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"Monopoly Pricing under Demand Uncertainty:
Final Sales versus Introductory Offers"

by

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Abstract

We study rationing as a tool of the monopolist's pricing strategy when demand is uncertain. Three pricing strategies are potentially optimal in our environment: uniform pricing, final sales, and introductory offers. The final sales strategy consists in charging a high price initially, but then lowering the price while committing to a total capacity. Consumers with high valuations to pay may decide to buy at the high price since the endogenous probability of rationing is higher at the lower price. The introductory offers strategy consists in selling a limited quantity at a low price initially, and then raising price. Those consumers with high valuations who were rationed initially at the lower price may find it optimal to buy the good at the higher price.

We show that while the introductory offers strategy may dominate uniform pricing, it is never optimal if the monopolist can use the final sales strategy.

Keywords: rationing, priority pricing, sales, demand uncertainty, introductory offer, price dispersion

JEL-Classification: L12, M31

1 Introduction

Firms frequently charge different prices for the same good at different points in time (and sometimes even contemporaneously). Such (intertemporal) price dispersion is often generated by *priority pricing* or *final sales*: a firm initially charges a high price and subsequently lowers the price for any remaining items. Some consumers (namely those with a high willingness to pay or those who believe that demand will be high) may prefer to purchase the good at the high price (so as to obtain the good with certainty) rather than buying the good at a lower price and risking that the good may be sold out. Other consumers (namely those with a low willingness to pay or those who believe that demand will be low) may resolve the trade off by opting for a lower price and a higher probability of being rationed. Under demand certainty, a final sales strategy may thus allow a firm to screen between different consumers with high and low valuations. In the presence of demand uncertainty, setting different prices may in addition allow a firm to discriminate between demand states.

The pricing strategies of many opera houses, theaters, and concert venues involve priority pricing: advance ticket sales are complemented by lower priced “community rush tickets”, “day seats”, or standby tickets. Similarly, holiday tour operators and airlines typically offer both regular and last-minute deals. (In the case of last-minute holiday packages, the consumer may not obtain the destination or hotel of choice, while in the case of stand-by airline tickets, the consumer buys the ticket in advance but risks to be forced to take a later flight, which he views as an inferior substitute.) Winter or summer sales for fashion goods may also partially be explained by the idea of priority pricing. A less obvious example of priority pricing concerns season tickets for sporting events (such as baseball or soccer) or cultural events (such as concerts or operas); see Ferguson (1994). Some consumers may decide to buy a season ticket knowing that they will miss many events so that the season ticket is likely to turn out to be more expensive than buying single tickets only for those events the consumer actually attends. However, if consumers choose to buy tickets only shortly prior to the event, they risk not obtaining the desired ticket.[1]

In this paper, we consider the pricing (and capacity) policy of a monopolist who faces uncertain demand. Before the state of demand is realized, the monopolist has to commit to prices and capacities for each period. Then, consumers (who want to buy only one unit of the good) learn the state of demand (or, at least, their own willingness to pay) and decide when to buy the good. Consumers rationally anticipate the behavior of other consumers and thus the endogenous probabilities of rationing in each period. Three pricing strategies are potentially optimal for the monopolist: uniform pricing, final sales, and introductory offers. *Uniform pricing* means that the monopolist commits to charging the same price in each period. As explained above, the *final sales strategy* consists in charging a high price initially, but then lowering the price while committing to a total capacity. High valuation consumers may decide to buy at the high price since the endogenous probability of rationing is higher at the lower price. The *introductory offer strategy* consists in selling a limited quantity at a low price initially, and then raising price. Those consumers with high valuations who were rationed at the lower price may find it optimal to buy the good later at the higher price.

In our model, there are two types of consumers (with high and low valuations, respectively), and two demand states (a good and a bad state). We analyze two orthogonal cases of demand uncertainty: vertical demand shifts (where a good demand shock increases the willingness to pay for all consumers) and horizontal demand shifts (where a good demand shock increases the number of high valuation consumers and the total number of consumers). For both types of demand shifts, we show that introductory

[1] This means that product bundling can implement priority pricing for those spectators who are only interested in certain events for which rationing may occur. While living in England, the first author of this article often bought a season ticket for the famous BBC Promenade Concert Series at the Royal Albert Hall, London, knowing that he would only attend a small fraction of the more than seventy concerts. His rationale was to make sure that he could attend some of the more popular concerts.

offers are always dominated by final sales or uniform pricing strategies. In the case of vertical demand shifts, final sales always dominate all other pricing strategies. In the case of horizontal demand shifts, final sales and uniform pricing may both be optimal in different circumstances.[2]

An example. Suppose there are two states of the world, a good demand state and a bad demand state, which are equally likely. Consumers have unit demand and may either have a high or a low valuation for the good. High types have a willingness to pay of 7, and low types a valuation of 1, independently of the demand state. In the good demand state, there is a mass 1 of high types, and a mass 5 of low types. In the bad demand state, there are no high types, and a mass 4 of low types. A consumer who learns that he has a high valuation can thus infer that the demand state must be good. (In this example, it is therefore immaterial whether the consumers directly learn the demand state or only learn their own valuation before making their purchasing decision.) The monopolist produces at zero marginal cost and may set prices and capacities for two periods. In the case of excess demand in one period, consumers are rationed randomly. There is no discounting.

First, consider uniform pricing and introductory offers. Conditional on charging a single price, the monopolist will optimally set a price of 1. This yields expected profits of $0.5 \times 4 + 0.5 \times 6 = 5$. Alternatively, the monopolist can make a limited introductory offer and charge a higher price to consumers who are rationed in period 1. The monopolist's optimal introductory offer strategy is to offer 4 units at a price of 1 in the first period, and to serve any unserved consumers at a price of 7 in the second period. Consumers arrive at random, and so are rationed with probability 1/3 in period 1, provided the state of demand is good. Hence, consumers of mass 1/3 buy the good at the high price in the good demand state. In expected terms, the monopolist makes a profit of $0.5 \times 4 + 0.5 \times (4 + (1/3) \times 7) = 5.167$. This strategy dominates uniform pricing.

Second, consider final sales. The optimal strategy with final sales consists in setting total capacity equal to 4, charging a first-period price p_1 with $7 > p_1 > 1$, and offering all remaining units in the second period at a price of $p_2 = 1$. In the bad demand state, there are no high valuation consumers, and all low valuation consumers purchase the good in the second period. In the high demand state, all high type consumers buy the good in the first period, while the low types demand the good in the second period (and hence are rationed with probability 0.4). Indeed, high type consumers weakly prefer to demand the good in the first period rather than in the second period if $7 - p_1 \geq 0.6 \times (7 - 1)$, where 0.6 is the probability of not being rationed in the second period. Hence, the monopolist will optimally set $p_1 = 3.4$, which results in an expected profit of $0.5 \times 4 \times 1 + 0.5 \times (1 \times 3.4 + 3 \times 1) = 5.2$.

Comparing profits, we observe that final sales perform better than introductory offers. Final sales involve screening between high and low valuation consumers in the good demand state so that the monopolist does not extract all of the surplus from high type consumers. Nevertheless, the introductory offers strategy performs worse since, in the good demand state, a share of high type consumers obtain the good at the low second-period price.

Related Literature. Our paper complements the existing literature on pricing strategies with commitment and provides a stronger theoretical underpinning for the use of final sales strategies. Earlier work has considered the use of introductory offer and final sales strategies under demand certainty. Wilson (1988) analyzes the problem of a monopolist who wants to sell a given quantity q of a good. He shows that an introductory offer strategy may be more profitable than a uniform pricing strategy (namely if and only if there exists a neighborhood around q where the single-price revenue function is non-concave in quantity). However, if the monopolist can choose the quantity q she wants to sell, and the marginal cost of production is non-increasing, then uniform pricing is always optimal.[3] Ferguson (1994) shows the revenue equivalence between the best final sales and the best introductory offer strategy, for any

[2] In one case, a hybrid strategy which combines elements of final sales and introductory offers may be optimal.

[3] As Denicolo and Garella (1999) have shown, introductory offers are also a useful strategy if the monopolist lacks

given quantity q. Hence, these results do not allow us to predict when a monopolist should prefer a final sales strategy over an introductory offer strategy, or *vice versa*. Moreover, the results indicate that non-uniform pricing should only be observed when the single-price revenue function is non-concave in quantity *and* there are decreasing returns to scale in production.

In a model similar to ours, Dana (2001) considers a monopolist who faces uncertain demand and can serve any demand at constant marginal cost of production. He shows that introductory offers may dominate uniform pricing. However, Dana does not allow for final sales strategies.[4]

Harris and Raviv (1981) also analyze monopoly pricing under demand uncertainty. They show that priority pricing is an optimal strategy for a monopolist who produces at constant marginal costs but faces a (binding) capacity constraint. However, in their model, demand uncertainty is of a very special kind: there are a finite number of (large) buyers with i.i.d. valuations. Hence, as the number of buyers increases, demand uncertainty vanishes in the limit. Furthermore, Harris and Raviv show that if the monopolist can costlessly choose capacity (and thus serve any demand at constant marginal cost), then uniform pricing dominates other pricing schemes (see also Riley and Zeckhauser (1983)).

To summarize, in a world where a monopolist can serve any demand at non-increasing marginal costs, the literature so far lacks an explanation for the use of final sales strategies which purely relies on rationing. In our model, we show the non-optimality of introductory offers and the potential optimality of final sales. This implies that the revenue equivalence between the two pricing strategies breaks down when demand is uncertain.[5]

Our plan of the paper is as follows. In section 2, we present our simple model. In sections 3 and 4, we then consider horizontal demand shifts. In section 3 we assume that consumers do not observe the state of demand, but only learn their own type (and update their beliefs about aggregate demand accordingly). In section 4, we assume that consumers observe the demand state before making their purchasing decision. In both sections, we show that final sales and uniform pricing strategies dominate introductory offer strategies. In section 5, we consider vertical demand shifts and show that final sales strategies dominate all other strategies, including introductory offer strategies. Finally, in section 6, we discuss our key assumptions and conclude.

2 The Model

We consider a monopolist producing a homogeneous product and facing heterogeneous consumers with uncertain demand. The monopolist can sell the good over T periods. For most of the paper, we assume $T = 2$. There is no discounting.

Consumers. There is a mass M of potential consumers. There are two demand states (or "states of the world") $\sigma \in \{G, B\}$: a good demand state, G, and a bad demand state, B. In each demand state, each consumer gets a random draw of his willingness to pay. Consumer can have a high valuation or a low valuation for the good. We also allow for the possibility that some consumers do not value the good at all; we call such consumers "null types". High types are denoted by H, low types by L, null types by $\varnothing$, and the generic consumer type by $\theta \in \{H, L, \varnothing\}$. In demand state σ, the probability that a consumer's type is θ is given by $m(\theta|\sigma)/M$. Here, $m(\theta|\sigma)$ denotes the mass of consumers of type θ in state σ. We assume that there are (weakly) more high types in the good demand state than in the bad

commitment power beyond the first period. Clearly, final sales strategies can only be used if commitment for two periods is possible.

[4] Dana (1998) provides a related model of advanced purchase discounts in competitive markets.

[5] We also want to mention some other less closely related work, namely Chao and Wilson (1987), Che and Gale (2000), and Gale and Holmes (1992, 1993). For a discussion of that work, see Dana (2001). We clarify the relation of our paper to other work in the last section.

demand state; that is,

$$m(H|G) \geq m(H|B).$$

Moreover, the total mass of consumers with positive valuation is at least as large in the good demand state as in the bad state:

$$m(H|G) + m(L|G) \geq m(H|B) + m(L|B).$$

Consumers have unit demand. Conditional on buying one unit of the product at price p, a consumer of type θ in demand state σ has (indirect) utility

$$v(\theta|\sigma) - p,$$

where $v(\theta|\sigma)$ is the consumer's willingness to pay. Since high types have a higher willingness to pay than low types,

$$v(H|\sigma) > v(L|\sigma) \text{ for } \sigma \in \{G, B\}.$$

Furthermore, the willingness to pay of a high or low type consumer is weakly increasing in the state of demand; i.e.,

$$v(\theta|G) \geq v(\theta|B) \text{ for } \theta \in \{H, L\}.$$

In contrast, the willingness to pay of a null type is equal to zero, independently of the state of demand:

$$v(\varnothing|\sigma) = 0 \text{ for } \sigma \in \{G, B\}.$$

To reduce the number of parameters in our model, we focus attention on two polar cases: vertical and horizontal demand shifts.

Horizontal Demand Shift. In this case, each consumer's willingness to pay is independent of the demand state, but the total mass of high types is strictly larger in the good demand state than in the bad state. That is, $v(\theta|\sigma)$ is independent of σ, and $m(H|G) > m(H|B)$. We can thus normalize the willingness to pay of the high type (in both demand states) to 1, and denote the willingness to pay of the low type by $v(L) \in (0, 1)$.

Vertical Demand Shift. In this case, the mass of each consumer type is independent of the demand state, but each type's willingness to pay is strictly higher in the good demand state than in the bad state. That is, $m(\theta|\sigma)$ is independent of σ, and $v(\theta|G) > v(\theta|B)$ for $\theta \in \{H, L\}$. We can thus normalize the total mass of consumers with positive valuation to 1, and denote the mass of high types (in both demand states) by $m(H)$. The mass of low types is then given by $1 - m(H)$.

Before making his purchasing decision, each consumer observes a signal s about the underlying aggregate demand state. We consider two information structures:

1. Each consumer only observes his own valuation, i.e., $s = v(\theta|\sigma)$, and updates his beliefs about the demand state σ using Bayes' rule. This information structure is explored in section 3.

2. Each consumer directly observes the true state of demand, i.e., $s = \sigma \in \{G, B\}$. This information structure is explored in sections 4 and 5.

The Monopolist's Strategies. The monopolist can produce any amount of the homogeneous good at constant marginal cost c. For simplicity, we set $c = 0$. The monopolist can sell the product over two periods, $t = 1, 2$. (We will also discuss the more general case, where the good can be sold over $T \geq 2$ periods.) Before the demand state is realized, she sets prices p_1 and p_2 for periods 1 and 2, respectively. In addition, she can commit to a capacity for period 1, k_1, and to a total capacity (for both periods),

$k \geq k_1$. Ex ante, the monopolist thus sets an overall capacity k, and may commit not to sell more than a certain fraction of this capacity (namely, k_1 units) in $t = 1$. Any capacity unsold in the first period, is then available in $t = 2$. (One extreme interpretation, consistent with our assumptions, is that the monopolist has to produce all k units before the state of the world is realized.) By setting a price p_t and a capacity, the monopolist commits to serving all demand up to capacity at price p_t. There are no capacity costs. (This is, admittedly, a strong assumption, which we discuss further in the conclusion.) Following Dana (2001), we assume that prices and capacities cannot be conditioned on the state of demand σ. Our model thus only applies to those industries, where the identity of consumers is unknown (to the monopolist) ahead of time, and so forward contracts with consumers cannot be written.

Depending on the intertemporal profile of prices, we can distinguish between three different types of pricing strategies:

Uniform pricing. The monopolist sets prices such that, with probability 1, all items are sold at the same price. In particular, setting the same price in both periods, $p_1 = p_2$, is a uniform pricing strategy.

Introductory Offers. The monopolist sets a lower price in the first period, $p_1 < p_2$, and some units are sold in each period with positive probability (that is, in at least one demand state).

Final Sales. The monopolist sets a lower price in the second period, $p_1 > p_2$, and some units are sold in each period with positive probability (that is, in at least one demand state).

Since consumers clearly prefer to purchase the good at the lowest possible price, an introductory offer strategy must have the property that first-period capacity k_1 is binding in at least one demand state (otherwise all units would be sold at the low price in the first period). Similarly, a final sales strategy must have the property that total capacity k is binding with positive probability (otherwise, all consumers would always prefer to buy the good at the low price in the second period).

Consumer Rationing. Since the monopolist can commit to capacities, consumers may be rationed in period 1, period 2, or both periods. Again following Dana (2001), we assume that rationing is *proportional* (or *random*). Under the proportional rationing rule (see, for example, Beckmann (1965), and Davidson and Deneckere (1986)), each consumer who is willing to purchase the good has the same probability of obtaining the good. That is, if a mass m of consumers demand the good, but only a quantity $k < m$ is available, then each consumer – independently of his type – is served with probability k/m. Note that this rationing rule is consistent with a queuing model, where consumers arrive in random order, and consumers who arrive first are served first.

Consumer Equilibrium. Observing the monopolist's strategy (p_1, k_1, p_2, k) and their (private) signals about the demand state σ, consumers make their purchasing decisions. For any (p_1, k_1, p_2, k) and distribution of signals, consumers thus play an anonymous game with discrete actions. A consumer equilibrium is a (Bayesian) Nash equilibrium of this (sub-)game. For $p_1 \leq p_2$, consumers have a (weakly) dominant strategy: "demand the good in the first period if and only if your willingness to pay is equal to or higher than p_1; if you are rationed in the first period, demand the good in the second period, provided your valuation is at least p_2." Moreover, any consumer equilibrium is revenue equivalent for the monopolist. In contrast, if the monopolist chooses a final sales strategy (and so $p_1 > p_2$), a consumer may not have a dominant strategy. The only reason why consumers may be willing to buy the good at the higher price in the first period is that they expect to be rationed with a higher probability at the lower price in the second period. However, if consumers expect that more consumers postpone their purchase until $t = 2$, they expect a lower probability of rationing in the second period (as the monopolist will sell all unsold units in the second period), and hence buying in the second period becomes more attractive. This may give rise to the existence of multiple consumer equilibria with different revenues.

The best consumer equilibrium from the monopolist's point of view (and the worst from the consumers' point of view) is the one that maximizes sales at the high price (in $t = 1$).

The Monopolist's Maximization Problem. The monopolist optimally chooses her strategy (p_1, k_1, p_2, k) assuming that, in each subgame, consumers' purchasing decisions form a (Bayesian) Nash equilibrium. To obtain a unique solution, we select, for each (final sales) strategy of the monopolist, the best consumer equilibrium (from the monopolist's point of view).[6]

Remark 1 *To understand the role of demand uncertainty for non-uniform pricing, suppose there are no demand shocks, i.e., $v(\theta|\sigma)$ and $m(\theta|\sigma)$ are independent of the state of demand σ. To simplify notation, we can then write consumer type θ's valuation as $v(\theta)$, the mass of consumer type θ as $m(\theta)$, and normalize the total mass of consumers with positive valuation to 1, so that $m(L) = 1 - m(H)$. Consider first uniform pricing (i.e., $p_1 = p_2$). The optimal uniform price is equal to $v(H)$ if $m(H)v(H) \geq v(L)$, and equal to $v(L)$ if $m(H)v(H) \leq v(L)$. The equilibrium profit from uniform pricing is thus given by*

$$\pi^U = \max\{m(H)v(H), v(L)\}.$$

Next, consider introductory offers (i.e., $p_1 < p_2$). The only candidate equilibrium prices are $p_1 = v(L)$ and $p_2 = v(H)$, and total capacity $k \geq 1$. The profit with this pricing strategy – as a function of first-period capacity k_1 – is then given by

$$\pi^{IO}(k_1) = v(L)k_1 + v(H)(1 - k_1)m(H),$$

which is linear in k_1. That is, the optimal choice of first-period capacity is $k_1 \in \{0, 1\}$, which is equivalent to uniform pricing. Hence, the introductory offer strategy is (weakly) dominated by uniform pricing. Finally, consider the final sales pricing strategy (i.e., $p_1 > p_2$). Clearly, it is optimal to set $p_2 = v(L)$, $k_1 = k \in [m(H), 1]$, and p_1 such that the high types are just indifferent between buying in period 1 (without being rationed), and buying in period 2 (and being rationed with probability $1 - [k - m(H)]/[1 - m(H)]$). The high type's indifference condition can be written as

$$v(H) - p_1 = \frac{k - m(H)}{1 - m(H)} (v(H) - p_2),$$

where $p_2 = v(L)$. The optimal first period price (as a function of total capacity k) is then given by

$$p_1(k) = \frac{1 - k}{1 - m(H)} v(H) + \frac{k - m(H)}{1 - m(H)} v(L),$$

and the profit by

$$\pi^{FS}(k) = p_1(k)k + v(L)(k - m(H)).$$

Since the final sales profit is linear in k, and we assumed $m(H) \leq k \leq 1$, the optimal capacity k is either $k = m(H)$ or $k = 1$. However, if $k = m(H)$, then all capacity is sold at price $v(H)$, and this strategy is equivalent to setting a uniform price $v(H)$. If, on the other hand, $k = 1$, then $p_1 = p_2 = v(L)$, which is a uniform pricing strategy. Hence, the uniform pricing strategy (weakly) dominates the final sales strategy.

[6] In Section 6, we consider a perturbation of the model, which introduces heterogeneity in consumers' valuations. We show that such a perturbation can lead to a unique consumer equilibrium.

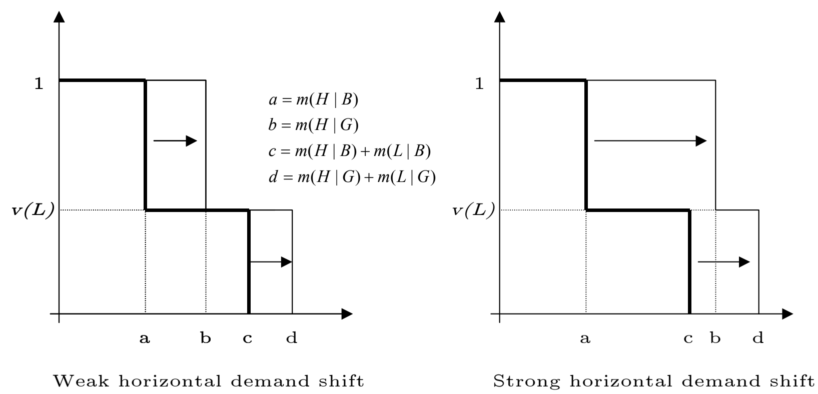

Figure 1: Weak and strong horizontal demand shifts.

3 Pricing under Horizontal Demand Shifts when Consumers only Learn their own Valuation

In this section, we analyze demand uncertainty when consumers do not directly observe the demand state, but only their own valuation. That is, in demand state σ, each consumer of type θ observes the signal $s = v(\theta|\sigma)$. Observing his own valuation, the consumer then updates his beliefs about the demand state σ, using Bayes' rule.

Vertical demand shifts. Under vertical demand shifts, each consumer's private signal s fully reveals the state of demand σ (assuming that $v(H|B) \neq v(L|G)$). The analysis is thus identical to the one when consumers directly observe the demand state. We analyze the case of vertical demand shifts in section 5.

Horizontal demand shifts. In this section, we thus restrict attention to horizontal demand shifts, where a consumer's private signal s does not fully reveal the demand state as $v(\theta|\sigma)$ is independent of σ. Independently of the demand state, the willingness to pay of high type consumers is given by $v(H)$, which we normalize to 1, and the willingness to pay of low types by $v(L) < 1$. The number of consumers of either type depend on the demand state. By assumption, the mass of high types and the total mass of consumers is greater in the good demand state than in the bad demand state: $m(H|G) \geq m(H|B)$ and $m(H|G) + m(L|G) \geq m(H|B) + m(L|B)$ with at least one strict inequality.

In our analysis, we first follow Dana (2001) in restricting attention to non-decreasing price paths, and compare introductory offers with uniform pricing. Then, we also allow for decreasing price paths and compare final sales strategies with introductory offers and uniform pricing. When analyzing final sales strategies, it is useful to distinguish between two demand regimes; see figure 1 for a graphic illustration.

- *Weak horizontal demand shifts.* In this case, the rightward shift of the demand curve is sufficiently small in the sense that the number of high type consumers in the good state is less than the total number of consumers in the bad state, i.e., $m(H|G) < m(L|B) + m(H|B)$.
- *Strong horizontal demand shifts.* In this case, the rightward shift of the demand curve is sufficiently large in the sense that the number of high type consumers in the good state is greater than the total number of consumers in the bad state, i.e., $m(H|G) > m(L|B) + m(H|B)$.

Uniform pricing versus introductory offers. Let us first consider uniform pricing, where the monopolist sets the same price p in both periods. Under uniform pricing, the monopolist has no incentive to ration consumers, and will thus set capacities $k = k_1 = m(H|G) + m(L|G)$ so that demand can always be met. Independently of his beliefs about the demand state (and the behavior of other consumers), a consumer will optimally purchase the good (in either period 1 or 2) if and only if the price his lower than his willingness to pay. We therefore do not need to consider consumers' belief formation at this point. Clearly, the monopolist will optimally extract all of the surplus from one of the two consumer types. Hence, we can confine attention to two uniform prices, $p = 1$ and $p = v(L)$. Expected profits are

$$\pi^U(p) = \begin{cases} \rho m(H|G) + (1-\rho) m(H|B) & \text{if } p = 1 > v(L), \\ \begin{array}{l} v(L)[\rho(m(H|G) + m(L|G)) \\ \quad +(1-\rho)(m(H|B) + m(L|B))] \end{array} & \text{if } p = v(L). \end{cases}$$

The profit maximizing uniform price is $p = v(L)$ if

$$\begin{aligned} v(L)\left\{\rho[m(H|G) + m(L|G)] + (1-\rho)[m(H|B) + m(L|B)]\right\} \quad > \\ \rho m(H|G) + (1-\rho) m(H|B), \end{aligned} \tag{1}$$

and $p = 1$ if the reverse inequality holds.

Next, let us consider introductory offers, where $p_1 < p_2$. Independently of his beliefs, each consumer has a dominant strategy, namely to demand the good at the low price in period 1, provided the price is not higher than his willingness to pay. If the consumer is rationed at the low price, his dominant strategy is to demand the good at the high price in period 2, provided again this price is less than his valuation. In each period, the monopolist optimally extracts all of the surplus of some consumer type. Under introductory offers, the monopolist will therefore set prices $p_1 = v(L)$ and $p_2 = 1$. Without loss of generality, we can assume that first-period capacity $k_1 \leq m(H|G) + m(L|G)$. Expected profits are then given by

$$\begin{aligned} & \pi^{IO}(v(L), 1, k_1) \\ = \quad & (1-\rho)\left[v(L)\min\{k_1, m(H|B) + m(L|B)\}\right. \\ & \left. +1\left(\frac{\max\{0, m(H|B) + m(L|B) - k_1\}}{m(H|B) + m(L|B)}\right) m(H|B)\right] \\ & +\rho\left[v(L)k_1 + 1\left(\frac{m(H|G) + m(L|G) - k_1}{m(H|G) + m(L|G)}\right) m(H|G)\right]. \end{aligned}$$

Hence, the unique candidate for an optimal introductory offer strategy is $p_1 = v(L)$, $p_2 = 1$ and

$k_1 = m(H|B) + m(L|B)$. In this case, profits are

$$\begin{aligned}
& \pi^{IO}(v(L), 1, m(H|B) + m(L|B)) \\
= \;& v(L)[m(H|B) + m(L|B)] \\
& + \rho \left(1 - \frac{m(H|B) + m(L|B)}{m(H|G) + m(L|G)}\right) m(H|G). \qquad (2)
\end{aligned}$$

Note that introductory offers are dominated by uniform pricing if total demand does not expand in the good demand state, i.e., $m(H|G) + m(L|G) = m(H|B) + m(L|B)$. If $m(H|G) + m(L|G) > m(H|B) + m(L|B)$, however, then introductory offers may be more profitable than uniform pricing strategies. We find that $\pi^{IO}(v(L), 1, m(H|B) + m(L|B)) > \pi^U(1)$ is equivalent to

$$\rho \frac{m(H|G)}{m(H|G) + m(L|G)} + (1 - \rho) \frac{m(H|B)}{m(H|B) + m(L|B)} < v(L). \qquad (3)$$

Furthermore, we find that $\pi^{IO}(v(L), 1, m(H|B) + m(L|B)) > \pi^U(v(L))$ is equivalent to

$$m(H|G) > [m(H|G) + m(L|G)]v(L). \qquad (4)$$

The latter condition says that, in the good state, uniform pricing with the high price ($p = 1$) is more profitable than with the low price ($p = v(L)$). Clearly, for these two conditions to hold simultaneously it is necessary that

$$m(H|B) < [m(H|B) + m(L|B)]v(L) \qquad (5)$$

which says that in the bad demand state uniform pricing with the low price ($p = v(L)$) is more profitable than with the high price ($p = 1$). Conditions (4) and (5) are thus necessary (but not sufficient) for introductory offers to dominate uniform pricing.[7]

Remark 2 *Introductory offer strategies are optimal among the set of strategies with $p_1 \leq p_2$ if conditions (3) and (4) are satisfied.*

Hence, as in the case of vertical demand shifts, the monopolist operates in an environment in which introductory offers can be an optimal strategy if final sales strategies are excluded from the analysis, as in Dana (2001).

Consumer learning. So far, we have only considered non-decreasing price paths, where consumers have a (weakly) dominant strategy that is independent of their beliefs about the state of demand. If $p_1 > p_2$, however, those consumers whose willingness to pay exceeds the high first-period price do not have a dominant strategy. Since the distribution of consumer types varies with the state of demand, a consumer's best reply to the purchasing strategies of other consumers will generally depend on his beliefs about the state of demand. Learning his own willingness to pay, a Bayesian consumer uses this information to update his beliefs about the underlying state of the world. However, in the case of horizontal demand shifts, a consumer's private signal is not perfectly revealing.[8]

[7] If (4) and (5) are satisfied and the probability of the good demand state ρ is chosen sufficiently small (holding all other variables fixed), then uniform pricing is dominated by introductory offers.

[8] Since the total mass of high and low type consumers may be larger in the good demand state than in the bad demand state, $m(H|G) + m(L|G) \geq m(H|B) + m(L|B)$, we introduce the construct of a "null type" so as to be able to use Bayes' rule. Recall that a null type has a valuation of zero, and is thus not willing to buy at any (positive) price. The mass of null types in demand state σ is the difference between the total mass and the mass of high and low type consumers, $m(\varnothing|\sigma) = M - [m(H|\sigma) + m(L|\sigma)] \geq 0$. That is, while the total mass of high, low, and null types is independent of the demand state, the shares of the different types are state-dependent

The probability of being a consumer of type $\theta \in \{L, H, \varnothing\}$, given that the demand state is $\sigma \in \{G, B\}$, can then be written as

$$Q(\theta|\sigma) = \frac{m(\theta|\sigma)}{M}.$$

The unconditional probability of a good (bad) demand state is given by $Q(G) = \rho$ ($Q(B) = 1 - \rho$), and the unconditional probability of being a high type is

$$\begin{aligned} Q(H) &= Q(G)Q(H|G) + Q(B)Q(H|B) \\ &= \rho\frac{m(H|G)}{M} + (1-\rho)\frac{m(H|B)}{M}. \end{aligned}$$

A consumer who learns that he has a high valuation, will then (using Bayes' rule) compute the probability of the state of demand being good as

$$\begin{aligned} Q(G|H) &= \frac{Q(H|G)Q(G)}{Q(H)} \\ &= \frac{\rho m(H|G)}{\rho m(H|G) + (1-\rho)m(H|B)}. \end{aligned}$$

Final sales. Let us now consider final sales strategies, where $p_1 > p_2$. Since consumers cannot condition their purchasing decision on the state of the world, but only on their own valuation, any optimal final sales strategy must have the property that all high type consumers demand the good in the first period, while all low types demand the good in the second period (and are rationed with positive probability). Hence, it is sufficient to consider the family of final sales strategies (parameterized by capacity k), $(\widehat{p}_1(k), v(L), k)$, where $\widehat{p}_1(k)$ is set so as to make high type consumers just indifferent between demanding the good in the first period at price $\widehat{p}_1(k)$, and postponing the purchase (so as to demand the good in the second period at price $v(L)$). For $k \geq m(H|G)$, the indifference condition can be written as

$$\begin{aligned} 1 - \widehat{p}_1(k) &= [1 - v(L)]\left[Q(G|H)\left(\frac{k - m(H|G)}{m(L|G)}\right)\right. \\ &\quad \left.+(1 - Q(G|H))\min\left\{1, \frac{k - m(H|B)}{m(L|B)}\right\}\right], \end{aligned} \tag{6}$$

where $\min\{[k - m(H|\sigma)/m(L|\sigma)], 1\}$ is the probability of obtaining the good at the low price in demand state σ. (For $k < m(H|G)$, rationing occurs even at the high price. This case is considered in the proof of proposition 5.)

Strong horizontal demand shifts. Suppose first that horizontal demand shifts are strong, and so $m(H|G) \geq m(H|B) + m(L|B)$. In section 5, we have shown that the final sales strategy $(p_1, p_2, k) = (1, v(L), m(H|G))$ dominates any introductory offer strategy that itself is not dominated by a uniform pricing strategy. Under the information structure considered here, however, a final sales strategy with $p_1 = 1$ and $k > m(H|B)$ is not viable: if a consumer who has learnt that that he has a high valuation ($v(H) = 1$) buys in the first period, he gets zero rents; if, however, he waits and demands the good at a lower price in the second period, he will obtain the good with positive probability, even if all other high types buy the good in the first period (in which case, $k - m(H|B)$ units remain for sale at the low price in the bad demand state). Let us therefore now consider the final sales strategy $(\widehat{p}_1(m(H|G)), v(L), m(H|G))$. From equation (6), the first-period price $\widehat{p}_1(m(H|G))$ is then equal to

$$\widehat{p}_1 = 1 - \frac{(1-\rho)m(H|B)}{\rho m(H|G) + (1-\rho)m(H|B)}[1 - v(L)].$$

The monopolist expected profit is given by

$$\begin{aligned}
& \pi^{FS}(\widehat{p}_1(m(H|G)), v(L), m(H|G)) \\
= \;& \widehat{p}_1(m(H|G))[\rho m(H|G) + (1-\rho)m(H|B)] + (1-\rho)v(L)m(L|B) \\
= \;& \rho m(H|G) + (1-\rho)m(H|B) - (1-\rho)m(H|B)[1-v(L)] + (1-\rho)v(L)m(L|B) \\
= \;& \rho m(H|G) + (1-\rho)v(L)[m(L|B) + m(H|B)] \qquad (7)
\end{aligned}$$

Note that the final sales strategy $(\widehat{p}_1(m(H|G)), v(L), m(H|G))$ induces intertemporal price dispersion in the bad demand state: all high type consumers buy the good in the first period, and all low type consumers demand the good in the second period. (The same happens in the good demand state, but there is no residual supply at the low price.)

We can now compare the final sales strategy $(\widehat{p}_1(m(H|G)), v(L), m(H|G))$ with uniform prices $p = 1$ and $p = v(L)$. The final sales strategy is more profitable than the uniform price of $v(L)$ if and only if equation (4) holds. This condition says that, conditional on demand being in the good state, the best uniform price is 1. The final sales strategy is more profitable than the uniform price of 1 if and only if equation (5) holds. This condition says that, conditional on demand being in the bad state, the best uniform price is $v(L)$. Recall that equations (4) and (5) are necessary conditions for introductory offers to be more profitable than uniform pricing. The results can be summarized as follows.

Lemma 1 *Under strong horizontal demand shifts, uniform pricing is optimal amongst all pricing strategies if conditions (4) or (5) do not hold. If both conditions hold, then uniform pricing is dominated by the final sales strategy* $(\widehat{p}_1(m(H|G)), v(L), m(H|G))$.

The non-optimality of introductory offers. Comparing the introductory offer strategy with the final sales strategy, we find that $\pi^{FS}(\widehat{p}_1(m(H|G)), v(L), m(H|G)) > \pi^{IO}(v(L), 1, m(H|B) + m(L|B))$ is equivalent to $m(H|G) > [m(H|G) + m(L|G)]v(L)$, which is exactly condition (4). On the other hand, if condition (4) does not hold, then we have already seen that the uniform price of $v(L)$ is more profitable than the introductory offers strategy $(v(L), 1, m(H|B) + m(L|B))$. Hence, introductory offers are dominated by either the uniform pricing strategy with $p = v(L)$ or the final sales strategy $(1, v(L), m(H|G))$.

Lemma 2 *Under strong horizontal demand shifts, the profit-maximizing pricing strategy involves a non-increasing price path,* $p_1 \geq p_2$*. If conditions (4) or (5) hold, final sales are optimal, and hence* $p_1 > p_2$.

Weak horizontal demand shifts. Suppose now that horizontal demand shifts are weak, and so $m(H|G) < m(H|B) + m(L|B)$. Let us consider the final sales strategy $(\widehat{p}_1(m(H|B) + m(L|B)), v(L), m(H|B) + m(L|B))$, where first-period price $\widehat{p}_1(m(H|B) + m(L|B))$ is given by

$$\begin{aligned}
& \widehat{p}_1(m(H|B) + m(L|B)) \\
= \;& v(L) + [1 - v(L)]Q(G|H)\left(\frac{m(H|G) + m(L|G) - m(H|B) - m(L|B)}{m(L|G)}\right).
\end{aligned}$$

Expected profits are

$$\begin{aligned}
& \pi^{FS}(\widehat{p}_1(m(H|B) + m(L|B)), v(L), m(H|B) + m(L|B)) \\
= \;& \rho\{\widehat{p}_1(m(H|B) + m(L|B))m(H|G) + v(L)[m(H|B) + m(L|B) - m(H|G)]\} \\
& +(1-\rho)[\widehat{p}_1(m(H|B) + m(L|B))m(H|B) + v(L)m(L|B)] \\
= \;& v(L)[m(H|B) + m(L|B)] \\
& +\rho[1 - v(L)]\left(\frac{m(H|G)}{m(L|G)}\right)[m(H|G) + m(L|G) - m(H|B) - m(L|B)]\,. \qquad (8)
\end{aligned}$$

The optimal pricing strategy under weak horizontal demand shifts. Next, we show that introductory offer strategies are never optimal under weak demand shifts, assuming that the monopolist can use also use final sales strategies. Recall that the only possibly optimal introductory offer strategy is $(v(L), 1, m(H|B) + m(L|B))$. As we have shown above, it is (weakly) dominated by the uniform price $p = v(L)$ if and only if equation (4) does *not* hold, i.e., if and only if $m(H|G) \leq [m(H|G) + m(L|G)]v(L)$. We now analyze under which condition the final sales strategy with intertemporal price dispersion dominates the introductory offer strategy. It is straightforward to show that $\pi^{FS}(\widehat{p}_1(m(H|B) + m(L|B)), v(L), m(H|B) + m(L|B)) > \pi^{IO}(v(L), 1, m(H|B) + m(L|B))$ if and only if equation (4) holds, i.e., if and only if $m(H|G) > [m(H|G) + m(L|G)]v(L)$. Hence, the introductory offer strategy is either dominated by uniform pricing with $p = v(L)$ or by the final sales strategy $(p_1^G, v(L), m(L|B) + m(H|B))$.

Lemma 3 *Under weak horizontal demand shifts, the profit-maximizing pricing strategy involves a non-increasing price path, $p_1 \geq p_2$.*

Horizontal demand shifts: main results. We summarize our findings in the following proposition.

Proposition 4 *Suppose that consumers only learn their own type before making their purchasing decision. Then, under horizontal demand shifts, the optimal strategy of the monopolist involves non-increasing price paths, $p_1 \geq p_2$.*

Above, we have only considered two final sales strategies, namely $(\widehat{p}_1(m(H|G)), v(L), m(H|G))$ in the case of strong horizontal demand shifts, and $(\widehat{p}_1(m(H|B) + m(L|B)), v(L), m(H|B) + m(L|B))$ in the case of weak horizontal demand shifts. Are these the only potentially optimal final sales strategies? This is addressed in the following proposition.

Proposition 5 *Suppose that consumers only learn their own type before making their purchasing decision. Under strong horizontal demand shifts, the only potentially optimal final sales strategy is $(\widehat{p}_1(m(H|G)), v(L), m(H|G))$. Under weak horizontal demand shifts, the only potentially optimal final sales strategy are $(\widehat{p}_1(m(H|G)), v(L), m(H|G))$ and $(\widehat{p}_1(m(H|B) + m(L|B)), v(L), m(H|B) + m(L|B))$.*

Proof. See Appendix. ■

In the model considered here, any final sales strategy with $k > m(H|B)$ induces intertemporal price dispersion: if $k > m(H|B)$, a measure of at least $m(H|B)$ of consumers buys at the high price; in addition, when demand is in the bad state, a measure $k - m(H|B)$ purchases the good at the low price. Hence, any optimal final sales strategy leads to intertemporal price dispersion in the bad demand state. Furthermore, under weak horizontal demand shifts, final sales strategy $(\widehat{p}_1(m(H|B) + m(L|B)), v(L), m(H|B) + m(L|B))$ induces price dispersion in both demand states.

Strategy space. It is possible to show that the monopolist cannot do better by charging more than two prices. Essentially, the argument is that a consumer of a given type cannot condition his purchasing decision on the state of demand. Moreover, all consumers of the same type have the same willingness to pay and the same beliefs about the state of demand. Since there are two consumer types with positive valuation, the monopolist does not need to charge more than two prices.

4 Pricing under Horizontal Demand Shifts when Consumers Know the Demand State

In the previous section, we have assumed that consumers only observe their own valuation before making their first-period purchasing decision. Here we consider an alternative information structure:

Consumers learn the state of the world before making their purchasing decisions, i.e., each consumer observes the signal $s = \sigma$. We maintain our assumption that the monopolist has to commit to prices and capacities before the state of the world is realized.[9] This new information structure implies a strong (informational) asymmetry between the monopolist and the consumers. This asymmetry may be motivated by the existence of a time lag between the monopolist's pricing and capacity decision and consumers' purchasing decisions. Below, we show that our previous results are (qualitatively) robust to such a change in the information structure.

Uniform pricing and introductory offer strategies are not affected by the change in the information structure since each consumer has a (weakly) dominant strategy which does not depend on the state of demand. It remains to analyze final sales strategies.

Final sales strategies. Let us now consider final sales strategies, where $p_1 > p_2$. Recall that the monopolist has no incentive to restrict sales at the high price; this implies $k_1 = k$. Assume that the monopolist sets capacity k such that $m(H|\sigma) < k < m(H|\sigma) + m(L|\sigma)$ for some demand state σ Furthermore, suppose that prices are such that, in demand state σ, all high type consumers buy the good in the first period, while all low type consumers demand the good in the second period. Hence, in state σ, those consumers who decide to purchase in the second period are rationed with probability $1 - [k - m(H|\sigma)]/m(L|\sigma)$. A consumer with reservation value $\widehat{v}$ is then indifferent between purchasing the good in period 1 and delaying the purchase if

$$\widehat{v} - p_1 = \frac{k - m(H|\sigma)}{m(L|\sigma)}(\widehat{v} - p_2).$$

Clearly, the monopolist will optimally set prices such that, in some demand state σ, high type consumers are just indifferent between purchasing the good at the high price and trying to purchase at the low price (but risking not to obtain the good), i.e., $\widehat{v} = 1$. Moreover, any optimal final sales strategy has the property that the monopolist extracts all the rents from the low types: $p_2 = v(L)$. (To see this, note that if the monopolist charged a higher price in the second period, she would never serve any low type consumers; but for serving only high types, it would be optimal to charge a uniform price of 1. If the monopolist charged a price $p_2 < v(L)$, then she could increase her profit by raising p_2: all low types would still demand the good in the second period, and the high types would find it more profitable to purchase at the high first-period price.) Hence, if the monopolist wants to make high type consumers indifferent between purchasing and delaying purchase in demand state σ, she will optimally set the following first-period price (as a function of capacity k):

$$p_1^\sigma(k) \equiv \begin{cases} v(L) & \text{if } k \geq m(H|\sigma) + m(L|\sigma), \\ \left(1 - \frac{k - m(H|\sigma)}{m(L|\sigma)}\right) + \frac{k - m(H|\sigma)}{m(L|\sigma)} v(L) & \text{if } k \in (m(H|\sigma), m(H|\sigma) + m(L|\sigma)), \\ 1 & \text{otherwise.} \end{cases} \tag{9}$$

Note that $p_1^\sigma(k)$ is (linearly) decreasing in k, provided that $m(H|\sigma) + m(L|\sigma) > k > m(H|\sigma)$. As the monopolist raises total capacity, the probability of rationing in the second period decreases. Hence, the first-period price has to be reduced to make high type consumers indifferent between purchasing in period 1 and postponing demand to period 2.

Suppose that the monopolist sets $p_1 = p_1^\sigma(k)$ and $p_2 = v(L)$, and thus makes, in demand state σ, all high types indifferent between purchasing now and purchasing later. Since she cannot condition the first-period price on the demand state, the following question then arises: In demand state $\sigma' \neq \sigma$, will high type consumers purchase the good in the first period or delay purchase?

[9] If we allowed the monopolist to condition prices and capacities on the state of demand, we could analyze the pricing problem for each demand state separately. In this case, the results from the monopoly problem under demand certainty apply. In particular, from Ferguson (1994), we know that the optimal pricing strategy with non-decreasing price paths is revenue equivalent to the optimal pricing strategy with non-increasing price paths. Hence, this model does not predict as to when we should observe final sales rather than introductory offers (and vice versa).

Lemma 6 *Suppose capacity $k \in (m(H|B), m(H|G) + m(L|G))$. Then, $p_1^G(k) > p_1^B(k)$. That is, if the monopolist sets prices such that the high types are just willing to buy the good in the first period when demand is good, then the high types will delay consumption in the bad state. On the other hand, if prices are such that the high types are just willing to buy the good in the first period when the demand state is bad, then they will do so also when the demand state is good.*

Proof. See Appendix. ■

Intuitively, demand is "greater" in the good demand state than in the bad demand state, which implies that the probability of being rationed in the second period (assuming that all high types buy in the first period, and all low types in the second) is higher in the good demand state. (This holds even though the mass of low types may be greater in the bad demand state.)

Strong horizontal demand shifts. Suppose that demand shocks are strong in that $m(H|G) \geq m(H|B) + m(L|B)$.

Uniform pricing versus final sales. Consider the final sales strategy where the high consumer type is made indifferent between purchasing in the first period and delaying purchase; that is, suppose $p_1 = p_1^G(k)$ and $p_2 = v(L)$. The expected profit of this strategy (as a function of capacity k) is given by

$$\widetilde{\pi}^{FS}(k) = \begin{cases} \rho m(H|G)p_1^G(k) \\ +\rho[k - m(H|G)]v(L) & \text{if } m(H|G) + m(L|G) \geq k \\ +(1-\rho)[m(H|B) + m(L|B)]v(L) & \geq m(H|G) \\ \rho k + (1-\rho)\left[m(H|B) + m(L|B)\right]v(L) & \text{if } m(H|G) \geq k \\ & \geq m(H|B) + m(L|B) \\ \rho k + (1-\rho)kv(L) & \text{if } m(H|B) + m(L|B) \geq \\ & k > m(H|B) \\ k & \text{if } k \leq m(H|B) \end{cases}$$

The profit function $\widetilde{\pi}^{FS}$ is piece-wise linear in capacity k; it is continuous, except at $k = m(H|B)$, where it has a downward jump. To understand the different pieces of the profit function, note that (i) all low types always demand the good in the second period at price $v(L)$; (ii) in the good state, the high types demand the good in the first period at price $p_1^G(k)$ (which is equal to 1 if $k \leq m(H|G)$); (iii) in the bad state, high type consumers demand the good in the second period at price $v(L)$, provided that $k > m(H|B)$. However, if $k \leq m(H|B)$, then all high types demand the good in the first period at price $p_1^G(k) = 1$ even when the state of demand is bad (since a high type who decided to deviate and purchase the good in the second period would be rationed with probability 1). This gives rise to the discontinuity of the profit function at $k = m(H|B)$.

Note that $\widetilde{\pi}^{FS}$ is increasing on each of the four linear pieces, except possibly on the piece where $k \geq m(H|G)$. Consequently, there are only two possible interior solutions: $k = m(H|B)$ and $k = m(H|G)$. However, for $k = m(H|B)$, all items are sold in the first period at price $p_1^G(k) = 1$; this strategy is (strictly) dominated by a uniform price of 1. The unique candidate is thus $k = m(H|G)$; the associated prices are $p_1 = 1$ and $p_2 = v(L)$. This final sales strategy yields expected profits of

$$\widetilde{\pi}^{FS}(m(H|G)) = \rho m(H|G) + (1-\rho)[m(H|B) + m(L|B)]v(L). \tag{10}$$

An alternative final sales strategy consists in charging $p_1^B(k)$ in the first period so as to make the high type indifferent between purchasing in the first period and postponing purchase. It can be shown that this strategy is dominated by either the final sales strategy considered above or uniform pricing. We thus have the following result.

Lemma 7 *Under strong horizontal demand shifts, the unique candidate for an optimal final sales strategy is $p_1 = 1$, $p_2 = v(L)$, and $k = m(H|G)$.*

Proof. See Appendix. ■

This profit is the same as for the final sales strategy $(\widehat{p}_1(m(H|G)), v(L), m(H|G))$ analyzed in section 3 (see equation (7)). We can thus conclude that introductory offer strategies are never optimal under strong horizontal demand shifts.

Rationing rule. To what extent are our results driven by the assumption of proportional rationing? Suppose instead that consumers are rationed according to a different rule when the monopolist uses a final sales strategy. Recall that, in our model with vertical demand shifts, any optimal final sales strategy has the property that the rent of some consumer type is fully extracted at the high price. Consequently, this consumer type is only willing to purchase at the high price if delaying consumption leads to rationing with probability 1. Hence, the chosen rationing rule is immaterial to the profitability of the relevant final sales strategies in the model with vertical demand shift. In contrast, profits derived from the use of introductory offer strategies are typically not neutral to the rationing rule. For example, if rationing is efficient introductory offers cannot dominate uniform pricing. More generally, any increase in the probability of serving low types relative to high types reduces the profit from a given introductory offer strategy. We call a rationing rule *positively selective* if a high type consumer is at least as likely as a low type consumer to obtain the good in case of excess demand. Under such a rationing rule, introductory offers are always dominated.[10]

Weak horizontal demand shifts. Suppose now that demand shocks are "weak" in that $m(H|G) < m(H|B) + m(L|B)$.

As in the case of strong demand shifts, the optimal second period price is $p_2 = v(L)$ and the optimal first period price is set such that in one of the demand states high type consumers are just willing to purchase in the first period. Hence, we can confine attention to two families of final sales strategies (parameterized by capacity k), namely $\left(p_1^G(k), v(L), k\right)$ and $\left(p_1^B(k), v(L), k\right)$, where first period prices $p_1^G(k)$ and $p_1^B(k)$ are given by equation (9).

Uniform pricing versus final sales. Observing that profits from final sales strategies are piecewise linear in k, we can reduce the number of potentially optimal final sales strategies to three: $(1, v(L), m(H|G))$, $(p_1^G(m(L|B)+m(H|B)), v(L), m(L|B)+m(H|B))$, and $\left(p_1^B(m(H|G)), v(L), m(H|G)\right)$. Comparing the third of these strategies with uniform pricing, we find that the final sales strategy is always dominated. That is, it is never optimal to make the high types indifferent between purchasing in the first period and delaying purchase when demand is in the *bad* state. We thus obtain the following result.

Lemma 8 *Under weak horizontal demand shifts, all final sales strategies except* $(1, v(L), m(H|G))$ *and* $(p_1^G(m(L|B) + m(H|B)), v(L), m(L|B) + m(H|B))$ *are never optimal.*

Proof. See Appendix ■

Recall that the first of these two final sales strategies, $(1, v(L), m(H|G))$, is the only potentially optimal final sales strategy under *strong* horizontal demand shifts. It results in an expected profit of $\pi^{FS}(1, v(L), m(H|G)) = m(H|G)\,[\rho + (1-\rho)v(L)]$. Note that this strategy induces price dispersion of posted prices but not of prices at which transactions occur: in the good demand state, all items are sold at price 1, while in the bade demand state, all items are sold at price $v(L)$. In contrast, the final sales strategy $(p_1^G(m(L|B) + m(H|B)), v(L), m(L|B) + m(H|B))$ induces true intertemporal price dispersion in the good demand state (when $m(H|G)$ units are sold at price $p_1^G(m(L|B) + m(H|B))$ in the first period, and $m(L|B)+m(H|B)-m(H|G)$ units at price $v(L)$ in the second period). We call this strategy

[10] Under a *negatively* selective rationing rule, introductory offers may be optimal. For example, in the extreme case where low types are served first, an introductory offer strategy could perfectly price discriminate between the two types of consumers when demand is certain. By continuity, if one of the states is highly likely, introductory offers would be optimal also under demand uncertainty.

the *final sales strategy with intertemporal price dispersion*. The expected profit from this strategy is

$$\begin{aligned}
& \pi^{FS}(p_1^G(m(L|B)+m(H|B)), v(L), m(L|B)+m(H|B)) \\
= \ & [m(L|B)+m(H|B)]\, v(L) \\
& +\rho m(H|G) \left[p_1^G(m(L|B)+m(H|B)) - v(L)\right],
\end{aligned}$$

where $p_1^G(m(L|B) + m(H|B))$ is given by (9). This is the same profit as for final sales strategy $\left(p_1^G(m(H|B)+m(L|B)), v(L), m(H|B)+m(L|B)\right)$ under the information structure analyzed in section 3, where consumers do not directly observe the state of the world before making their purchasing decision (see equation (8)). We can thus conclude that introductory offer strategies are never optimal under weak horizontal demand shifts.

Next we give conditions which ensure that the profit-maximizing price path is *strictly* decreasing. Final sales strategy $(1, v(L), m(H|G))$ dominates the uniform price $p = 1$ if and only if

$$m(H|G)v(L) > m(H|B), \tag{11}$$

and the uniform price $p = v(L)$ if and only if

$$\begin{aligned}
& \rho \left\{m(H|G) - [m(H|G)+m(L|G)]v(L)\right\} \\
> \ & (1-\rho)\left\{[m(H|B)+m(L|B)] - m(H|G)\right\} v(L).
\end{aligned} \tag{12}$$

Note that (4) is a necessary condition for the latter equation to hold. *Ceteris paribus*, condition (12) is more difficult to be satisfied, the smaller is ρ and the larger is $m(H|B) + m(L|B) - m(H|G)$. Final sales strategy $(p_1^G, v(L), m(L|B) + m(H|B))$ dominates the uniform price with $p = v(L)$ if and only if condition (4) holds. Hence, for $(p_1^G, v(L), m(L|B)+m(H|B))$ to dominate all uniform pricing strategies, it is sufficient that (i) condition (4) holds, and (ii) that the uniform price $p = v(L)$ is more profitable than the uniform price $p = 1$; (ii) holds if (1) is satisfied.[11] We thus have the following sufficient conditions for the non-optimality of uniform pricing.

Remark 3 *Suppose horizontal demand shifts are weak. If either conditions (11) and (12) or conditions (4) and (1) hold, then the profit-maximizing price path is strictly decreasing, $p_1 > p_2$.*

Combining lemma 2 and remark 3 thus gives sufficient conditions for the optimality of final sales strategies.

Horizontal demand shifts: main results. Summarizing our results on decreasing price paths under strong and weak demand shifts (Lemmas 2 and 3), we can state the following proposition.

Proposition 9 *In the model with horizontal demand shifts, the optimal strategy of the monopolist involves non-increasing price paths, $p_1 \geq p_2$.*

[11] A sufficient condition (which is also necessary if $m(H|G) + m(L|G) = m(H|B)] + m(L|B))$ for the final sales strategy to dominate the uniform price $p = 1$ is given by

$$[m(L|B) + m(H|B)]v(L) > (1-\rho)m(H|B) + \rho m(H|G).$$

The necessary and sufficient condition is rather involved and not very helpful. Note that the left-hand side of the above inequality is less than or equal to

$$(1-\rho)[m(L|B) + m(H|B)]v(L) + \rho[m(L|G) + m(H|G)]v(L).$$

Hence, the necessary condition implies that the uniform price $p = v(L)$ dominates the uniform price $p = 1$ (see inequality (1)). For the final sales strategy to dominate both uniform prices, (4) and (1) are therefore weaker necessary conditions than (4) and the above equation.

Under horizontal demand shifts, introductory offers cannot be optimal if one allows for final sales strategies.

Conditions for the optimality of intertemporal price dispersion Concluding our analysis of horizontal demand shifts, we now establish under which conditions the optimal pricing strategy induces intertemporal price discrimination. To this end, we have to show that the final sales strategy $(p_1^G\left(m(L|B)+m(H|B)\right),$ $v(L), m(L|B)+m(H|B))$ is indeed optimal for some parameter configurations.

Proposition 10 *In the model with horizontal demand shifts, the monopolist's optimal pricing involves intertemporal price dispersion if the following conditions hold:*

1. $m(H|B)+m(L|B)>m(H|G)$,
2. $m(H|G)>[m(H|G)+m(L|G)]v(L)>m(H|G)\left[\rho+(1-\rho)v(L)\right]$, *and*
3. $(1-\rho)\left\{[m(L|B)+m(H|B)]v(L)-m(H|B)\right\}\geq\rho\left\{m(H|G)-[m(L|G)+m(H|G)]v(L)\right\}$.

Proof. First, recall that $(p_1^G\left(m(L|B)+m(H|B)\right), v(L), m(L|B)+m(H|B))$ can only be optimal under weak demand shifts. (This is the first condition.) Second, the final sales strategy with intertemporal price dispersion dominates the introductory offer strategy if and only if equation (4) holds, i.e., $m(H|G)>$ $[m(H|G)+m(L|G)]v(L)$. Moreover, it dominates the final sales strategy $(1, v(L), m(H|G)$ if and only if

$$[m(H|G)+m(L|G)]v(L)>m(H|G)\left[\rho+(1-\rho)v(L)\right].$$

(The two inequalities are summarized by the second condition.) Third, the uniform price $p=v(L)$ performs better than the uniform price $p=1$ if and only if the third condition holds. The uniform price $p=v(L)$ is in turn dominated by the final sales strategy $(p_1^G\left(m(L|B)+m(H|B)\right), v(L), m(L|B)+$ $m(H|B))$ if and only if the first inequality in the second condition holds. ■

The necessary conditions for intertemporal price dispersion can be understood as follows. The first condition says that demand shifts are weak. The first inequality in the second condition says that, in the good demand state, the uniform price $p=1$ is more profitable than the uniform price $p=v(L)$. The second inequality says that it is more profitable to serve $m(H|G)+m(L|G)$ consumers at price $p=v(L)$ than to serve only $m(H|G)$ consumers at price $p=1$ with probability ρ and at price $p=v(L)$ with the remaining probability. The third condition ensures that uniform pricing with $p=v(L)$ dominates uniform pricing with $p=1$. *Ceteris paribus*, the smaller is the probability of the good demand state ρ, the "more likely" is it that the second and third conditions are satisfied. It is easily verified that the example in the introduction satisfies all of these conditions.

Strategy space. In our model, we assumed that prices and capacities could only be set for two periods. That is, we restricted the monopolist to selecting prices p_1 and p_2, first-period capacity k_1, and total capacity $k\geq k_1$. An interesting question is whether our results still hold for an extended strategy space, where the monopolist can set price p_t and capacity $k_t\geq k_{t-1}$ in any period $t=1,...,T$, where $T\geq 2$. In the case of strong horizontal demand shifts, the answer is affirmative:

Lemma 11 *Consider an extended strategy space in which for any finite number of periods T the monopolist sets prices and capacities. Then, under strong horizontal demand shifts, an optimal strategy in the two-period model remains optimal in the T-period extension, $T>2$.*

Proof. See Appendix. ■

However, in the case of weak horizontal demand shifts, allowing for more-than-two price strategies changes the picture. In addition to final sales or uniform pricing, a hybrid strategy which combines final sales with introductory offers can be optimal.

Lemma 12 *Consider an extended strategy space in which for any finite number of periods T the monopolist sets prices and capacities. Then, under weak horizontal demand shifts, there exists a potentially optimal strategies with more than two prices: $(p_1 = 1, p_2 = v(L), p_3 = 1, k_1 = k_2 = m(H|G), k \geq m(H|G) + \widetilde{m})$, where*

$$\widetilde{m} = \frac{[m(H|B) + m(L|B) - m(H|G)]m(H|B)}{m(L|B) + m(H|B)}.$$

All other strategies with more than two different prices (at which transactions occur with positive probability) cannot be optimal.[12]

Proof. See Appendix. ■

This three-price strategy is a hybrid between a final sales and an introductory offer strategy: in the good state, all high-type consumers buy in period 1, while in the bad state, all consumers demand the good in period 2. Among the consumers rationed in the second period, those with a high valuation buy the good in period 3. Because of the use of introductory offers in the bad state, we observe intertemporal price dispersion.

This means that there exist only one potentially optimal strategy with more than two prices: $(p_1 = 1, p_2 = v(L), p_3 = 1, k_1 = k_2 = m(H|G), k \geq m(H|G) + \widetilde{m})$. Clearly this strategy dominates the final sales strategy $(1, v(L), m(H|G))$, which was shown to be optimal under some parameter constellations in the two-period model. We thus have established the following result.

Proposition 13 *Consider an extended strategy space in which for any finite number of periods T, the monopolist sets prices and capacities. Under weak horizontal demand shifts, the optimal strategy is either uniform pricing, final sales with intertemporal price dispersion, or a hybrid strategy with intertemporal price dispersion.*

The conditions for the optimality of a strategy with intertemporal price dispersion are weaker than in the two-period model. Note that our analysis reveals that the optimal number of periods needed to implement an optimal strategy can be larger than the number of demand states. This is in contrast to Dana (2001), where price paths are restricted to be non-decreasing.

5 Pricing under Vertical Demand Shifts

In this section, we analyze demand uncertainty in the form of vertical demand shifts. Under vertical demand shifts, the reservation values of the different consumer types depend on the state of the world, whereas the number of consumers of each type is independent of the state of demand. Assuming that $v(H|B) \neq v(L|G)$, a consumer who learns that his own valuation is given by $v(\theta|\sigma) > 0$, $\theta \in \{H, L\}$, will rationally infer that the demand state is σ with probability 1. Consequently, under vertical demand

[12] A different three price strategy, $(p_1 = 1, p_2 = p_1^B(m(H|G)), p_3 = v(L), k_1 = k_2 = k = m(H|G))$, may also seem to be potentially optimal. This three-price strategy is a sophisticated final sales strategy: in the good state all high type consumers buy in period 1 because any deviating consumer is rationed with probability 1. In the bad state, all high-type consumers buy in period 2. Any high-type consumer is indifferent between buying in period 2 at the price $p_1^B(m(H|G))$ and buying at a lower price in period 3 in which there is rationing with positive probability. Clearly, all low type consumer demand the good in period 3. Although both strategies generate the same revenues in the good state (so that one might think that this state is irrelevant for a profit comparison) the two strategies are not revenue equivalent (which might be a bit surprising in light of Ferguson, 1994). The reason is that the presence of the good state imposes restrictions on the capacities. With the introductory offer more units can be sold in the bad state than with a final sales strategy targeted for the bad state, when respecting the capacity restriction arising from selling at the high price in the good state. It can be shown that the hybrid strategy gives higher profits than the three-price final sales strategy if and only if $[m(H|B) + m(L|B)]v(L) > m(H|B)$. This is equivalent to the condition that uniform pricing with $p = 1$ is dominated by any of the two strategies. Hence, the three-price final sales strategy cannot be optimal (for details see the proof of Lemma 12).

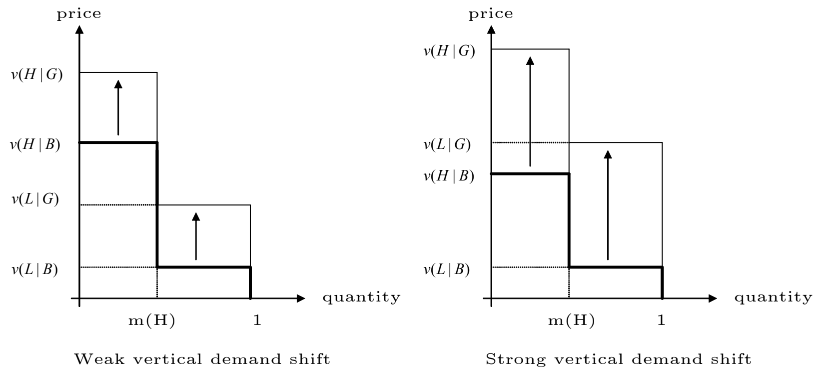

Figure 2: Weak and strong vertical demand shifts.

shifts, private signals perfectly reveal the state of the world (except for null types), and our analysis does not depend on whether consumers directly observe the demand state, or only their own valuation.

We normalize the total mass of consumers to 1. The mass of high type consumers is $m(H)$ and the mass of low type consumers $m(L) = 1 - m(H)$. By definition, the reservation value of each consumer type is higher in the good demand state than in the bad demand state, i.e., $v(\theta|G) > v(\theta|B)$, $\theta = L, H$. Moreover, in each demand state, high type consumers have a higher willingness to pay than low type consumers: $v(H|\sigma) > v(L|\sigma)$, $\sigma = B, G$.

It is useful to distinguish between two demand regimes; see figure 2 for an illustration.

- *Weak vertical demand shifts.* In this case, the upward shift of the demand curve is sufficiently small in the sense that high type consumers in the bad state have a higher willingness to pay than low type consumers in the good state, i.e., $v(H|B) > v(L|G)$.
- *Strong vertical demand shifts.* In this case, the upward shift of the demand curve is sufficiently large in the sense that high type consumers in the bad state have a lower willingness to pay than low type consumers in the good state, i.e., $v(H|B) < v(L|G)$.

Weak vertical demand shifts. Suppose that demand shocks are sufficiently weak so that $v(H|B) > v(L|G)$.

Uniform pricing versus introductory offers. First, we consider uniform pricing, where the monopolist sets the same price p in both periods. Since the monopolist does not attempt to discriminate between consumer types or demand states, she has no incentive to ration consumers. Under uniform pricing, the monopolist thus sets capacities $k_1 = k = 1$ so as to always meet demand. Moreover, the monopolist will set the price so as to fully extract the rent of some consumer type in some demand state. We can thus restrict attention to prices $p = v(\theta|\sigma)$ for some θ and σ. Expected profits are

$$\pi^U(p) = \begin{cases} \rho m(H)v(H|G) & \text{if } p = v(H|G), \\ m(H)v(H|B) & \text{if } p = v(H|B), \\ [(1-\rho)m(H)+\rho]\, v(L|G) & \text{if } p = v(L|G), \\ v(L|B) & \text{if } p = v(L|B). \end{cases}$$

Second, we consider introductory offers, where $p_1 < p_2$. Clearly, consumers will only be willing to purchase at the higher price in the second period if they have been unable to obtain the good in the first period. An introductory offer strategy is thus based on consumer rationing at the low price in some demand state. This requires the monopolist to set first-period capacity $0 < k_1 < 1$. On the other hand, the monopolist has no incentive to ration consumers at the high price. She thus sets total capacity $k = 1$ so as to always meet demand in the second period. An introductory offer strategy can therefore be summarized by the triplet (p_1, p_2, k_1), where we suppress capacity $k = 1$ for notational simplicity. Note that we can restrict attention to prices $v(L|B)$, $v(H|B)$, $v(L|G)$, and $v(H|G)$ since, in each period, the monopolist optimally extracts all of the surplus from some consumer type in some demand state.

Lemma 14 *Suppose vertical demand shifts are weak. Introductory offers different from* $(v(L|G), v(H|G), m(H))$ *cannot be optimal among the set of strategies with non-decreasing price paths.*

Proof. See Appendix. ■

The lemma says that either a uniform price or the introductory offer strategy $(v(L|G), v(H|G), m(H))$ is optimal among the set of strategies with $p_1 \leq p_2$. Under the introductory offer strategy, the first-period capacity $k_1 = m(H)$ is always sold at price $v(L|G)$. In the good demand state, those high type consumers who did not obtain the good in the first period, purchase it in the second period at price $v(H|G)$; since the probability of rationing in the first period is $m(H)$ and there are $m(H)$ high type consumers, demand for the good at the high price is equal to $[1 - m(H)]\, m(H)$. The expected profit from the introductory offer strategy is thus given by

$$\pi^{IO}(v(L|G), v(H|G), m(H)) = v(L|G)m(H) + \rho v(H|G)\,[1 - m(H)]\, m(H). \tag{13}$$

This introductory offer strategy dominates any uniform price if[13]

$$\begin{aligned} v(H|G)m(H) &> v(L|G) > v(H|G)m(H)\rho, \\ v(L|B) &< v(L|G)m(H) + v(H|G)m(H)(1-m(H))\rho, \text{ and} \\ v(H|B) &< v(L|G) + v(H|G)(1-m(H))\rho, \end{aligned}$$

Clearly, there exist parameter constellations such that these conditions are satisfied simultaneously.

Final sales versus uniform pricing. We now consider final sales strategies, where $p_1 > p_2$. Clearly, the monopolist has no incentive to ration demand at the high price. She will therefore set first-period capacity $k_1 = k$. A final sales strategy can therefore be summarized by the triplet (p_1, p_2, k), where we suppress first-period capacity k_1 for notational simplicity. For consumers to be willing to purchase the good in the first period, there must exist a positive probability that consumers are rationed in the second period.

Consider the final sales strategy $(v(H|G), v(H|B), m(H) - \varepsilon)$. Facing these prices and capacity commitment, all high type consumer will, in the good demand state, purchase the good in the first

[13] The first inequality ensures that the introductory offer strategy dominates the uniform price $p = v(L|G)$; the second that it dominates $p = v(H|G)$; the third that it performs better than $p = v(L|B)$; and the last that it is more profitable than $p = v(H|B)$.

period at price $v(H|G)$. Given that all of the high types demand the good in the first period, consumers are rationed with probability 1 in the second period, and so no high type consumer can profitably deviate by delaying the purchase. In the bad state, however, all high types demand the good in the second period. Expected profits are then given by

$$\begin{aligned} & \pi^{FS}(v(H|G), v(H|B), m(H) - \varepsilon) \\ = \quad & [(1-\rho)v(H|B) + \rho v(H|G)]\,[m(H) - \varepsilon]\,, \end{aligned}$$

which is decreasing in ε. Taking the limit as $\varepsilon \to 0$, profits become

$$\pi^{FS}(v(H|G), v(H|B), m(H)) = [(1-\rho)v(H|B) + \rho v(H|G)]\, m(H). \tag{14}$$

Note that this strategy implements quantity setting by a monopolist with output $q = m(H)$, where prices are determined by a Walrasian auctioneer.[14] It is immediate to see that this final sales strategy dominates uniform prices $p = v(H|B)$ and $p = v(H|G)$. The final sales strategy performs better than $p = v(H|B)$ as the monopolist sells the same quantity in each demand state (namely, $m(H)$), but charges a higher price in the good demand state. It performs better than $p = v(H|G)$ as the monopolist makes the same profit in the good demand state, but a strictly higher profit in the bad state (where the monopolist does not sell anything if she charges the uniform price $v(H|G)$).

Consider now the final sales strategy $(v(L|G), v(L|B), 1-\varepsilon)$. In the good demand state, this strategy induces both high and low type consumers to demand the good in the first period at price $v(L|G)$ (as the probability of rationing in the second period is 1). In the bad demand state, all consumers demand the good in the second period. Expected profits are thus $[(1-\rho)v(L|B) + \rho v(L|G)]\,(1-\varepsilon)$. In the limit as $\varepsilon \to 0$, profits converge to

$$\pi^{FS}(v(L|G), v(L|B), 1) = (1-\rho)v(L|B) + \rho v(L|G)$$

Note that this strategy implements quantity setting by a monopolist with output $q = 1$, where prices are determined by a Walrasian auctioneer. Clearly this strategy dominates the uniform pricing strategies $p = v(L|B)$ and $p = v(L|G)$.

Since uniform prices $p = v(H|B)$ and $p = v(H|G)$ are dominated by final sales strategy $(v(H|G), v(H|B), m(H))$, and uniform prices $p = v(L|B)$ and $p = v(L|G)$ by $(v(L|G), v(L|B), 1)$, uniform pricing cannot be optimal.

Lemma 15 *Under weak vertical demand shifts, uniform pricing strategies are less profitable than final sales strategies.*

It is possible to show that the two final sales strategies described above, $(v(H|G), v(H|B), m(H))$ and $(v(L|G), v(L|B), 1)$, are the only optimal ones. To see this, note that if consumers are rationed in the second period, there exists a marginal consumer with willingness to pay $\widehat{v}$ who is indifferent between purchasing the good in the first period and delaying the purchase. Suppose that, in some demand state, consumer beliefs are such that all high types purchase the good at price p_1 in the first period, and all remaining capacity is sold in the second period at price p_1. Then, the marginal consumer $\widehat{v}$ satisfies

$$(\widehat{v} - p_2)\left(\frac{k - m(H)}{1 - m(H)}\right) = \widehat{v} - p_1 \text{ for } k \in [m(H), 1]\,.$$

[14] Let us make two remarks. (i) Due to the discontinuity in demand (at $m(H)$), there is a continuum of prices which clear the market for a fixed quantity (in a given demand state). Our assertion holds true if one selects the highest price which clears the market. (ii) While it is not the aim of our analysis to compare price and quantity setting strategies, some of our comparisons can be interpreted in this way. An early contribution on price versus quantity setting in a central planner's problem is Weitzman (1974). Klemperer and Meyer (1986) compare price and quantity setting in an oligopoly model.

Rewriting the equation, we obtain the first-period price

$$p_1(\widehat{v}, p_2; k) = \left(\frac{1-k}{1-m(H)}\right)\widehat{v} + \left(\frac{k-m(H)}{1-m(H)}\right)p_2, \quad k \in [m(H), 1],$$

which is linear in capacity k. Assuming the high types demand the good at the high price only in the good demand state, but at the low price in the bad demand state, the monopolist's expected profit can be written as $\rho\left[p_1(\widehat{v}, p_2; k)m(H) + p_2(k - m(H))\right] + (1-\rho)p_2$. Since the profit is linear in k, there can only be two potentially optimal capacity levels: $k = m(H)$ and $k = 1$. Clearly, for $k = m(H)$, the monopolist does best with strategy $(v(H|G), v(H|B), m(H))$, where in each demand state, the monopolist extracts all of the rents from the high type consumers. For capacity $k = 1$, note that $p_1(\widehat{v}, p_2; 1) = p_2$. Hence, with capacity $k = 1$, the monopolist cannot discriminate between different types. If the monopolist wishes to serve low types in some demand state, she does therefore best by using the final sales strategy $(v(L|G), v(L|B), 1)$, where in each demand state, she extracts all of the rents from the low type consumers.

Lemma 16 *Under weak vertical demand shifts, the only potentially optimal final strategies are* $(v(H|G), v(H|B), m(H))$ *and* $(v(L|G), v(L|B), 1)$*.*

Final sales versus introductory offers. Since uniform pricing strategies are never optimal if one allows for final sales strategies, the monopolist uses either final sales or introductory offer strategies. Comparing equations (13) and (14), we find that the final sales strategy $(v(H|G), v(H|B), m(H))$ gives higher expected profits than the introductory offer strategy $(v(L|G), v(H|G), m(H))$ if and only if

$$(1-\rho)v(H|B) + \rho m(H)v(H|G) > v(L|G).$$

Since $v(H|B) > v(L|G)$, this inequality is implied by $m(H)v(H|G) > v(L|G)$, which is a necessary condition for the introductory offer strategy to dominate uniform pricing. Since uniform pricing is always dominated by one final sales strategy, introductory offers cannot be an optimal strategy in our setting. Hence, any optimal pricing strategy involves final sales.

Lemma 17 *Under weak vertical demand shifts, any optimal pricing strategy is a final sales strategy, and so* $p_1 > p_2$*.*

Strong vertical demand shifts. Suppose now that demand shocks are sufficiently strong so that $v(H|B) \leq v(L|G)$.

Uniform pricing versus introductory offers. First, we consider uniform pricing, where the monopolist sets the same price p in both periods. The monopolist optimally sets capacities $k_1 = k = 1$ so as to always meet demand. We can restrict attention to prices $p = v(\theta|\sigma)$ for some θ and σ. Expected profits are

$$\pi^U(p) = \begin{cases} \rho m(H)v(H|G) & \text{if } p = v(H|G), \\ \rho v(L|G) & \text{if } p = v(L|G), \\ \left[(1-\rho)m(H) + \rho\right]v(H|B) & \text{if } p = v(H|B), \\ v(L|B) & \text{if } p = v(L|B). \end{cases}$$

Second, we consider introductory offers, where $p_1 < p_2$. Since the monopolist optimally sets total capacity $k = 1$ so as to always meet demand in the second period, an introductory offer strategy can be summarized by the triplet (p_1, p_2, k_1), where $k_1 \in (0, 1)$. Exploiting the (piecewise) linearity of $\pi^{IO}(p_1, p_2, k_1)$ in k_1, we can reduce the number of potentially optimal introductory offer strategies.

Lemma 18 *Suppose vertical demand shifts are strong.*

1. *Assume $m(H)v(H|G) < v(L|G)$. Among the set of pricing strategies with a nondecreasing price path, the only potentially optimal introductory offer strategy is $(v(H|B), v(L|G), m(H))$.*

2. *Assume $m(H)v(H|G) > v(L|G)$. Among the set of pricing strategies with a nondecreasing price path, the only potentially optimal introductory offer strategy is $(v(H|B), v(H|G), m(H))$.*

Proof. See Appendix. ■

The lemma implies that either a uniform price or the introductory offer strategies $(v(H|B), v(L|G), m(H))$ or $(v(H|B), v(H|G), m(H))$ are optimal among the set of strategies with $p_1 \leq p_2$. The expected profit from $(v(H|B), v(L|G), m(H))$ is

$$\pi^{IO}(v(H|B), v(L|G), m(H)) = m(H)v(H|B) + \rho\,[1 - m(H)]\,v(L|G), \tag{15}$$

and the profit from $(v(H|B), v(H|G), m(H))$ is

$$\pi^{IO}(v(H|B), v(H|G), m(H)) = m(H)v(H|B) + \rho m(H)\,[1 - m(H)]\,v(H|G). \tag{16}$$

Comparing the profits with those under uniform pricing, we find that introductory offer strategy $(v(H|B), v(L|G), m(H))$ is optimal if

$$\begin{aligned} v(L|G) > m(H)v(H|G) > v(H|B) \quad > \quad & \max\left\{\rho v(L|G), \right. \\ & \left. \frac{v(L|B) - \rho\,[1 - m(H)]\,v(L|G)}{m(H)}\right\} \end{aligned}$$

Introductory offer strategy $(v(H|B), v(H|G), m(H))$ is optimal if

$$\begin{aligned} m(H)v(H|G) > v(L|G) \geq v(H|B) \quad > \quad & \max\left\{\rho m(H)v(H|G), \right. \\ & \left. \frac{v(L|B)}{m(H)} - \rho\,[1 - m(H)]\,v(H|G)\right\}. \end{aligned}$$

Final sales versus uniform pricing. We now turn to final sales strategies, where $p_1 > p_2$. Since the monopolist optimally sets the first-period capacity $k_1 = k$, a final sales strategy can be summarized by the triplet (p_1, p_2, k).

Let us first consider those final sales strategies, which are potentially optimal when demand shocks are weak (see our analysis above), namely $(v(H|G), v(H|B), m(H))$ and $(v(L|G), v(L|B), 1)$. Expected profits from the former strategy are

$$\pi^{FS}(v(H|G), v(H|B), m(H)) = [\rho v(H|G) + (1 - \rho)v(H|B)]\,m(H). \tag{17}$$

Note that this final sales strategy dominates the uniform price $p = v(H|G)$, but (in contrast to the case of weak demand shifts) not necessarily the uniform price $p = v(H|B)$. Expected profits from final sales strategy $(v(L|G), v(L|B), 1)$ are

$$\pi^{FS}(v(L|G), v(L|B), 1) = \rho v(L|G) + (1 - \rho)v(L|B).$$

As in the case of weak demand shifts, this strategy dominates the uniform prices $p = v(L|G)$ and $p = v(L|B)$. In contrast to the case of weak demand shifts, there is a third final sales strategy which can be optimal, namely $(v(L|G), v(H|B), 1)$. Note that this final sales strategy does not correspond to any quantity setting strategy (since the monopolist sells a mass 1 in the good state but only a mass $m(H)$ in the bad demand state). This strategy yields profits

$$\pi^{FS}\,(v(L|G), v(H|B), 1) = \rho v(L|G) + (1 - \rho)m(H)v(H|B), \tag{18}$$

and thus dominates the uniform price $p = v(H|B)$. There are no other profitable final sales strategies. Hence, we have the following result.

Lemma 19 *Under strong vertical demand shifts, uniform pricing strategies are less profitable than final sales strategies.*

Final sales versus introductory offers. Since uniform prices are always dominated by final sales strategies (and sometimes by introductory offers), any optimal pricing strategy must either be a final sales or an introductory offer strategy.

If $m(H)v(H|G) > v(L|G)$, the only introductory offer strategy which can be optimal in the space of strategies with nondecreasing price paths is $(v(H|B), v(H|G), m(H))$. Comparing equations (16) and (17), we find that this strategy is (weakly) dominated by the final sales strategy $(v(H|G), v(H|B), m(H))$ if and only if $m(H)v(H|G) \geq v(L|G)$, which is the necessary condition for the introductory offer strategy to dominate other nondecreasing pricing strategies.

If $m(H)v(H|G) < v(L|G)$, the only introductory offer strategy which can be optimal in the space of strategies with nondecreasing price paths is $(v(H|B), v(L|G), m(H))$. Comparing equations (15) and (18), we find that this strategy is (weakly) dominated by the final sales strategy $(v(L|G), v(H|B), 1)$ if and only if $m(H)v(H|G) \leq v(L|G)$, which is the necessary condition for the introductory offer strategy to dominate other nondecreasing pricing strategies.

Hence, any introductory offer strategy is dominated by some final sales strategy.

Lemma 20 *Under strong vertical demand shifts, any optimal pricing strategy is a final sales strategy, and so $p_1 > p_2$.*

Vertical demand shifts: Main results. Summarizing our results on optimal pricing under strong and weak demand shifts (lemmas 17 and 20), we can state the following proposition.

Proposition 21 *In the model with vertical demand shifts, the optimal strategy of the monopolist involves final sales and hence a strictly decreasing price path, $p_1 > p_2$.*

Rationing rule. Using the same arguments as in the previous section (see the discussion of strong horizontal demand shifts), we can show that our results are robust to introducing any positively selective rationing rule. We summarize this observation in the following remark.

Remark 4 *Proposition 21 holds for any positively selective rationing rule.*

Strategy space. As before, we derived our results assuming that the monopolist can set prices and quantities in only two periods. Our results remain unchanged if we allow for more than two periods.

Remark 5 *Consider an extended strategy space in which, for any finite number of periods T, the monopolist sets prices and capacities. Any optimal strategy in the two-period model remains optimal in the model with $T > 2$ periods.*[15]

Proof. The proof is rather lengthy and is available from the authors upon request. ■

6 Discussion and Conclusion

In this paper we have shown that introductory offer strategies are never optimal if the monopolist can use final sales strategies. Furthermore, we have shown that, under horizontal demand shifts, the optimal pricing policy can generate intertemporal price dispersion.

[15] To write a two-period final sales strategy in the T-period extension, we can set $p_3, ..., p_T = \infty$ and $k_2 = k_3 = ... = k_T = k$.

Below, we discuss our key assumptions. We distinguish between assumptions concerning market characteristics, demand side characteristics, and supply side characteristics.

Market characteristics: Rationing. In our analysis, we have assumed random or proportional rationing (where high type consumers are rationed with the same probability as low types). In general, the optimal strategy of the monopolist depends on the particular rationing rule. Clearly, any change in the rationing rule toward efficient (or parallel) rationing (so that high type consumers are rationed with a lower probability than low types) makes it more difficult for the monopolist to use rationing as a tool of her optimal strategy. This will reduce the profitability of final sales and introductory offer strategies. However, if the optimal strategy involves rationing with probability 1, then the rationing rule becomes irrelevant. In particular, any final sales strategy which is optimal under proportional rationing and involves price dispersion across demand states, but not within demand states, remains optimal under any rationing rule where low type consumers are rationed with a higher probability than high types (see Section 4).

Demand side: Multiplicity of consumer equilibrium. As pointed out before, for any given final sales strategy, there may be a multiplicity of consumer equilibria. In our analysis, we have selected the consumer equilibrium that is most favorable for the monopolist. We now want to argue that the best consumer equilibrium (from the monopolist's point of view) is often the unique consumer equilibrium if one introduces some heterogeneity amongst consumers of a particular type (in our example below, amongst high valuation consumers). We illustrate this observation in Appendix 2.

Demand side: Consumer decision making. For final sales strategy to work, consumers have to form beliefs about the likelihood of being rationed in the future. Since the probability of rationing depends on the behavior of other consumers, high type consumer do not have a dominant strategy. This requires consumers to be quite sophisticated in their decision-making. In contrast, when facing an introductory offer strategy (or a uniform price), consumers have a dominant strategy (and can follow a simple decision rule): "demand the good at the low price; if it is sold out at this price, buy it at the higher price (provided the price is less than your valuation)." In a world where consumers are not sophisticated decision makers, this implies that a firm may favor an introductory offer strategy (or a uniform price) over a final sales strategy.

Supply side: Capacity costs. We have assumed that the monopolist faces zero costs of capacity. Would introductory offers still be dominated if we allowed for positive capacity costs? In our analysis, we have shown that whenever an introductory offer strategy performs better than uniform pricing, it is dominated by some final sales strategy. One can show that any such final sales strategy can be implemented with a smaller capacity k than the dominated introductory offer strategy. Holding the capacity choices fixed, an increase in the costs of capacity thus makes the introductory offer strategy even less profitable relative to the final sales strategy. This suggests that introductory offer strategies are still dominated in the presence of positive capacity costs.

Supply side: Fixed capacity. In our analysis, we have assumed that the monopolist chooses total capacity k. Suppose now instead that the monopolist faces the restriction $k \leq \overline{k}$, where $\overline{k}$ is an exogenous capacity limit. This hypothesis may apply well to the case of ticket sales for concerts and the like, where the maximum number of seats in the concert hall is fixed. How does this assumption affect the profitability of the different pricing strategies? To address this question, let us consider the case of horizontal demand shifts when consumers only know their own valuation. In the absence of an exogenous capacity limit, the (maximum) output sold in the good demand state is lower under the best final sales strategy than under the best introductory offer strategy. We can then easily show that whenever there exists a final sales strategy which dominates introductory offers in the absence of exogenous capacity limits, there exists some final sales strategy which dominates introductory offers in the presence of exogenous capacity limits.[16] Hence, final sales strategies perform even better - relative to introductory

[16] Note that for sufficiently small exogenous capacities, namely $\overline{k} \leq m(H|B)$, optimal final sales and introductory offer

offer strategies - when the monopolist faces an (exogenous) capacity constraint.

However, this does not mean that introductory offer strategies cannot be optimal in this case. To see this, note that the monopolist may operate in an environment in which introductory offer strategies dominate final sales strategies. In section 3, we have shown that, in this case, introductory offer strategies are dominated by the uniform price $p = v(L)$. This may no longer hold when the monopolist faces an exogenous capacity limit. To be precise, without exogenous capacity limits, introductory offers are dominated by final sales strategies if $m(H|G) > [m(H|G) + m(L|G)]v(L)$, and by the uniform price $p = v(L)$ if the reverse inequality holds. The best introductory offer strategy involves, in the good demand state, a total quantity of

$$m(H|B) + m(L|B) + \rho\left(1 - \frac{m(H|B) + m(L|B)}{m(H|G) + m(L|G)}\right)m(H|G) \equiv \widehat{k},$$

which is always less than $m(H|G) + m(L|G)$. Hence, for $\overline{k} \in [\widehat{k}, m(H|G) + m(L|G))$, the exogenous capacity limit is not binding when the monopolist uses this strategy. On the other hand, for $\overline{k} < m(H|G) + m(L|G)$, the capacity constraint would be binding if the monopolist used the uniform price $p = v(L)$. Therefore, there exist parameter constellations under which introductory offer strategies are optimal in the presence of exogenous capacity limits. In particular, at $k = \widehat{k}$, the best introductory offer strategy and the uniform price $p = v(L)$ induce the same quantity sold in both demand states, but in the high demand state some units are sold at the high price when the monopolist uses the introductory offer.

To summarize, exogenous capacity limits make final sales more attractive relative to introductory offers. However, there exist parameter constellations under which introductory offer strategies are optimal since a (low) uniform price becomes less attractive when the monopolist faces an exogenous capacity constraint.

Supply side: The monopolist's decision making. In our model, we have assumed that the monopolist ex ante commits to a price for each period, a first-period capacity, and a total capacity. Not all of the available selling strategies require such commitment, however.

- *Price commitment.* Introductory offer strategies may require less (intertemporal) commitment power than final sales strategies. When using an introductory offer strategy, the monopolist has no incentive to change her price (or capacity) ex post in period 2. (In fact, it can be shown that an introductory offer strategy does not require commitment the second-period price.) In contrast, when using a final sales strategy, the monopolist has ex post an incentive to raise her capacity or price in period 2.

- *Capacity commitment.* A final sales strategy requires a commitment to total capacity. Such commitment can be implemented if total production is determined ex ante and if the production of additional units is sufficiently costly (high marginal costs or high fixed costs for an additional run). An introductory offer strategy requires a commitment to first-period capacity. This can be implemented if the monopolist produces in each period and has limited production capacity per period.

Hence, while our (simple) model predicts that we should not observe introductory offers (but rather final sales and uniform prices), the demanding commitment requirements for final sales strategies may give a rationale, within our framework, for the use of introductory offer strategies.[17]

strategies degenerate to uniform pricing.

[17] In particular, introductory offer strategies can be used in durable goods monopoly with a Coasian commitment problem (see Denicolo and Garella, 1999).

Another reason for the use of introductory offers strategies is that, in contrast to final sales, goods can be offered *concurrently* at different prices, and consumers can freely choose at which price they want to buy the good. Clearly, consumers will select the cheaper units first, and once these items are stocked out, high valuation consumers purchase the high priced units.

Our paper is also connected to several general themes in the industrial organization and microeconomics literature.

The economics of rationing. In this paper, we provide a justification for the use of rationing as part of the optimal strategy of a monopolist. However, ours is not the first paper pointing out that consumer rationing may be an equilibrium phenomenon. Apart from consumer segmentation, reasons for rationing include sunk investments by consumers (Gilbert and Klemperer, 2000), buying frenzies (DeGraba, 1995), bundling (DeGraba and Mohammed, 1999), and direct demand externalities (e.g. Becker, 1991, Karni and Levin, 1994). In our model, we do not need any of these demand-side considerations to generate rationing as an equilibrium outcome.

Price dispersion. Our paper contributes to the literature on price dispersion, initiated by Salop (1977). In our model, we distinguish between price dispersion across states and within states. Price dispersion across demand states means that the monopolist posts different prices for different periods but, in each demand state, trade occurs in only one period. Price dispersion within states means that, in at least one demand state, actual trade occurs at different prices. We have shown that, under vertical demand shifts, final sales induce price dispersion across states but not within states. In contrast, under horizontal demand shifts, final sales can give rise to price dispersion within states. (This holds for some final sales strategies when consumers learn the state of the world before purchasing, and for all optimal final sales strategies when consumers only learn their own type.) The difference between horizontal and vertical demand shifts may be explained as follows. Under vertical demand shifts, the number of high and low type consumers is the same across states. Hence, it is optimal for the monopolist not to separate between types, but rather between demand states. Consequently, any rationing occurs with probability 1. Under horizontal demand shifts, on the other hand, the number of high and low types depends on the demand state. The optimal final sales strategy may, therefore, be such that all consumers are served (at the low price) in the bad demand state, whereas rationing occurs with probability less than 1 in the good demand state. In our model, intertemporal price dispersion (within states) may thus arise in the absence of discounting (see, for instance, Stokey, 1979, for an analysis of price dispersion with discounting). We may interpret the probability of rationing in our model as a “discount rate”, which is endogenously determined.

Endogenous quality. Our paper is also loosely connected to the literature on product differentiation. In the context of final sales, we may interpret the probability of obtaining the good as the quality of the good. If the price of the good is adjusted by this probability, the model corresponds to a model of quality differentiation such as Mussa and Rosen (1978). In contrast to Mussa and Rosen (and the literature on vertical product differentiation in general), the good’s quality under a priority pricing scheme is endogenously determined by demand (and thus ultimately by prices and capacity). In the literature on product differentiation, on the other hand, the firm directly controls quality. The use of different qualities or classes of service is common in the pricing of tickets (see Rosen and Rosenfield, 1997, for an economic analysis).

Final sales as a marketing strategy. More generally, our paper contributes to the literature on selling strategies of a firm with market power. We have shown that a final sales strategy, which involves restricting total capacity and thus leads to consumer rationing, may effectively separate between demand states and consumers types. Sophisticated price-capacity strategies can thus be used by a monopolist to segment the market for a homogeneous good. Price-capacity strategies may be superior to other non-price strategies such as product differentiation, which may be more costly to implement or may reduce consumers’ reservation values (as in the case of damaged goods). Introductory offer strategies are less “sophisticated” than final sales strategies in that they do not fully segment the market by consumer types. In our world with commitment and rational consumers, we have shown that introductory offer strategies are never the optimal marketing strategy. Nevertheless, as argued above, since they require less sophisticated consumer behavior (and perhaps less commitment power by the seller), introductory offers may sometimes be the preferred selling strategy.

Appendix 1: Proofs

Proof of Proposition 5

As pointed out in the main text, any optimal final sales strategy must be such that all high types are just willing to demand the good at the high price. Hence, any optimal final sales strategy is of the form $(\widehat{p}_1(k), v(L), k)$, where $\widehat{p}_1(k)$ is chosen so as to make high type consumers just indifferent between purchasing at $\widehat{p}_1(k)$ and delaying the purchase.

Step 1. Suppose $k \in [m(H|B), m(H|G)]$. In this case, rationing occurs even at the high first-period price (when demand is in the good state). If $k \leq \min\{m(H|G), m(H|B) + m(L|B)\}$, a high type consumer is indifferent between purchasing at $\widehat{p}_1(k)$ and postponing his purchase if

$$\begin{aligned} & \left\{Q(G|H)\frac{k}{m(H|G)} + 1 - Q(G|H)\right\}[1 - \widehat{p}_1(k)] \\ = \quad & [1 - Q(G|H)]\left(\frac{k - m(H|B)}{m(L|B)}\right)[1 - v(L)]\,, \end{aligned}$$

and so

$$\widehat{p}_1(k) = 1 - \frac{[1 - Q(G|H)]\left(\frac{k - m(H|B)}{m(L|B)}\right)}{1 - \left(\frac{m(H|G) - k}{m(H|G)}\right)Q(G|H)}[1 - v(L)]\,.$$

The monopolist's expected profit is then

$$\begin{aligned} & \pi^{FS}(\widehat{p}_1(k), v(L), k) \\ = \quad & \rho\widehat{p}_1(k)k + (1 - \rho)[\widehat{p}_1(k)m(H|B) + v(L)[k - m(H|B)]], \end{aligned}$$

which is non-linear in k.

Similarly, if $k \in [\min\{m(H|G), m(H|B) + m(L|B)\}, m(H|G)]$, the first-period price is equal to

$$\widehat{p}_1(k) = 1 - \frac{[1 - Q(G|H)]}{1 - \left(\frac{m(H|G) - k}{m(H|G)}\right)Q(G|H)}[1 - v(L)]\,,$$

and the expected profit is given by

$$\begin{aligned} & \pi^{FS}(\widehat{p}_1(k), v(L), k) \\ = \quad & \rho\widehat{p}_1(k)k + (1 - \rho)[\widehat{p}_1(k)m(H|B) + v(L)m(L|B)], \end{aligned}$$

which again is non-linear in k.

Let

$$\overline{p}_1(k) \equiv 1 - [1 - Q(G|H)]\max\left\{\frac{k - m(H|B)}{m(L|B)}, 1\right\}[1 - v(L)]\,,$$

and note that $\overline{p}_1(k) > \widehat{p}_1(k)$ for all $k < m(H|G)$, and $\overline{p}_1(m(H|G)) = \widehat{p}_1(m(H|G))$. Next, let

$$\begin{aligned} & \overline{\pi}^{FS}(\overline{p}_1(k), v(L), k) \equiv \\ & \rho\overline{p}_1(k)m(H|G) + (1 - \rho)\left\{\overline{p}_1(k)m(H|B) + v(L)\min\{k - m(H|B)\}, m(L|B)\right\}, \end{aligned}$$

and note that $\overline{\pi}^{FS}(\overline{p}_1(k), v(L), k) > \pi^{FS}(\widehat{p}_1(k), v(L), k)$ for all $k < m(H|G)$, and $\overline{\pi}^{FS}(\overline{p}_1(m(H|G)), v(L), m(H|G)) = \pi^{FS}(\widehat{p}_1(m(H|G)), v(L), m(H|G))$. Moreover, observe that $\overline{\pi}^{FS}(\overline{p}_1(k), v(L), k)$ is linear in k for $k \leq \min\{m(H|G), m(H|B) + m(L|B)\}$, and independent of k on $[m(H|B) + m(L|B), m(H|G)]$.

We now claim that $\overline{\pi}^{FS}(\overline{p}_1(m(H|B)), v(L), m(H|B))$ is equal to $\pi^U(1)$, the profit from the uniform price $p = 1$. To see this, note that $\overline{p}_1(m(H|B)) = 1$, and

$$\overline{\pi}^{FS}(1, v(L), m(H|B)) = \rho m(H|G) + (1-\rho)m(H|B) = \pi^U(1).$$

Since $\overline{\pi}^{FS}(\overline{p}_1(k), v(L), k)$ is linear for $k \le \min\{m(H|G), m(H|B)+m(L|B)\}$, it follows that an optimal final sales strategy must have $k \ge \min\{m(H|G), m(H|B)+m(L|B)\}$. Moreover, since $\overline{\pi}^{FS}(\overline{p}_1(k), v(L), k)$ is constant on $[m(H|B)+m(L|B), m(H|G)]$, strictly larger than $\pi^{FS}(\widehat{p}_1(k), v(L), k)$ for all $k < m(H|G)$, and $\overline{\pi}^{FS}(\overline{p}_1(m(H|G)), v(L), m(H|G)) = \pi^{FS}(\widehat{p}_1(m(H|G)), v(L), m(H|G))$, an optimal final sales strategy must have $k \ge m(H|G)$. Hence, there cannot be rationing at the high price.

Step 2. Suppose $k \in [m(H|G), m(H|G)+m(L|G)]$. In this case, rationing can only occur at the low price. The indifference condition for high type consumers can now be written as

$$\begin{aligned} 1-\widehat{p}_1(k) &= \left\{Q(G|H)\left(\frac{k-m(H|G)}{m(L|G)}\right)\right. \\ &\quad \left.+[1-Q(G|H)]\min\left[1, \frac{k-m(H|B)}{m(L|B)}\right]\right\}[1-v(L)]. \end{aligned}$$

The expected profit is then

$$\begin{aligned} & \pi^{FS}(\widehat{p}_1(k), v(L), k) \\ = \; & \rho\left\{\widehat{p}_1(k)m(H|G) + v(L)\left[k-m(H|G)\right]\right\} \\ & +(1-\rho)[\{\widehat{p}_1(k)m(H|B) + v(L)\min\left[m(L|B), k-m(H|B)\right]\}, \end{aligned}$$

which is linear in k on $[m(H|G), m(H|B)+m(L|B)]$, provided this interval is non-empty (i.e., when horizontal demand shifts are weak), and on $[\max\{m(H|G), m(H|B)+m(L|B)\}, m(H|G)+m(L|G)]$.

Hence, under strong horizontal demand shifts (where $m(H|G) \ge m(H|B)+m(L|B)$), the unique candidate for an interior optimum is at capacity $k = m(H|G)$. Under weak horizontal demand shifts (where $m(H|G) < m(H|B)+m(L|B)$), there are two candidates: $k = m(H|G)$ and $k = m(H|B)+m(L|B)$. ■

Proof of Lemma 6

Under horizontal demand shifts, $m(H|G) \ge m(H|B)$ and $m(H|G)+m(L|G) \ge m(H|B)+m(L|B)$ with at least one strict inequality. We prove the assertion under the assumption that $m(H|G) > m(H|B)$.(The proof for the case when $m(H|G) = m(H|B)$ and $m(H|G)+m(L|G) > m(H|B)+m(L|B)$ proceeds analogously.) First, note that if $k \in (m(H|B), m(H|G)]$, then $p_1^G(k) = 1 > p_1^B(k)$, and so the assertion holds true. Second, if $k \in [m(H|B)+m(L|B), m(H|G)+m(L|G)]$, then $p_1^B(k) = v(L) < p_1^G(k)$ Third, assume that $k \in (m(H|G), m(H|B)+m(L|B))$. It is immediate to see that $p_1^G(k) > p_1^B(k)$ if and only if

$$\frac{k-m(H|G)}{m(L|G)} < \frac{k-m(H|B)}{m(L|B)}.$$

Since $m(H|G) > m(H|B)$, this inequality is satisfied (and hence the claim is shown) if $m(L|G) \ge m(L|B)$. Suppose now that $m(L|G) < m(L|B)$. Then, $p_1^G(k) > p_1^B(k)$ if and only if

$$k < \frac{m(H|G)m(L|B) - m(L|G)m(H|B)}{m(L|B) - m(L|G)}.$$

The r.h.s. of this inequality is larger than or equal to $m(H|G)+m(L|G)$ if and only if $m(H|G)+m(L|G) \ge m(H|B)+m(L|B)$, which we assumed. Hence, $p_1^G(k) > p_1^B(k)$ for all $k \in (m(H|B), m(H|G)+m(L|G))$. ■

Proof of Lemma 7

In the main text, we have shown that the final sales strategy $(1, v(L), m(H|G))$ is the only potentially optimal one amongst the family of final sales strategies $(p_1^G(k), v(L), k)$. We now consider final sales strategies of the family $(p_1^B(k), v(L), k)$, where high types are just induced to buy in the first period when demand is in the bad state. We have to show that these strategies are dominated either by the final sales strategy $(1, v(L), m(H|G))$ or by a uniform price. First, note that we can exclude capacities $k \leq m(H|B)$ as, in this case, $p_1^\sigma(k) = 1$ for $\sigma = G, B$, and in both demand states all capacities is sold in the first period at a price of 1, and the monopolist earns profit k. (The monopolist would then be strictly better off by charging a uniform price of 1 and serving all demand at that price.) Second, note that we can exclude capacities $k \geq m(H|B) + m(L|B)$ as this would result in uniform pricing: $p_1^B(k) = v(L)$ for $k \geq m(H|B) + m(L|B)$. It remains to consider capacities $k \in (m(H|B), m(H|B) + m(L|B))$; the associated expected profit is

$$\tilde{\tilde{\pi}}^{FS}(k) = \rho k p_1^B(k) + (1-\rho)\left\{m(H|B)p_1^B(k) + [k - m(H|B)]v(L)\right\},$$

which is quadratic in capacity k. We now show that, for $k \in (m(H|B), m(H|B) + m(L|B))$, this strategy is dominated by the final sales strategy $(1, v(L), m(H|G))$ or the uniform price $p = 1$.

For $k \in (m(H|B), m(H|B) + m(L|B))$, the profit from final sales strategy $(p_1^B(k), v(L), k)$ in the bad state is $m(H|B)p_1^B(k) + [k - m(H|B)]v(L)$. Since this expression is linear in k, it is bounded above by $\max\{m(H|B), (m(L|B) + m(H|B))v(L)\}$. The profit in the good state is $kp_1^B(k)$.

Step 1. Suppose that $(m(L|B) + m(H|B))v(L) \geq m(H|B)$. Then, the strategy $(p_1^B(k), v(L), k)$ is dominated by the final sales strategy $(1, v(L), m(H|G))$. To see this, note that the latter strategy gives weakly higher profit in the bad state (namely, $(m(L|B)+m(H|B))v(L)$). It is therefore sufficient to show that, in the good state, the profit from $(1, v(L), m(H|G))$, which is $m(H|G)$, is greater or equal to the profit from $(p_1^B(k), v(L), k)$, which is $kp_1^B(k)$. Since $p_1^B(k) \leq 1$ and $k \leq m(H|B) + m(L|B) < m(H|G)$, this profit ranking holds good.

Step 2. Suppose now that $(m(L|B) + m(H|B))v(L) < m(H|B)$. Then, the strategy $(p_1^B(k), v(L), k)$ is dominated by the uniform price $p = 1$. To see this, note that the uniform price gives a weakly higher profit in the bad state, namely $m(H|B)$. Moreover, uniform pricing gives a profit of $m(H|G)$ in the good state, which is strictly higher than the corresponding profit from $(p_1^B(k), v(L), k)$, which is $kp_1^B(k) \leq k < m(H|G)$.

This shows that a final sales strategy of the family $(p_1^B(k), v(L), k)$ can never be optimal. ■

Proof of Lemma 8

Step 1. We consider two families of final sales strategies (parameterized by capacity k), $\left(p_1^G(k), v(L), k\right)$ and $\left(p_1^B(k), v(L), k\right)$. As argued in the proof of lemma 7, we can exclude capacities $k \leq m(H|B)$ and $k \geq m(H|\sigma) + m(L|\sigma)$ for strategy $(p_1^\sigma(k), v(L), k)$, $\sigma = G, B$, since for these capacities all items would always be sold at the same price. For capacity $k \in (m(H|B), m(H|G) + m(L|G))$, the expected profit from final sales strategy $\left(p_1^G(k), v(L), k\right)$ is given by

$$\begin{aligned}\pi^{FS}(p_1^G(k), v(L), k) &= (1-\rho)\min\{k, m(H|B) + m(L|B)\}v(L) \\ &\quad + \rho\left[p_1^G(k)\min\{m(H|G), k\}\right. \\ &\quad \left. + \max\{k - m(H|G), 0\}v(L)\right].\end{aligned}$$

Since the profit function is piecewise linear in k, there are two potentially optimal capacity levels: $k = m(H|G)$ and $k = m(H|B) + m(L|B)$. For capacity $k \in (m(H|B), m(H|B) + m(L|B))$, the expected profit from final sales strategy $\left(p_1^B(k), v(L), k\right)$ is

$$\begin{aligned}\pi^{FS}(p_1^B(k), v(L), k) &= (1-\rho)\left[p_1^B(k)m(H|B) + [k - m(H|B)]v(L)\right] \\ &\quad +\rho\left[p_1^B \min\{m(H|G), k\} + \max\{k - m(H|G), 0\}\, v(L)\right].\end{aligned}$$

Since the profit function is piecewise linear in k, there is only one potentially optimal capacity level: $k = m(H|G)$.

We thus have three potentially optimal final sales strategies.

- FS 1: $(1, v(L), m(H|G))$ with expected profit

$$\pi^{FS}(1, v(L), m(H|G)) = (1-\rho)m(H|G)v(L) + \rho m(H|G).$$

- FS 2: $(p_1^G\,(m(H|B) + m(L|B))\,, v(L), m(H|B) + m(L|B))$ with expected profit

$$\begin{aligned}& \pi^{FS}(p_1^G(m(L|B) + m(H|B)), v(L), m(L|B) + m(H|B)) \\ =\ & \rho\left[m(H|G)p_1^G(m(L|B) + m(H|B)) + [m(L|B) + m(H|B) \right. \\ & \left. -m(H|G)]v(L)\right] + (1-\rho)[m(L|B) + m(H|B)]v(L),\end{aligned}$$

  where

$$p_1^G(m(L|B) + m(H|B)) = 1 - \left(\frac{m(L|B) + m(H|B) - m(H|G)}{m(L|G)}\right)[1 - v(L)]\,.$$

- FS 3: $(p_1^B\,(m(H|G))\,, v(L), m(H|G))$ with expected profit

$$\begin{aligned}& \pi^{FS}(p_1^B(m(H|G)), v(L), m(H|G)) \\ =\ & [\rho m(H|G) + (1-\rho)m(H|B)]\, p_1^B(m(H|G)) \\ & +(1-\rho)[m(H|G) - m(H|B)]v(L),\end{aligned}$$

  where

$$p_1^B(m(H|G)) = 1 - \left(\frac{m(H|G) - m(H|B)}{m(L|B)}\right)[1 - v(L)]\,.$$

Step 2. We now show that FS 3 is dominated by uniform pricing. FS 3 is (weakly) more profitable than the uniform price $p = 1$ if and only if

$$p_1^B(m(H|G)) \geq 1 - \frac{(1-\rho)[m(H|G) - m(H|B)]v(L)}{\rho m(H|G) + (1-\rho)m(H|B)}. \tag{19}$$

Inserting the expression for $p_1^B(m(H|G))$, this inequality can be rewritten as

$$\begin{aligned}& \rho[1 - v(L)]m(H|G) \\ \leq\ & (1-\rho)\left\{[m(L|B) + m(H|B)]v(L) - m(H|B)\right\}\end{aligned} \tag{20}$$

That is, for FS 3 to (weakly) dominate the uniform price $p = 1$, equation (20) must hold good.

FS 3 is (weakly) more profitable than the uniform price $p = v(L)$ if and only if

$$p_1^B(m(H|G)) - v(L) \geq \frac{\rho m(L|G) + (1-\rho)\left[m(H|B) + m(L|B) - m(H|G)\right]}{\rho m(H|G) + (1-\rho)m(H|B)} v(L).$$

Inserting the expression for $p_1^B(m(H|G))$ and multiplying both sides by $m(L|B)\times[\rho m(H|G)+(1-\rho)m(H|B)]$, we obtain

$$\begin{aligned} & [\rho m(H|G)+(1-\rho)m(H|B)]\,[m(H|B)+m(L|B)-m(H|G)]\,[1-v(L)] \\ \geq\; & \rho m(L|G)m(L|B)+(1-\rho)m(L|B)\,[m(H|B)+m(L|B)-m(H|G)]\,v(L). \end{aligned}$$

This inequality can be rewritten as

$$\begin{aligned} & \rho m(L|G)m(L|B)+[m(H|B)+m(L|B)-m(H|G)] \\ & \times\{(1-\rho)m(L|B)v(L)-[\rho m(H|G)+(1-\rho)m(H|B)]\,[1-v(L)]\} \\ \leq\; & 0. \end{aligned}$$

Hence, for FS 3 to (weakly) dominate the uniform price $p=v(L)$ the second term must be (strictly) negative, and so

$$(1-\rho)m(L|B)v(L)<[\rho m(H|G)+(1-\rho)m(H|B)]\,,$$

or equivalently

$$\begin{aligned} & \rho[1-v(L)]m(H|G) \\ >\; & (1-\rho)\{[m(L|B)+m(H|B)]v(L)-m(H|B)\} \end{aligned} \tag{21}$$

Note that condition (21) is identical to (20), except that the inequality is reversed. Hence, FS 3 is dominated by either the uniform price $p=1$ or the uniform price $p=v(L)$. ■

Proof of Lemma 11

First note that we can restrict ourselves to 4 periods because of 2 states and 2 types. Furthermore, $p\geq v(L)$ in all periods with positive sales. Note that if $p>v(L)$ we are in the case of demand uncertainty with a single type because low-type consumers never buy. In this case, a one-price or two-price strategy is optimal (compare example 1). Hence we can restrict attention to strategies with $p_t=v(L)$ in at least one period. Furthermore note that we only need to consider prices $v(L)$, 1, and prices such that high-type consumers in the good or in the bad state are indifferent between buying in that period or delaying their purchase.

- Suppose $p_1=1$. We now argue that for such a price a more-than-two-price strategy cannot be optimal. Note that one must have $k\leq m(H|G)$ to have positive sales in period 1. Hence, the low type does not buy in the good state. Then the pricing in periods later than period 1 can only target consumers in the bad demand state. From the remark in Section 3 we know that under demand certainty a uniform price dominates. Hence, there cannot exist optimal more-than-two-price strategies with $p_1=1$.

- Suppose $p_1=p_1^G$. This means that the first period price is set such that high-type consumers in the good state are made indifferent between buying in period 1 or in some later period. Clearly, $k>m(H|G)$ because otherwise $p_1=1$. For $k>m(H|G)$ and $p_t=v(L)$ in some period t all low-type consumers will buy in period t in the good state. Also all consumers in the bad state will buy at this price. This is not a more-than-two-price strategy.

- Suppose $p_1=p_1^B$. This means that the first period price is set such that high-type consumers in the bad state are made indifferent between buying in period 1 or in some later period. Clearly, $k>m(H|B)$ and $p_t=v(L)$ in some period t.

  First, suppose that no units are sold between periods 1 and t. Then we can set $t=2$. For certain quantities k_2 the monopolist can sell additional units in the good state at a price $p=1$. This then constitutes a three-price strategy. For any given $k_2 \in [m(H|B), m(L|B) + m(H|B)]$ it is optimal to set $k_1 = m(H|B)$. Profits then are linear in k_2 so that the optimal first-period price is either $v(L)$ or 1. Consequently, a more-than-two-price strategy cannot be optimal.

Second, suppose that some units are sold between periods 1 and t. Then we can set $t = 3$ so that $p_3 = v(L)$. Since $m(H|G) > k > m(H|B)$ and $k_1 = m(H|B)$ the monopolist can sell up to $k - m(H|B)$ units in period 2 at $p_2 = 1$. This means that the monopolist sells k_1 units at price p_1^B, $k - k_1$ units at price $p_2 = 1$ in the good demand state and $k - k_1$ units at price $p_3 = v(L)$ in the bad demand state. Such a strategy cannot be optimal because, considering only the bad state, we know that under certain demand uniform pricing weakly dominates (see the remark in Section 3) and, considering the good demand state, profits are dominated by $p = 1$ and $k = m(H|G)$. Corresponding profits can be obtained through a uniform price or two-price strategy.

- Suppose $p_1 = v(L)$. We distinguish two cases: k_1 is less or greater than $m(L|B) + m(H|B)$.

  Suppose $k_1 \in [m(L|B)+m(H|B), m(L|G)+m(H|G))$. Then all consumers in the bad state obtain the good in period 1 at $p_1 = v(L)$. Hence, there is unsatisfied demand only in the good state. For this unsatisfied demand we know that the optimal strategy from period 2 onward is uniform pricing (because uncertainty does not play any role so that the remark in Section 3 applies). Overall, uniform pricing or a two-price strategy is optimal.

  Suppose $k_1 < m(L|B) + m(H|B)$. In this case there is unsatisfied demand in both states. For given $p_1 = v(L)$ and k_1 we can look at the truncated problem from period 2 onward (because all consumers try to buy in period 1 in any case). We then know that either $p_2 = v(L)$ or a two-price strategy is optimal. Clearly, we only need to consider the latter case. Here we know that either uniform pricing or final sales are optimal in the truncated problem (as characterized in the main text). Overall we either find that a two-price strategy is optimal (in uniform pricing is optimal in the truncated problem) or that a strategy with $p_1 = v(L)$, $p_2 = 1$, $p_3 = v(L)$ can be optimal. Clearly, if some consumers buy in period 3 it can be only consumers in the bad state but then no consumer in the bad state will buy in period 2. Then profits in the bad state are less or equal to $v(L)[m(L|B) + m(H|B)]$. Profits in the good state are less than profits under uniform pricing. Consequently, the above three-price strategy is dominated by an uniform price or two-price (final sales) strategy ($p = v(L)$ or $p_1 = 1$ and $p_2 = v(L)$, $k = m(H|G)$). ■

Proof of Lemma 12

With the same argument as in Lemma 11 we can restrict ourselves to 4 periods, $p \geq v(L)$ in all periods with positive sales, and $p_t = v(L)$ in at least one period. Furthermore prices are $v(L)$, 1, or such that high-type consumers in the good or in the bad state are indifferent between buying in that period or delaying their purchase.

- Suppose $p_1 = 1$. If the total capacity for goods subsequently priced at less than 1 does not exceed $m(H|G)$, all high-type consumers buy in the good state in period 1. Otherwise, no consumer would buy in period 1. Hence, we only have to consider total capacity $k \leq m(H|G)$ or the constellation $p_n = 1$ and $k_{n-1} \leq m(H|G)$. Start with the first case. Then no low-type consumer buys in the good state so that we only have to consider prices p_2 and p_3 which target consumers in the bad state. If $k \leq m(H|B)$ there are only sales in the first period so that we can restrict attention to k with $m(H|B) < k \leq m(H|G)$. If there was rationing in the bad state in period 2 the monopolist would set $p_3 = 1$ so that we were in the second case. Therefore, suppose that there is no rationing in the bad state in period 2. This implies $p_2 \geq p_3$. The best two-price strategy with $p_2 \geq p_3$ and $k \leq m(H|G)$ then is $(p_2 = p_1^B(m(H|G)), p_3 = v(L), k = m(H|G))$. Taken together this gives rise to the three-price final sales strategy ($p_1 = 1$, $p_2 = p_1^B(m(H|G))$,

$p_3 = v(L)$, $k_1 = k_2 = k = m(H|G)$). Consider now the second case. High type consumers who are rationed until the last period buy the good at $p_n = 1$. Since $k_{n-1} \leq m(H|G)$ no low-type consumer buys in the good state. Hence we can restrict attention to three price strategies with $p_1 = 1$, $p_3 = 1$ and $k_2 \leq m(H|G)$. Clearly, $p_2 = v(L)$ because otherwise no low-type consumer would ever buy so that under this restriction a uniform price would be optimal. Then $k_1 = k_2$. We do not need to place any restriction on k except that it is large enough to meet all unsatisfied demand in the last period. If $k_2 \leq m(H|B)$ there are no sales in period 2. Hence the monopolist has to choose $k_2 \in (m(H|B), m(H|G)]$. Since profits are linear, we obtain as the only candidate for an optimal strategy in the second case a strategy with $k_2 = m(H|G)$. In this case demand is rationed in period 2 in the bad state with positive probability (and with probability 1 in the good state). The number $m(H|B) + m(L|B) - m(H|G)$ of consumer did not obtain the good in the bad state in period 2. A share $m(H|B)/[m(L|B) + m(H|B)]$ is of the high type. Hence, the monopolist must offer at least $\widetilde{m}$ extra units in period 3 to meet all unsatisfied demand at a price $p_3 = 1$. Taken together this gives rise to the three price strategy $(p_1 = 1, p_2 = p_1^B(m(H|G)), p_3 = v(L), k_1 = k_2 = k = m(H|G))$. This strategy dominates the above three-price final sales strategy if and only if $[m(H|B) + m(L|B)]v(L) > m(H|B)$. However, the three-price strategy dominates uniform pricing with $p = 1$ if and only if $[m(H|B) + m(L|B)]v(L) > m(H|B)$. Therefore, the strategy ($p_1 = 1$, $p_2 = p_1^B(m(H|G))$, $p_3 = v(L)$, $k_1 = k_2 = k = m(H|G)$) cannot be optimal.

- Suppose $p_1 = p_1^G$. This means that the first period price is set such that high-type consumers in the good state are made indifferent between buying in period 1 or in some later period. Clearly, the capacity for goods sold at a price less than 1 $\widetilde{k}$ has to be greater than $m(H|G)$ because otherwise $p_1 = 1$. For $\widetilde{k} \geq m(H|B) + m(L|B)$ and $p_t = v(L)$ in some period t all low-type consumers will buy in period t in the good state. Also all consumers in the bad state will buy at this price. This is not a more-than-two-price strategy. Consider therefore $\widetilde{k}$ with $m(H|G) < \widetilde{k} < m(H|B) + m(L|B)$. No consumer will buy in period 1 in the bad state. The monopolist can then choose between final sales and introductory offer for consumers in the bad state (a mixture cannot be optimal). In the first case the monopolist sets $p_1 = p_1^G(\widetilde{k})$, $p_2 = p_1^B(\widetilde{k})$, $p_3 = v(L)$, $k_1 = k_2 = m(H|G)$, $k_3 = \widetilde{k}$. Because profits are linear in $\widetilde{k}$ there does not exist such an optimal three-price strategy. In the second case the monopolist sets $p_1 = p_1^G(\widetilde{k})$, $p_2 = v(L)$, $p_1 = 1$, $k_1 \in [m(H|G), \widetilde{k}]$, $k_2 = \widetilde{k}$, $k_3 \in [\widetilde{k} + \widetilde{m}(\widetilde{k}), \infty)$ where

$$\widetilde{m}(\widetilde{k}) = [m(H|B) + m(L|B) - \widetilde{k}]\frac{m(H|B)}{m(L|B) + m(H|B)}.$$

Profits are

$$\begin{aligned}\pi(\widetilde{k}) \quad &= \quad \rho\{p_1^G(\widetilde{k})m(H|G) + v(L)[\widetilde{k} - m(H|G)]\} \\ &\quad\ +(1-\rho)[v(L)\widetilde{k} + 1 \cdot \widetilde{m}(\widetilde{k})]\end{aligned}$$

which are linear in $\widetilde{k}$. Hence such a three-price strategy cannot be optimal.

- Suppose $p_1 = p_1^B$. This means that the first period price is set such that high-type consumers in the bad state are made indifferent between buying in period 1 or in some later period. Clearly, $k > m(H|B)$ and $p_t = v(L)$ in some period t. We do not need to consider first-period capacities $k_1 < m(H|B)$ because $p_1^B(k_1)k_1$ is convex in k_1 on $[0, m(H|B)]$ so that profits are also strictly convex in k_1 on this set.

First, suppose that no units are sold between periods 1 and t. Then we can set $t = 2$. For certain quantities k_2 the monopolist can sell additional units in the good state at a price $p = 1$. This then constitutes a three-price strategy. Profits then are linear in k_2 on $[m(H|B), m(H|G)]$ so that the optimal second-period capacity is either $m(H|B)$ or $m(H|G)$. This implies that either there are no sales in period 2 or that there are no sales in period 3. Consequently, a more-than-two-price strategy cannot be optimal.

Second, suppose that some units are sold between periods 1 and t. Then we can set $t = 3$ so that $p_3 = v(L)$. If $m(H|G) > k > m(H|B)$ and $k_1 = m(H|B)$ the monopolist can sell up to $k - m(H|B)$ units in period 2 at $p_2 = 1$. This means that the monopolist sells $m(H|B)$ units at price p_1^B, $k - m(H|B)$ units at price $p_2 = 1$ in the good demand state and $k - m(H|B)$ units at price $p_3 = v(L)$ in the bad demand state. Such a strategy cannot be optimal because, considering only the bad state, we know that under certain demand uniform pricing weakly dominates (see the remark in Section 3) and, considering the good demand state, profits are dominated by $p = 1$ and $k = m(H|G)$. Corresponding profits can be obtained through a uniform price or two-price strategy. If $m(L|B) + m(H|B) > k > m(H|G)$ and $k_1 = m(H|B)$, the monopolist sells $m(H|B)$ units at price p_1^B, $m(H|G) - m(H|B)$ units at price $p_2 = p_1^G(k)$ in the good demand state and $k - m(H|\sigma)$ units at price $p_3 = v(L)$ in demand state σ. By setting $k_1 = k_2 = m(H|G)$ the monopolist can increase profits because it sells $m(H|B)$ units at price $p_1^G(k)$ instead of $p_1^B(k)$. Hence, this cannot constitute an optimal three-price strategy.

- Suppose $p_1 = v(L)$. We distinguish two cases: (i) $m(L|B)+m(H|B) < k_1 < m(L|G)+m(H|G)$,(ii) $k_1 \leq m(L|B) + m(H|B)$, .

  In the first case all consumers in the bad state obtain the good in period 1 at $p_1 = v(L)$. Hence, there is unsatisfied demand only in the good state. For this unsatisfied demand we know that the optimal strategy from period 2 onward is uniform pricing (because uncertainty does not play any role so that the remark in Section 3 applies). Overall, uniform pricing or a two-price strategy is optimal.

  In the second case, there is rationing in both demand states. For given $p_1 = v(L)$ and k_1 we can look at the truncated problem from period 2 onward (because all consumers try to buy in period 1 in any case). We then know that either $p_2 = v(L)$ or a strategy with $p_2 > v(L)$ is optimal. Clearly, we only need to consider the latter case. In the truncated problem, we have characterized the optimal strategy (as characterized in the main text and the lemma). Clearly, we only need to consider possibly optimal strategies in the truncated problem which are not uniform pricing. First consider the truncated model with three-price strategies of the second type: Setting $k_1 > 0$ reduces the number of units sold at the high price in the good state while increasing the total number of units sold. In the bad state no unit will be sold in period 2 in any case so that a change in k_1 is without effect. Hence, writing $\alpha = k_1/[m(H|G) + m(L|G)]$ one has that in the truncated problem the new mass is $\widehat{m}(H|G) = (1 - \alpha)m(H|G)$. Clearly profits are linear in α on $[0, [m(L|B) + m(H|B)]/[m(L|G) + m(H|G)]]$. Hence, such a strategy with α in the interior cannot be optimal. Second consider the truncated model with three price strategies of the first type. A change in k_1 changes the composition of demand in each state: we can write $k_1 = \alpha[m(H|G) + m(L|G)] = \beta[m(H|B) + m(L|B)]$ which implicitly defines $\beta(\alpha)$, which is linear in α. It then straightforward to check that profits depending on α and β are linear in these two variables (for $k_1 \in [0, m(L|B) + m(H|B)]$) so that a strategy with k_1 in the interior cannot be optimal. Third consider, the truncated model with final sales strategy (starting in period 2) $p_2 = p_1^G(\widehat{m}(H|B) + \widehat{m}(L|B))$, $p_3 = v(L)$, $k = k_1 + \widehat{m}(H|B) + \widehat{m}(L|B)$. Profits are linear in α and β (as defined above) for $k_1 \in [0, m(L|B) + m(H|B)]$. Hence, such a three-price strategy cannot be optimal. We do not consider the truncated model with final sales strategy $p_2 = 1$, $p_3 = v(L)$, $k = k_1 + \widehat{m}(H|G)$ because this final sales strategy is dominated in the n-period extension of the model, as argued in the main text below the lemma. ■

Proof of Lemma 14. We have to consider the following price pairs (p_1, p_2): $(v(L|B),(L|G))$, $(v(L|B), v(H|B))$, $(v(L|B), v(H|G))$, $(v(L|G), v(H|B))$, $(v(L|G), v(H|G))$, and $(v(H|B), v(H|G))$.

We first consider prices $(v(L|B), v(L|G))$. Expected profits are

$$\begin{aligned}
& \pi^{IO}(v(L|B), v(L|G), k_1) \\
= \;& (1-\rho)[v(L|B)k_1 + v(L|G)(1-k_1)m(H)] \\
& +\rho[v(L|B)k_1 + v(L|G)(1-k_1)] \\
= \;& v(L|B)k_1 + v(L|G)(1-k_1)((1-\rho)m(H)+\rho).
\end{aligned}$$

The expression on the right-hand side is linear in k_1. Hence, any introductory offer strategy with $k_1 \in (0,1)$ is dominated by a uniform price.

Next, consider $(v(L|B), v(H|B))$. Expected profits are

$$\begin{aligned}
\pi^{IO}(v(L|B), v(H|B), k_1) &= (1-\rho)[v(L|B)k_1 + v(H|B)(1-k_1)m(H)] \\
& \quad +\rho[v(L|B)k_1 + v(H|B)(1-k_1)m(H)] \\
&= v(L|B)k_1 + v(H|B)(1-k_1)m(H).
\end{aligned}$$

Again, the expression on the right-hand side is linear in k_1. Hence, any introductory offer strategy with $k_1 \in (0,1)$ is dominated by a uniform price.

Next, consider $(v(L|B), v(H|G))$. Expected profits are

$$\begin{aligned}
& \pi^{IO}(v(L|B), v(H|G), k_1) \\
= \;& (1-\rho)v(L|B)k_1 + \rho[v(L|B)k_1 + v(H|G)(1-k_1)m(H)] \\
= \;& v(L|B)k_1 + v(H|G)(1-k_1)\rho m(H).
\end{aligned}$$

Again, the expression on the right-hand side is linear in k_1. Hence, any introductory offer strategy with $k_1 \in (0,1)$ is dominated by a uniform price.

Next, consider $(v(L|G), v(H|B))$. Expected profits are

$$\begin{aligned}
& \pi^{IO}(v(L|G), v(H|B), k_1) \\
= \;& (1-\rho)[v(L|G)\min\{k_1, m(H)\} + v(H|B)\max\{0, m(H)-k_1\}] \\
& +\rho[v(L|G)k_1 + v(H|B)(1-k_1)m(H)]
\end{aligned}$$

Since expected profits are piecewise linear, the only potentially optimal introductory offer strategy with $k_1 \in (0,1)$ has first-period capacity $k_1 = m(H)$. The expected profit from introductory offer strategy $(v(L|G), v(H|B), m(H))$ is given by

$$\pi^{IO}(v(L|G), v(H|B), m(H)) = v(L|G)m(H) + v(H|B)m(H)(1-m(H))\rho. \tag{22}$$

Next, consider $(v(L|G), v(H|G))$. Expected profits are

$$\begin{aligned}
& \pi^{IO}(v(L|G), v(H|G), k_1) \\
= \;& (1-\rho)[v(L|G)\min\{k_1, m(H)\}] \\
& +\rho[v(L|G)k_1 + v(H|G)(1-k_1)m(H)]
\end{aligned}$$

Since expected profits are piecewise linear, the only potentially optimal introductory offer strategy with $k_1 \in (0,1)$ has first-period capacity $k_1 = m(H)$. The expected profits from the introductory offer strategy $(v(L|G), v(H|B), m(H))$ is given by (13). Comparing equations (22) and (13), we find that $\pi^{IO}(v(L|G), v(H|G), m(H)) > \pi^{IO}(v(L|G), v(H|B), m(H))$.

Finally, consider $(v(H|B), v(H|G))$. Expected profits are

$$\begin{aligned}
& \pi^{IO}(v(H|B), v(H|G), k_1) \\
= \; & (1-\rho)v(H|B)\min\{k_1, m(H)\} \\
& +\rho[v(H|B)\min\{k_1, m(H)\} + v(H|G)\max\{0, m(H)-k_1\}].
\end{aligned}$$

Since expected profits are linear in k_1 on $[0, m(H)]$ and constant on $[m(H), 1]$, any introductory offer strategy with $k_1 \in (0,1)$ is weakly dominated by a uniform price. ■

Proof of Lemma 18. We start with prices $p_1 = v(L|B), p_2 = v(H|B)$. Expected profits are

$$\begin{aligned}
& \pi^{IO}(v(L|B), v(H|B), k_1) \\
= \; & (1-\rho)[v(L|B)k_1 + v(H|B)(1-k_1)m(H)] \\
& +\rho[v(L|B)k_1 + v(H|B)(1-k_1)] \\
= \; & v(L|B)k_1 + v(H|B)(1-k_1)((1-\rho)m(H)+\rho).
\end{aligned}$$

The expression on the right-hand side is linear in k_1. Therefore, the introductory offer strategy is dominated by a uniform price.

Next take $p_1 = v(L|B), p_2 = v(L|G)$. Expected profits are

$$\begin{aligned}
\pi^{IO}(v(L|B), v(L|G), k_1) &= (1-\rho)v(L|B)k_1 + \rho[v(L|B)k_1 + v(L|G)(1-k_1)] \\
&= v(L|B)k_1 + v(L|G)\rho(1-k_1).
\end{aligned}$$

Again, the expression on the right-hand side is linear in k_1. Therefore this introductory offer strategy is dominated by a uniform price.

Next take $p_1 = v(L|B), p_2 = v(H|G)$. Expected profits are

$$\begin{aligned}
& \pi^{IO}(v(L|B), v(H|G), k_1) \\
= \; & (1-\rho)v(L|B)k_1 + \rho[v(L|B)k_1 + v(H|G)(1-k_1)m(H)] \\
= \; & v(L|B)k_1 + v(H|G)(1-k_1)\rho m(H).
\end{aligned}$$

Again, the expression on the right-hand side is linear in k_1. Therefore this introductory offer strategy is dominated by a uniform price.

Next take $p_1 = v(H|B), p_2 = v(L|G)$. Expected profits are

$$\begin{aligned}
& \pi^{IO}(v(H|B), v(L|G), k_1) \\
= \; & (1-\rho)v(H|B)\min\{k_1, m(H)\} + \rho[v(H|B)k_1 + v(L|G)(1-k_1)] \\
= \; & \begin{cases} v(H|B)k_1 + v(L|G)\rho(1-k_1) & \text{if } k_1 < m(H) \\ v(H|B)[\rho k_1 + (1-\rho)m(H)] + v(L|G)\rho(1-k_1) & \text{if } k_1 \geq m(H). \end{cases}
\end{aligned}$$

Because expected profits are piecewise linear, the first-period quantity $k_1 = m(H)$ can be the only interior solution to profit maximization. Hence, with above prices and first-period quantity $k_1 = m(H)$ expected profits are given by (15).

Next take $p_1 = v(H|B), p_2 = v(H|G)$. Expected profits are

$$\begin{aligned}
& \pi^{IO}(v(H|B), v(H|G), k_1) \\
= \; & (1-\rho)[v(H|B)\min\{k_1, m(H)\}] \\
& +\rho[v(H|B)k_1 + v(H|G)(1-k_1)m(H)] \\
= \; & \begin{cases} v(H|B)k_1 + v(H|G)\rho(1-k_1)m(H) & \text{if } k_1 < m(H) \\ v(H|B)[\rho k_1 + (1-\rho)m(H)] + v(H|G)\rho(1-k_1)m(H) & \text{if } k_1 \geq m(H). \end{cases}
\end{aligned}$$

Expected profits with the introductory offer strategy ($k_1 = m(H)$) are given by (16). Note that $\pi^{IO}(v(H|B), v(H|G), m(H)) > \pi^{IO}(v(H|B), v(L|G), m(H))$ if and only if $m(H)v(H|G) > v(L|G)$ (compare equations (15) and (16)).

Finally take $p_1 = v(L|G), p_2 = v(H|G)$. Expected profits are

$$\begin{aligned} & \pi^{IO}(v(L|G), v(H|G), k_1) \\ = \quad & \rho[v(L|G)k_1 + v(H|G)(1-k_1)m(H)] \end{aligned}$$

which are linear in k_1. Hence, any $k_1 \in (0,1)$ is weakly dominated by a uniform price. ■

Appendix 2

Consider the final sales strategy $(v(H|G), v(H|B), m(H))$ under strong vertical demand shifts (where $v(H|B) \leq v(L|G)$). Suppose now there is a continuous distribution of high valuation consumers (in the good demand state). Specifically, assume that there are $m(H)$ consumers whose valuations in the good demand state are uniformly distributed on $[v(H|G), v(H|G)+\gamma]$, where $\gamma > 0$.

Since consumers are atomistic, in any consumer equilibrium there must exist a threshold $\lambda^* \in [0,1]$ such that, in the good demand state, all consumers with valuation above $v(H|G) + \lambda^*\gamma$ buy the good in the first period, while all consumers with valuations less than $v(H|G) + \lambda^*\gamma$ demand the good in the second period. The monopolist's capacity is $k = m(H)$, and so the probability of not being rationed in the second period is $\lambda^* m(H)/[1-(1-\lambda^*)m(H)]$. In the good demand state, a consumers with valuation $v(H|G) + \lambda\gamma$ weakly prefers to demand the good in the second period rather than to purchase it in the first period if and only if

$$\begin{aligned} \lambda\gamma &\leq \frac{\lambda^* m(H)}{1-(1-\lambda^*)m(H)}[v(H|G) - v(H|B) + \lambda\gamma] \\ \iff \lambda\gamma &\leq \lambda^*\gamma^* \equiv \lambda^* \left(\frac{m(H)}{1-m(H)}\right)[v(H|G) - v(H|B)]\,. \end{aligned}$$

In consumer equilibrium, this inequality must hold for all $\lambda \leq \lambda^*$. Hence, if $\gamma > \gamma^*$, the unique consumer equilibrium is such that $\lambda^* = 0$: in the good demand state, all consumers with valuations in $[v(H|G), v(H|G)+\gamma]$ purchase the good in the first period (while all low types demand the good in the second period and are rationed with probability 1).

Finally, we have to show that there exists a $\gamma > \gamma^*$ such that the monopolist has no incentive to charge a price $p_1 > v(H|G)$. For this, a consumer with valuation $v(H|G) + \lambda^*\gamma$ is indifferent (in the good demand state) between purchasing the good at price p_1 in the first period and demanding the good at price $v(H|B)$ in the second period if

$$v(H|G) + \lambda^*\gamma - p_1 = \left(\frac{k-(1-\lambda^*)m(H)}{1-(1-\lambda^*)m(H)}\right)[v(H|G) - v(H|B) + \lambda^*\gamma]\,,$$

where $k \leq m(H|G)$ denotes again total capacity. Solving this equation for the first-period price, we obtain

$$p_1(\lambda^*, k) = v(H|G) + \lambda^*\gamma - \left(\frac{k-(1-\lambda^*)m(H)}{1-(1-\lambda^*)m(H)}\right)[v(H|G) - v(H|B) + \lambda^*\gamma]\,.$$

Instead of choosing k and p_1, the monopolist can be thought of as selecting k and λ^*. For a given capacity k, the monopolist chooses the marginal consumer λ^* so as to maximize her profit in the good demand state (as the profit in the bad demand state is $kv(H|B)$, independently of λ^*):

$$\max_{\lambda^*} \pi_G(\lambda^*; k) = (1-\lambda^*)m(H)p_1(\lambda^*, k) + [k-(1-\lambda^*)m(H)]\,v(H|B).$$

Taking the partial derivative with respect to λ^*, we obtain that

$$\begin{aligned}\frac{\partial}{\partial \lambda^*}\pi_G(\lambda^*; k) &\leq 0 \\ \Longleftrightarrow \psi(\lambda^*) \equiv \gamma \left\{(1-\lambda^*)\left[1-(1-\lambda^*)m(H)\right]-\lambda^*\right\} &\leq v(H|G)-v(H|B).\end{aligned}$$

Since $\psi(0) = \gamma$ and $\psi'(\lambda^*) < 0$ for all $\lambda^* \in [0,1]$, we have $\psi(\lambda^*) < 0$ for all $\lambda^* \in [0,1]$ if $\gamma < v(H|G) - v(H|B)$. Note that $v(H|G) - v(H|B) > \gamma^*$ if and only if $m(H)/\left[1 - m(H)\right] < 1$, i.e., if and only if $m(H) < 1/2$. That is, if $m(H) < 1/2$, there exists a nonempty interval $[\gamma^*, v(H|G) - v(H|B)]$ such that if γ falls into this interval, the monopolist has no incentive to charge a first-period price different from $v(H|G)$, and all high valuation consumers will buy the good at this price when demand is in the good state.

References

[1] Becker, G. (1991), A Note on Restaurant Pricing and other Examples on Social Influences on Price, *Journal of Political Economy* 99, 1109–1116.

[2] Beckmann, M. (1965), Edgeworth-Bertrand Duopoly Revisited, in: R. Henn (ed.), Operations Research Verfahren III, Meisenheim: Verlag Anton Hain.

[3] Chiang and Spatt (1982), Imperfect Price Discrimination and Welfare, *Review of Economic Studies* 49, 155–181.

[4] Chao, H.-P. and R. Wilson (1987), Priority Service Pricing: Pricing, Investment, and Market Organization, *American Economic Review* 77, 899–916.

[5] Che, Y.-K. and I. Gale (2000), The Optimal Mechanism for Selling to Budget-Constrained Buyer, *Journal of Economic Theory* 92, 198–233.

[6] Dana, J. (1998), Advanced Purchase Discounts and Price Discrimination in Competitive Markets, *Journal of Political Economy* 106, 395-422.

[7] Dana, J. (2001), Monopoly Price Dispersion Under Demand Uncertainty, *International Economic Review* 42, 649–670.

[8] Davidson, C. and R. Deneckere (1986), Long-Run Competition in Capacity, Short-Run Competition in Price and the Cournot Model, *Rand Journal of Economics* 17, 404–415.

[9] DeGraba, P. (1995), Buying Frenzies and Seller-Induced Excess Demand, *Rand Journal of Economics* 26, 331–342.

[10] DeGraba, P. and R. Mohammed (1999), Intertemporal Mixed Bundles and Buying Frenzies, *Rand Journal of Economics* 30, 694–718.

[11] Denicolo, V. and P. Garella (1999), Rationing in a Durable Goods Monopoly, *Rand Journal of Economics* 30, 44–55.

[12] Ferguson, D.G. (1994), Shortages, Segmentation, and Self-Selection, *Canadian Journal of Economics* 27, 183–197.

[13] Gale, I. and T. Holmes, (1992), The Efficiency of Advance Purchase Discounts in the Presence of Aggregate Demand Uncertainty, *International Journal of Industrial Organization* 10, 413-37.

[14] Gale, I. and T. Holmes, (1993), Advanced Purchase Discounts and Monopoly Allocation of Capacity, *American Economic Review* 83, 135-146.

[15] Gilbert, R. and P. Klemperer (2000), An Equilibrium Theory of Rationing, *Rand Journal of Economics* 31, 1–21.

[16] Harris, M. and A. Raviv (1981), A Theory of Monopoly Pricing Schemes with Demand Uncertainty, *American Economic Review* 71, 347–365.

[17] Karni, E. and D. Levin (1994), Social Attributes and Strategic Equilibrium: A Restaurant Pricing Game, *Journal of Political Economy* 102, 822–840.

[18] Klemperer, P. D. and M. A. Meyer (1986), Price Competition vs. Quantity Competition: The Role of Uncertainty, *Rand Journal of Economics* 17, 618-638.

[19] Mussa, M. and S. Rosen (1978), Monopoly and Product Quality, *Journal of Economic Theory* 18, 301–307.

[20] Riley, J. and R. Zeckhauser (1983), Optimal Selling Strategies: When to Haggle, When to Hold Firm, *Quarterly Journal of Economics* 98, 267-289.

[21] Rosen, S. and A. Rosenfield (1997), Ticket Pricing, *Journal of Law and Economics* 40, 315–376.

[22] Salop, S. (1977), The Noisy Monopolist: Imperfect Information, Price Dispersion and Price Discrimination, *Review of Economic Studies* 44, 393–406.

[23] Salop, S. and J. Stiglitz (1982), A Theory of Sales: A Simple Model of Equilibrium Price Dispersion with Identical Agents, *American Economic Review* 72, 1121–1130.

[24] Stokey, N. (1979), Intertemporal Price Discrimination, *Quarterly Journal of Economics* 93, 355–371.

[25] Weitzman, M. (1974), Prices vs. Quantities, *Review of Economic Studies* 41, 477-491.

[26] Wilson, C. (1988), The Optimal Pricing Policy of a Monopolist, *Journal of Political Economy* 96, 164–176.