a message.[3]

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[3]Contracts in which both parties send messages may be of value if equilibria in mixed message

This result holds for any renegotiation procedure that achieves an ex post efficient outcome, and is continuous in the disagreement outcome.

We then revisit, in the general setting, the case in which the entrepreneur has all the bargaining power. Our main result here is that no contract outperforms debt (again restricting attention to pure strategies). As in the simpler setting, the basis of the result is that debt provides the strongest incentives of all feasible contracts, and no feasible contract provides enough incentives to achieve an efficient effort. Of course, an investor-option contract containing debt may be payoff-equivalent to debt. Convertible debt is such a contract: it is an investor-option contract that consists of a debt contract and the simple contract to which, in equilibrium, it is renegotiated after the effort is chosen. In the equilibrium of a convertible debt contract, the entrepreneur takes the same effort as he would have given just the debt contract, and then the investor selects the alternative simple contract instead of the debt. The entrepreneur is deterred from shirking by the credible threat that it would cause the investor to select the debt contract. This is like "converting" to equity some or all of the debt in a real convertible debt contract, if the entrepreneur is observed to have performed well. The convertible debt contract, unlike the simple debt contract, is not renegotiated in equilibrium; in this sense it is the renegotiation-proof equivalent of the debt contract.

We next turn to the case in which the investor has bargaining power. We show that then debt still provides the strongest incentives. However, the incentives provided by debt may be too strong if the entrepreneur is risk averse. This is easiest to see when the investor has all the bargaining power. In this case the entrepreneur does not gain at all from renegotiation, and so cares about the riskiness of the initial contract. Debt is very risky for him, since it gives him a zero return if the realized output is low. He may therefore over-exert himself in order to lower the probability of low outputs. The possibility that he might over-supply effort may seem surprising; the standard view is that he should under-supply effort because he ignores the positive externality his effort

strategies can be implemented. This is considered in Appendix B.

has on investors.[4] Here, however, part of the entrepreneur's motivation to raise effort is that doing so reduces the riskiness of the debt contract. This has no social value, since the contract will anyway be renegotiated to one that shares risk efficiently. His effort can thus increase his payoff by reducing risks that are not socially costly.

Our main result for when the investor has all the bargaining power is that either a simple contract that is not debt achieves an efficient outcome, or a debt contract is optimal in the set of deterministic general contracts (again with the pure strategy proviso). In the former case, the non-contractibility of effort does not lower welfare. Debt is thus optimal whenever the non-contractibility of effort matters. We prove this under strong but standard separability and concavity-like assumptions.

Lastly, we show that when both parties have bargaining power, debt still provides the strongest incentives, given a simple "ray" bargaining solution and further separability assumptions on the entrepreneur's utility.

1.2. Links with the Literature

We have mentioned the connection between this paper and Hermalin and Katz (1991) and Innes (1990). Other related papers consider renegotiation of incentive contracts when the principal does not observe the agent's effort. Fudenberg and Tirole (1990), Ma (1991, 1994), and Matthews (1995) study such models without liability or monotonicity restrictions. Matthews (2001) studies a model with these restrictions, in an environment that is the same as in this paper except that his investor cannot observe the effort. Restricting attention to simple contracts and to the case in which the entrepreneur has all the bargaining power, Matthews (2001) shows that debt is optimal. The asymmetric information make this result less robust than ours: multiple, non-equivalent equilibria may exist, simple contracts that are not debt may also be optimal, and message game contracts with pure strategy equilibria may outperform debt.

Our results also relate to the broader literature on renegotiation. The fact that a

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[4]See, e.g., Jensen and Meckling (1976) or Myers (1977), and the ensuing literature on the "outside equity" and "debt overhang" problems.

simple contract without messages can be optimal is also true in Hart and Moore (1988), and in some parts of Segal and Whinston (2002). Even the null contract is optimal in Che and Hausch (1999), Segal (1999), Hart and Moore (1999), and Reiche (2001). It is renegotiation that causes simple contracts to be optimal in these models as well as in ours, for two reasons. First, equilibrium renegotiation "completes" the initial contract, since the renegotiated contract can depend on observable but non-contractible variables. Second, renegotiation makes any message game strictly competitive, and therefore of limited use, because it ensures ex post efficiency. In our paper the simple contract that emerges, debt, does so because it maximizes incentives. In the other papers, either a simple profit-sharing rule or the null contract is optimal because contracting is unable to strengthen incentives.[5]

Finally, our paper contributes to the corporate finance literature by developing a simple theory of capital structure dynamics. It can be seen as describing an entrepreneur who first obtains debt finance from a bank, but then later adopts equity finance by going public. It also fits the case of an entrepreneur who issues convertible debt to a venture capitalist. Real-world contracts are of course more complicated than the ones we consider here, but the model nonetheless generates a dynamic pattern of financial contracting, with debt first and (after conversion or renegotiation) a more equity-like structure later on, that is fairly realistic despite its simplicity.[6] Of course, this paper is essentially a theoretical contribution. It would be interesting in future work to consider the ideas it explores in a setting with more detailed firm financing and dynamics, as in the literature on convertible securities in venture capital finance: Berglof (1994), Bergemann and Hege (1997), Cornelli and Yosha (1997), Repullo and Suarez (1999), Casamatta (2000), Schmidt (2000) or Dessi (2002).

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[5]The optimal contract in Hart-Moore (1988) is a simple profit-sharing rule because trade cannot be enforced ex post. The null contract is optimal in the other papers, either because of the presence of direct investment externalities (Che-Hausch, 1999), or because the nature of the good to be traded cannot be specifed (Segal, 1999, Hart and Moore, 1999, and Reiche, 2001).

[6]See Diamond (1991), Sahlman (1990) or Kaplan and Stromberg (2000, 2002) for facts on firm financing patterns.

1.3. Structure of the Paper

The paper is organized as follows. The environment is described in Section 2. The special case in which the investor is risk neutral, and the entrepreneur has the renegotiation bargaining power, is studied in Section 3. The case of general contracts and renegotiation is analyzed in Section 4. Section 5 contains results for the general model when the entrepreneur has all the bargaining power. Section 6 considers the case in which the investor has some or all the bargaining power. Section 7 concludes. Appendix A contains proofs. Appendix B shows how non-debt contracts may be better than debt if third parties are introduced, or mixed message strategies can be implemented.

2. Preliminaries

An entrepreneur (agent) must contract with an investor (principal) to obtain the K dollars required to start a project. After contracting, the entrepreneur chooses an effort level e from an interval $E = [\underline{e}, \bar{e}] \subset \mathbb{R}$. His effort determines a probability distribution, $g(e) = (g_1(e), \ldots, g_n(e))$, over the set of possible (monetary) *outputs*, $\{\pi_1, \ldots, \pi_n\}$. We assume $n > 1$ and $\pi_i < \pi_{i+1}$. Each g_i is twice continuously differentiable and positive on E. Output increases stochastically with effort in the sense of the strict monotone likelihood ratio property:

$$(\text{MLRP}) \quad \frac{g_i'(e)}{g_i(e)} \text{ increases in } i \text{ for any } e \in E.$$

The only contractible variable is output. Accordingly, a *simple contract* is a vector $r = (r_1, \ldots, r_n)$ specifying a payment from the entrepreneur to the investor for each possible output. An *allocation* is a pair (r, e).

Given an allocation (r, e), the entrepreneur's utility if π_i is realized is $u(\pi_i - r_i, e)$. His *payoff* (expected utility) from an allocation is

$$U(r, e) \equiv \sum g_i(e) u(\pi_i - r_i, e).^7$$

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[7] We omit the summation index if it is $i = 1, \ldots, n$.

The function u is twice continuously differentiable. With respect to income, the entrepreneur's utility increases, $u_1 > 0$, and he is weakly risk averse: $u_{11} \leq 0$. His utility decreases with effort at all interior efforts: $u_2(\cdot, e) < 0$ for $e \in (\underline{e}, \bar{e})$. Corner solutions are eliminated by assuming $u_2(\cdot, \underline{e}) = 0$ and $u_2(\cdot, \bar{e}) = -\infty$.

The investor's net utility is $v(y)$ if she makes the start-up investment and receives y dollars in return.[8] The function v has continuous derivatives $v' > 0$ and $v'' \leq 0$. The investor's payoff from an allocation is

$$V(r, e) \equiv \sum g_i(e)v(r_i).$$

We assume at least one party is risk averse: $u_{11} < 0$ or $v'' < 0$.

The timing and information structure of the game are as follows. After a contract is adopted, the entrepreneur chooses effort. The investor observes the effort immediately. The parties then send any messages that the contract may require. As a function of these messages, the contract specifies a (possibly random) simple contract that, together with the effort, determines a status quo allocation. The parties then renegotiate to another simple contract. Finally, output is realized and payments made according to the renegotiated contract.

At the heart of our model is a set of restrictions on what makes a simple contract feasible. The first is a limited liability constraint for the entrepreneur:

$$\text{(LE)} \quad r_i \leq \pi_i \text{ for } i \leq n.$$

This standard constraint reflects the reality that because of their limited wealth, entrepreneurs often cannot pay back more than the project earns. If the start-up investment satisfies $K > \pi_1$, then LE rules out the contract that pays back K after any output.

The second important restriction is a monotonicity constraint for the investor that requires her income to weakly increase with the project's output:

$$\text{(MI)} \quad r_i \leq r_{i+1} \text{ for } i < n.$$

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[8]This v is a normalization of the investor's utility function for income, $\hat{v}$. If she keeps the K dollars, her utility is $\hat{v}(K)$. If she invests it and receives y in return, her utility is $\hat{v}(y)$. So $v(y) \equiv \hat{v}(y) - \hat{v}(K)$ is her net utility from making the investment.

Introduced by Innes (1990), MI should be viewed as a result of various ex post moral hazards we have not modeled explicitly. For example, MI is easily shown to be satisfied by any implementable contract if the investor can engage in sabotage to distort the apparent π_i downwards. Alternatively, it is satisfied if the entrepreneur can borrow secretly from another lender after a contract has been signed, thereby distorting the apparent π_i upwards.[9] Note that the expected payback of any r satisfying MI increases with effort: MLRP implies $\sum g_i'(e) r_i \geq 0$, and the inequality is strict if the contract is risky (so at least one of the inequalities $r_i \leq r_{i+1}$ is strict).

We denote the set of feasible simple contracts as C, and assume it is defined by LE, MI, and one other constraint:

$$C \equiv \{ r \in \mathbb{R}^n \mid r \text{ satisfies LE, MI, and LI} \}.$$

The additional constraint,

$$\text{(LI)} \quad r_i \geq \underline{r} \text{ for } i \leq n,$$

is a limited liability constraint for the investor that imposes a lower bound (which can be arbitrarily low) on how much she can be paid back. Its only role is to simplify the analysis by insuring that C is compact. We assume $\underline{r} < \pi_1$, so that C has an interior. It is also convex.

Debt contracts have a central role in this paper. A *debt contract,* $\delta(D)$, is defined, for any *face value* $D \leq \pi_n$, by

$$\delta_i(D) \equiv \min(D, \pi_i) \text{ for } i \leq n.$$

For simplicity we often denote $\delta(D)$ as δ. Note that $\delta \in C$ if and only if $D \geq \underline{r}$. The debt is *risky* if $\delta_1 < \delta_n$, which is equivalent to $D > \pi_1$.

We define a *riskless debt contract* to be a contract that pays the investor the same amount after any output. The one that pays an amount V is denoted $\delta^V \equiv (V, \ldots, V)$.

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[9]It may also be likely that the entrepreneur can destroy output, or the investor can inject cash to inflate apparent profit. These moral hazards lead to the constraints $\pi_i - r_i \leq \pi_{i+1} - r_{i+1}$. Since debt satisfies them, our Propositions $2 - 6$ on debt carry over if these constraints are added. So does Proposition 1 on investor-option contracts, as it does not rely on the specific nature of a feasible contract.

Note that $\delta^V \in C$ if and only if $\underline{r} \leq V$ so that it satisfies LI, and $V \leq \pi_1$ so that it satisfies LE.

An *efficient risk-sharing contract* for a fixed effort e is a contract in C that solves the following program, for some investor payoff $\hat{V}$:

$$H(\hat{V}, e) \equiv \max_{r \in C} U(r, e) \text{ such that } V(r, e) \geq \hat{V}. \tag{1}$$

This is a "constrained efficiency" notion, taking as given the constraints that define C. (We reserve the modifier "first-best" for outcomes that are efficient in the full, unconstrained sense.) Any solution of (1) is unique, since at least one party is strictly risk averse. The graph of $H(\cdot, e)$ is the Pareto frontier of possible payoff pairs given the fixed effort. Lemma A1 in Appendix A shows that $H(\cdot, e)$ is concave, and has a negative derivative, $H_1(\cdot, e)$, on its domain.

An allocation (r^*, e^*) is *efficient* if e^* maximizes $H(\hat{V}, \cdot)$ for some $\hat{V}$, and r^* solves (1) when $e = e^*$. Such allocations set the welfare benchmark: they determine the achievable Pareto frontier if effort as well as output were to be contractible, the parties could commit not to renegotiate, and constraints MI, LE, and LI had to be respected.

3. The Case of a Risk Neutral Investor and Entrepreneur-Offer Bargaining

We now give the key arguments for a simple canonical case defined by two restrictions. First, the investor is risk neutral. Second, the entrepreneur has all the renegotiation bargaining power, as though he can offer a new contract as an ultimatum.

As the investor is risk neutral, an efficient risk-sharing contract pays the entrepreneur a fixed wage. The *wage contract* that pays wage w is denoted r^w and defined by

$$r_i^w \equiv \pi_i - w \text{ for } i \leq n.^{10}$$

Since renegotiation occurs after both parties observe the effort, it yields an efficient risk-sharing contract. So, in the present case, the entrepreneur renegotiates to a wage

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[10]Because of the liability constraints, r^w is feasible if and only if $w \in [0, \pi_1 - \underline{r}]$.

contract, i.e., he sells his entire stake in the firm to the investor.[11] If it were adopted initially, a wage contract would not be renegotiated, since it shares risk efficiently for any effort. Wage contracts thus provide no incentives: they pay the entrepreneur a fixed amount regardless of output, and so induce him to take the lowest effort.

3.1. Simple Contracts

Suppose the parties initially adopt a contract $r \in C$. The entrepreneur will then, after he has chosen an effort e, offer the investor a wage contract r^w that has the highest wage she will agree to pay, i.e., the largest w satisfying

$$\sum g_i(e)\pi_i - w \geq \sum g_i(e)r_i.$$

This constraint binds – the investor does not gain from the renegotiation. The resulting wage is given by a *wage function* defined by

$$w^*(r, e) \equiv \sum g_i(e)(\pi_i - r_i). \tag{2}$$

When he chooses effort, the entrepreneur knows his ultimate wage will be given by $w^*(r, \cdot)$. An equilibrium outcome of r is thus a solution, (e^*, w^*), of this program:

$$\max_{e,w} u(w, e) \text{ subject to } w = w^*(r, e). \tag{3}$$

The contract r provides incentives by determining the slope of $w^*(r, \cdot)$. Renegotiation allows the two functions of contracts to be separated: the initial contract provides the incentives, and the final contract provides the risk sharing.

It is now easy to see that an equilibrium outcome of any riskless debt contract, $\delta^V = (V, \ldots, V)$, is first-best efficient.[12] Simply observe that $w = w^*(\delta^V, e)$ is the equation for the indifference curve of pairs (e, w) that give the investor utility V.[13]

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[11]If the investor were risk averse, the final contract would be more like equity, since efficient risk-sharing would require both parties' earnings to increase in output (linearly if they had CARA utility).

[12]This result is buried in the proof of Proposition 3 in Hermalin and Katz (1991). We show in Section 5 that if the investor is risk averse, riskless debt still achieves efficient (but not first-best) allocations.

[13]The (e, w) pairs that give the investor utility V are those that satisfy $\sum g_i(e)\pi_i - w = V$. As this can be rewritten as $w = w^*(\delta^V, e)$, the graph of $w^*(\delta^V, \cdot)$ is the investor's indifference curve.

Thus, if the initial contract is δ^V, program (3) is the Pareto program that yields a first-best outcome giving the investor utility V.

The non-contractibility of effort may therefore be irrelevant. Even if any (e, w) could be directly enforced, it is impossible to make both parties better off than when a riskless debt contract is adopted and renegotiated. The problem with this argument, however, is that a feasible riskless debt contract may not compensate the investor enough for investing K. (Recall that δ^V satisfies the entrepreneur's liability constraint only if $V \leq \pi_1$.) In this case the only feasible contracts to which she might agree are risky.

If the contract must be risky, the non-contractibility of effort does prevent the attainment of an efficient allocation. Specifically, feasible risky contracts give inefficiently low incentives. Recall that the expected payback to the investor of any risky $r \in C$ strictly increases in effort: $\sum g_i'(e) r_i > 0$. This diminishes the entrepreneur's incentive to raise effort. Let V be the investor's payoff from an equilibrium of r. As we noted above, the riskless debt contract δ^V provides efficient incentives. The marginal incentives that δ^V and r provide the entrepreneur to raise effort are given by the wage derivatives $w_e^*(\delta^V, \cdot)$ and $w_e^*(r, \cdot)$, respectively. Those provided by δ^V are higher, since for any $e \in E$,

$$w_e^*(\delta^V, e) - w_e^*(r, e) = \sum g_i'(e) r_i > 0.$$

From this it is easy to show that the effort achieved by r is less than the effort in any efficient allocation that gives the investor the same payoff V.

A generalization of this argument from riskless to risky debt shows that within the feasible set of contracts, debt provides the greatest incentives. Consider a non-debt contract $r \in C$, and a debt contract δ, such that neither contract always pays more than the other. Since r satisfies LE, $r_i \leq \delta_i$ for low outputs π_i. But since r satisfies MI, $r_i \geq \delta_i$ for high outputs. That is, δ pays the entrepreneur less for low outputs and more for high outputs. It thus gives him a greater incentive to shift probability from low to high outputs, which by MLRP he accomplishes by increasing effort. Formally, if the wage curves $w^*(\delta, \cdot)$ and $w^*(r, \cdot)$ ever cross, the former has a greater slope at the

point of crossing.[14] This key single-crossing property implies that of all the contracts in C that give the investor some equilibrium payoff V, it is a debt contract that achieves the largest effort.

We use Figure 1 to now show the Pareto dominance of debt.

[INSERT FIGURE 1 HERE]

Contract $r \in C$ is a non-debt contract, and (e^*, w^*) is an equilibrium outcome of it. Contract δ is the debt contract satisfying $w^*(\delta, e^*) = w^*$. By the single-crossing property, $w^*(\delta, \cdot)$ is steeper than $w^*(r, \cdot)$ at (e^*, w^*). Let V be the investor's payoff at this outcome. As shown above, $w^*(\delta^V, \cdot)$ is the investor's indifference curve at (e^*, w^*), and it is there the steepest of the three curves. An equilibrium outcome of δ must be on the thick portion of $w^*(\delta, \cdot)$, which is in the lens between the parties' indifference curves. Thus, any outcome of δ Pareto dominates the outcome (e^*, w^*) of r.

3.2. More General Contracts

We now turn to contracts that require messages to be sent. Convertible debt, a standard way of financing venture capital, is a prominent example. It is a debt security that the investor has the option of converting to equity in the future. It is a contract that only requires the investor to send a message. In this section we restrict attention to such *investor-option contracts,* and assume the number of options is finite. (This is nearly without loss of generality, as we show in Section 4.)

Such an investor-option contract can be represented as a finite set $R \subset C$. After a contract R has been signed and the effort chosen, the investor selects a simple contract from R. The entrepreneur may then offer a new simple contract to supplant it. (The same results obtain if renegotiation instead occurs before the investor selects from R.)

Suppose the investor selects $r \in R$ after effort e is chosen. The entrepreneur's equilibrium renegotiation offer is then the wage contract that gives the investor the same payoff as would r, namely, $\sum g_i(e) r_i$. Foreseeing this, the investor selects r to

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[14] This is a special case of Lemma A4 in Appendix A.

maximize this expression. The resulting wage curve the entrepreneur faces is the lower envelope of the wage curves generated by the simple contracts in R :

$$w^*(R, e) \equiv \sum g_i(e)\pi_i - \max_{r \in R} \sum g_i(e)r_i$$

$$= \min_{r \in R} w^*(r, e).$$

An equilibrium outcome (e, w) of R maximizes $u(w, e)$ subject to $w = w^*(R, e)$.

The possible value of an investor-option contract can be seen in Figure 2.

[INSERT FIGURE 2 HERE]

Contract r^a leads to the low-effort outcome $\hat{a}$. But the investor-option contract $R = \{r^a, r^b\}$ yields the high-effort outcome a. Given R, the investor selects r^b if the entrepreneur chooses a low effort; as r^b then results in a low wage, the entrepreneur works hard so that the the investor will select r^a instead. Thus, packaging r^b with r^a results in a higher effort than either simple contract would alone.

However, an investor-option contract cannot improve on debt. The argument is basically the same as before. In Figure 1, replace r by R, so that the curve $w^*(r, \cdot)$ becomes $w^*(R, \cdot)$. Let (e^*, w^*) be the outcome of R, and δ be the debt contract satisfying $w^*(\delta, e^*) = w^*$. Our single-crossing property still implies that $w^*(\delta, \cdot)$ and $w^*(R, \cdot)$ can cross only at (e^*, w^*), and that $w^*(\delta, \cdot)$ is then the steeper of the two curves at this point. Hence, δ induces the entrepreneur to choose an effort, say e^δ, no less than e^*. If $e^\delta = e^*$, the outcome of δ is the same as that of R. If $e^\delta > e^*$, the entrepreneur must be better off with the debt contract (by revealed preference, as he could have chosen e^*), and the investor is also better off because her indifference curve through (e^*, w^*) is at least as steep as $w^*(\delta, \cdot)$. So δ Pareto dominates R, at least weakly.

Of course, an investor-option contract containing debt may achieve the same outcome as would the debt alone. A striking example is convertible debt. Let δ be debt,[15] with an equilibrium outcome (e, w). Consider the investor-option contract $R^\delta = \{\delta, r^w\}$, where r^w is the wage contract with wage w. This R^δ can be interpreted as convertible

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[15] Assume the face value of δ is less than π_n, so that $w^*(\delta, e)$ strictly increases in e.

debt, i.e., a security that executes the debt contract δ unless the investor exercises her option of "converting" it to r^w.[16] Since the two wage curves $w^*(\delta, \cdot)$ and $w^*(r^w, \cdot)$ intersect at (e, w), this outcome is on $w^*(R^\delta, \cdot)$. In addition, since (e, w) is the entrepreneur's optimal point on $w^*(\delta, \cdot)$, which is everywhere at least as high as the lower envelope $w^*(R^\delta, \cdot)$, (e, w) is also his optimal point on the latter curve. Thus, (e, w) is an equilibrium outcome of R^δ : the entrepreneur takes effort e, the investor then selects r^w, and it is not renegotiated. The convertible debt contract is in this sense the renegotiation-proof equivalent to the debt contract.[17]

4. The General Model

We now consider general "message game" contracts, in the general model in which both parties may be risk averse. We make no assumptions here about the distribution of bargaining power. Furthermore, the results of this section do not depend on our specific definition of a feasible simple contract: they hold for any feasible set $C \subset \mathbb{R}$ that is non-empty and compact, and leads to a downward sloping Pareto function $H(\cdot, e)$. The main result is that any pure strategy equilibrium outcome of a general contract is also an equilibrium outcome of an investor-option contract.

A *general contract (game form, mechanism)* is a function
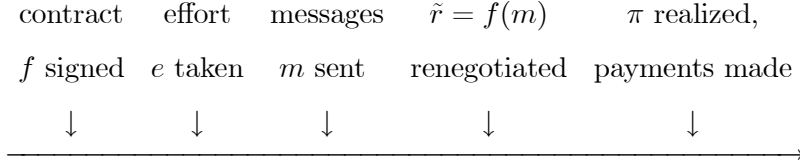
$$f : M_E \times M_I \to \Delta C,$$

where M_E and M_I are sets of messages that the entrepreneur and investor can respec-

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[16] One way R^δ differs from convertible debt is that r^w is not equity. This is due in part to the investor's assumed risk neutrality. If she too were risk averse, the relevant investor-option contract would be $\{\delta, r\}$, where is r is the efficient risk-sharing contract to which δ would be renegotiated. This r would be linear in output, i.e., equity, if both parties had CARA utility.

[17] Renegotiation occurs if the entrepreneur takes an effort $\hat{e} < e$. Since $w^*(\delta, \hat{e}) < w^*(r^w, \hat{e}) = w$, this effort choice causes the investor to select δ instead of r^w from R^δ. (It is this threat that in equilibrium deters the entrepreneur from taking efforts less than e.) As δ does not share risk efficiently, it would be renegotated to a wage contract (with a lower wage than w).

tively send, and ΔC is the space of probability distributions on C.[18] Let $M = M_E \times M_I$, and denote a message pair as $m = (m_E, m_I)$. When m is sent, the contract prescribes a random simple contract, $\tilde{r} = f(m) \in \Delta C$ that would, if it were not renegotiated, determine the entrepreneur's payment to the investor.

Bargaining and renegotiation occur according to the following time line:

contract	effort	messages	$\tilde{r} = f(m)$	π realized,
f signed	e taken	m sent	renegotiated	payments made
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$

Two features are noteworthy. First, renegotiation takes place *ex post,* after messages are sent. This is assumed only for simplicity. So long as the parties cannot commit to not renegotiate at this *ex post* date, our results still hold if renegotiation is also possible at the *interim* date that occurs after effort is chosen but before messages are sent. This is made clear below. Second, renegotiation occurs before the randomness in the mechanism's prescribed $\tilde{r}$ is realized. This is the same convention as in Segal and Whinston (2002), but differs from that in Maskin and Moore (1999).[19]

We let $\hat{V}(\tilde{r}, e)$ and $\hat{U}(\tilde{r}, e)$ denote the post-renegotiation payoffs of the investor and entrepreneur, respectively, when effort e has been taken and messages m have been sent, where $\tilde{r} = f(m)$. We assume that renegotiation is efficient,

$$\hat{U}(\tilde{r}, e) = H(\hat{V}(\tilde{r}, e), e) \text{ for all } (\tilde{r}, e) \in \Delta C \times E, \tag{4}$$

and that the post-renegotiation payoffs are continuous in the prescribed outcome:

$$\hat{V}(\cdot, \cdot) \text{ and } \hat{U}(\cdot, \cdot) \text{ are continuous on } \Delta C \times E. \tag{5}$$

The efficient renegotiation assumption (4) implies that $\tilde{r}$ is renegotiated to an efficient risk sharing contract; any randomness in $\tilde{r}$ has no efficiency consequence. The con-

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[18] Endow ΔC with the topology of weak convergence. It is compact, since C is compact in $\mathbb{R}^n$.

[19] In Maskin and Moore (1999), the parties can commit not to renegotiate during the time interval between the sending of messages and the realization of the contract's random outcome.

tinuity assumption (5) is weaker than the continuity and differentiability assumed in the quasilinear framework of Segal and Whinston (2002), and it holds fairly generally. It requires the bargaining powers of the parties in the renegotiation game not to shift discontinuously in $(\tilde{r}, e)$, the allocation that determines their disagreement payoffs.

Given a contract f, the *message game following effort* e is the game in which the strategies are messages, and the payoff functions are $\hat{U}(f(\cdot), e)$ and $\hat{V}(f(\cdot), e)$. This game is "strictly competitive", which means that the two players have opposing preferences on the set of message pairs. This is because renegotiation is efficient, and so any message profile results in a post-renegotiation payoff pair on the downward-sloping Pareto frontier for the given effort. In particular, since

$$\hat{U}(f(m), e) = H(\hat{V}(f(m), e), e),$$

the entrepreneur's best reply to any m_I minimizes the investor's payoff $\hat{V}(f(\cdot, m_I), e)$. Until Appendix B, we restrict attention to pure strategy equilibria of the message game.

Consider an equilibrium $m^*(e)$ of the message game. Denote the corresponding equilibrium payoffs as $V^*(e)$ and $U^*(e) = H(V^*(e), e)$. Because the message game is strictly competitive, $m^*(e)$ is also an equilibrium of the zero-sum game in which the investor's payoff is $\hat{V}(f(m), e)$ and the entrepreneur's is $-\hat{V}(f(m), e)$. (This is not true of mixed strategy equilibria, as we discuss below.) Therefore, by a standard "maxmin" argument,

$$V^*(e) = \sup_{m_I} \inf_{m_E} \hat{V}(f(m_I, m_E), e).^{20} \tag{6}$$

A *(subgame perfect) equilibrium of (the game generated by) contract* f is a pair $(e^*, m^*(\cdot))$, where e^* is an effort that maximizes the entrepreneur's equilibrium continuation payoff in the message game:[21]

$$e^* \in \arg\max_{e \in E} U^*(e). \tag{7}$$

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[20] Since $m^*(e)$ is an equilibrium, the "sup" in (6) can be replaced by "max".

[21] Given our goal of characterizing the best equilibria, our focus on equilibria in which the entrepreneur uses a pure effort strategy is without loss of generality. Suppose an equilibrium of f is $(\sigma, m^*(\cdot))$, where σ is a mixed effort strategy with compact support. The continuation equilibrium payoffs are $U^*(e)$ and $V^*(e)$ for any e. Let e^* maximize $V^*(\cdot)$ on the support of σ. Then, $(e^*, m^*(\cdot))$ is another equilibrium,

We now prove that an investor-option contract performs as well as any general contract. For a quasilinear model the result is Proposition 9 in Segal and Whinston (2002). The heuristic argument is the following. Consider an equilibrium $(e^*, m^*(\cdot))$ of a contract f. Define an investor-option contract $f^I : M_I \to \Delta C$ by holding the entrepreneur's message fixed at $m_E^*(e^*)$:

$$f^I(m_I) = f(m_E^*(e^*), m_I).$$

Given this option contract, after any effort the investor can obtain a payoff at least as large as she would get from the equilibrium of the message game determined by f. This is because, as we discussed above, the entrepreneur chooses a message to minimize the investor's payoff when the contract is f. But f^I does not allow him to choose a message to harm the investor in this way. Hence, if f^I generates equilibrium payoffs $V^I(e)$ and $U^I(e)$, we have $V^I(\cdot) \geq V^*(\cdot)$, with equality at e^* because $m_I^*(e^*)$ is a best reply to $m_E^*(e^*)$. Efficient renegotiation then implies $U^I(\cdot) \leq U^*(\cdot)$, with equality at e^*. Thus, since it maximizes $U^*(\cdot)$, e^* indeed maximizes $U^I(\cdot)$.

The unwarranted assumption in this heuristic proof is that f^I has an equilibrium. A correct proof is given in Appendix A.

Proposition 1. *Given any equilibrium of any contract, an investor-option contract exists that has the same equilibrium payoffs and effort.*

Proposition 1 also holds if the parties can renegotiate at the interim stage, after effort is chosen but before messages are sent, so long as they can also renegotiate ex post. This is because the Proposition refers to equilibria that are in pure strategies, and so yield continuation payoffs $(V^*(e), U^*(e))$ on the Pareto frontier given the chosen e. Knowing that these payoffs will obtain when the contract is not renegotiated, every interim renegotiation proposal by one party will be rejected by the other. Whether the parties can commit not to renegotiate at the interim date is thus irrelevant.

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with a pure effort strategy, since the entrepreneur is indifferent between all effort levels in the support of σ. And $(e^*, m^*(\cdot))$ weakly Pareto dominates $(\sigma, m^*(\cdot))$, since the entrepreneur is indifferent between them, and $V^*(e^*) \geq V^*(e)$ for all e in the support of σ.

5. Entrepreneur-Offer Renegotiation

We now show in the general model that if the entrepreneur has all the bargaining power in the renegotiation stage, then any general contract is weakly Pareto dominated by debt. Furthermore, a debt contract is a limit point of the set of simple contracts prescribed by any Pareto optimal general contract as its messages vary; only such generalized convertible debt contracts are optimal.

Since the entrepreneur has the bargaining power, the investor receives the same payoff regardless of whether she agrees to renegotiate. Thus, after an effort e is taken and a message pair m is sent, renegotiation of the prescribed $\tilde{r} = f(m)$ yields an efficient risk-sharing contract for e that gives the investor the same payoff as does $\tilde{r}$. Her post-renegotiation payoff is

$$\hat{V}(\tilde{r}, e) = V(\tilde{r}, e) = \mathcal{E}_{\tilde{r}} \left\{ \sum g_i(e) v(\tilde{r}_i) \right\}, \tag{8}$$

and the entrepreneur's is

$$\hat{U}(\tilde{r}, e) = H(V(\tilde{r}, e), e). \tag{9}$$

The two assumptions made in Section 4 are satisfied: renegotiation is efficient, and the post-renegotiation payoffs are continuous in $\tilde{r}$.

We first dispense with random contracts. The investor's certainty equivalent for $\tilde{r} \in \Delta C$ is the $r^c \in \mathbb{R}^n$ defined by $v(r_i^c) \equiv \mathcal{E}_{\tilde{r}} v(\tilde{r}_i)$. Since $V(r^c, \cdot) = V(\tilde{r}, \cdot)$, we see from (8) and (9) that for any effort, r^c and $\tilde{r}$ yield the same post-renegotiation payoffs. Thus, for any contract f, an equivalent deterministic contract $\bar{f}$ is defined by letting $\bar{f}(m)$ be the investor's certainty equivalent for $f(m)$. The contracts f and $\bar{f}$ have the same equilibrium efforts and payoffs. Since the certainty equivalent of any $\tilde{r} \in \Delta C$ is in C,[22] we have proved the following.

Lemma 1. *The equilibrium efforts and payoffs of any contract $f : M \to \Delta C$ are the same as those of a contract $\bar{f} : M \to C$ defined by letting $\bar{f}(m)$ be the investor's certainty equivalent for $f(m)$.*

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[22] In particular, r^{ci} satisfies MI because $v(r_{i+1}^{ci}) - v(r_i^{ci}) = \mathcal{E}_{\tilde{r}} \left[v(\tilde{r}_{i+1}) - v(\tilde{r}_i) \right] \geq 0$, since any realization of $\tilde{r}$ satisfies MI because it is in C.

In light of Proposition 1 and Lemma 1, we can restrict attention to deterministic investor-option contracts. The revelation principle allows us to further restrict attention to revelation mechanisms for the investor, $r^* : E \to C$, that are incentive compatible. Given such an r^*, its truthful equilibrium yields post-renegotiation payoffs

$$V^*(e) = V(r^*(e), e) \text{ and } U^*(e) = H(V^*(e), e). \tag{10}$$

Any maximizer of $U^*(\cdot)$ is an equilibrium effort.

It is now easy to see that when the entrepreneur has the bargaining power, an equilibrium of a riskless debt contract is efficient. Suppose that for all possible reports, $r^*(\cdot)$ specifies a riskless debt contract, $\delta^D \equiv (D, \dots, D)$. By (10), the investor's post-renegotiation payoff is then $V(\delta^D, e) = v(D)$, which is independent of e. The equilibrium effort maximizes $U^*(\cdot) = H(v(D), \cdot)$, and is hence the effort component of the efficient allocation that gives the investor payoff $v(D)$. This efficient allocation is the equilibrium outcome, since renegotiation is efficient and does not benefit the investor.

Of course, as we observed in Section 3, a riskless debt contract that is acceptable to the investor may not be feasible. We accordingly turn to debt contracts that may be risky. The following lemma establishes a single-crossing property which will imply that debt provides the greatest incentives of all contracts in C.

Lemma 2. *For any $(r, e) \in C \times E$ such that r is not debt, a unique debt contract $\delta \in C$ exists for which $V(r, e) = V(\delta, e)$. Furthermore,*

(i) $V_e(r, e) > V_e(\delta, e)$, *and*

(ii) $(e - \hat{e})(V(r, \hat{e}) - V(\delta, \hat{e})) < 0$ *for all $\hat{e} \neq e$.*

We now prove the first main result of this section: any equilibrium of a general contract is weakly Pareto dominated by an equilibrium of a debt contract. Again considering the investor-option incentive-compatible revelation mechanism $r^*(\cdot)$ and its equilibrium effort e^*, the desired debt contract is defined by

$$V(\delta, e^*) = V(r^*(e^*), e^*). \tag{11}$$

If δ is adopted and effort e taken, the equilibrium post-renegotiation payoffs are

$$V^\delta(e) = V(\delta, e) \text{ and } U^\delta(e) = H(V(\delta, e), e). \tag{12}$$

It follows from $(10) - (12)$ that when δ is adopted, e^* yields the same payoffs as it does when $r^*(\cdot)$ is adopted:

$$V^\delta(e^*) = V^*(e^*) \text{ and } U^\delta(e^*) = U^*(e^*). \tag{13}$$

The entrepreneur therefore weakly prefers any equilibrium of δ to the given one of $r^*(\cdot)$, since any equilibrium effort of δ maximizes $U^\delta(\cdot)$. The investor has the same preference, provided that the equilibrium effort of δ, say e^δ, is not less than e^*. This is because

$$V^\delta(e^\delta) = V(\delta, e^\delta) \geq V(\delta, e^*) = V^*(e^*),$$

where the inequality follows from the monotonicity of δ, MLRP, and $e^\delta \geq e^*$. The proof is complete once $e^\delta \geq e^*$ is proved; this is done in Appendix A using Lemma 2.

Proposition 2. *Assume entrepreneur-offer renegotiation. Then, given any equilibrium of any general contract, a debt contract exists that has an equilibrium with a weakly greater effort, and which both parties weakly prefer.*

Proposition 2 leaves open the possibility that a contract quite unlike debt has an equilibrium with a Pareto optimal outcome. The following proposition shows this is not true. It shows that in an equilibrium of any optimal general contract, the equilibrium messages following any effort prescribe a simple contract that converges to either a debt contract, or to a probability distribution over riskless debt contracts, as the effort converges to the equilibrium effort from below. Any optimal investor-option contract is, in this sense, a generalized convertible debt contract. One implication is that if the contract specifies only a finite number of simple contracts, in equilibrium it must prescribe a debt contract following the choice of any effort in some interval that has the equilibrium effort as its upper endpoint. If the contract is simple, it must be debt.

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Proposition 3. *Assume entrepreneur-offer renegotiation. Suppose an equilibrium, $(e^*, m^*(\cdot))$, of a general contract f is not Pareto dominated by an equilibrium of a debt contract, and $e^* \in int(E)$. Then the left hand limit,*

$$\tilde{r}^* = \lim_{e \to e^*-} f(m^*(e)),$$

exists in ΔC, and it puts all probability either on a debt contract δ, or on a set of riskless debt contracts.

6. Investor Bargaining Power

In this section we suppose the investor has some bargaining power in the renegotiation. As we shall see, in this case the riskiness of the initial contract matters for incentives. This is most starkly true when the investor has *all* the bargaining power, so that the entrepreneur does not gain at all from the renegotiation to an efficient risk-sharing contract. His choice of effort is then dictated entirely by the direct consequences of the initial contract for himself, including its riskiness.

A debt contract is very risky for the entrepreneur: it gives him no income if output is below its face value, and it gives him the entire residual above the face value if output is high. He has therefore a large incentive to lower this risk by taking a high effort, thereby decreasing the probability of low outputs and increasing that of high outputs (by the MLRP). But efficiency would require the risk properties of the initial contract to be ignored. It is thus possible for debt to lead to excessive effort relative to an efficient allocation. We provide such an example in Appendix B. The example also suggests an upcoming result, namely, that an efficient allocation can be achieved, by a simple non-debt contract, when debt leads to excessive effort.

Debt may induce excessive effort because it allows the entrepreneur to improve his pre-renegotiation payoff by more than it raises total surplus.[23] The entrepreneur's incentive to provide effort is too high because by raising his effort, he reduces risks that

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[23] This is reminiscent of the over-investment result in the hold-up literature when a party's investment raises his own 'default option', as in, e.g., Aghion et al (1994).

are not socially costly, since they are subsequently removed by renegotiating with the risk-neutral investor. This excessive effort result may seem at odds with the conventional wisdom that external funding reduces managerial effort because the manager ignores its positive externality on investors – the "outside equity" and "debt overhang" problems that the corporate finance literature has dwelt upon since Jensen and Meckling (1976) and Myers (1977). Here, however, because higher effort reduces the risk of debt to the entrepreneur, it imposes a *negative* externality on the investor. It is not debt *per se* that causes excessive effort, but the fact that the entrepreneur behaves as if he were not insured: even if he did not need external funding if he had no access to insurance debt could still cause him to work too hard to reduce the probability of zero income.

We now examine more generally the nature of optimal contracts when the investor has bargaining power. We first consider the case in which she has all the bargaining power, and then turn to the case in which the parties share the bargaining power. Because the investor has bargaining power, it is now convenient to let $J(\cdot, e) \equiv H^{-1}(\cdot, e)$, so that $V = J(U, e)$ describes the Pareto frontier given e.[24]

6.1. Investor Has All Bargaining Power

Assuming the investor has all the renegotiation bargaining power, we now give conditions under which two results hold: (i) debt maximizes the entrepreneur's incentives to provide effort; and (ii) either debt is optimal or, as in the example above, an efficient allocation is obtainable by another simple contract.

The first new condition, often made so that the entrepreneur's risk attitude does not depend on effort, is that his utility function be separable:

$$\text{(SEP)} \quad u(w, e) = a(e)\bar{u}(w) - c(e),$$

where $a(\cdot) > 0$. We now have another single-crossing result for debt, like Lemma 2.

Lemma 3. *For any $(r, e) \in C \times E$ such that r is not debt, a unique debt contract $\delta \in C$ exists for which $U(\delta, e) = U(r, e)$. If* SEP *holds, then*

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[24] As $H(\cdot, e)$ is continuous and decreasing, J is well-defined and has the same properties as H.

(i) $U_e(\delta, e) > U_e(r, e)$, and

(ii) $(e - \hat{e})\left(U(\delta, \hat{e}) - U(r, \hat{e})\right) < 0$ for all $\hat{e} \neq e$.

Our second restriction is to deterministic contracts, $f : M_E \times M_I \to C$, that assign to each message pair m a non-random simple contract.[25] When such a contract specifies $r \in C$ after messages have been sent, the post-renegotiation payoffs are

$$\hat{U}(r, e) = U(r, e) \text{ and } \hat{V}(r, e) = J(U(r, e), e),$$

since the investor has the bargaining power. The following proposition establishes that debt again maximizes incentives.

Proposition 4. *Assume investor-offer renegotiation and SEP. Then, given any equilibrium of any deterministic general contract, a debt contract exists that has an equilibrium with a weakly greater effort, and it gives the same payoff to the entrepreneur.*

We now give two conditions under which debt is optimal if and only if an efficient allocation is unobtainable. The argument is roughly the following. The effort component of an efficient allocation that gives the entrepreneur some utility U^* maximizes $V = J(U^*, \cdot)$. Suppose $J(U^*, \cdot)$ is strictly concave (we weaken this below). Then $J(U^*, \cdot)$ is maximized by a unique effort, say e^F, and it increases with effort to the left of e^F. Suppose no feasible contract which gives the entrepreneur utility U^* results in an efficient allocation. Then, since the minimal effort $\underline{e} < e^F$ can always be induced (by a wage contract), a continuity argument shows that every contract which yields the entrepreneur utility U^* induces him to take an effort less than e^F. Hence, as $J(U^*, \cdot)$ is concave, it is maximized by adopting a contract that induces the largest possible effort. By Proposition 4, this contract is debt.

The first of the two conditions is that $J(\hat{U}, \cdot)$ is pseudoconcave ('single peaked'):

(SP) For any feasible U^*, maximizer e^{**} of $J(U^*, \cdot)$, and

$$e \in E: \ (e - e^{**})J_2(U^*, e) \leq 0.$$

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[25]Random contracts when the investor has the bargaining power are problematic. They cannot be eliminated by appeal to the entrepreneur's certainty equivalent contract, as we did in Lemma 1 with respect to the investor, because it can violate MI. See Lemma 6 in Matthews (2001).

The second is that the "first-order approach" is valid:[26]

(FOA) For any $r \in C$, the effort maximizing $U(r, \cdot)$ is unique.

Proposition 5. [27] *Assume investor-offer renegotiation, SEP, FOA, and SP. Suppose an equilibrium of a deterministic general contract is not Pareto dominated by an equilibrium of any other deterministic contract, and it gives the entrepreneur payoff $U^* \leq u(\pi_1 - \underline{r}, \underline{e})$. Then, either* (i) *this equilibrium achieves an efficient allocation, which is also attained by a simple contract, or* (ii) *a debt contract achieves the same payoffs.*

6.2. Intermediate Bargaining Powers

A new complication arises if both parties have bargaining power. When one has all the bargaining power, the entrepreneur's equilibrium payoff is a function of only one of the payoffs the parties would receive if renegotiation did not occur. That is, if the initial contract yields a simple contract r when effort e is taken, the entrepreneur's payoff depends on only one of the status quo payoffs, $U_0 = U(r, e)$ or $V_0 = V(r, e)$, of the ensuing bargaining game. (His payoff is $H(V_0, e)$ if he has the bargaining power, and U_0 if the investor has the bargaining power.) But if they both have bargaining power, his ultimate payoff depends on both status quo payoffs. The allocation of risk between the parties determined by the initial contract is thus of importance for incentives, despite the fact that it will be renegotiated to an efficient risk-sharing contract.

Nonetheless, there is still some reason to expect debt to maximize incentives. General bargaining solutions give the entrepreneur a payoff that increases in his status quo payoff U_0, and decreases in the investor's V_0. An increase in effort changes these status quo payoffs. The single-crossing properties of Lemmas 2 and 3 suggest that the increase in effort will increase U_0 the most, and simultaneously increase V_0 the least, when the

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[26] Various properties of the primitives imply FOA. See, e.g., Rogerson (1985) and Jewitt (1988).

[27] Proposition 5 applies only to equilibria that give the entrepreneur utility no more than $u(\pi_1 - \underline{r}, \underline{e})$, where $\underline{r}$ is the investor's liability bound. The lower is $\underline{r}$, the more equilibria satisfy this inequality; it holds vacuously if $\underline{r} = -\infty$.

initial contract is debt. If so, the entrepreneur's incentive to raise effort so as to put himself in a good bargaining position is maximized by debt.

The difficulty with this argument is that given a debt contract δ, the single-crossing properties say something about the relative slopes of $U(r, \cdot)$ and $U(\delta, \cdot)$ only at an effort where they are equal, and similarly for $V(r, \cdot)$ and $V(\delta, \cdot)$. There may not be an effort at which both equalities hold. (Previously, when only one party had bargaining power, we needed only one of these equalities to hold.)

However, under two further assumptions we can reduce these two equalities to one and apply our previous arguments. The first new assumption is a stronger, but still standard, separability condition:

$$(\text{SEP}') \quad u(w, e) = \bar{u}(w) - c(e).$$

The second is that the bargaining outcome is given by a simple bargaining solution we call *ray bargaining.* According to this solution, the investor's bargaining power is measured by a parameter $\theta \geq 0$, where $\theta = 0$ ($\theta = \infty$) is the case in which the investor has none (all) of the bargaining power. The solution specifies that if the initial contract, after an effort e has been taken and messages sent, yields status quo utilities (U_0, V_0), then renegotiation yields the utility pair $(\hat{U}, \hat{V})$ on the Pareto frontier $\hat{V} = J(\hat{U}, e)$ where it intersects the ray that emanates from (U_0, V_0) with slope θ.[28] The post-renegotiation utilities thus satisfy

$$\hat{V} - V_0 = \theta \left(\hat{U} - U_0 \right). \tag{14}$$

We now proceed by inserting $\hat{V} = J(\hat{U}, e)$ and SEP′ into (14) to obtain

$$J(\hat{U}, e) - \sum g_i(e) v(r_i) = \theta \left[\hat{U} - \sum g_i(e) \bar{u}(\pi_i - r_i) + c(e) \right].$$

Rearrangement of this yields

$$K(\hat{U}, e) = \sum g_i(e) w_i(r_i), \tag{15}$$

[28]Ray renegotiation is generalized Nash bargaining (see, e.g., Myerson 1991, p.390) if the Pareto frontier has slope -1, i.e., if utility is transferable. It is not otherwise, as generalized Nash bargaining would require γ to be a function of the status quo utilities and effort.

where the functions K and w_i are defined by

$$K(\hat{U}, e) \equiv \theta \left(\hat{U} + c(e) \right) - J(\hat{U}, e),$$

$$w_i(y) \equiv \theta \bar{u}(\pi_i - y) - v(y).$$

Since $K(\cdot, e)$ is increasing, its inverse $K^{-1}(\cdot, e)$ is well defined. Solving (15) for $\hat{U}$ yields an expression for the entrepreneur's post-renegotiation payoff in terms of r and e :

$$\hat{U}(r, e) = K^{-1} \left(\sum g_i(e) w_i(r_i), \ e \right).$$

The entrepreneur's post-renegotiation payoff is thus a function of the payoffs both parties would receive if r were not renegotiated. But these two status quo payoffs are combined into a single expected utility-like term, $\sum g_i(e) w_i(r_i)$. We can establish the single-crossing property of debt for it, and so again show that debt maximizes incentives.

Lemma 4. *For any $(r, e) \in C \times E$ such that r is not debt, a unique debt contract $\delta \in C$ exists for which*

$$\sum g_i(e) w_i(\delta_i) = \sum g_i(e) w_i(r_i). \tag{17}$$

Furthermore,

(i) $\sum g_i'(e) w_i(\delta_i) > \sum g_i'(e) w_i(r_i)$, *and*

(ii) $(e - \hat{e}) \left(\sum g_i(\hat{e}) w_i(\delta_i) - \sum g_i(\hat{e}) w_i(r_i) \right) < 0$ *for all $\hat{e} \neq e$.*

Proposition 6. *Assume ray renegotiation and SEP'. Then, given any equilibrium of any deterministic general contract, a debt contract exists that has an equilibrium with a weakly greater effort, and it gives the same payoff to the entrepreneur.*

As before, the debt contract of Proposition 6 may induce the entrepreneur to take such a high effort that it makes the investor worse off. If it does not generate excessive effort in this sense, the debt contract Pareto dominates the given general contract.

7. Conclusions

We have analyzed a dynamic entrepreneurial incentive problem with: (i) observable but nonverifiable effort; (ii) renegotiation between the effort choice and the output realization; and (iii) two contractual constraints: limited entrepreneurial liability, and monotonicity of the investor's return with the firm's performance. Fairly generally, when the entrepreneur holds the renegotiation bargaining power, debt and, equivalently, convertible debt, are optimal contracts. Their optimality stems from their inducing maximal effort from the entrepreneur, as in Innes (1990). Unlike in Innes' setup, the parties here are risk averse. Debt is nonetheless optimal because renegotiation allows a separation of effort from insurance provision. This result is similar to that obtained in Matthews (2001), for the case in which entrepreneurial effort is privately observed.

If one accepts the underlying assumptions, these results provide a simple theory of capital structure dynamics: debt is adopted initially in order to generate maximum effort from the entrepreneur. Once effort is chosen, the parties switch to optimal insurance. The investor takes on more risk by transforming her claim into something closer to equity. As is well known, standard equity shares risk optimally if both parties have CARA preferences. The model then delivers a simple prediction: the firm starts as an all-debt firm, and later becomes an all-equity firm by, for example, going public.

One might object that the model requires the investor to be risk averse for the entrepreneur to hold equity after renegotiation, since otherwise he is given a fixed-wage contract. However, in our view it is not implausible for the investor to be risk averse, especially if she is a specialized venture capitalist. Alternatively, though outside the model, a post-renegotiation compensation for the entrepreneur that varies with output could be due to a need to give him incentives for subsequent effort provision. More generally, the model's simplicity allows it to serve as a theoretical benchmark that may, for example, be of use for evaluating the more specific features of venture capital mentioned in the introduction.

We end by discussing three situations in which debt is not optimal. The first is when the investor has renegotiation bargaining power, considered in Section 6 and

Appendix B. In this case the entrepreneur cares about the riskiness of the initial contract. Since debt gives him no income if output is low, it may cause him to take excessive effort. Under certain assumptions, Proposition 5 shows that when the investor has the bargaining power, either some simple contract induces a large enough effort that an efficient allocation is achievable – in which case effort's non-contractibility does not lower welfare – or debt remains optimal.

Second, we also show in Appendix B that debt may be suboptimal if a third party can join the contract. The three-person contract we consider induces the investor to give money to the third party instead of the entrepreneur if the latter takes too low an effort. It illustrates the general principle that unverifiability is not a binding constraint when a third-party 'budget breaker' is available to obviate renegotiation constraints. We note, however, that third parties can be used in this way only if somehow collusion between the entrepreneur and either the investor or the third party can be prevented.

Third, the final example in Appendix B shows that mixed strategy equilibria of two-sided message games may also outperform debt. The underlying reason is the following.[29] When at least one party is risk averse, the Pareto frontier given by $H(\cdot, e)$ is strictly concave. A mixed strategy equilibrium of the message game (with subsequent renegotiation) that is played after the choice of e may then generate a convex combination of frontier payoffs that itself lies below the frontier. In this way, both players can be punished at the same time. This alleviates the central difficulty of the implementation with renegotiation problem, which is how to punish one player for deviating without rewarding the other player so much that he deviates.

Our instinctive response to this is that contracts which require the parties to simultaneously and randomly send messages do not seem very realistic. We conjecture in Appendix B that perhaps such schemes are ruled out if the parties are able to engage in pre-play espionage or interference, or if the courts are unable to verify messages that the parties wish ex post to rescind. We welcome further research that sheds light on why such contracts do not emerge in the real world.

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[29]Maskin and Moore (1999) and Maskin and Tirole (1999) expound on this logic.

A. Appendix A: Proofs Missing from the Text

Lemma A1. *For all $e \in E$, $H(\cdot, e)$ is continuous and concave on $[v(\underline{r}), V(\pi, e)]$, its domain.[30] For $\hat{V} \in (v(\underline{r}), V(\pi, e))$, $H_1(\hat{V}, e)$ exists and is negative. If $e \in int(E)$, then $H(\cdot, \cdot)$ is differentiable and has continuous partial derivatives at $(\hat{V}, e)$.*

Proof. Fix $e \in E$ and $\hat{V} \in [v(\underline{r}), V(\pi, e)]$. Then some convex combination of $(\underline{r}, \ldots, \underline{r})$ and π, say r, satisfies $\hat{V}(r, e) = \hat{V}$. Since C is convex and contains π and $\underline{r}$, $r \in C$. So the constraint set of (1) is nonempty. As it is also compact, and the objective is continuous, the program has a solution. As stated in the text, its solution is unique; denote it as r^*. We thus see that $H(\cdot, e)$ is well-defined on $[v(\underline{r}), V(\pi, e)]$. It is continuous on this interval by the maximum theorem. It is concave on this interval by a direct argument using Jensen's inequality, the concavity of $U(\cdot, e)$ and $V(\cdot, e)$, and the convexity of C.

Now let $\hat{V} \in (v(\underline{r}), V(\pi, e))$. Then some convex combination of $(\underline{r}, \ldots, \underline{r})$ and π, say r, satisfies $\hat{V}(r, e) > \hat{V}$. So Slater's condition holds. Thus, $\lambda^* \geq 0$ exists such that (r^*, λ^*) is a saddle point in $C \times \mathbb{R}_+$ of the Lagrangian

$$\mathcal{L}(r, \lambda, e, \hat{V}) \equiv U(r, e) + \lambda \left[V(r, e) - \hat{V} \right].$$

We claim (r^*, λ^*) is the only saddle point of $\mathcal{L}(\cdot, \cdot, e, \hat{V})$. Since r^* is the unique solution of (1), any other saddle point takes the form (r^*, λ), with $\lambda \neq \lambda^*$. The following argument shows that λ^* is determined by r^*, and so (r^*, λ^*) is unique.

Note first that $\lambda^* > 0$. If $\lambda^* = 0$, the saddle point property would imply that r^* maximizes $U(\cdot, e) = \mathcal{L}(\cdot, 0, e, \hat{V})$ on C. But then $r^* = \underline{r}$, contrary to $\hat{V} > v(\underline{r})$.

Instead of maximizing $\mathcal{L}(\cdot, \lambda^*, e, \hat{V})$ on C, consider the relaxed problem obtained by deleting MI. A solution r to this relaxed program satisfies the Kuhn-Tucker condition

$$-u_1(\pi_i - r_i, e) + \lambda^* v'(r_i) = (\beta_i - \alpha_i) / g_i(e) \tag{A1}$$

for each $i = 1, \ldots, n$, where $\alpha_i \geq 0$ and $\beta_i \geq 0$ are the multipliers for (LI) $r_i \geq \underline{r}$ and (LE) $r_i \leq \pi_i$, respectively. If $r_i > r_{i+1}$ for some $i < n$, then $r_i > \underline{r}$ and $r_{i+1} < \pi_i$, and

---

[30] No feasible investor payoff is less than $v(\underline{r})$, or greater than $V(\pi, e) \equiv \sum g_i(e) v(\pi_i)$.

in turn $\alpha_i = 0$ and $\beta_{i+1} = 0$ by complementary slackness. Hence, (A1) would imply

$$\beta_i/g_i(e) = -u_1(\pi_i - r_i, e) + \lambda^* v'(r_i)$$

$$< -u_1(\pi_{i+1} - r_{i+1}, e) + \lambda^* v'(r_{i+1}) = -\alpha_{i+1}/g_{i+1}(e),$$

where the inequality follows from $u_{11} \leq 0$ and $v'' \leq 0$, with one strict, $\lambda^* > 0$, $r_i > r_{i+1}$, and $\pi_i - r_i < \pi_{i+1} - r_{i+1}$. But this is contrary to $\alpha_{i+1} \geq 0$ and $\beta_i \geq 0$. We conclude that any solution of the relaxed problem satisfies the neglected constraint MI. Hence, the solution of the relaxed problem is the unique solution r^* of the unrelaxed problem. So r^* satisfies (A1). Now, suppose there is no $i \leq n$ such that $\underline{r} < r_i^* < \pi_i$. Then by MI and $\hat{V} \in (v(\underline{r}), V(\pi, e))$, $1 \leq k < n$ exists such that $r_k^* = \underline{r}$ and $r_{k+1}^* = \pi_{k+1}$. Hence, $\beta_k = 0$ and $\alpha_{k+1} = 0$, and (A1) implies

$$-\alpha_k/g_k(e) = -u_1(\pi_k - \underline{r}, e) + \lambda^* v'(\underline{r})$$

$$> -u_1(0, e) + \lambda^* v'(\pi_{k+1}) = \beta_{k+1}/g_{k+1}(e).$$

This is contrary to $\alpha_k \geq 0$ and $\beta_{k+1} \geq 0$. We conclude that $\underline{r} < r_i^* < \pi_i$ for some $i \leq n$. For this i we have $\alpha_i = \beta_i = 0$, and (A1) implies

$$\lambda^* = \frac{u_1(\pi_i - r_i^*, e)}{v'(r_i^*).} \tag{A2}$$

This proves that (r^*, λ^*) is the unique saddle point of $\mathcal{L}(\cdot, \cdot, e, \hat{V})$.

This uniqueness implies that a general envelope theorem, Corollary 5 of Milgrom and Segal (2002), now applies. The derivative $H_1(\hat{V}, e)$ therefore exists, with $H_1(\hat{V}, e) = -\lambda^* < 0$. If also $e \in int(E)$, then $H_2(\hat{V}, e)$ exists and is given by

$$H_2(\hat{V}, e) = \mathcal{L}_e(r^*, \lambda^*, e, \hat{V}) = U_e(r^*, e) + \lambda^* V_e(r, e).$$

Now, since the solution r^* of (1) is unique, Berge's maximum theorem implies that it is a continuous function of $(\hat{V}, e)$. In turn, (A2) implies that λ^* is a continuous function of $(\hat{V}, e)$. Thus, both H_1 and H_2 are continuous at any interior point, i.e., at any $(\hat{V}, e)$ satisfying $e \in int(E)$ and $\hat{V} \in (v(\underline{r}), V(\pi, e))$. So H is differentiable at such points. ∎

Proof of Proposition 1. Let f be a contract with an equilibrium $(e^*, m^*(\cdot))$. Simplify notation by denoting $m_E^*(e^*)$ as m_E^*. For all $e \in E$, define

$$V^I(e) \equiv \sup_{m_I \in M_I} \hat{V}(f(m_E^*, m_I), e). \tag{A3}$$

Let $\{m_I^k(e)\}_{k=1}^\infty$ be a sequence in M_I such that $\hat{V}(\tilde{r}^k(e), e) \to V^I(e)$ as $k \to \infty$, where $\tilde{r}^k(e) = f(m_E^*, m_I^k(e))$. Since ΔC is compact, there is a subsequence, which for simplicity we take to be $\{\tilde{r}^k(e)\}$ itself, that converges to some $\tilde{r}(e) \in \Delta C$. The continuity of $\hat{V}(\cdot, e)$ implies

$$V^I(e) = \hat{V}(\tilde{r}(e), e). \tag{A4}$$

For any $e' \neq e$, (A3) implies

$$V^I(e) \geq \hat{V}(f(m_E^*, m_I^k(e')), e) = \hat{V}(\tilde{r}^k(e'), e).$$

Taking limits, $\tilde{r}^k(e') \to \tilde{r}(e')$ and the continuity of $\hat{V}(\cdot, e)$ imply

$$V^I(e) \geq \hat{V}(\tilde{r}(e'), e). \tag{A5}$$

By (A4) and (A5), $\tilde{r}(\cdot) : E \to \Delta C$ is an incentive compatible revelation mechanism for the investor. Thus, $f^I(\cdot) \equiv \tilde{r}(\cdot)$ defines an investor-option contract with message set E, and an equilibrium of it following any $e \in E$ is given by the identity function, $\iota(e) \equiv e$. We now show that $(e^*, \iota(\cdot))$ is an equilibrium of f^I.

Given f^I and $e \in E$, the equilibrium $\iota(e)$ gives the investor payoff $V^I(e)$, and it gives the entrepreneur payoff $U^I(e) \equiv H(V^I(e), e)$. From (A3),

$$V^I(e) \equiv \sup_{m_I} \hat{V}(f(m_E^*, m_I), e)$$

$$\geq \sup_{m_I} \inf_{m_E} \hat{V}(f(m_E, m_I), e) = V^*(e),$$

using (6). Hence, (4) and the presumption that each $H(\cdot, e)$ is a decreasing function imply that $U^I(e) \leq U^*(e)$ for all $e \in E$, with equality at $e = e^*$ because $V^I(e^*) = V^*(e^*)$. Thus, e^* maximizes $U^I(\cdot)$ because it maximizes $U^*(\cdot)$. This proves that $(e^*, \iota(\cdot))$ is an equilibrium of f^I. The equality of the equilibrium payoffs, $(V^I(e^*), U^I(e^*)) = (V^*(e^*), U^*(e^*))$, is obvious. ∎

The following lemma is used to prove Lemmas 2 and 3.

Lemma A2. *For any $(r, e) \in C \times E$ and nonnegative constants α and β, not both zero, a unique debt contract $\delta \in C$ exists such that*

$$\alpha U(\delta, e) - \beta V(\delta, e) = \alpha U(r, e) - \beta V(r, e). \tag{A6}$$

Proof. By LI, the RHS of (A6) is not more than

$$\bar{w} \equiv \alpha \sum g_i(e) u(\pi_i - \underline{r}, e) - \beta v(\underline{r}).$$

By LE, the RHS of (A6) is not less than

$$\underline{w} \equiv \alpha u(0, e) - \beta \sum g_i(e) v(\pi_i).$$

For any $D \in [\underline{r}, \pi_n]$ define $W(D) \equiv \alpha U(\delta(D), e) - \beta V(\delta(D), e)$, where $\delta(D)$ is the debt contract with face value D. This $W(\cdot)$ is continuous and, since α and β are not both zero, strictly decreasing on $[\underline{r}, \pi_n]$. Observe that $W(\pi_n) = \underline{w}$ and, as $\underline{r} < \pi_1$, $W(\underline{r}) = \bar{w}$. A unique $D \in [\underline{r}, \pi_n]$ thus exists for which $\delta(D)$ satisfies (A6). ∎

The following lemmas establish the single-crossing property of debt. Define $x \in \mathbb{R}^n$ to be *quasi-monotone* if and only if $k \in \{1, \ldots, n\}$ exists such that $x_i \leq 0$ for $i < k$ and $x_i \geq 0$ for $i > k$ (Karamardian and Schaible, 1990). Equivalently, x is quasi-monotone if and only if $x_i > 0$ implies $x_j \geq 0$ for all $j > i$. The crucial property of a quasi-monotone vector is that by MLRP, its expectation is a quasi-monotone function of effort:[31]

Lemma A3. *For any $(x, e) \in \mathbb{R}^n \times E$, if $x \neq 0$ is quasi-monotone and $\sum g_i(e) x_i = 0$, then (i) $\sum g_i'(e) x_i > 0$, and (ii) $(e - \hat{e}) \sum g_i(\hat{e}) x_i < 0$ for all $\hat{e} \neq e$.*

Proof. Routine calculus proves (ii) if (i) holds for all e such that $\sum g_i(e) x_i = 0$. To prove (i), note that

$$
\begin{aligned}
\sum g_i'(e) x_i &= \sum \left(\frac{g_i'(e)}{g_i(e)} \right) g_i(e) x_i \\
&> \sum_{i < k} \left(\frac{g_k'(e)}{g_k(e)} \right) g_i(e) x_i + \sum_{i \geq k} \left(\frac{g_k'(e)}{g_k(e)} \right) g_i(e) x_i \\
&= \left(\frac{g_k'(e)}{g_k(e)} \right) \sum g_i(e) x_i = 0.
\end{aligned}
$$

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[31] Versions of this lemma are proved, e.g., by Innes (1990), Matthews (2001), and Athey (2002).

The inequality follows from MLRP; it is strict because $x \neq 0$ implies $k < n$, and $x_i > 0$ for some $i > k$. ∎

Lemma A4. *For each $i = 1, \ldots, n$, let h_i be a decreasing function on $\mathbb{R}$. Then, for any $(r, e) \in C \times E$ such that r is not debt, if δ is a debt contract for which*

$$\sum g_i(e) \left(h_i(\delta_i) - h_i(r_i) \right) = 0,$$

the following hold:

    (i) $\sum g_i'(e) \left(h_i(\delta_i) - h_i(r_i) \right) \geq 0$*, and*

    (ii) $(e - \hat{e}) \sum g_i(\hat{e}) \left(h_i(\delta_i) - h_i(r_i) \right) < 0$ *for all $\hat{e} \neq e$.*

Proof. Define $x \in \mathbb{R}^n$ by $x_i \equiv h_i(\delta_i) - h_i(r_i)$. Then $\sum g_i(e) x_i = 0$, and $x \neq 0$, as r is not debt. Assume $x_i > 0$. Then $\delta_i < r_i$. Since $\delta_i = \min(\pi_i, D)$, and $r_i \leq \pi_i$ by LE, we have $\delta_i = D < \pi_i$. Hence, $\delta_{i+1} = D < r_i \leq r_{i+1}$, using MI. This yields $x_{i+1} > 0$. Continuing in this fashion proves that $x_j > 0$ for all $j > i$, and so x is quasi-monotone. Both *(i)* and *(ii)* now follow from Lemma A3. ∎

Proof of Lemma 2. This is a direct implication of Lemmas A2 and A4, setting $\alpha = 0$ in the former and $h_i(y) = -v(y)$ in the latter. ∎

Proof of Proposition 2. Continuing from the text, now let e^δ be the largest equilibrium effort of δ, i.e., the largest maximizer of $U^\delta(\cdot)$.[32] In the text we proved that $U^\delta(e^\delta) \geq U^*(e^*)$, and that the Proposition is proved once we show that $e^\delta \geq e^*$. Consider any $e < e^*$. Let δ' be the debt contract determined by

$$V(r^*(e), e) = V(\delta', e). \tag{A7}$$

Then

$$V(\delta', e^*) \leq V(r^*(e), e^*) \leq V(r^*(e^*), e^*) = V(\delta, e^*),$$

where the first inequality follows from Lemma 2 *(ii)* and $e < e^*$; the second from the incentive compatibility of $r^*(\cdot)$ for the investor; and the third is (11). The face value

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[32] This e^δ exists, since $U^\delta(\cdot)$ is continuous on the compact set $E = [\underline{e}, \bar{e}]$.

of δ' is thus no more than that of δ. Hence, $V(\delta', \cdot) \leq V(\delta, \cdot)$. This and (A7) imply $V(r^*(e), e) \leq V(\delta, e)$. Therefore, since $V^*(e) = V(r^*(e), e)$, we have proved that

$$V^*(e) \leq V(\delta, e) \text{ for all } e < e^*. \tag{A8}$$

Now, if $e^\delta < e^*$, then

$$U^*(e^*) \geq U^*(e^\delta) = H(V^*(e^\delta), e^\delta) \geq H(V(\delta, e^\delta), e^\delta) = U^\delta(e^\delta),$$

where the second inequality comes from (A8) and $H_1 < 0$. But then $U^\delta(e^*) = U^\delta(e^\delta)$, contrary to e^δ being the largest maximizer of $U^\delta(\cdot)$. This proves that $e^\delta \geq e^*$. ∎

The following lemma is used to prove Proposition 3. The background assumptions are those of the general model in Section 4; in particular, the lemma does not assume entrepreneur-offer renegotiation.

Lemma A5. *Given a contract f and any $e \in E$, let $m^*(e)$ be an equilibrium of the message game following e. Denote the corresponding equilibrium payoff of the investor as $V^*(e)$. Then, for any sequence $\{e^k\}$ converging to some e^*, and any limit point $\tilde{r}$ of the sequence $\{\tilde{r}^k\} = \{f(m^*(e^k))\}$, we have $\hat{V}(\tilde{r}, e^*) = V^*(e^*)$.*

Proof. To simplify notation, let $m^* = m^*(e^*)$ and $\tilde{r}^* = f(m^*)$. Message $m_I^*(e^k)$ is a best reply to m_E^* in the message game following e^k. Hence,

$$\hat{V}(\tilde{r}^k, e^k) \geq \hat{V}(\bar{r}^k, e^k),$$

where $\bar{r}^k = f(m_E^*(e^k), m_I^*)$. Similarly, m_E^* is a best reply to m_I^* in the game following e^*. Since the entrepreneur wishes to minimize $\hat{V}$, this implies

$$\hat{V}(\tilde{r}^*, e^*) \leq \hat{V}(\bar{r}^k, e^*).$$

Reversing the "k" and "$*$" in this argument yields two more inequalities:

$$\hat{V}(\tilde{r}^*, e^*) \geq \hat{V}(\hat{r}^k, e^*) \text{ and } \hat{V}(\tilde{r}^k, e^k) \leq \hat{V}(\hat{r}^k, e^k),$$

where $\hat{r}^k = f(m_E^*, m_I^*(e^k))$. Combine these four inequalities to obtain

$$\hat{V}(\bar{r}^k, e^k) - \hat{V}(\bar{r}^k, e^*) \leq \hat{V}(\tilde{r}^k, e^k) - \hat{V}(\tilde{r}^*, e^*) \leq \hat{V}(\hat{r}^k, e^k) - \hat{V}(\hat{r}^k, e^*). \tag{A9}$$

Since C is compact, there exists a subsequence $\{\tilde{r}^{k_j}\}$ that converges to $\tilde{r}$, and for which $\{\bar{r}^{k_j}\}$ and $\{\hat{r}^{k_j}\}$ both converge. Taking limits in (A9) along this subsequence, and using the continuity of $\hat{V}(\cdot, \cdot)$, we conclude that $\hat{V}(\tilde{r}, e^*) = \hat{V}(\tilde{r}^*, e^*) = V^*(e^*)$. ∎

Proof of Proposition 3. Consider the certainty equivalent contract $\bar{f}$ that has the same equilibrium $(e^*, m^*(\cdot))$ and corresponding payoffs as f. We need only show that $\lim_{e \to e^*-} \bar{f}(m^*(e))$ exists, and that it is a debt contract δ. This is because the certainty equivalent of any $\tilde{r} \in \Delta C$ is debt if and only if either the support of $\tilde{r}$ is that debt contract alone (when the face value of δ exceeds π_1), or the support of $\tilde{r}$ contains only riskless debt contracts. To simplify notation, we henceforth assume $f = \bar{f}$, i.e., f specifies only non-random simple contracts.

To simplify notation more, let $m^* = m^*(e^*)$ and $r^* = f(m^*)$. The equilibrium payoffs are $V^*(e^*) = V(r^*, e^*)$ and $U^*(e^*) = H(V^*(e^*), e^*)$. Let δ be the debt contract determined by (11). Given δ the entrepreneur's post-renegotiation payoff following any $e \in E$ is $U^\delta(e) = H(V(\delta, e), e)$. Hence, using (11),

$$U^\delta(e^*) = H(V^*(e^*), e^*) = U^*(e^*). \tag{A10}$$

Let e^δ be the maximizer of $U^\delta(e) = H(V(\delta, e), e)$ for which Proposition 2 implies $V(\delta, e^\delta) \geq V^*(e^*)$ and $U^\delta(e^\delta) \geq U^*(e^*)$. By assumption, neither of these inequalities is strict, and so

$$U^\delta(e^\delta) = U^*(e^*). \tag{A11}$$

Since $e^* \in int(E)$, the derivative $U^{\delta\prime}(e^*)$ exists and is given by

$$U^{\delta\prime}(e^*) = H_1(V(\delta, e^*), e^*)V_e(\delta, e^*) + H_2(V(\delta, e^*), e^*). \tag{A12}$$

Now, let $\{e^k\}$ be a sequence converging from below to e^*, and set $r^k = f(m^*(e^k))$. Let r be a limit point of $\{r^k\}$. We shall show that $r = \delta$. Since C is compact, this will imply $r^k \to \delta$, proving the Proposition.

Note first that Lemma A5 implies $V(r, e^*) = V(r^*, e^*)$. Thus, (11) implies $V(\delta, e^*) = V(r, e^*)$. From (A12), therefore, we have

$$U^{\delta\prime}(e^*) = H_1(V(r, e^*), e^*)V_e(\delta, e^*) + H_2(V(r, e^*), e^*). \tag{A13}$$

Now, observe that since e^* maximizes $U^*(\cdot)$,

$$0 \le U^I(e^*) - U^I(e^k) = H(V(r^*, e^*), e^*) - H(V(r^k, e^k), e^k). \qquad \text{(A14)}$$

We also have

$$V(r^*, e^*) = V(f(m^*), e^*) \ge V(f(m_E^*, m_I^*(e^k)), e^*)$$

$$\ge V(f(m_E^*, m_I^*(e^k)), e^k)$$

$$\ge V(f(m_E^*(e^k), m_I^*(e^k)), e^k) = V(r^k, e^k),$$

where the first inequality comes from m_I^* being a best reply to m_E^* following e^*; the second inequality comes from MLRP, $e^* > e^k$, and the simple contract $f(m_E^*, m_I^*(e^k))$ satisfying MI; and the third inequality comes from $m_E^*(e^k)$ being a best reply to $m_I^*(e^k)$ following e^k, and the fact that this best reply minimizes $V(f(\cdot, m_I^*(e^k)), e^k)$. Thus, from (A14) and $H_1 < 0$ we obtain

$$0 \le H(V(r^k, e^*), e^*) - H(V(r^k, e^k), e^k).$$

Since H and $V(r^k, \cdot)$ are differentiable, the mean value theorem applied twice yields

$$0 \le \left\{ H_1(V(r^k, \bar{e}^k), \bar{e}^k) V_e(r^k, \hat{e}^k) + H_2(V(r^k, \bar{e}^k), \bar{e}^k) \right\} (e^* - e^k), \qquad \text{(A15)}$$

where both $\bar{e}^k$ and $\hat{e}^k$ are in (e^k, e^*). Since H_1, H_2, and V are continuous functions, dividing (A15) by $e^* - e^k > 0$, and taking the limit along the subsequence for which $r^k \to r$, yields

$$0 \le H_1(V(r, e^*), e^*) V_e(r, e^*) + H_2(V(r, e^*), e^*). \qquad \text{(A16)}$$

Assume now that $r \ne \delta$. Then, since $V(\delta, e^*) = V(r, e^*)$, Lemma 2 (i) implies $V_e(r, e^*) > V_e(\delta, e^*)$. This, $H_1 < 0$, and (A16) imply

$$0 < H_1(V(r, e^*), e^*) V_e(\delta, e^*) + H_2(V(r, e^*), e^*) = U^{\delta\prime}(e^*),$$

using (A12) for the equality. But $U^{\delta\prime}(e^*) > 0$, contradicts (A10) and (A11). So $r = \delta$, and the proof is finished. ∎

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Proof of Lemma 3. Lemma A2 implies the existence of the debt contract δ satisfying $U(r, e) = U(\delta, e)$. Given SEP, let $h_i(y) = \bar{u}(\pi_i - y)$ and $x_i = h_i(\delta_i) - h_i(r_i)$. Then

$$\sum g_i(e)x_i = a(e)^{-1}\left(U(\delta, e) - U(r, e)\right) = 0. \tag{A17}$$

The conclusions of Lemma A4 therefore follow. Lemma A4 (i) implies

$$U_e(\delta, e) - U_e(r, e) = a'(e)\sum g_i(e)x_i + a(e)\sum g_i'(e)x_i > 0,$$

using (A17) and $a(e) > 0$. This proves (i). Lemma A4 (ii) implies

$$(e - \hat{e})\left(U(\delta, \hat{e}) - U(r, \hat{e})\right) = a(\hat{e})(e - \hat{e})\sum g_i(\hat{e})x_i < 0$$

for $\hat{e} \neq e$, which proves (ii). ∎

Proof of Proposition 4. By the argument of Proposition 1 and the revelation principle, we can restrict attention to deterministic revelation mechanisms for the investor that are incentive compatible. Let $r^* : E \to C$ be such a mechanism. Its truthful equilibrium gives the entrepreneur a post-renegotiation payoff of $U(r^*(e), e)$. Let e^* maximize this. The entrepreneur's equilibrium payoff is then $U^* \equiv U(r^*(e^*), e^*)$. By Lemma 3, a debt contract $\delta^* = \delta(D^*)$ exists such that

$$U(\delta^*, e^*) = U^*. \tag{A18}$$

By the maximum theorem, the function defined by $\bar{U}(D) \equiv \max_{e \in E} U(\delta(D), e)$ is continuous. By (A18), $\bar{U}(D^*) \geq U^*$. Since $r^*(e^*)$ satisfies LE,

$$\bar{U}(\pi_n) = u(0, \underline{e}) \leq U(r^*(e^*), \underline{e}) \leq U^*.$$

Hence, $\hat{D} \in [D^*, \pi_n]$ exists such that $\bar{U}(\hat{D}) = U^*$. The desired debt contract is $\hat{\delta} \equiv \delta(\hat{D})$. Letting $\hat{e}$ be the largest maximizer of $U(\hat{\delta}, \cdot)$, we have

$$U(\hat{\delta}, \hat{e}) = U^*. \tag{A19}$$

The entrepreneur's equilibrium payoff is thus the same from $\hat{\delta}$ as from $r^*(\cdot)$. We now need only to prove that $\hat{e} \geq e^*$.

Assume otherwise, so that $\hat{e} < e^*$. For any e, define $\gamma(e) \equiv U(\hat{\delta}, e) - U(r^*(\hat{e}), e)$. Observe that

$$\gamma(\hat{e}) = U(\hat{\delta}, \hat{e}) - U(r^*(\hat{e}), \hat{e})$$
$$= U^* - U(r^*(\hat{e}), \hat{e})$$
$$= U(r^*(e^*), e^*) - U(r^*(\hat{e}), \hat{e}) \geq 0,$$

using (A19) and the fact that e^* maximizes $U(r^*(\cdot), \cdot)$. Furthermore,

$$\gamma(e^*) = U(\hat{\delta}, e^*) - U(r^*(\hat{e}), e^*)$$
$$< U^* - U(r^*(\hat{e}), e^*)$$
$$\leq U^* - U(r^*(e^*), e^*) = 0.$$

(The first inequality is due to $\hat{e} < e^*$ being the largest maximizer of $U(\hat{\delta}, \cdot)$. The second is due to $r^*(\cdot)$ being incentive compatible for the investor and the renegotiation game being strictly competitive, so that $U(r^*(\cdot), e^*)$ is minimized by e^*.) Hence, since $\gamma(\cdot)$ is continuous, $e \in [\hat{e}, e^*)$ exists such that $\gamma(e) = 0$ and $\gamma(e') < 0$ for all $e' \in (e, e^*]$. But by Lemma 3, this is impossible. ∎

Proof of Proposition 5.   Let e^* be the effort taken in the general contract's equilibrium. Let $\hat{\delta} = \delta(\hat{D})$ be the debt contract of Proposition 4, so that it has an equilibrium in which the effort is some $\hat{e} \geq e^*$, and the entrepreneur's payoff is $U(\hat{\delta}, \hat{e}) = U^*$. The investor's corresponding payoffs in the two equilibria are $J(U^*, e^*)$ and $J(U^*, \hat{e})$. Since the equilibrium of the debt does not Pareto dominate the initial equilibrium, $J(U^*, e^*) \geq J(U^*, \hat{e})$. If this is an equality, case (ii) holds. So assume $J(U^*, e^*) > J(U^*, \hat{e})$. Then $e^* < \hat{e}$. These two inequalities, together with SP, imply $e^{**} < \hat{e}$. We now show that (i) holds.

Let $e^{**} \in E$ be a maximizer of $J(U^*, \cdot)$. We present a simple contract $r^* \in C$ that has an equilibrium in which the entrepreneur chooses e^{**}, and $U(r^*, e^{**}) = U^*$. This equilibrium achieves an efficient allocation that gives the entrepreneur payoff U^*, since the renegotiation provides first-best risk-sharing given e^{**}, and the entrepreneur does not gain from the renegotiation. Since the equilibrium of the general contract that has

effort e^* is not Pareto dominated by an equilibrium of r^*, and the entrepreneur has the same payoff in both equilibria, so must the investor. Thus, the equilibrium of the general contract also achieves an efficient allocation.

It remains to prove this r^* exists. First, note that a wage $w \in [0, \pi_n - \hat{D}]$ is defined by $u(w, \underline{e}) = U^*$.[33] Since U^* satisfies WA, $w \leq \pi_1 - \underline{r}$, and so $r^w \in C$. Now define, for any $b \in B \equiv [0, w]$ and $t \in T \equiv [0, \pi_n - \hat{D} - w]$, a simple contract $r(b, t)$ by

$$r_i(b,t) \equiv \begin{cases} \pi_i - b & \text{for } \pi_i \leq b + \hat{D} \\ \hat{D} & \text{for } b + \hat{D} < \pi_i \leq \pi_n - t \\ \pi_i + \hat{D} + t - \pi_n & \text{for } \pi_n - t < \pi_i. \end{cases}$$

Because $r(b, t)$ satisfies MI, LE (as $b \geq 0$), and LI (as $b \leq w \leq \pi_1 - \underline{r}$), we have $r(b, t) \in C$. Note that $r(0, 0) = \hat{\delta}$, and $r(w, \pi_n - \hat{D} - w) = r^w$.

For each $(b, t) \in B \times T$, let

$$e(b, t) \equiv \arg \max_{e \in E} U(r(b, t), e).$$

As E is compact, the maximum theorem and FOA imply that $e(\cdot, \cdot)$ is a well-defined continuous function on $B \times T$, as is the maximized utility,

$$\bar{U}(b, t) \equiv U(r(b, t), e(b, t)).$$

Now, as is easy to show,[34] for each $t \in (0, \pi_n - \hat{D} - w]$ there exists a unique $b(t) \in B$ such that $\bar{U}(b(t), t) = U^*$. Thus, since $\bar{U}(\cdot, \cdot)$ is continuous, $b(\cdot)$ is continuous on $(0, \pi_n - \hat{D} - w]$. Define $b(0)$ so that $b(\cdot)$ is continuous on T. Then $\bar{U}(b(t), t) = U^*$ for all $t \in T$. Furthermore, $e(b(t), t)$ is continuous on T. Because $r(b(0), 0) = \hat{\delta}$, we have $e(b(0), 0) = \hat{e} > e^{**}$. Because $r(b(\pi_n - \hat{D} - w), \pi_n - \hat{D} - w) = r^w$, we have

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[33] As $0 \leq \pi_i - \hat{\delta}_i$ for each i, we have $u(0, \underline{e}) \leq U(\hat{\delta}, \underline{e}) \leq U(\hat{\delta}, \hat{e}) = U^*$. Since $\pi_n - \hat{D} \geq \pi_i - \hat{\delta}_i$, $u(\pi_n - \hat{D}, \underline{e}) \geq u(\pi_n - \hat{D}, \hat{e}) \geq U(\hat{\delta}, \hat{e}) = U^*$. So a unique $w \in [0, \pi_n - \hat{D}]$ satisfying $u(w, \underline{e}) = U^*$ exists.

[34] For example, consider the case $\hat{D} \geq \pi_1$. Then $r_1(\cdot, t)$ strictly decreases on B, and $r_i(\cdot, t)$ is non-increasing on B for each $i > 1$. It follows that $\bar{U}(\cdot, t)$ strictly increases on B. Since $\bar{U}(\cdot, t)$ is also continuous, and $r_i(0, t) \geq \hat{\delta}_i$ implies $\bar{U}(0, t) \leq U(\hat{\delta}, e(0, t)) \leq U(\hat{\delta}, \hat{e}) = U^*$, and $r_i(w, t) \leq r_i^w$ implies $\bar{U}(w, t) \geq U(r(w, t), \underline{e}) \geq U(r^w, \underline{e}) = U^*$, for each $t \in T$ there exists one and only one $b \in B$ such that $\bar{U}(b, t) = U^*$. The argument is similar for the case $\hat{D} < \pi_1$, although then $b(t)$ is unique only for $t > 0$.

$e(b(\pi_n - \hat{D} - w), \pi_n - \hat{D} - w) = \underline{e} \leq e^{**}$. Thus, $t^* \in T$ exists such that $e(b(t^*), t^*) = e^{**}$. The desired simple contract is $r^* \equiv r(b(t^*), t^*)$. ∎

Proof of Lemma 4. This is implied by Lemmas A2 and A4, setting $(\alpha, \beta) = (\theta, 1)$ in the former and $h_i(y) = w_i(y)$ in the latter. ∎

Proof of Proposition 6. Most of this proof is like that of Proposition 4, replacing $U(r, e)$ there by the function $\hat{U}$ we now define by

$$\hat{U}(r, e) \equiv K^{-1}\left(\sum g_i(e)w_i(r_i),\ e\right). \tag{A20}$$

We can again restrict attention to a deterministic revelation mechanism $r^* : E \to C$ for the investor that is incentive compatible. Let e^* maximize $\hat{U}(r^*(\cdot), \cdot)$, and denote the entrepreneur's equilibrium payoff as $U^* \equiv \hat{U}(r^*(e^*), e^*)$. By (A20) and Lemma 4, a debt contract $\delta^* = \delta(D^*)$ exists such that

$$\hat{U}(\delta^*, e^*) = U^*. \tag{A21}$$

Since $\hat{U}$ and $\delta(\cdot)$ are continuous, the function defined by

$$\bar{U}(D) \equiv \max_{e \in E} \hat{U}(\delta(D), e)$$

is continuous. By (A21), $\bar{U}(D^*) \geq U^*$. Since $\delta_i(\pi_n) = \pi_i$, LE implies that for any e, the maximum feasible payoff for the investor is $V(\delta(\pi_n), e)$. Thus, $\delta(\pi_n)$ will not be renegotiated, and $\hat{U}(\delta(\pi_n), e) = U(\delta(\pi_n), e)$. It follows that $\bar{U}(\pi_n) = u(0, \underline{e})$. As this is the entrepreneur's smallest feasible payoff, $\bar{U}(\pi_n) \leq U^*$. Thus, $\hat{D} \in [D^*, \pi_n]$ exists such that $\bar{U}(\hat{D}) = U^*$. The desired debt contract is $\hat{\delta} \equiv \delta(\hat{D})$.

Lett $\hat{e}$ be the largest maximizer of $\hat{U}(\hat{\delta}, \cdot)$. Then

$$\hat{U}(\hat{\delta}, \hat{e}) = U^*. \tag{A22}$$

We now show $\hat{e} \geq e^*$. Assume $\hat{e} < e^*$, and define $\gamma(\cdot) \equiv \hat{U}(\hat{\delta}, \cdot) - \hat{U}(r^*(\hat{e}), \cdot)$. Then

$$\gamma(\hat{e}) = \hat{U}(\hat{\delta}, \hat{e}) - \hat{U}(r^*(\hat{e}), \hat{e})$$
$$= U^* - \hat{U}(r^*(\hat{e}), \hat{e})$$
$$= \hat{U}(r^*(e^*), e^*) - \hat{U}(r^*(\hat{e}), \hat{e}) \geq 0,$$

using (A22) and the fact that e^* maximizes $\hat{U}(r^*(\cdot), \cdot)$. Furthermore,

$$\gamma(e^*) = \hat{U}(\hat{\delta}, e^*) - \hat{U}(r^*(\hat{e}), e^*)$$

$$< U^* - \hat{U}(r^*(\hat{e}), e^*)$$

$$\leq U^* - \hat{U}(r^*(e^*), e^*) = 0.$$

(The first inequality is due to $\hat{e} < e^*$ being the largest maximizer of $U(\hat{\delta}, \cdot)$. The second is due to $r^*(\cdot)$ being incentive compatible for the investor and the renegotiation game being strictly competitive, so that e^* minimizes $\hat{U}(r^*(\cdot), e^*)$.) Hence, since $\gamma(\cdot)$ is continuous, $e \in [\hat{e}, e^*)$ exists such that $\gamma(e) = 0$ and $\gamma(e') < 0$ for all $e' \in (e, e^*]$. However, by (A20) and the fact that $K^{-1}(\cdot, e)$ is increasing, $\gamma(e) = 0$ implies

$$\sum g_i(e)w_i(\hat{\delta}_i) = \sum g_i(e)w_i(r_i^*(\hat{e})).$$

Thus, Lemma 4 (ii) implies that for $e' > e$,

$$\sum g_i(e')w_i(\hat{\delta}_i) > \sum g_i(e')w_i(r_i^*(\hat{e})).$$

Hence, for any $e' > e$, the fact that $K^{-1}(\cdot, e')$ is increasing implies

$$\gamma(e') = K^{-1}\left(\sum g_i(e')w_i(\hat{\delta}_i),\ e'\right) - K^{-1}\left(\sum g_i(e)w_i(r_i^*(\hat{e})),\ e'\right) > 0.$$

This contradiction proves $\hat{e} \geq e^*$. ∎

B. Appendix B: When Debt is Not Optimal

In this appendix we give examples in which non-debt contracts outperform debt when (i) the investor has all bargaining power, or (ii) a third party can be involved, or (iii) a mixed strategy of a two-sided message game can be implemented.

Investor Bargaining Power and Excessive Effort with Debt

In this example the investor has all the renegotiation bargaining power. She is risk neutral, with a liability bound $\underline{r}$ so low that it never binds for the contracts we consider. The entrepreneur's utility function is $u(w, e) = \bar{u}(w) - .5e^2$, where

$$\bar{u}(w) = \min(w, 11w - 1).$$

The entrepreneur is thus risk averse, but risk neutral with respect to any gamble for which all the payments to him are on one side of .1. The possible outputs are $(\pi_1, \pi_2, \pi_3) = (0, .5, 1)$. The interval of possible efforts is $[0, 1]$. The probability that π_i occurs given effort e is

$$g_i(e) = \begin{cases} .5 - .5e & \text{for } i = 1 \\ .5 & \text{for } i = 2 \\ .5e & \text{for } i = 3. \end{cases}$$

Given a simple contract r, let $u_i = \bar{u}(\pi_i - r_i)$. The entrepreneur's expected payoff given r and an effort e can be written as

$$U(r, e) = .5 \left[(u_3 - u_1)e - e^2 + u_1 + u_2 \right]. \tag{B1}$$

Consider the allocations (r, e) that, for some $\bar{U}$, satisfy three conditions: (i) $U(r, e) = \bar{U}$; (ii) each payment $\pi_i - r_i$ to the entrepreneur exceeds .1, so that he is effectively risk neutral; and (iii) $e = .5$, the effort that maximizes expected output net of effort cost:

$$\sum g_i(e)\pi_i - .5e^2 = .5\left(e - e^2 + .5\right).$$

Such allocations exist for any $\bar{U} \geq -.025$, and they are the first-best efficient allocations that give the entrepreneur payoff $\bar{U}$. One such allocation is $(r^w, .5)$, where r^w is the wage contract with wage $w = .\bar{U} + .125$. (Note that $w \geq .1$, since $\bar{U} \geq -.025$.)

As the investor has all the bargaining power, the entrepreneur does not gain from renegotiation; his post-renegotiation payoff given (r, e) is $U(r, e)$. His optimal effort is hence $e = .5(u_3 - u_1)$, if it is in $[0, 1]$.

Consider a debt contract δ for which the face value is $D \in [0, .28516]$. For this contract, $(u_1, u_2, u_3) = (-1, .5 - D, 1 - D)$. The entrepreneur's best effort is $e^\delta = .5(2 - D)$. His equilibrium utility satisfies $U(\delta, e^\delta) \geq -.025$, as is easily shown. But we have just seen that any first-best allocation that gives the entrepreneur utility in this range must have effort equal to .5. Since $e^\delta > .5$, these debt contracts provide incentives that are too strong.

We now show that each of these debt contracts is Pareto dominated by a non-debt contract, and the latter achieves the first best. Simplify by setting $U^\delta = U(\delta, e^\delta)$, and

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recall that $U^\delta \geq -.025$. The desired contract r is defined implicitly by

$$u_i = \begin{cases} U^\delta - .375 & \text{for } i = 1 \\ U^\delta + .125 & \text{for } i = 2 \\ U^\delta + .625 & \text{for } i = 3. \end{cases} \tag{B2}$$

The contract so defined satisfies the LE and MI constraints, and is not debt. Given this r, the entrepreneur chooses the first-best effort $e = .5(u_3 - u_1) = .5$, and obtains utility $U(r, .5) = U^\delta$. The renegotiation of r when $e = .5$ yields a first-best allocation that gives the entrepreneur utility U^δ, and is therefore strictly preferred by the investor to the allocation obtained when δ is renegotiated and the effort is $e^\delta > .5$.

The Value of Third Parties

As is common in contract theory, introducing a risk neutral third party can be beneficial – if she is not susceptible to collusion. The third party does not even need to be able to observe the effort. Consider the following example.

The investor is risk neutral. Let (r^{w*}, e^*) be an efficient allocation, where r^{w*} is the wage contract that pays the entrepreneur a wage w^*. Let $R = \{s^E, s^T\}$ be an investor-option contract with two schemes. According to scheme s^E, the entrepreneur is paid the fixed wage w^* for any output, and the third party is paid nothing. The resulting incomes when π_i is realized are then w^*, $\pi_i - w^*$, and 0 for the entrepreneur, investor, and third party, respectively. According to s^T, the entrepreneur is paid nothing and the third party is paid an amount r_i^T if output π_i is realized. So s^T yields, when π_i is realized, incomes of 0, $\pi_i - r_i^T$, and r_i^T for the entrepreneur, investor, and third party, respectively. The payments r_i^T are given by

$$r_i^T = a + b\pi_i,$$

where $b \in (0, 1)$ and $a + b\sum g_i(e^*)\pi_i = w^*$.

Both s^E and s^T can easily satisfy our monotonicity and liability constraints. They each make every party's income nondecreasing in output. They each satisfy LE, and LI

so long as $\underline{r} \leq (1-b)\pi_1 - a$. Any liability constraint for the third party is satisfied if her liability limit is less than $a + b\pi_1$.

Because of MLRP, the expected payment to the third party under s^T,

$$\sum g_i(e)r_i^T = a + b\sum g_i(e)\pi_i,$$

increases with the entrepreneur's effort e. It exceeds the wage w^* the investor must pay the entrepreneur under s^E if and only if $e > e^*$. So the investor selects from R the scheme s^T that pays the entrepreneur nothing if she sees an effort less than e^*. She selects the scheme s^E that pays him w^* if he chooses any $e > e^*$. Thus, so long as $u(w^*, e^*)$ is not less than the entrepreneur's minimal possible utility of $u(0, \underline{e})$, this investor-option contract has an equilibrium in which the entrepreneur takes effort e^* and the investor selects scheme s^E. It achieves the efficient allocation (w^*, e^*). Note that R is renegotiation proof: regardless of which effort the entrepreneur chooses or scheme the investor selects, there is no scheme that will make all three parties better off.

Let us not overemphasize this example. It is well known that adding a third party can improve on renegotiation-proof schemes between two parties, since making a third party a contingent claimant can eliminate ex-post inefficiencies. However, if the entrepreneur and investor have an informational advantage over the third party about the effort, they may be able to collude so as to misreport the effort. The three-person contract also breaks down if the entrepreneur and third party can collude, whereby the entrepreneur shirks in return for an under-the-table compensation from the third party. Moreover, third parties may have costs of their own, such as the cost of acquiring information about the environment, etc. In any case, the problem of third parties is not specific to this paper, but applies to the contract literature generally.

The Value of Mixed Message Strategies

We give below an example in which a mixed strategy equilibrium of a two-sided message game contract performs better than debt. Its basic logic is discussed in the concluding section of the text. The message game has, for any effort, a mixed strategy equilibrium that achieves an efficient contract for that effort. Thus, despite the fact that the players

can foresee that they will play a mixed strategy equilibrium of the upcoming message game, they have no desire to renegotiate the contract before they send their messages, regardless of the effort taken. The contract is renegotiation proof, both on and off the equilibrium path, and both before and after the messages are sent.

Again, we do not wish to overemphasize this example. Mixed strategy equilibria can be problematic. First, if they are complicated (as they are below), their plausibility relies perhaps too heavily on the extreme rationality and knowledge assumptions of game theory. Second, in a mixed strategy equilibrium players will have "ex post regret", i.e., after some realizations of their message strategies, one player will want to change his message once he learns the other's message.[35] The parties will thus want to engage in espionage to determine the other's message before sending his own, obviating the rules of the message game. In addition, the veracity of the messages can be subject to interference, as in Legros and Newman (2002). And the dates at which the messages are sent must be certified to the contract-enforcing court, perhaps by using certified mail. The certifying and espionage-preventing burden of implementing a message-game equilibrium are certainly lower if it is in pure strategies, as then there is no incentive to engage in espionage or lie about one's message.

Another caveat to the message game below is that it requires the investor to make large payments to the entrepreneur, off the equilibrium path. This will violate the investor's liability limit, unless $\underline{r}$ is sufficiently low.

The example is for the following case of our general model. The bargaining power in the renegotiation stage is shared in any way. The investor is risk neutral. Her liability limit is so small it will not bind (e.g., $\underline{r} = -\infty$.) The entrepreneur's utility function is $u(w) - c(e)$, where $u' > 0$, $u'' < 0$, and

$$u'(w) \to 0 \text{ as } w \to \infty. \tag{B3}$$

Two necessary conditions for (r^*, e^*) to be an equilibrium allocation is for r^* to be a wage contract, and for it to give the entrepreneur a payoff no less than if he were

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[35]This is not true of a pure strategy equilibrium, as then each player's message is a best reply to the actual message the other sends.

paid nothing and chose the minimal effort, $u(0) - c(\underline{e})$. We show that these conditions together are also sufficient. Any wage contract itself implements the minimal effort, and so we restrict attention to non-minimal efforts. Accordingly, let $e^* \in (0, \bar{e}]$ and $w^* \in (0, \infty)$ be an effort and wage that satisfy

$$u(w^*) - c(e^*) > u(0) - c(\underline{e}). \tag{B4}$$

We construct a two-sided message game contract that achieves (w^*, e^*) via a mixed strategy equilibrium.

The message of player $i = E, I$ in this contract is a two-tuple, $(e_i, x_i) \in E \times [0, 1] \equiv M_i$. The player's effort report is e_i. If the effort reports agree, say $e_E = e_I = e$, the contract requires the investor to pay the entrepreneur the wage

$$w_a(e) \equiv \begin{cases} w^* & \text{if } e = e^* \\ \underline{w} & \text{if } e \neq e^*, \end{cases} \tag{B5}$$

where $\underline{w} \in (0, w^*)$ and

$$u(w^*) - c(e^*) \geq u(\underline{w}) - c(\underline{e}). \tag{B6}$$

(By (B4), this $\underline{w}$ exists.) The numbers x_E and x_I form a "jointly controlled lottery" (Aumann et al., 1968) that comes into play if the reported efforts are not the same. Let $y \equiv x_E + x_I - [|x_E + x_I|]$ be the fractional part of $x_E + x_I$. Then, if $e_E \neq e_I$, the investor pays the wage

$$w_d(x_E, x_I) \equiv \begin{cases} 0 & \text{if } y \leq p \\ \hat{w} & \text{if } y > p, \end{cases} \tag{B7}$$

where $\hat{w}$ and $p \in (0, 1)$ are numbers to be determined.

This defines a deterministic contract, $f : M_E \times M_I \to C$. It prescribes a wage contract for any message pair. Thus, since here a wage contract shares risk efficiently for any effort, renegotiation will not occur after the messages are sent, on or off the equilibrium path, regardless of how the bargaining power in the renegotiation stage is shared. The prescribed wage contract determines both parties' payoffs.

Simple arguments show that the strategies of choosing x_E and x_I from uniform distributions are mutual best replies. Furthermore, it is simple to show the existence of

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numbers $\hat{w}$ and p satisfying

$$(1 - p)\hat{w} \geq w^*, \tag{B8}$$

$$u(\underline{w}) \geq pu(0) + (1 - p)u(\hat{w}), \tag{B9}$$

where (B8) and (B9) are the truthtelling conditions for the investor and for the investor respectively. It then follows that e^* is an equilibrium effort, and the equilibrium contract is the wage contract with wage w^*. ∎

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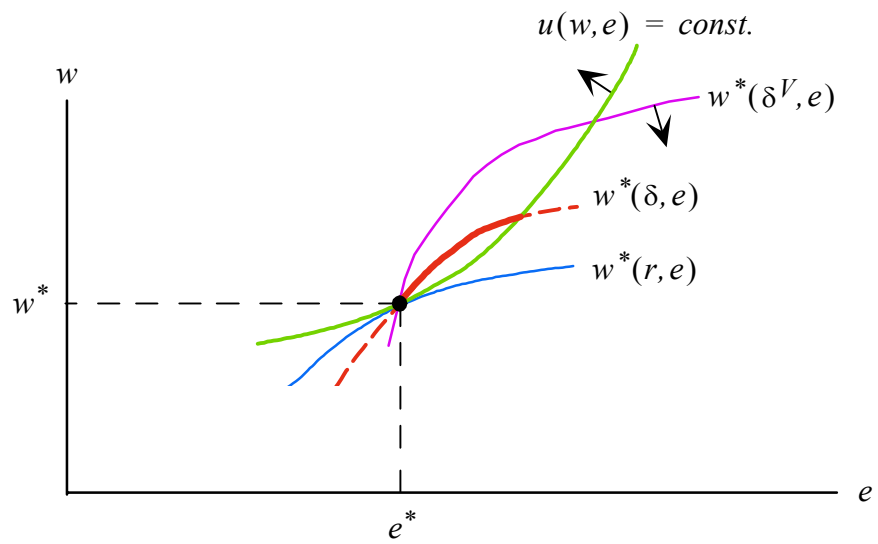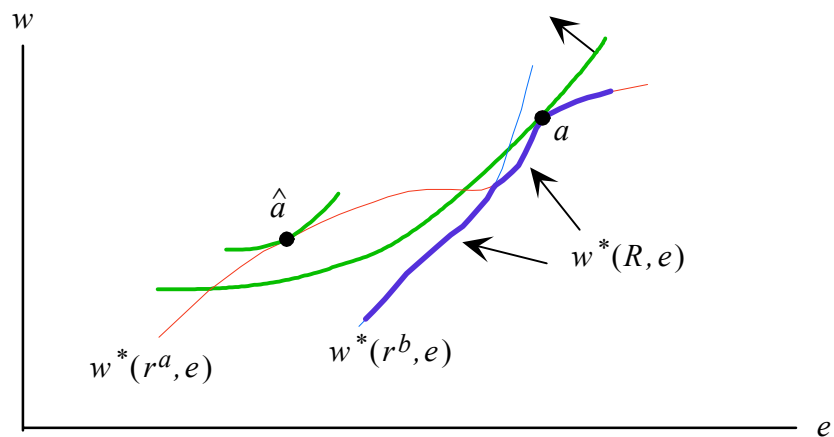
Figure 1:

Figure 2:

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