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PIER Working Paper 03-001

"Foreign Direct Investment and Exports with Growing Demand"

by

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January 13, 2003

Abstract

We explore entry into a foreign market with uncertain demand growth. A multinational can serve the foreign demand by two modes, or by a combination thereof: it can export its products, or it can create productive capacity via Foreign Direct Investment. The advantage of FDI is that it allows for lower marginal cost than exporting does. The disadvantage is that FDI is irreversible and, hence, entails the risk of creating underutilized capacity in the case that the market turns out to be small. The presence of demand uncertainty and irreversibility gives rise to an interior solution, where the multinational, under certain conditions, both exports its products and does FDI.

JEL classification numbers: D8, D92, F2

Keywords: Foreign Direct Investment, Entry, Exports, New Markets

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1 Introduction

Overview and Results. When a multinational establishes new business in a foreign market, it typically starts out by exporting its product. Then, depending on the results, it may open production facilities in the foreign market - do Foreign Direct Investment (FDI for short) - and start satisfying some of the local demand from these facilities. After this initial FDI, the multinational may continue to export its product and gradually expand its local production facilities. Vernon's (1966) celebrated "product cycle" paper is the first to have drawn attention to such patterns, and a vast theoretical and empirical literature followed. In this paper, we attempt a dynamic modeling of this phenomenon, with an emphasis on demand uncertainty and irreversibility of investments. Our aim is to generate the time-paths of exports and FDI, and relate them to the observed behavior of multinationals and to economic fundamentals. Our setting is described in the following paragraph.

We consider a foreign market where demand is growing stochastically over time. This market can be served through exports from an existing facility (already established in a home market), through investment in the foreign market, or a combination of the two. The variable cost of serving the market through FDI is lower than the variable cost of serving it through exports. This is due to lower transportation costs, lower taxes, or to labor and materials being relatively inexpensive in the foreign market. However, FDI requires an entry cost, which becomes irreversible as soon as resources are sunk. Hence, if demand turns out to be large, the savings on variable costs that result from FDI are more than enough to cover the entry cost. Otherwise, the multinational is better off exporting the product. Given this trade-off and the way demand fluctuates, the multinational picks - at each point in time - an optimal combination of exports and FDI.

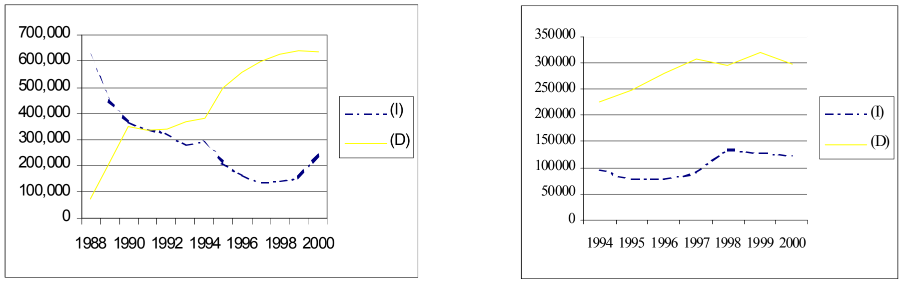
Some qualitative features of the optimal solution, as it emerges from our analysis, are as follows. Since the entry cost represents an irreversible investment, the seller will typically wait and enter the market only when demand has reached a sufficiently high level. In the pre-entry stage, exports are increasing as demand is growing. Once the seller enters the market, the initial investment is relatively high. Then, the seller adds to invested capacity as demand grows over time. In this post-entry stage, the seller may use only FDI or use a combination of FDI and exports, depending on the parameters. In the case that the seller uses a combination of FDI and exports, the two play complementary roles. FDI is used to satisfy proven demand, whereas exports are used to explore uncertain demand.[1]

Empirical Support. These features are broadly consistent with several "stylized facts" of the international trade and investment literature. Case studies that show that exports lead FDI include the China beer market (INSEAD, 1998) and the China automobile market (see *Time*, May 22, 2000). For the latter, some companies like General Motors have chosen to invest early in China, while others like Ford and DaimlerChrysler have chosen to initially reach the market via exports. Interestingly, after the initial exports period, Ford has recently announced a decision to build a plant (see *Automotive News,* July 10, 2000). Survey data assembled by Nicholas *et al.* (1994) suggests a similar pattern. 69% of the firms in their sample first exported to Australia before directly investing in that country.

On the empirical side, several studies have documented the co-existence of exports and FDI, and have tried to establish whether the two are "complements" or "substitutes," i.e., whether a high level of exports is associated (contemporaneously) with a high or a low level of FDI. The literature on this topic includes Lipsey and Weiss (1984), Yamawaki (1991), Brainard (1997), Swenson (1999), Clausing (2000), Head and Ries (2001), and Blonigen (2001). The findings of these studies typically depend on the level of product aggregation. For studies that use data on multi-product firms or sectorial or country-level data, there is evidence of complementarity between exports and FDI. On the other hand, for studies that use more disaggregated data, the evidence is usually in favor of substitutability.

However, even in the studies that use disaggregated data, the product is not entirely narrowly-defined. Since our model deals with a single product, we assembled data on a narrowly-defined product to illustrate the empirical features on which our model focuses. The data, which come from *Automotive News*, pertains to the car industry, where several Japanese and European car companies both export to the U.S. and use their domestic production capacity in that country. One such company is Toyota, which produces Camry cars both in Georgetown, Kentucky and in Tsutsumi, Japan. Figure 1A presents the pattern of Toyota car sales over the last few years, while Figure 1B focuses attention on one particular model, the Toyota Camry. Both Figures 1A and 1B show that FDI and exports co-exist; i.e., that Toyota is serving the U.S. market via a combination of exports and FDI. Figure 1B exhibits an additional feature: as demand increases, both FDI and exports are increased;

[1] This follows because FDI is less costly but requires greater commitment. Hence, for proven demand it is better to take advantage of the low cost of FDI. For uncertain demand it is better to take advantage of exports, which involve less commitment.



1A

1B

Figure 1: All Toyota models (A) and Camry (B) sales in the U.S.

the two move together. These findings, which regard the geographic, horizontal allocation of production between domestic and foreign facilities, appear related to the dynamics of our model (by "geographic, horizontal" we mean the *same* product is produced in geographically separated locations).

Related Literature. As stated in the opening paragraph, the choice of exports versus FDI has been the subject of numerous studies. An early contribution is Caves (1971), who emphasized scale economies and other cost factors. Subsequent contributions, which formalize these factors, include Buckley and Casson (1981), Smith (1987), and Horstmann and Markusen (1987), (1996). Similar to what we do here, these studies consider the decision to do FDI as being driven by a trade-off between incurring entry costs and economizing on variable costs. However, in all of these studies, once a multinational decides to do FDI, it no longer exports.[2] By contrast, in our model multinationals sometimes do both. This is due (as we show in Section 5) to demand uncertainty, investment irreversibility and the dynamics, which make 'diversification' optimal.

Another literature to which our paper is naturally related is the one on investments under uncertainty; see, e.g., Dixit and Pindyck (1994). We differ from that literature in that

[2]This follows from the nature of the production technology in those models, i.e., fixed cost plus constant marginal cost or, more generally, increasing returns to scale.

we consider features that are specific to the FDI problem. In particular, we consider two qualitatively different instruments: FDI, which involves commitment and low variable cost, and exports, which involve no commitment and high variable cost. By analyzing a model with these two instruments, we generate specific predictions regarding how FDI and exports are combined and how that relates to economic fundamentals. Smets (1993, chapter 1), using the same methodology as in Dixit and Pindyck (1994), determines the timing of FDI. However, his analysis is predicated on FDI replacing exports and, hence, does not deliver predictions regarding how FDI and exports are combined.

Disclaimer. To focus on the main questions of our paper and avoid excessively complicating the presentation, we have either simplified or abstracted away from several aspects of the problem. In particular, we study the choice between exports and FDI only, whereas in reality there are additional ways of entering a foreign market, such as licensing or joint ventures. Thus, what we do here can, at one level, be viewed as a first stab at the analysis of a more general scenario in which multiple modes of entry into a foreign market exist. At another level, there are other factors which determine the mode of entry. One such factor is private information and other agency problems, which determine whether a seller enters via licensing, joint ventures, FDI, exports, or some combination thereof. Studies that take that approach include Dunning (1977, 1981) that puts "internalization" at the center of the FDI decision, as well as Ethier (1986) - see also the review and discussion in Markusen (1995). In this paper we abstract away from agency considerations.

In addition, our focus here is on the geographic, horizontal division of production among facilities at home and abroad. When a multinational operates in several countries, there are, obviously, other ways of dividing production, for example vertically or functionally. For instance, in the case of a multinational like Nike most of the R&D takes place at home, most of the manufacturing is abroad, and most of the product is re-exported to the home and world markets. In this paper we abstract away from vertical and functional divisions of production.

Finally we abstract away from strategic considerations and analyze the optimal strategy of a single seller.

Preview. The remainder of the paper is organized as follows: Section 2 presents the model and introduces the basic notation. Section 3 sets up the dynamic programming problem and characterizes the timing of initial FDI into the foreign market. Section 4 derives the optimal exports path in the pre-entry stage. Section 5 characterizes the optimal exports and investments paths in the post-entry stage. Section 6 presents comparative statics results. Section 7 discusses the qualitative features of the dynamics. Section 8 discusses the robustness of our findings and how they may extend to other settings. Section 9 provides numerical illustrations of the solution. Section 10 contains concluding remarks. All proofs are found in a Technical Appendix on the website of this *Review*, http://www.restud.org.uk/supplements.htm.

2 The model

Time is discrete and the horizon is infinite.

One-consumer demand. Let $p = D(q)$ denote the one-period, one-consumer inverse demand function, and let $R(q) \equiv qD(q)$ be the corresponding revenue function. We assume that $R(q)$ is strictly concave. Further, by re-scaling units, we assume that $R(q)$ is maximized at $q = 1$, that $D(1) = 1$ and, consequently, that $R(1) = 1$. Thus, $q = 1$ is the maximizer of monopoly profits when marginal cost is zero.

Market demand. The number of consumers is denoted by $A \geq 0$. In a period with A consumers and sales q, the inverse demand is $p = D(q/A)$ and the revenue is $AR(q/A)$.

Demand dynamics. The number of consumers, A, grows over time until the market has been saturated. An initial A, A_0, is given. Then, at the beginning of each period (starting with period 1) one of two things happens. Either growth stops (forever), which occurs with probability s. Or, growth continues, which occurs with probability $1 - s$. If growth continues a new consumers arrive, so the number of consumers becomes $A' = A + a$. a is a random variable drawn from the p.d.f. $f(a)$ with a corresponding c.d.f. $F(a)$. s and $f(\cdot)$ are the same every period, and independent of that period's A. The support of f is contained in $(0, \infty)$. Therefore, A increases or stays put; A never decreases.

Investments and Exports. The seller's discount factor is $\delta \in (0, 1)$. The seller can serve the market either through exports or by installing productive capacity through FDI. The state variables at the end of a generic period are the number of consumers, A, and the productive capacity, X. When productive capacity is X the seller can produce up to X units per period at zero variable cost. The initial value of X is zero. We assume, for simplicity, that capacity does not depreciate over time.

At the end of a generic period, the seller is making the following decisions. If some capacity has already been installed, $X > 0$, the seller is choosing the levels of addition to

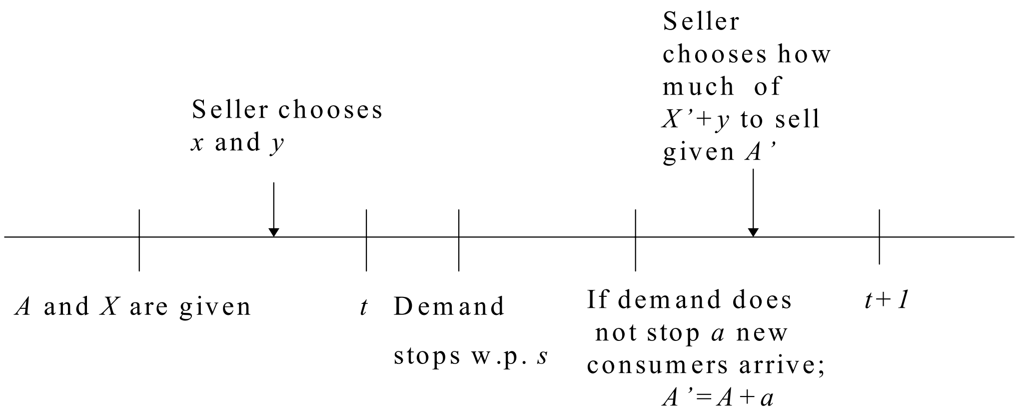


Figure 2: Timing of Events within a Single Period

capacity, denoted by x, and exports, denoted by y. If capacity has not been installed, the seller decides whether to install capacity for the first time or not. In the former case, the seller chooses x and y as above. In the latter, the seller chooses y only. These decisions are made prior to next period's demand realization, and the costs associated with them are non-recoverable. Given X and the seller's end-of-period decisions, he is able to sell $X' = X + x$ units, using foreign capacity, and y units, using exports. Altogether the seller is able to sell $X' + y$.

Sales. Once demand is realized the seller has to decide how much of $X' + y$ to sell given the new value of A. At that point he is a seller with zero variable cost. Therefore, according to our normalization, he optimally sells $\min\{X' + y, A'\}$. The timing of events is summarized by Figure 2.

Costs. The cost of adding x units of productive capacity is kx, while the cost of exporting y units is cy. The cost of entry into the market, paid the first time the seller does FDI, is $e > 0$.

Maintained assumptions. We assume that the following parameter restrictions hold:

$$D(0) > c > (1 - \delta)k. \tag{1}$$

The left inequality says that the market is viable, and the right inequality says that, in the absence of demand uncertainty, it is less expensive to serve the market through FDI as compared to exports. If $c < (1-\delta)k$, FDI is a dominated instrument so the seller only exports.

Let q_k be the maximizer of $R(q) - (1-\delta)kq$ and let $\pi_k \equiv R(q_k) - (1-\delta)kq_k$. Also, let q_c be the maximizer of $R(q) - cq$ and let $\pi_c \equiv R(q_c) - cq_c$. From the assumptions and parameter restrictions above, it follows that $q_c < q_k < 1$ and that $\pi_c < \pi_k < 1$.

3 Value function and characterization of entry

We set up now the dynamic programming problem facing the seller and determine the point of entry into the foreign market. We first consider the case where the seller already paid e, and call the value function over this domain $v(X, A)$. Then we consider the case where the seller has not paid e yet, and call the value function over that domain $u(A)$.

The post-entry problem. Consider the beginning of a period and assume that this period's s and a have already been realized, but the seller has not sold any output yet. We distinguish between two cases.

Case I: Demand did not stop growing. Then the seller sells $\min\{X + y, A\}$.

Thus, if $X+y > A$, the seller collects revenue A in the current period and the continuation payoff is $v(X, A)$; altogether the value is $A + \delta v(X, A)$.

On the other hand, if $X+y < A$, the seller collects revenue $AR((X+y)/A)$ in the current period,[3] the continuation payoff is $v(X, A)$, and the value is $AR((X + y)/A) + \delta v(X, A)$.

Case II: Demand stopped growing. Then the seller either keeps the level of FDI intact or he makes a final adjustment to it. Let us call the seller's terminal value, which reflects this final adjustment, $H(X, y, A)$. This value is as follows:[4]

- if $X + y \geq A$:

[3] In this case the monopolist sells $X + y$ at the price that the market will bear for it, $D((X + y)/A)$, so his revenue is $AR((X + y)/A)$.

[4] In the post-entry stage and in periods following the one where demand stopped growing, the marginal cost of supplying the market through FDI is lower than that of exports and, thus, no exports take place. In particular, the current-period payoff is A, when the quantity available for sale, $X + y$, is at least A, and is equal to $AR((X + y)/A)$ otherwise. The payoff in each of the following periods depends on the invested capacity, X. If $X \geq Aq_k$, the seller never invests again and the per-period future payoff is either A or $AR(X/A)$, depending of whether X exceeds A or not. If $X < Aq_k$, the seller invests in the following period the quantity required to bring the level of total capacity to Aq_k.

$$H(X,y,A)=\begin{cases}\frac{A}{1-\delta} & \text{if } X\geq A\\ A+\frac{\delta}{1-\delta}AR(\frac{X}{A}) & \text{if } A>X\geq Aq_k\\ A+\delta[kX+\frac{A\pi_k}{1-\delta}] & \text{if } Aq_k>X,\end{cases} \tag{2}$$

- and, if $X+y<A$:

$$H(X,y,A)=\begin{cases}AR(\frac{X+y}{A})+\frac{\delta}{1-\delta}AR(\frac{X}{A}) & \text{if } A>X\geq Aq_k\\ AR(\frac{X+y}{A})+\delta[kX+\frac{A\pi_k}{1-\delta}] & \text{if } Aq_k>X.\end{cases} \tag{3}$$

Between cases I and II, the post-entry value function is:

$$v(X,A)=\max_{(x,y)\in\Re_+^2}\{-kx-cy+sH(X+x,y,A)+ \tag{4}$$

$$(1-s)[\int_0^{X+x+y-A}(A+a)f(a)da+\int_{X+x+y-A}^{\infty}(A+a)R(\frac{X+x+y}{A+a})f(a)da$$

$$+\delta\int_0^{\infty}v(X+x,A+a)f(a)da]\}$$

$$\equiv \max_{(x,y)\in\Re_+^2}\{\widehat{\phi}(x,y,X,A)+(1-s)\delta\int_0^{\infty}v(X+x,A+a)f(a)da\}. \tag{5}$$

Since the marginal cost of investments is constant, v satisfies the following:

$$v(X+\overline{x},A)=v(X,A)+k\overline{x} \tag{6}$$

for any $\overline{x}$ that is no bigger than the right-hand-side maximizer of (4).

The pre-entry problem. Now consider the pre-entry stage, that is, when $X=0$.

Assume first that the seller chooses exports only. Then, if demand stops, he gets:[5]

$$G(y,A) = \begin{cases} A + \delta \max[\frac{A\pi_c}{1-\delta}, -e + \frac{A\pi_k}{1-\delta}] & \text{if } y \geq A \\ AR(y/A) + \delta \max[\frac{A\pi_c}{1-\delta}, -e + \frac{A\pi_k}{1-\delta}] & \text{if } y < A. \end{cases}$$

Therefore, the expected payoff - predicated on exports only - is equal to:

$$\max_{y \in \Re_+} \{-cy + sG(y,A) +$$

$$(1-s)[\int_0^{y-A} (A+a)f(a)da + \int_{y-A}^{\infty} (A+a)R(\frac{y}{A+a})f(a)da + \delta \int_0^{\infty} u(A+a)f(a)da]\}$$

$$\equiv \max_{y \in \Re_+} \{\psi(y,A) + (1-s)\delta \int_0^{\infty} u(A+a)f(a)da\}. \tag{7}$$

On the other hand, if the seller installs capacity his payoff is:

$$-e + v(0,A) = -e + \phi(x^*(A), y^*(A), A) + (1-s)\delta \int_0^{\infty} v(x^*(A), A+a)f(a)da, \tag{8}$$

where $\phi(x,y,A) \equiv \widehat{\phi}(x,y,0,A)$ and $x^*(A), y^*(A)$ are the maximizers of $v(0,A)$. Using (6), (8) is equal to:

$$-e + \phi(x^*(A), y^*(A), A) + (1-s)\delta \int_0^{\infty} [v(0, A+a) + kx^*(A)]f(a)da. \tag{9}$$

The pre-entry value function, $u(A)$, is the maximum of (7) and (9).

Point of Entry. The seller exclusively exports whenever $v(0,A) - e < u(A)$, and enters whenever $v(0,A) - e \geq u(A)$. This suggests that a cutoff rule characterizes the point of entry: there exists an A^*, possibly zero, for which the seller is indifferent between exporting and entering (i.e., A^* is such that $v(0,A^*) - e = u(A^*)$) and entry occurs if and only if $A \geq A^*$.

[5] This terminal value is calculated as follows. Once demand growth stops, the seller collects the current period payoff, A or $AR(y/A)$ depending on the current level of exports, and then chooses whether to cover future demand via exports (if $A\pi_c/(1-\delta) > -e + A\pi_k/(1-\delta)$) or via FDI (otherwise).

We prove in the Appendix that this is indeed the case and show that if A^* is positive this indifference condition boils down to:

$$[1-(1-s)\delta]e = (1-s)\delta k x^*(A) - \psi(\bar{y}(A), A) + \phi(x^*(A), y^*(A), A), \tag{10}$$

where $\bar{y}(A)$ is the maximizer of (7). We state now our first result:

Proposition 1 *(i) Entry obeys a cutoff rule. Either the RHS of (10) is no less than its LHS for all $A \geq 0$, in which case the seller does FDI from the very start. Otherwise, there exists a unique $A^* > 0$ so that (10) is satisfied. The seller only exports if $A < A^*$ and does FDI and, possibly, exports as soon as $A \geq A^*$. When the seller does FDI for the first time, he chooses a positive x.*
(ii) A^* satisfies

$$A^* < \overline{A} \equiv \frac{e(1-\delta)}{\pi_k - \pi_c}. \tag{11}$$

(iii) A^* is increasing in e.

The reason why it is potentially optimal for the seller to wait, that is, why A^* might be positive, is that there is an option value to waiting. If the entry cost e is large, A must be big enough to justify paying this cost. By waiting the seller gets more information, or, "learns" about A and is, thereby, able to avoid paying e when A is not big enough.

When A reaches the point where the seller pays e, it is optimal to install positive capacity, x. Paying e and not installing capacity is dominated by delaying entry for one period and implementing the same investment plan thereafter. Then the seller saves at least the one-period interest cost on e, $(1-\delta)e$.

The reason A^* is strictly smaller than $\overline{A}$ is the following. The return on spending e is the discounted savings on variable costs. When the market size is $\overline{A}$, these savings exactly equal e, and when the market size is bigger than $\overline{A}$ these savings exceed e. Hence, since A can only increase, the seller would certainly spend e when $A = \overline{A}$ and, by continuity, would spend e at an A which is somewhat smaller than $\overline{A}$.

The degree to which entry is delayed, that is, how large A^* is, depends on all parameters of the model. For instance, even if e is large but the prospects for demand growth are good (small s, or large expected value of a) the seller may optimally enter in the initial period, $A^* = 0$.

Finally, the flexibility to use both exports and FDI affects the entry point A^*. Namely, the firm enters at a smaller A than in a formulation where exports are not possible after the first FDI. This follows from the fact that not allowing for the possibility of exports in the post-entry stage decreases the value in that stage (with both instruments the seller can achieve a higher expected profit), while it leaves unchanged the value in the pre-entry stage.

4 Pre-entry behavior: exports only

Before we proceed to the characterization of the post-entry optimal behavior, we derive the optimal pre-entry export path. This can be found by maximizing the RHS of (7). Since u does not depend on y, this is a static maximization program. Hence, equating the derivative of the RHS of (7) with respect to y to zero, we find:

Proposition 2 *(i) If $A > 0$ or if $(1-s)D(0) > c$ the optimal level of exports in the pre-entry stage is positive, and can be found by solving :*

$$c = sG_1(y, A) + (1 - s) \int_{y-A}^{\infty} R'(\frac{y}{A + a})f(a)da, \tag{12}$$

where

$$G_1(y, A) = \begin{cases} 0 & \text{if } y \geq A \\ R'(y/A) & \text{if } y < A \end{cases}$$

is the derivative of $G(y, A)$ with respect to y. Furthermore, the quantity exported is at least Aq_c.

(ii) Otherwise, the LHS of (12) is no less than the RHS, and exports are zero.

Further, condition (12) shows that the optimal exports level is increasing in the current level of A, and decreasing in s. Therefore, in the pre-entry stage, the level of exports is increasing over time, as A increases.

5 Post-entry behavior

We proceed now to characterize the optimal path of exports and FDI in the post-entry stage. Two qualitatively different possibilities arise here: either the optimum is "interior," with positive quantities of both exports and FDI; or, it is "corner," with zero exports and positive FDI. Which of these possibilities is the optimum and how to characterize the optimum depend on parameter values and the state of the market. In this section we show these dependencies by analyzing the first-order conditions to the firm's problem.

5.1 Preliminaries

The first order condition for FDI is:

$$k \geq sH_1(X+x,y,A) + (1-s) \int_{X+x+y-A}^{\infty} R'(\frac{X+x+y}{A+a})f(a)da + (1-s)\delta \int_0^{\infty} v_1(X+x,A+a)f(a)da \tag{13}$$

and for exports:

$$c \geq sH_2(X+x,y,A) + (1-s) \int_{X+x+y-A}^{\infty} R'(\frac{X+x+y}{A+a})f(a)da, \tag{14}$$

where H_1 and H_2 are the derivatives of H with respect to its first and second variables and v_1 is the derivative of v with respect to its first variable. Equality holds in (13) and/or (14) whenever the corresponding variable takes on a positive value.

As Proposition 1 states, the initial x in the post-entry stage is positive. Relying on the assumption that A can only increase, the next Lemma states that all subsequent x's are positive as well.

Lemma 3 *If x is positive in some period, then it is positive in all subsequent periods (until demand stops growing).*

Given that x is positive in every period we use the envelope theorem to establish:

$$v_1(X+x,A+a) = k. \tag{15}$$

Then we insert (15) into (13), and eliminate the value function from its RHS. This gives:

$$k = sH_1(X+x,y,A) + (1-s)[\int\limits_{X+x+y-A}^{\infty} R'(\frac{X+x+y}{A+a})f(a)da + \delta k]. \tag{16}$$

If y is positive, (14) is satisfied with equality, so we write it as:

$$c = sH_2(X+x,y,A) + (1-s)\int\limits_{X+x+y-A}^{\infty} R'(\frac{X+x+y}{A+a})f(a)da. \tag{17}$$

Equations (16) and (17) suggest a technique for pinning down the optimum. Let $x^o(y)$ be the solution to equation (16) (where "solution" means we choose some y, solve for x and do this for every non-negative value of y) and let $y^o(x)$ be the solution to equation (17). The curves corresponding to these solutions are shown in Figure 3 below. As this Figure shows (and as Lemma 4 states) the curves are downward sloping and the $x^o(y)$ curve is steeper than the $y^o(x)$ curve. Hence, either the two curves intersect uniquely in the interior of $\Re^2_{++}$, which corresponds to an interior optimum; or, one of the curves lies entirely above the other curve, which corresponds to a corner optimum. Therefore, finding the optimal exports-FDI combination when the market is in state (X,A) amounts to drawing the curves that correspond to (16) and (17) at (X,A) and finding the (generalized) intersection point. The convenient feature of this technique is that the value function is no longer present in (16) and (17) and hence one is able to find the optimal combination (x,y) at each and every value of (X,A), using only the primitive data.

The technique described above relies on the following result.

Lemma 4 *(i) $x^o(y)$ and $y^o(x)$ are downward sloping. (ii) Let us fix a value of x and look at the y points above it, one on $x^o(y)$ and one on $y^o(x)$. Then, the curve corresponding to $x^o(y)$ is steeper than the curve corresponding to $y^o(x)$.*

The next sub-section provides further details concerning how this technique works, it simplifies the first-order conditions (16) and (17), interprets them in economic terms and shows how to extend the technique to the case of a corner optimum.

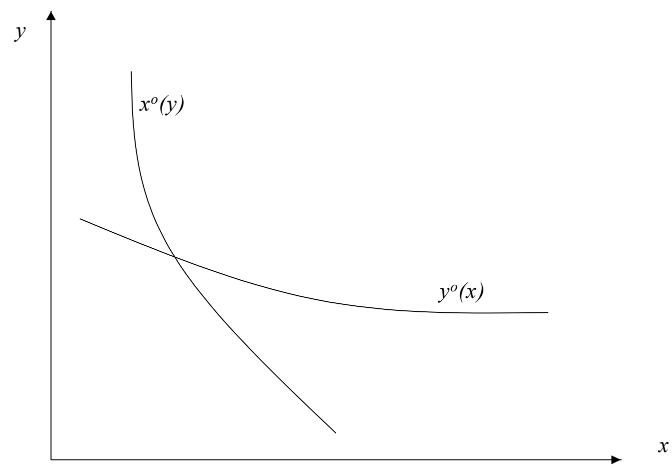


Figure 3: First order conditions for an interior optimum

5.2 Characterization of the solution

Let us first show how to compute the optimal level of FDI in the event of an interior optimum. Substituting equation (17) into (16) one gets:

$$k = c + (1 - s)\delta k + s[H_1(X + x, y, A) - H_2(X + x, y, A)]. \tag{18}$$

In the Appendix we compute the derivatives of H and show that the optimal FDI is such that $Aq_k < X + x < A$. Given that $X + x$ is in this range we substitute the relevant expression for $H_1 - H_2$ into (18) and obtain:

$$k = c + (1 - s)\delta k + s\,\frac{\delta}{1-\delta} R'(\frac{X + x}{A}). \tag{19}$$

The characterization of an interior optimum has now been reduced to solving one equation, (19), in one unknown, x. This equation has the interpretation that the marginal cost of FDI, k, is equated to the marginal benefit (which appears on the RHS). This marginal benefit consists of three terms. The first term is the expected marginal revenue in the current period and the reason it equals c is that c itself is equated - by of the optimality condition for exports - to the expected marginal revenue. The remaining two terms equal the discounted marginal benefit of FDI in all subsequent periods. The first of these two terms is the marginal benefit in the event that demand continues to grow; it equals $(1-s)\delta k$ because, if demand continues to grow, the seller saves himself the cost k, which he would have incurred in the next period (if he were not to incur it now). The second of these terms is the marginal benefit in the event that demand stops growing. This marginal benefit is the discounted value of all subsequent marginal revenues - predicated on the seller not investing anymore in capacity (which is true because $X + x > Aq_k$). Altogether the RHS of (19) is the marginal benefit of FDI, taking into account all contingencies and all time periods.

Having computed the optimum level of FDI, x, one inserts this value of x into equation (17) and solves that equation for the optimal level of exports, y. Equation (17) has a similar, marginal cost equal to marginal benefit, interpretation.

All this assumes that the optimum is interior. How does one establish whether the optimum is interior and, if it is not, how does one extend the solution to the case of a corner optimum? The first observation here is that a corner optimum in which FDI is zero is ruled out by Lemma 3. Hence, it remains to reckon with a corner optimum in which exports are zero. Let us assume then that $y = 0$. Then substituting $y = 0$ into equation (16) one obtains:

$$k = (1-s)[\int_{X+x-A}^{\infty} R'(\frac{X+x}{A+a})f(a)da + \delta k] + sH_1(X+x, 0, A). \tag{20}$$

Solving (20) for x, one finds $x^o(0)$ and substitutes it into the RHS of equation (17). If the resulting RHS (which is the marginal benefit of exports) exceeds the LHS (which is the marginal cost of exports), our assumption that $y = 0$ is wrong and one is back to the case of an interior optimum, which has already been worked out above. Otherwise, i.e., if the marginal benefit of exports is lower than the marginal cost of exports, our assumption that $y = 0$ is right and, in the process, we have found the optimal level of FDI, $x^o(0)$. After some manipulations, the condition to determine whether the optimum is interior or corner is:

$$k[1-(1-s)\delta] - c > s[H_1(X + x^o(0), 0, A) - H_2(X + x^o(0), 0, A)]. \tag{21}$$

In summary, we have the following result.

Proposition 5 *In the post-entry stage and as long as demand keeps growing: (i) FDIs are positive in each and every period, and are found as follows. (ii) If (21) holds, exports are positive. The optimal x is found from (19), and is such that $X + x$ is between Aq_k and A and is proportional to A. The optimal y is determined by equation (17), once we substitute the optimal x. (iii) If (21) does not hold, exports are zero, and the optimal x is found by solving equation (20) in x.*

Proposition 5, which is the paper's central result, shows that exports might be used in conjunction with local productive capacity to serve a foreign market. This Proposition rests on the assumptions that demand is uncertain and that the market is served over multiple periods. We show now that if either assumption does not hold, the foreign market is served instead through FDI or exports, but not both.

Let us first consider a static, one period scenario, but retain the assumption that demand is uncertain. Then, since FDI is used over a single period, FDI and exports are perfect substitutes so the only thing that matters is whether $c > k$ or $c < k$. In the first case the market is exclusively served through FDI; in the second case - through exports.

Second, let us consider an infinite horizon scenario, but assume that demand is deterministic. More specifically, assume that demand grows with probability 1 until some deterministic date, say $\overline{T}$, and thereafter grows with probability 0, and that a deterministic number, say $\overline{a}$, of new consumers arrives in the market in each period that the demand is growing.[6] Consider the post-entry stage and suppose that assumption (1) holds. Then, inspection of condition (21) (when $s = 0$) along with assumption (1) reveals that one has a corner optimum with zero exports.[7]

Thus, uncertainty and dynamics are necessary for an interior optimum. However, they are not sufficient. Going back to the model with uncertainty and dynamics, assume that $c > k$. Then, the cost of exports is so high (compared to FDI) that the foreign market is served exclusively through FDI. This holds true not just for $c > k$ but for any c that is above some threshold value. How this threshold is determined and how it depends on primitives of the problem is shown by Proposition 5, see condition (21). Thus, one thing that Proposition 5 does is to delineate the set of circumstances under which the optimum is interior.

[6] To make the models "equivalent" we could set $\overline{T} = \frac{1}{s}$ and $\overline{a} = Ea$.

[7] The optimum, more precisely, is such that the level of cumulative FDI "tracks" the level of demand.

Let us also note that Proposition 5 shows that a combination of exports and FDI may be optimal only *as long as demand continues to grow.* Once demand stops growing, a combination of exports and FDI is no longer optimal. Instead, there are only two patterns of serving the market at that point. Either demand stopped growing during the post-entry stage, in which case the market is served exclusively by FDI. Or, demand stopped growing during the pre-entry stage, in which case the market is served exclusively by exports. An interior solution is possible only along the growth path.

Proposition 5 describes the solution to the seller's problem under the assumptions $X_0 = 0$ and $e > 0$. The solution would have to be modified along the following lines if these assumptions do not hold.

- If $X_0 > 0$, the pre-entry stage is eliminated (by assumption) so one does not have to determine the point of entry. The solution differs from the one above in that there may be several periods (at the beginning) without FDI because the initial capacity is already high enough.

- If $e = 0$ and $A_0 > 0$ entry occurs right away. If $e = 0$ and $A_0 = 0$ entry may occur right away, or, it may be delayed by one period; but it is never delayed by more than one period. This is true because $A > 0$ in the second period.

6 Comparative statics

We now consider how the solution depends on the parameters of the model, specifically on c, k, δ, s, and the current number of consumers, A. We do this for a fixed value of X, i.e., we consider a generic period and determine how that period's x and y change when each of the parameters change. The results are summarized in the following Proposition.

Proposition 6 *For a given level of X, the optimal solution responds to changes in the values of the parameters as follows:*

(i) An increase in k (1) decreases x and increases y if there is an interior solution, (2) decreases x if there is a corner solution, and (3) tends to move the solution from a corner to an interior point.

(ii) An increase in c (1) increases x and decreases y if there is an interior solution, (2) leaves x unchanged if there is a corner solution, and (3) tends to move the solution from an interior point to a corner.

(iii) An increase in δ (1) increases x and decreases y if there is an interior solution, (2) increases x if there is a corner solution, and (3) tends to move the solution from an interior point to a corner.

(iv) An increase in s (1) decreases x if there is an interior solution, while its effect on y is ambiguous. (2) It decreases x if there is a corner solution, and (3) tends to move the solution from a corner to an interior point.

(v) An increase in A (1) increases x if there is an interior solution, while its effect on y is ambiguous. (2) It increases x if there is a corner solution, and it (3) either tends to move the solution from an interior point to a corner (if $A > X + x^o(0) \geq Aq_k$) or it leaves the boundary unaffected (otherwise).

This result is useful for relating the volume of trade vis à vis the volume of FDI to external conditions and to changes in these conditions. For example, if trade barriers are lowered (making c smaller) as a result of a regional trade agreement (say NAFTA), the model predicts that exports will increase at the expense of FDI. On the other hand, if legal barriers to foreign investments are lowered (making k smaller), for example as a result of governmental efforts to partake in globalization, the model predicts that FDI will increase at the expense of exports.

Proposition 6 is also useful for making cross country comparisons. For example, consider a corporation that exports to and does FDI in two foreign countries, say A and B. Assume exports are sold at the same price in both countries but that FDI is less costly in country A than in country B; for example country A might be less developed and labor costs to build new capacity in it are lower. Then, the model predicts that FDI carries a higher weight in the FDI-exports combination of country A than of country B. That is, the less developed country produces more domestically and imports less than the developed country.

Note that in cases *(i)*, *(ii)* and *(iii)*, a parametric change results in x and y moving in opposite directions. However, in cases *(iv)* and *(v)*, x and y may move in the same direction. Consider, for instance, an increase in A. Part *(iv)* of Proposition 6 says that x increases (to satisfy the larger demand). However, as regards y, there are two opposing effects. On the one hand, there is a direct effect to increase y (again because of the larger demand); on the other hand, there is an indirect effect: the seller may decrease y because x and y are substitutes and x is increased. Because of these opposing effects the effect that an increase in A has on y is, in general, ambiguous. It depends on various details of the model - the demand elasticity, the distribution over consumer arrival, etc.[8]

7 Qualitative features of the dynamics

The solution we derived and its comparative statics properties show the qualitative features of the dynamics; i.e., they tell us what the model predicts will happen to the mix of exports and FDI over time. We now summarize these features with a view towards how some of these features may be tested using actual data. In Section 9, we further illustrate these features by working out a particular example. The main features of the solution are as follows.[9]

First, due to the existence of the fixed cost of entry, e, the seller typically does not do FDI right away. Instead, he waits until A has become large enough to enter the market. Therefore, the use of FDI follows a period of exporting. Likewise, after the seller enters and in the case of an interior solution, the seller "replaces" last period's exports by current period FDI. So again the use of FDI follows exports. In this sense, the model predicts a lagged relationship between exports and FDI; i.e., if one is to regress FDI on lagged values of exports the model predicts a positive relationship.[10]

At the time of entry, the level of investments is large as a consequence of the accumulated demand. The waiting period for the initial entry is longer when each of the entry cost, e, the cost of investment, k, and the probability that growth stops, s, are large and when the cost of exports, c, is small. Of course, if e, k and s are small and c is large, or if the expected value of a is large, the seller may serve the market via FDI right away and skip the initial stage of exports-only.

In the post-entry stage, and following the large initial investment, the seller's investments grow gradually. The seller adds to invested capacity each period the demand is growing. Since demand growth is stochastic, the level of these additional investments are sometimes high and sometimes low. Investments stop once demand stops growing.

[8] In some parametrizations of the problem y increases in A. For example, this is true with linear demand and uniform distribution - see the website of this *Review* http://www.restud.org.uk/supplements.htm. In other parametrizations, y is constant or decreasing in A.

[9] Our analysis helps characterize stages of what, following Vernon (1966), has been known as product cycle. Still, our analysis does not correspond to the entire cycle of an exported product's economic life, as it does not include a stage of shrinking demand or replacement of the product by a newer version.

[10] This relationship is not a causality relationship. Rather, exports and FDI are simultaneously determined by all underlying parameters. Nonetheless, the optimum mix exhibits this positive lagged relationship.

Furthermore, as A increases, exports and FDI may or may not move together (contemporaneously); FDI always increases in A, but exports may increase or decrease. Thus, if "move together" is interpreted as FDI and exports being complements, and "not move together" as them being substitutes, our model predicts that either can happen.[11] The "resolution" of this issue requires dynamic econometric analysis, using data on narrowly-defined products.

A couple of caveats are in order at this point. First, as discussed in the introduction, these features of the model pertain to a particular scenario: the product is homogenous, sold in the foreign market only, and the multinational decides how to allocate production between a home and a foreign facility. If production is vertically or functionally related or if exports from the foreign market are allowed, these features need not hold.

Second, there are other ways of obtaining the result that multinationals divide production between a home facility and a foreign facility. One way is to continue to consider horizontal division, but assume increasing marginal cost curves. Then, overall costs are lowered by dividing production; see, e.g., Horst, (1971). Another way is to consider vertical division. Then, if there are increasing returns, a multinational may optimally export from home some components and combine them with other components produced in the foreign destination; see, e.g., Venables, (1996, 1999) and Markusen and Venables, (1998).

There are two other avenues to the result that FDI and exports are combined. Aversion to actual and threatened trade restrictions is one. It has been widely discussed in the literature on political economy and international trade (and is certainly pertinent to the car industry discussed in the Introduction), see e.g. Baldwin (1984). Another avenue is that multinationals manage their exposure to exchange rate fluctuations by setting up facilities in several countries; see Kogut and Kulatilaka (1994).

Likewise, there are other avenues to the result that exports lead FDI. Saggi (1998) studies a two-period model where initial exporting can be used to gather information about the demand, with the option to invest in the second period.

[11] This, again, is not a causality relationship. The co-variation in x and y is an optimum relationship, driven by changes in underlying parameters.

8 Robustness

To make the logic of the model transparent, we have made several simplifying assumptions. We now assess in what ways our results would have to be adjusted if these assumptions are changed. In this section, we maintain our basic set-up and examine the assumptions made within it. In the concluding section, we discuss more extensive variations of the basic set-up.

As shown in Section 5, much of the analysis hinges on marginal conditions. Hence, we discuss the effect of changing an assumption by determining what effect that change might have on the relevant marginal condition(s).

1. *Partial investment irreversibility.* Our model assumes that FDI is irreversible, in particular that there is no re-sale market for used capacity. Alternatively, it may be assumed that foreign capacity can be sold at some per unit scrap value, k', presumably less than k. The effect of this is to lower the cost of FDI by the amount the seller is able to recoup in case demand stops growing, $\delta sk'$. Since the adjusted cost of FDI is lower, more FDI will be used relative to exports. In fact, the seller may even switch from an interior mix of exports and FDI to FDI only. For similar reasons, the threshold of entry, A^*, is lower when capacity can be re-sold.

2. *Capacity constraints in the home country.* The model has been solved under the assumption that there are no binding capacity constraints at home; exports can always take place at a marginal cost of c. Generalizing the model to allow for capacity constraints at home is not expected to qualitatively change our main results. As long as the start-up cost, e, has been paid at home but not at the new market, there will be a delay until FDI is started. And, once FDI starts, the weight that is placed on FDI (in the FDI-exports mix) is bigger because capacity has to be expanded not only abroad but also at home. This makes exports more costly and hence less attractive.

3. *Exports not sold in the current period can be carried forward.* The model could be modified so that exports can be stored (perhaps at some cost) and brought to the market in subsequent periods. Then, the product could be sold without excessively depressing the current-period price. In this scenario, a third state variable has to be introduced - the amount of carried-over stock of exports - and there will be another margin - choosing a “trigger point” beyond which the seller is better off carrying over

exported stock to the future. Such a re-formulation will have the effect of lowering the effective cost of exports and, as a result, shift the solution towards more exports.

4. *Exports can be chosen after demand uncertainty is resolved.* In principle, export levels need not be chosen before hand; they can be chosen after the arrival of new consumers has been realized. In reality, waiting for demand uncertainty to be resolved and then deciding the level of exports is not practical because, typically, "shelves" have to be stocked prior to the arrival of consumers. To extend the model (nonetheless) to the case where exports are chosen after the arrival of new consumers, one would equate the marginal cost of exports, c, to the *actual* marginal revenue. Since exports are chosen ex-post in this way, only FDI has to be chosen ex-ante (so the ex-ante problem is simpler).

   A related possibility is one where production from FDI can be adjusted once the level of demand becomes known. This is relevant when there is a positive variable cost, which can be avoided (our model assumes, for simplicity, zero variable cost past the FDI stage). In this case, there is an additional decision variable - the seller has to decide how much to produce ex-post. However, if the reduced-form revenue function satisfies the same concavity assumptions imposed in our formulation, similar analysis and results apply.

5. *A fixed cost is paid each time capacity is expanded through FDI.* In addition to the initial entry cost, a fixed cost may have to be paid every time the seller decides to add capacity through FDI. This variation of the model is expected to modify the results by making the FDI path more volatile; as a result of the fixed cost, the seller may want to wait a few periods until he invests and then invest at a higher level. This behavior is directly analogous to the initial phase in our formulation where, because of the entry cost, the seller waits a few periods until demand reaches a certain threshold and only then does FDI.

6. *The probability that demand stops growing is not constant.* An interesting extension is where s increases in A; the probability of market saturation increases the more demand has increased. In this case, the seller may shift the balance towards more exports and, correspondingly, decrease FDI as A increases. The reason for this is that FDI becomes more risky so the seller is better off waiting to learn whether demand has actually

grown to justify further investment. An example corresponding to this case is explored in the following section.

7. *Demand may decline.* The result that FDI is positive in each and every period depends crucially on the assumption that demand never declines. Otherwise, if demand is allowed to decline, FDI may be zero in some periods. In particular, if demand declines in some period, FDI is stalled. FDI is resumed not just when new consumers start arriving again, but when the total number of consumers reaches a new high that exceeds the previous high by an endogenously determined margin. Thus, capacity expansion becomes more volatile. Furthermore, the point of entry into the foreign market (as discussed in Section 3) is delayed. In fact, A^* may even be above $\overline{A}$.

8. *Market Research.* Sometimes firms try to reduce demand uncertainty through marketing studies or by observing other firms that sell related products. To the extent that this affects firms' perception of consumers' arrival process, it will be reflected in an f that exhibits less variability. When f exhibits less variability the firm is expected to rely more on FDI and less on exports.

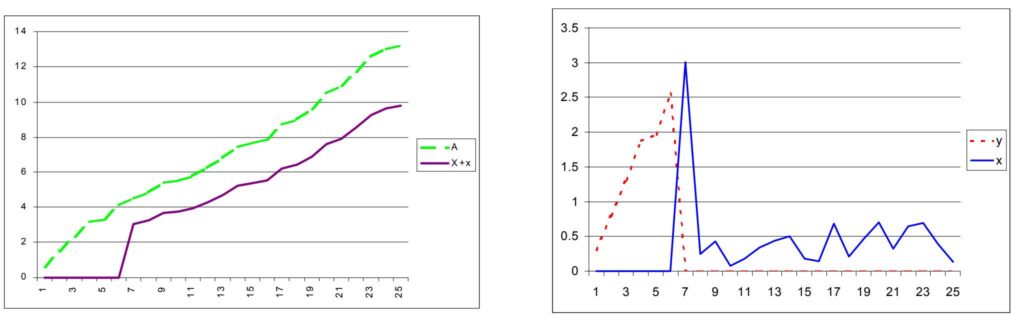
In summary, changes to the model along the lines we have discussed will have mostly a quantitative effect on the mix of FDI and exports. However, the comparative static results and the basic insights of the model would remain intact.

9 Numerical illustration of the results

To illustrate our approach, we now provide the solution for the particular case of linear demand, $D(q) = 2 - q$, and a uniform distribution over the arrival of new consumers, $f(a) = 1/\bar{a}$ and $1 - F(a) = (\overline{a} - a)/\overline{a}$. We have solved this case analytically, and the solution can be found on the website of this *Review,* http://www.restud.org.uk/supplements.htm. Here, we report results for the following parameter values: $e = 12$, $c = 1.5, k = 10$, $\delta = 0.9$, $\overline{a} = 1$. For these parameter values, $q_k = 0.5$ and the value of s that gives $k[1 - (1 - s)\delta] = c$ is $s \approx 0.0555$. Thus, as per Proposition 5, exports in the post-entry stage are zero for any $s < 0.0555$, whereas for $s > 0.0555$, exports are zero when s is small enough, but become positive as s is increased. Therefore, to enrich the set of possibilities, we have chosen only s

values that exceed 0.0555. The A sequence is constructed using a random-number generator for a.

We first assume that $s = 0.1$. The optimal pre-entry sequence (exports only) and post-entry sequences (FDI and exports) are then calculated as described in Section 5. The critical value of A for entry into the market is calculated to be $A^* \approx 4.4257$. Figures 4A and 4B present the optimal solution graphically for a particular realization of the A sequence. For $A < A^*$, one has only exports whose levels are increasing as A increases over time. For $A > A^*$, one has only FDI; the seller finds it optimal to proceed without exports.



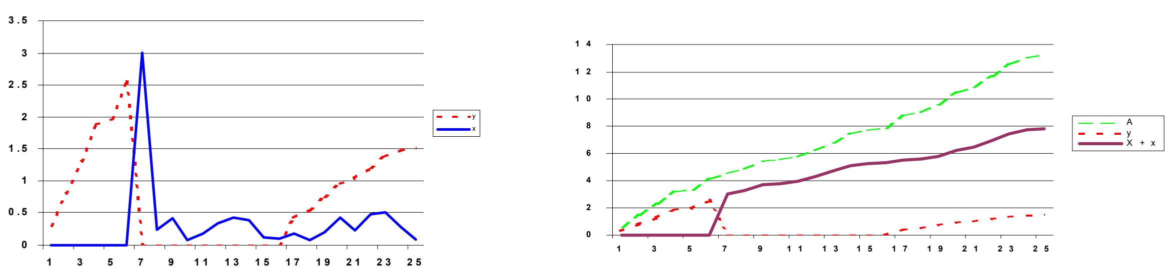
A - Stock   B - Flow

Figure 4: Example with fixed s and $D(q) = 2 - q$, $s = 0.1$, $e = 12$, $c = 1.5$, $k = 10$, $\delta = 0.9$ and f uniform on $[0, 1]$

We extended this example to other parameter values to illustrate how the solution varies with the parameters. In particular, as s increases from the level of 0.1, we observe two changes. First, the critical number of consumers required for entry, A^*, increases. So entry takes place later and does so on a larger scale. Second, higher values of s imply that, in the post-entry stage, the seller finds it optimal to have both investments and exports.

Finally, we have calculated a 'hybrid' example where s increases with A. This reflects the scenario where the probability of new consumer arrival diminishes as the size of proven demand increases (the idea being that, the more the demand that has been found, the less likely is the seller to find more demand). We have chosen $s = 0.1$ for $A \in [0, 7)$, $s = 0.0666A - 0.3666$ for $A \in [7, 10)$, and $s = 0.3$ for $A \in [10, \infty)$. The critical value A^* remains unchanged at the level calculated above ($A^* \approx 4.4257$). To make the comparison with the previous example transparent, we use the same realization of the A sequence as before. Figures 5A and 5B present the solution graphically. The time-paths of exports and FDI are divided into three phases. There is an initial phase, where only exports are used. This phase comes to an end when demand exceeds A^*. At that point, the seller enters the market via FDI, and the level of this initial FDI is high. In the second phase, the seller adds to the productive capacity by doing additional FDI, but there are no exports. In the third phase, the seller does both exports and FDI. Thus, the difference between Figure 4 and Figure 5 is the addition of the third phase, where FDI and exports are combined.

It is interesting to note that, while we have not "calibrated" the model to match actual data, there is a similarity between the third phase, shown in the right portion of figure 5B, and the actual Toyota Camry data, shown in the right portion of figure 1B. In both figures exports and FDI increase, i.e., they move together, as demand increases.



A-Flow

B-stock

Figure 5: Example with variable s and $D(q) = 2 - q$, $e = 12$, $c = 1.5$, $k = 10$, $\delta = 0.9$ and f uniform on $[0, 1]$

10 Concluding remarks

This paper characterizes optimal entry into a new market. The key features of the model are (i) stochastic demand growth, and (ii) the availability of two instruments to serve the market. The instruments are investment in capacity (which would be preferable if the demand were - in retrospect - large) and exports (which would be preferable if the demand were - in retrospect - small). Viewed somewhat more generally, our model explores the dynamics between a short-run and a long-run investment, or of the choice over time between a technology with lower marginal cost (FDI) and one with lower fixed cost (exports). The central result of the analysis is the characterization of how the presence of demand uncertainty and the dynamics may give rise to an interior solution, where both exports and FDI are used.

In addition to the variations of the model discussed in Section 8, other extensions of our analysis also appear of interest. First, we have focused on a seller with an existing production facility (at a "home" or "source" country) that enters a new ("target") market. We have not characterized the solution to the more general problem of a seller with multiple production facilities that can supply multiple markets. In particular, in our model we do not explore the possibility that, as demand in the target market grows, the seller may wish to move all production to the target market, and "reverse-export" from there back to the home market or to other world markets - one of Vernon's (1966) leading hypotheses.

Second, the entry considerations examined in this paper can also be studied within a strategic framework. In particular, one may explore entry by oligopolists into a market with growing and stochastic demand. Strategic considerations could affect the optimal choice between exports and FDI. We expect firms to do FDI quickly and at a high level compared to the single seller case, in order to obtain a strong position and deter rivals from investing in the following periods.[12]

Finally, additional aspects of entry, not present in our model, may further enrich the dynamics. For example, the seller may be able to learn more about the demand by penetrating the market faster (e.g. Rob, 1991). There may also be scope for experimentation

[12] This effect is similar to building capacity for strategic reasons, as in Dixit (1980), and appears empirically important (e.g. is consistent with patterns reported in Blonigen (2001), where firms tend to do FDI on a large scale). For related ideas and the role of uncertainty see, e.g., Maggi (1996). For strategic issues related to multinational activity also see Horstmann and Markusen (1987). Depending on the informational structure, in cases of strategic entry under uncertainty, informational externalities may also lead to free-riding among the entrants (and to slow expansion).

and strategic pricing (e.g. Bergemann and Välimäki, 1996), or entry itself may cause the demand to grow over time as a result of consumers' learning (e.g. Vettas, 1998).

Appendix

11 Proof of Proposition 1

In this Section we prove Proposition 1. Our approach is as follows. We define $E(A)$ as the value of exporting in the current period and committing to enter in the following period (although the policy that underlies this function is not optimal for certain A's, it is convenient to define and work with this function). We show that there exists an A^* where $u(A) = E(A) = v(0, A) - e$ and that the seller enters for every $A \geq A^*$ and exports for every $A < A^*$. Thus, one is able to determine A^* from the indifference condition $E(A) = v(0, A) - e$. The next step is to transform $E(A) = v(0, A) - e$ into equation (10):

$$[1 - (1 - s)\delta]e = (1 - s)\delta k x^*(A) - \psi(\bar{y}(A), A) + \phi(x^*(A), y^*(A), A), \tag{10}$$

and show that the RHS of the latter is strictly increasing. This implies that $E(A) = v(0, A) - e$ is uniquely solved. The details are as follows.

Let A^* be the supremum of the set of A's where exporting is optimal. Since this set is bounded from above by $\overline{A} < \infty$, we have $A^* < \infty$. (If this set is empty we define $A^* = 0$). By definition, entry is uniquely optimal for every $A > A^*$. By continuity it is also optimal to enter at A^* so the value of the value function at A^* is $v(0, A^*) - e$. Also, by definition, there exists a sequence (A_n) so that $A_n \nearrow A^*$ and so that it is optimal to export at A_n, i.e., the value at A_n is $u(A_n)$. Therefore, by continuity, it is also optimal to export at A^* and the value of the value function at A^* is $u(A^*)$. Finally, since it is uniquely optimal to enter at $A > A^*$, and since A is strictly increasing, an optimal policy at A^* is to export with a commitment to enter in the next period. The net result is that $u(A^*) = E(A^*) = v(0, A^*) - e$ so A^* is the solution to $E(A) = v(0, A) - e$.

This last equality is simplified as follows. The function we called $E(A)$ is obtained by substituting $u(A + a) = -e + v(0, A + a)$ into the RHS of (7). After some manipulations the equality $E(A) = v(0, A) - e$ is shown to be equivalent to equation (10) so it remains to show that the RHS of (10) is strictly increasing.

We prove this via 2 Lemmas.

Lemma A1: *$x^*(A) + y^*(A) \geq \overline{y}(A)$, with equality if and only if $y^*(A) > 0$.*

Proof. To alleviate the notation we use x, y and $\overline{y}$ instead of $x^*(A), y^*(A)$ and $\overline{y}(A)$.

In proving the Lemma, we use the optimality conditions in Propositions 2 and 3, which are derived without reliance on Lemma A1 that we are proving here.

The FOC for $\overline{y}$ is:

$$c = sG_1(\overline{y}, A) + (1-s) \int_{\overline{y}-A}^{\infty} R'(\frac{\overline{y}}{A+a}) f(a) da, \tag{A1}$$

where

$$G_1(\overline{y}, A) = \begin{cases} 0 & \text{if } \overline{y} \geq A \\ R'(\frac{\overline{y}}{A}) & \text{if } \overline{y} < A. \end{cases}$$

We now distinguish the following cases:

- If $c > k[1-(1-s)\delta]$, then $y = 0$ and x is determined via the FOC (20) once we substitute $X = 0$:

$$k[1-(1-s)\delta] = sH_1(x, 0, A) + (1-s) \int_{x-A}^{\infty} R'(\frac{x}{A+a}) f(a) da. \tag{A2}$$

Note that the RHS's of (A1) and (A2) are decreasing in $\overline{y}$ and x, respectively. Since the RHS of (A2) is bigger than the RHS of (A1) (because $H_1 > G_1$), while the LHS is smaller (because, in this case, $c > k - (1-s)\delta k$), we have $x > \overline{y}$.

- If $k > c + (1-s)\delta k$, then there are two possibilities: an interior solution or a corner solution (with $y = 0$). If we have an interior solution, then $x + y$ satisfies:

$$c = sH_2(x, y, A) + (1-s) \int_{x+y-A}^{\infty} R'(\frac{x+y}{A+a}) f(a) da.$$

We further see that $H_2 = G_1$, which implies that the two first-order conditions are identical and, hence, in this case, we have $x + y = \overline{y}$.

- The final case is when $k > c + (1-s)\delta k$ and the solution is $y = 0$. The optimal x is again determined via (A2). Then compared to the FOC for $\overline{y}$, (A1), both the RHS and the LHS of (A2) are larger. From (21) and the fact that $H_2 = G_1$ (and keeping in mind that we are exploring now the case $X = 0$), we know that we have a corner solution if and only if:

$$k[1-(1-s)\delta] - c < s[H_1(x^o(0), 0, A) - G_1(x^o(0), A)]. \tag{A3}$$

Arguing by contradiction, suppose that in the corner solution ($y = 0$) case we have ($x^o(0) =$) $x \leq \overline{y}$. Then the FOC under exports only holds with equality for $y = \overline{y}$:

$$c = sG_1(\overline{y}, A) + (1-s) \int_{\overline{y}-A}^{\infty} R'(\frac{\overline{y}}{A+a}) f(a) da.$$

Since the RHS is decreasing in its argument and we assume $x^o(0) \leq \overline{y}$, we have at $x^o(0)$:

$$c \leq sG_1(x^o(0), A) + (1-s) \int_{x^o(0)-A}^{\infty} R'(\frac{x^o(0)}{A+a}) f(a) da. \tag{A4}$$

Further, the FOC for x holds at $x^o(0)$:

$$k[1-(1-s)\delta] = sH_1(x^o(0), 0, A) + (1-s) \int_{x^o(0)-A}^{\infty} R'(\frac{x^o(0)}{A+a}) f(a) da. \tag{A5}$$

If we subtract (A5) from (A4), we obtain:

$$k[1-(1-s)\delta] - c \geq s[H_1(x^o(0), 0, A) - G_1(x^o(0), A)],$$

which contradicts (A3). We conclude that, in this case, we have ($x^o(0) =$) $x > \overline{y}$. ■

Based on Lemma A1, we now proceed to prove:

Lemma A2: The RHS of (10) is increasing in A.

Proof. Let us rewrite the value equation at $X = 0$:

$$v(0, A) = \max_{(x,y) \in \Re_+^2} \{\phi(x, y, A) + \delta(1-s) \int_0^{\infty} v(x, A+a) f(a) da]\}.$$

The first-order conditions, with respect to x and y, are:

$$\begin{aligned} \phi_1 + \delta(1-s)\int_0^{\infty} v_1(x, A+a)f(a)da &= 0, \\ \phi_2 &= 0. \end{aligned}$$

But $v_1 = k$ (which follows from $x > 0$ along the optimal path; this is shown in Section 2 of this Appendix). Thus, $\phi_1 = -\delta(1-s)k$. Therefore,

$$\phi_1 \frac{\partial x^*}{\partial A} + \phi_2 \frac{\partial y^*}{\partial A} + \phi_3 = -\delta(1-s)k\frac{\partial x^*}{\partial A} + 0 + \phi_3.$$

We also have:

$$\psi_1 \frac{\partial y^*}{\partial A} + \psi_2 = \psi_2.$$

Therefore,

$$\frac{d}{dA}\{(1-s)\delta k x^*(A) - \psi(\bar{y}(A), A) + \phi(x^*(A), y^*(A), A)\} = \phi_3 - \psi_2.$$

It remains to show that $\phi_3 > \psi_2$. We have:

$$\begin{aligned} &\phi_3 = sH_3(x,y,A) + \\ &\quad (1-s)[F(x+y-A) + \int_{x+y-A}^{\infty} [R(\frac{x+y}{A+a}) - \frac{x+y}{A+a}R'(\frac{x+y}{A+a})]f(a)da], \end{aligned} \tag{A6}$$

and

$$\begin{aligned} &\psi_2 = sG_2(\overline{y}, A) + \\ &\quad (1-s)[F(\overline{y}-A) + \int_{\overline{y}-A}^{\infty} [R(\frac{\overline{y}}{A+a}) - \frac{\overline{y}}{A+a}R'(\frac{\overline{y}}{A+a})]f(a)da]. \end{aligned} \tag{A7}$$

We will show that:

$$H_3(x,y,A) \geq G_2(\overline{y}, A),$$

that

$$F(x+y-A)+\int_{x+y-A}^{\infty}[R(\frac{x+y}{A+a})-\frac{x+y}{A+a}R'(\frac{x+y}{A+a})]f(a)da \geq$$

$$F(\overline{y}-A)+\int_{\overline{y}-A}^{\infty}[R(\frac{\overline{y}}{A+a})-\frac{\overline{y}}{A+a}R'(\frac{\overline{y}}{A+a})]f(a)da,$$

and that at least one of the above two inequalities is strict.

First note that the derivatives of the terminal values with respect to A are as follows:

$$G_2(\overline{y},A)=\begin{cases} 1+\frac{\delta}{1-\delta}\lambda & \text{if } \overline{y}\geq A \\ R(\frac{\overline{y}}{A})-\frac{\overline{y}}{A}R'(\frac{\overline{y}}{A})+\frac{\delta}{1-\delta}\lambda & \text{if } \overline{y}<A, \end{cases}$$

where

$$\lambda\equiv\begin{cases} \pi_c & \text{if } \frac{A\pi_c}{1-\delta}\geq -e+\frac{A\pi_k}{1-\delta} \\ \pi_k & \text{if } \frac{A\pi_c}{1-\delta}< -e+\frac{A\pi_k}{1-\delta}. \end{cases}$$

The above derivative is defined for any $A\neq\overline{A}\equiv e(1-\delta)/(\pi_k-\pi_c)$ ($G(\overline{y},A)$ is not differentiable at $A=\overline{A}$). Note, however, that we are now studying the pre-entry case and, as shown in part (ii) of Proposition 1, this means that the relevant A is lower than $\bar{A}$. Hence, we restrict attention to $A<\overline{A}$, and we have:

$$G_2(\overline{y},A)=\begin{cases} 1+\frac{\delta}{1-\delta}\pi_c & \text{if } \overline{y}\geq A \\ R(\frac{\overline{y}}{A})-\frac{\overline{y}}{A}R'(\frac{\overline{y}}{A})+\frac{\delta}{1-\delta}\pi_c & \text{if } \overline{y}<A. \end{cases}$$

We also have, for $x+y>A$:

$$H_3(x,y,A)=\begin{cases} \frac{1}{1-\delta} & \text{if } x>A \\ 1+\frac{\delta}{1-\delta}[R(\frac{x}{A})-\frac{x}{A}R'(\frac{x}{A})] & \text{if } A>x\geq Aq_k \\ 1+\frac{\delta}{1-\delta}\pi_k & \text{if } Aq_k>x. \end{cases}$$

And for $x+y<A$:

$$H_3(x,y,A)=\begin{cases} R(\frac{x+y}{A})-\frac{x}{A}R'(\frac{x+y}{A})+\frac{\delta}{1-\delta}[R(\frac{x}{A})-\frac{x}{A}R'(\frac{x}{A})] & \text{if } A>x\geq Aq_k \\ R(\frac{x+y}{A})-\frac{x}{A}R'(\frac{x+y}{A})+\frac{\delta}{1-\delta}\pi_k & \text{if } Aq_k>x. \end{cases}$$

Also note the following property that will be used in the remainder of this proof. The function $R(q) - qR'(q)$ is increasing, in particular, taking value 0 when $q = 0$ and value 1 when $q = 1$. This follows from the concavity of $R(q)$, and the fact that $R(q)$ is maximized at $q = 1$ (in particular, we have $\partial[R(q) - qR'(q)]/\partial q = -qR''(q) > 0$).

From Lemma A1, we know that $x + y \geq \bar{y}$. We now need to distinguish two cases:

- First consider the case $x + y = \bar{y}$.

In this case, the second terms at the RHS of (A6) and (A7) are equal. So we need to show that $H_3(x, y, A) > G_2(\overline{y}, A)$. First note that $\pi_c < \pi_k < 1$. Note also that $H_3(x, y, A)$ is increasing in x. Thus, when we show below that $H_3(x, y, A) > G_2(\overline{y}, A)$ holds for $x < Aq_k$ we also know that it holds for any x. There are two subcases to examine. If $\overline{y} \geq A$ then (for $x < Aq_k$) we have $H_3(x, y, A) = 1 + \frac{\delta}{1-\delta}\pi_k > 1 + \frac{\delta}{1-\delta}\pi_c = G_2(\overline{y}, A)$. If $\overline{y} < A$ then (for $x < Aq_k$) we have $H_3(x, y, A) = R(\frac{x+y}{A}) - \frac{x+y}{A}R'(\frac{x+y}{A}) + \frac{\delta}{1-\delta}\pi_k > R(\frac{\overline{y}}{A}) - \frac{\overline{y}}{A}R'(\frac{\overline{y}}{A}) + \frac{\delta}{1-\delta}\pi_c = G_2(\overline{y}, A)$.

- Now consider the case $x + y > \bar{y}$.

In this case, the term multiplied by $(1 - s)$ at the RHS of (A6) is strictly higher that the corresponding term of (A7). The proof is as follows:

$$F(x+y-A) + \int_{x+y-A}^{\infty} [R(\frac{x+y}{A+a}) - \frac{x+y}{A+a}R'(\frac{x+y}{A+a})]f(a)da =$$

$$F(\bar{y}-A) + [F(x+y-A) - F(\bar{y}-A)] + \int_{x+y-A}^{\infty} [R(\frac{x+y}{A+a}) - \frac{x+y}{A+a}R'(\frac{x+y}{A+a})]f(a)da =$$

$$F(\bar{y}-A) + \int_{\bar{y}-A}^{x+y-A} 1 \cdot f(a)da + \int_{x+y-A}^{\infty} [R(\frac{x+y}{A+a}) - \frac{x+y}{A+a}R'(\frac{x+y}{A+a})]f(a)da >$$

$$F(\bar{y}-A) + \int_{\bar{y}-A}^{x+y-A} [R(\frac{\overline{y}}{A+a}) - \frac{\overline{y}}{A+a}R'(\frac{\overline{y}}{A+a})]f(a)da +$$

$$\int_{x+y-A}^{\infty} [R(\frac{\overline{y}}{A+a}) - \frac{\overline{y}}{A+a}R'(\frac{\overline{y}}{A+a})]f(a)da =$$

$$F(\overline{y}-A)+\int_{\overline{y}-A}^{\infty}[R(\frac{\overline{y}}{A+a})-\frac{\overline{y}}{A+a}R'(\frac{\overline{y}}{A+a})]f(a)da,$$

where the inequality follows from the fact that $R(q)-qR'(q)$ is less than 1 and increasing in q, for $q\in[0,1]$.

Finally, in this case we have $H_3(x,y,A)\geq G_2(\overline{y},A)$. The proof follows the same steps as the proof of $H_3(x,y,A)>G_2(\overline{y},A)$ in the $x+y=\bar{y}$ case above (and, again, the property that $R(q)-qR'(q)$ increasing in q is used here). This completes the proof of Lemma A2. ■

Part (i) of Proposition 1 follows immediately from Lemma A2.

To prove part (ii) of Proposition 1 we first show that if demand stops growing at $\overline{A}$, the seller is indifferent between exports and FDI. The value under exports forever is $\frac{A\pi_c}{1-\delta}$, whereas the value under immediately doing FDI is $-e+\frac{A\pi_k}{1-\delta}$ (once the seller does FDI it is not optimal to do any exports.) Since $\overline{A}$ is defined by equality between these two, the seller is indifferent. The only remaining possibility is that the seller exports for a finite duration, say t, and then does FDI, and sticks with local production thereafter. However, since the above two terms are equal, that possibility gives the same value for any t.

Now consider $\overline{A}$, and assume demand has not stopped growing. Assume the monopolist does FDI, and chooses $x^{**}=Aq_k$ and a y^{**} for which $x^{**}+y^{**}=\overline{y}$. There are 2 possibilities. Either, demand stops growing in the next period. In this case, and as we have just shown, the seller attains the same value as under $u(\overline{A})$. Or, demand does not stop growing, in which case the seller attains a bigger value. So between these 2 cases the value under (x^{**},y^{**}) exceeds $u(\overline{A})$. Furthermore, since (x^{**},y^{**}) is not the optimal choice, $v(0,\overline{A})$ exceeds the value he attains with (x^{**},y^{**}) and, therefore, exceeds $u(\overline{A})$. Therefore, at $\overline{A}$ the seller is better off with FDI and, since $v(0,A)-u(A)$ is increasing in A, he prefers FDI for any $A>\overline{A}$.

To show part (iii) of Proposition 1, observe that the RHS of (10) is increasing in A and constant with respect to e, while the LHS is increasing in e.

12 Proof of Lemma 3

To prove Lemma 3, we start by proving the following Claim.

Claim: Assume the optimal x is positive in some period and let y be the optimal exports in that period. Then:

$$k-(1-s)\delta\int_{0}^{\infty}v_1(X+x,A+a)f(a)da-c\leq s[H_1(X,y,A)-H_2(X,y,A)]. \tag{A8}$$

Proof. Since x is positive, (13) is satisfied with equality, and we re-write it as:

$$k-(1-s)\delta\int_{0}^{\infty}v_1(X+x,A+a)f(a)da$$

$$=sH_1(X+x,y,A)+(1-s)\int_{X+x+y-A}^{\infty}R'(\frac{X+x+y}{A+a})f(a)da. \tag{A9}$$

We then use (14)

$$c\geq sH_2(X+x,y,A)+(1-s)\int_{X+x+y-A}^{\infty}R'(\frac{X+x+y}{A+a})f(a)da, \tag{14}$$

to eliminate $(1-s)\int_{X+x+y-A}^{\infty}R'(\frac{X+x+y}{A+a})f(a)da$ from (A9), and this proves the Claim.

Consider now some period t and assume x_t is positive (subscripted x and y are understood as optimal values). Assume, by way of contradiction, that $x_{t+1}=0$. If $y_{t+1}=0$ as well we get a contradiction to (13):

$$k\geq sH_1(X+x,y,A)+(1-s)\int_{X+x+y-A}^{\infty}R'(\frac{X+x+y}{A+a})f(a)da+$$

$$(1-s)\delta\int_{0}^{\infty}v_1(X+x,A+a)f(a)da. \tag{13}$$

Since (13) holds at t with equality and since the RHS is increasing in A, (13) is violated at $t+1$. Thus y_{t+1} must be positive. But then (14) is satisfied with equality, which we re-write as:

$$c = sH_2(X_{t+1}, y_{t+1}, A_{t+1}) + (1-s) \int\limits_{X_{t+1}+y_{t+1}-A_{t+1}}^{\infty} R'(\frac{X_{t+1}+y_{t+1}}{A_{t+1}+a}) f(a)da.$$

Also, since $x_{t+1} = 0$, we have:

$$\begin{aligned} & k - (1-s)\delta \int\limits_{0}^{\infty} v_1(X_{t+1}, A_{t+1}+a) f(a)da \\ > \quad & sH_1(X_{t+1}, y_{t+1}, A_{t+1}) + (1-s) \int\limits_{X_{t+1}+y_{t+1}-A_{t+1}}^{\infty} R'(\frac{X_{t+1}+y_{t+1}}{A_{t+1}+a}) f(a)da. \end{aligned}$$

We can now use the first equation to eliminate $(1-s) \int\limits_{X_{t+1}+y_{t+1}-A_{t+1}}^{\infty} R'(\frac{X_{t+1}+y_{t+1}}{A_{t+1}+a}) f(a)da$ from the second equation. This gives us:

$$k - (1-s)\delta \int\limits_{0}^{\infty} v_1(X_{t+1}, A_{t+1}+a) f(a)da - c > s[H_1(X_{t+1}, y_{t+1}, A_{t+1}) - H_2(X_{t+1}, y_{t+1}, A_{t+1})].$$

However, this is impossible because, by the above Claim, the reverse inequality holds for A_t, which is smaller than A_{t+1}, because $H_1 - H_2$ is increasing in A and because $H_1 - H_2$ is independent of y.

13 Proof of Lemma 4

Let us rewrite equation (16),

$$k = sH_1(X+x, y, A) + (1-s)[\int\limits_{X+x+y-A}^{\infty} R'(\frac{X+x+y}{A+a}) f(a)da + \delta k], \tag{16}$$

as

$$k[1-(1-s)\delta] = W(x, y),$$

and equation (17),

$$c = sH_2(X+x,y,A) + (1-s)\int_{X+x+y-A}^{\infty} R'(\frac{X+x+y}{A+a})f(a)da, \tag{17}$$

as

$$c = Z(x,y),$$

where the RHS expressions $W(x,y)$ and $Z(x,y)$ are determined by the RHS's of (16) and (17). We then have that the slope of $y^o(x)$ is $dy/dx = -(\partial Z/\partial x)/(\partial Z/\partial y) < 0$ and the slope of $x^o(y)$ is $dy/dx = -(\partial W/\partial x)/(\partial W/\partial y) < 0$. We need to show that:

$$(\partial W/\partial x)/(\partial W/\partial y) \geq (\partial Z/\partial x)/(\partial Z/\partial y).$$

Note that the term multiplied by $(1-s)$ is the same in the RHS of both (16) and (17), after one moves the constant $(1-s)\delta k$ to the LHS of (16). We also have that $\frac{\partial W}{\partial x}$, $\frac{\partial W}{\partial y}$, $\frac{\partial Z}{\partial x}$, and $\frac{\partial Z}{\partial y}$ are all negative. Therefore, it suffices to show:

$$H_{11} \leq H_{12} = H_{21} = H_{22} \leq 0$$

(where $H_{11} \equiv \partial H_1/\partial x$, and so on), which we prove next.

As a first step, we need to calculate the derivatives of H:

If $X+y \geq A$, we have:

$$H_1(X,y,A) = \begin{cases} 0 & \text{if } X \geq A \\ \frac{\delta}{1-\delta}R'(\frac{X}{A}) & \text{if } A > X \geq Aq_k \\ \delta k & \text{if } Aq_k > X \end{cases} \tag{A10}$$

and, if $X+y < A$, we have:

$$H_1(X,y,A) = \begin{cases} R'(\frac{X+y}{A}) + \frac{\delta}{1-\delta}R'(\frac{X}{A}) & \text{if } A > X \geq Aq_k \\ R'(\frac{X+y}{A}) + \delta k & \text{if } Aq_k > X. \end{cases} \tag{A11}$$

Also,

$$H_2(X,y,A) = \begin{cases} 0 & \text{if } X+y \geq A \\ R'(\frac{X+y}{A}) & \text{if } X+y < A. \end{cases} \tag{A12}$$

Then direct calculations, using (A10), (A11) and (A12), show that:

$$H_{12}(X+x,y,A) = H_{21}(X+x,y,A) = H_{22}(X+x,y,A) = \begin{cases} 0 & \text{if } X+x+y \geq A \\ \frac{1}{A}R''(\frac{X+x+y}{A}) & \text{if } X+x+y < A, \end{cases}$$

that, if $X+x+y \geq A$,

$$H_{11}(X+x,y,A) = \begin{cases} 0 & \text{if } X+x \geq A \\ \frac{\delta}{1-\delta}R''(\frac{X+x}{A})\frac{1}{A} & \text{if } A > X+x \geq Aq_k \\ 0 & \text{if } Aq_k > X+x, \end{cases}$$

and that, if $X+x+y < A$,

$$H_{11}(X+x,y,A) = \begin{cases} R''(\frac{X+x+y}{A})\frac{1}{A} + \frac{\delta}{1-\delta}R''(\frac{X+x}{A})\frac{1}{A} & \text{if } A > X+x \geq Aq_k \\ R''(\frac{X+x+y}{A})\frac{1}{A} & \text{if } Aq_k > X+x. \end{cases}$$

Using the fact that R is concave (and therefore that $\frac{\delta}{1-\delta}R''(\frac{X+x}{A})\frac{1}{A} < 0$), direct comparison of the above three terms shows that one indeed has $H_{11} \leq H_{12} = H_{21} = H_{22} \leq 0$.

14 Proof of Proposition 5

We are going to work with the first order conditions,

$$k = sH_1(X+x,y,A) + (1-s)[\int_{X+x+y-A}^{\infty} R'(\frac{X+x+y}{A+a})f(a)da + \delta k] \tag{16}$$

and

$$c = sH_2(X+x,y,A) + (1-s)\int_{X+x+y-A}^{\infty} R'(\frac{X+x+y}{A+a})f(a)da. \tag{17}$$

To work with these conditions we need the derivatives of H. These are calculated above as (A10), (A11), and (A12). We also need the difference, $H_1 - H_2$, which can be readily computed as:

$$H_1(X, y, A) - H_2(X, y, A) = \begin{cases} 0 & \text{if } X \geq A \\ \frac{\delta}{1-\delta} R'(\frac{X}{A}) & \text{if } A > X \geq Aq_k \\ \delta k & \text{if } Aq_k > X. \end{cases} \tag{A13}$$

Now, Lemma 4 implies that $x^o(y)$ and $y^o(x)$ cross at most once. So there are three cases to consider:

Case a: $x^o(y)$ intersects uniquely $y^o(x)$ in $\Re^2_{++}$. In this case (illustrated in Figure 3), we have an interior solution.

Case b: $x^o(y)$ is uniformly above $y^o(x)$. In that case, we have a corner solution with $y = 0$ and the x at which $x^o(y)$ intersects the horizontal axis, $x^o(0)$.

Case c: $x^o(y)$ is uniformly below $y^o(x)$. In this case, we have a corner solution with $x = 0$ and the y at which $y^o(x)$ intersects the vertical axis. Case (c) is ruled out by Lemma 3. It remains to consider cases (a) and (b).

Let us start with case (a). When an interior solution obtains, it can be calculated as follows. Substituting equation (17) into (16) one obtains equation (18):

$$k = c + (1 - s)\delta k + s[H_1(X + x, y, A) - H_2(X + x, y, A)]. \tag{18}$$

Consider $X + x \leq Aq_k$. Then, by (A13), we have $H_1 - H_2 = \delta k$. Therefore, (18) becomes $k = c + (1-s)\delta k + s\delta k$ or $c = (1-\delta)k$. But we maintain, from (1), the assumption $c > (1-\delta)k$. So this means that x should be increased to its maximum in this region, i.e., if x is to remain in this region we should set $X + x = Aq_k$.

Consider now an x such that $A > X + x > Aq_k$. Then, by (A13), $H_1 - H_2 = \frac{\delta}{1-\delta} R'(\frac{X}{A})$, so in this case (18) becomes:

$$k = c + (1 - s)\delta k + s \frac{\delta}{1 - \delta} R'(\frac{X + x}{A}). \tag{19}$$

The RHS is strictly decreasing in x. When $X + x = Aq_k$, the RHS equals $c + \delta k$ which, by assumption (1), is larger than k. So $X + x$ is optimally set above Aq_k. When $X + x = A$, the RHS equals $c + (1 - s)\delta k$, which is smaller than k - otherwise, as we shall shortly show, we have a corner solution. Thus the optimizing x must be such that $A > X + x > Aq_k$. Once we obtain x, we can solve for y from (17).

Let us turn now to case (b). We set $y = 0$ on the RHS of (16), which gives us (20), one equation in one unknown:

$$k = (1-s)[\int\limits_{X+x-A}^{\infty} R'(\frac{X+x}{A+a})f(a)da + \delta k] + sH_1(X+x,0,A). \tag{20}$$

The solution to this equation is $x^o(0)$. As in case (a), $x^o(0)$ satisfies $X + x^o(0) > Aq_k$. A value such that $X + x^o(0) \leq Aq_k$ would violate (20), since the RHS would exceed the LHS (recall that F puts probability 1 on strictly positive values of a). However, and unlike case (a), we may have $X + x^o(0) > A$.

It remains to determine when case (a) applies and when case (b) applies. Let us entertain the possibility that case (b) applies. Then we substitute $y = 0$ and $x = x^o(0)$ into the RHS of (17). If the resulting value is smaller than c, the marginal benefit of y is smaller than the marginal cost of y, so $y = 0$ is indeed optimal. On the other hand, if the resulting value is larger than c it pays to increase y, so we have $y > 0$. After some further manipulations cases (a) and (b) are distinguished as follows. The optimal y is positive if (21) holds and is equal to zero if the reverse inequality holds.

This concludes the proof. Note that since $H_1 - H_2 \geq 0$ (see equation (A13)), an implication of (21) is that if $c > k[1-(1-s)\delta]$, one has a corner solution, while if $c < k[1-(1-s)\delta]$, one can have either a corner or an interior solution.

15 Proof of Proposition 6

In this Appendix we prove our comparative statics results, Proposition 6. To alleviate the notation, for this proof we write $\hat{x}$ instead of $x^o(0)$.

In all cases, we proceed as follows. Let b be the parameter that is being examined (that is, b stands for c, k, δ, s, or A). In the case of an interior solution, we first obtain $\partial x/\partial b$ from equation (19). Then, we obtain $\partial y/\partial b$ from equation (17) (after we substitute the value $\partial x/\partial b$ from the previous step). If we have a corner solution ($y = 0$), we calculate $\partial x/\partial b$ ($= \partial\hat{x}/\partial b$) from equation (20). Finally, with respect to how the boundary between an interior and a corner solution changes, when the parameter b changes, we consider (21) with equality and see how the critical value changes with b. In this calculation, since the critical value depends on $\hat{x}$, we employ the value $\partial\hat{x}/\partial b$ from the previous step.

In the calculations described above, we use extensively the concavity of R and the normalization that R is maximized at 1. The details are as follows.

(i) Comparative statics with respect to k.

Consider first an interior solution. Then, from (19) we see that, as k increases, the RHS increases. For the equality to be restored, it is required that R' increases which implies a decrease in x. Next, we turn to equation (17). An increase in k only affects (17) through a decrease in x which means that, for the equality to be restored, we should have an increase in y.

Consider now a corner solution. Then, by implicitly differentiating (20), we obtain

$$\frac{\partial \hat{x}}{\partial k} = \frac{[1-(1-s)\delta]}{s\frac{\partial H_1}{\partial \hat{x}} + (1-s)[\partial(\int\limits_{X+\hat{x}-A}^{\infty} R'(\frac{X+\hat{x}}{A+a})f(a)da)/\partial \hat{x}]} < 0,$$

because the numerator is positive and the denominator is negative.

Finally, consider the boundary between interior and corner solutions that, by (20), can be written as

$$N \equiv k[1-(1-s)\delta] - c - s[H_1(X+\hat{x},0,A) - H_2(X+\hat{x},0,A)] = 0.$$

We then have

$$\frac{\partial N}{\partial k} = [1-(1-s)\delta] - s\frac{\partial(H_1 - H_2)}{\partial \widehat{x}}\frac{\partial \widehat{x}}{\partial k},$$

where

$$\frac{\partial(H_1 - H_2)}{\partial \widehat{x}} = \begin{cases} 0 & \text{if } X+\hat{x} \geq A \\ \frac{\delta}{1-\delta}R''(\frac{X+\hat{x}}{A})\frac{1}{A} & \text{if } A > X+\hat{x} \geq Aq_k \\ 0 & \text{if } Aq_k > X+\hat{x}. \end{cases}$$

Remember also that the corner solution satisfies $X+\hat{x} > Aq_k$. Now, for the case $X+\hat{x} \geq A$, we have $\partial N/\partial k = 1-(1-s)\delta > 0$. For the $A > X+\hat{x} \geq Aq_k$ case we have $\partial(H_1-H_2)/\partial\widehat{x} = \delta(\partial H_1/\partial\widehat{x}) = \frac{\delta}{1-\delta}R''(\frac{X+\hat{x}}{A})\frac{1}{A}$ and we obtain

$$\frac{\partial N}{\partial k} = [1-(1-s)\delta] - s\delta\frac{\partial H_1}{\partial \widehat{x}}\frac{[1-(1-s)\delta]}{s\frac{\partial H_1}{\partial \hat{x}} + (1-s)[\partial(\int\limits_{X+\hat{x}-A}^{\infty} R'(\frac{X+\hat{x}}{A+a})f(a)da)/\partial \hat{x}]} =$$

$$= [1-(1-s)\delta]\left\{1-\delta\frac{s\frac{\partial H_1}{\partial \widehat{x}}}{s\frac{\partial H_1}{\partial \widehat{x}}+(1-s)[\partial(\int\limits_{X+\hat{x}-A}^{\infty}R'(\frac{X+\hat{x}}{A+a})f(a)da)/\partial\hat{x}]}\right\}>0,$$

because both $[1-(1-s)\delta]$ and the expression in the braces are between 0 and 1. Thus, an increase in k tends to move the solution from a corner to an interior point.

(ii) Comparative statics with respect to c.

Consider first an interior solution. From (19) we see that, as c increases, the LHS decreases and, to restore the equality, we require a lower $R^{'}$ and, consequently, a higher x. Now turn to (17). An increase in c increases the LHS and also (through the increase is x) it decreases the RHS. Hence it is required that we have an increase in $R^{'}$, and hence a decrease in y.

Concerning a corner solution, clearly c does not affect (20) or the value of $\widehat{x}$

Consider now the boundary N. Since we have $\partial\widehat{x}/\partial c = 0$, we readily obtain $\partial N/\partial c = -1 < 0$ and hence an increase in the solution moves us from an interior to a corner.

(iii) Comparative statics with respect to δ.

Consider first an interior solution. An increase in δ decreases the LHS of (19) and increases its RHS. Thus, for the equality to be restored following an increase in s, we should have (a decrease in $R^{'}$ and) an increase in x. With respect to y, note that δ enters (17) only through x, therefore an increase in δ would decrease y.

Consider now a corner solution. From (20) we obtain

$$\frac{\partial\widehat{x}}{\partial\delta} = -\frac{(1-s)k+s\frac{\partial H_1}{\partial\delta}}{s\frac{\partial H_1}{\partial\widehat{x}}+(1-s)[\partial(\int\limits_{X+\hat{x}-A}^{\infty}R'(\frac{X+\hat{x}}{A+a})f(a)da)/\partial\hat{x}]}>0,$$

where the numerator is positive and the denominator is negative.

Concerning the boundary, we have $\partial N/\partial\delta = -(1-s)k < 0$ when $X+\hat{x}\geq A$, whereas when $A > X+\hat{x}\geq Aq_k$ we have

$$\frac{\partial N}{\partial\delta} = -(1-s)k - s\frac{\partial(H_1-H_2)}{\partial\widehat{x}}\frac{\partial\widehat{x}}{\partial\delta} - s\frac{\partial(H_1-H_2)}{\partial\delta} =$$

$$-(1-s)k+s\delta\frac{\partial H_1}{\partial\widehat{x}}\frac{(1-s)k+s\frac{\partial H_1}{\partial\delta}}{s\frac{\partial H_1}{\partial\widehat{x}}+(1-s)[\partial(\int\limits_{X+\hat{x}-A}^{\infty}R'(\frac{X+\hat{x}}{A+a})f(a)da)/\partial\hat{x}]}-s\frac{\partial H_1}{\partial\delta}=$$

$$=[(1-s)k+s\frac{\partial H_1}{\partial\delta}][\frac{\delta s\frac{\partial H_1}{\partial\widehat{x}}}{s\frac{\partial H_1}{\partial\widehat{x}}+(1-s)[\partial(\int\limits_{X+\hat{x}-A}^{\infty}R'(\frac{X+\hat{x}}{A+a})f(a)da)/\partial\hat{x}]}-1]<0.$$

Note that, to determine the sign of the above expression, we use

$$\frac{\partial(H_1-H_2)}{\partial\widehat{x}}=\delta\frac{\partial H_1}{\partial\widehat{x}}=\frac{\delta}{1-\delta}R''\frac{1}{A}<0,$$

$$\frac{\partial(H_1-H_2)}{\partial\delta}=\frac{\partial H_1}{\partial\delta}=\frac{1}{(1-\delta)^2}R'>0.$$

Thus, an increase in δ tends to move the solution from an interior point to a corner.

(iv) Comparative statics with respect to s.

Consider first an interior solution. Moving s to the LHS of (19) and differentiating, we obtain

$$\partial\left[\frac{k[1-(1-s)\delta]-c}{s}\right]/\partial s>0,$$

and thus, for the equality to be restored after an increase in s, we should have (an increase in R' and) a decrease in x. With respect to y, note that s enters (17) both directly and through x, with the two effects moving in opposite directions. Further manipulation shows that $\partial y/\partial s$ has an ambiguous sign.

Consider now a corner solution. From (20) we obtain,

$$\frac{\partial\widehat{x}}{\partial s}=-\frac{H_1-\int\limits_{X+\hat{x}-A}^{\infty}R'(\frac{X+\hat{x}}{A+a})f(a)da)-\delta k}{s\frac{\partial H_1}{\partial\widehat{x}}+(1-s)[\partial(\int\limits_{X+\hat{x}-A}^{\infty}R'(\frac{X+\hat{x}}{A+a})f(a)da)/\partial\hat{x}]}<0,$$

because both the numerator and the denominator are negative. While to see the sign of the denominator is immediate (given the concavity of R), the following argument can be used to establish the sign of the numerator. Recall that $\hat{x}$ solves (20). This equation can be rewritten as

$$s = [k(1-\delta) - \frac{\int\limits_{X+\hat{x}-A}^{\infty} R'(\frac{X+\hat{x}}{A+a})f(a)da]}{[H_1(X+\hat{x},0,A) - k\delta - \int\limits_{X+\hat{x}-A}^{\infty} R'(\frac{X+\hat{x}}{A+a})f(a)da]}].$$

Note that the numerator in the above expression is the same as the numerator in $\partial\widehat{x}/\partial s$. Now, since $s \in [0,1]$, both the numerator and the denominator in the above expression have the same sign. Moreover, note that $H_1(X+\hat{x},0,A) < k$. Then, arguing by contradiction, if the numerator and the denominator were positive then we would have $s > 1$, a contradiction. Thus the numerator (both in the above expression, as well as in $\partial\widehat{x}/\partial s$) is negative.

Turning now to the boundary, we have $\partial N/\partial s = \delta k - (H_1 - H_2) = \delta k > 0$ when $X + \hat{x} \geq A$, whereas when $A > X + \hat{x} \geq Aq_k$ we have

$$\frac{\partial N}{\partial s} = \delta k - (H_1 - H_2) + \frac{s\frac{\partial H_1}{\partial \widehat{x}}[H_1 - \int\limits_{X+\hat{x}-A}^{\infty} R'(\frac{X+\hat{x}}{A+a})f(a)da - \delta k]}{s\frac{\partial H_1}{\partial \widehat{x}} + (1-s)[\partial(\int\limits_{X+\hat{x}-A}^{\infty} R'(\frac{X+\hat{x}}{A+a})f(a)da)/\partial\hat{x}]} >$$

$$[H_1 - \int\limits_{X+\hat{x}-A}^{\infty} R'(\frac{X+\hat{x}}{A+a})f(a)da - \delta k]\left\{\frac{s\frac{\partial H_1}{\partial \widehat{x}}}{s\frac{\partial H_1}{\partial \widehat{x}} + (1-s)[\partial(\int\limits_{X+\hat{x}-A}^{\infty} R'(\frac{X+\hat{x}}{A+a})f(a)da)/\partial\hat{x}]} - 1\right\} > 0$$

where the first inequality is true because (remembering that in this case $A > X + \hat{x}$) the concavity of R implies

$$H_2 = R'(\frac{X+\hat{x}}{A}) > \int\limits_{X+\hat{x}-A}^{\infty} R'(\frac{X+\hat{x}}{A+a})f(a)da,$$

and, for the second inequality, we have shown earlier that the first factor is negative while the second factor is clearly negative, as well.

Thus, an increase in s tends to move the solution from a corner to an interior point.

(v) Comparative statics with respect to A.

Finally, we consider how the solution (for a given X) changes if we have a higher A.

Concerning an interior solution, an increase in A increases $R^{'}$ so, for the equality to be restored, we should have an increase in x. With respect to y, note that A enters (17) both directly and through x, with the two effects moving in opposite directions. Further manipulation shows that $\partial y/\partial A$ has an ambiguous sign.

Consider now a corner solution. From (20) we obtain

$$\frac{\partial \widehat{x}}{\partial A}=-\frac{s\frac{\partial H_1}{\partial A}+(1-s)[\partial(\int\limits_{X+\hat{x}-A}^{\infty} R^{'}(\frac{X+\hat{x}}{A+a})f(a)da)/\partial A]}{s\frac{\partial H_1}{\partial \widehat{x}}+(1-s)[\partial(\int\limits_{X+\hat{x}-A}^{\infty} R^{'}(\frac{X+\hat{x}}{A+a})f(a)da)/\partial \hat{x}]}>0,$$

where the numerator is positive and the denominator is negative.

Turning now to the boundary, we have $\partial N/\partial A=0$ in the $X+\hat{x}\geq A$ case (since N is affected by A only through H_1-H_2). We now turn to the $A>X+\hat{x}\geq Aq_k$ case. We have:

$$\frac{\partial N}{\partial A}<0 \Leftrightarrow \frac{\partial(H_1-H_2)}{\partial A}>0 \Leftrightarrow \frac{\partial[\frac{\delta}{1-\delta}R'(\frac{X+\hat{x}}{A})]}{\partial A}>0 \Leftrightarrow \frac{\partial(\frac{X+\hat{x}}{A})}{\partial A}<0 \Leftrightarrow$$
$$\frac{\partial \hat{x}}{\partial A}<\frac{X+\hat{x}}{A}.$$

Now, using $\partial\widehat{x}/\partial A$ that we have calculated above and rearranging, the above inequality is equivalent to

$$A[s\frac{\partial H_1}{\partial A}+(1-s)\frac{\partial(\int\limits_{X+\hat{x}-A}^{\infty} R^{'}(\frac{X+\hat{x}}{A+a})f(a)da)}{\partial A}]+$$
$$(X+\hat{x})[s\frac{\partial H_1}{\partial \widehat{x}}+(1-s)\frac{\partial(\int\limits_{X+\hat{x}-A}^{\infty} R^{'}(\frac{X+\hat{x}}{A+a})f(a)da)}{\partial \hat{x}}]<0$$

or

$$s[A\frac{\partial H_1}{\partial A}+(X+\hat{x})\frac{\partial H_1}{\partial \widehat{x}}]+$$
$$(1-s)[A\frac{\partial(\int\limits_{X+\hat{x}-A}^{\infty} R^{'}(\frac{X+\hat{x}}{A+a})f(a)da)}{\partial A}+(X+\hat{x})\frac{\partial(\int\limits_{X+\hat{x}-A}^{\infty} R^{'}(\frac{X+\hat{x}}{A+a})f(a)da)}{\partial \hat{x}}]<0.$$

We now have

$$\frac{\partial H_1}{\partial A} = -\frac{\delta}{1-\delta} R''(\frac{X+\hat{x}}{A})\frac{X+\hat{x}}{A^2} \qquad \text{and} \qquad \frac{\partial H_1}{\partial \widehat{x}} = \frac{\delta}{1-\delta} R''(\frac{X+\hat{x}}{A})\frac{1}{A},$$

and direct calculation shows that the first term in the above inequality is zero. The second term of the inequality is equal to

$$(1-s)(X+\hat{x}) \int_{X+\hat{x}-A}^{\infty} R''(\frac{X+\hat{x}}{A+a})\frac{a}{(A+a)^2} f(a)da < 0.$$

Thus, in the $A > X + \hat{x} \geq Aq_k$ case, we have $\partial N / \partial A < 0$, that is, an increase in A tends to move the solution from an interior point to a corner.

16 A parametric example: linear demand

In this section we flesh out the details of the parametric example presented in Section 9 of the paper. The example assumes linear demand, $D(q) = 2 - q$, and a uniform distribution over the number of new consumers, a. We first work out all the results that can be obtained without specifying a particular parametric form for f. Then, in Section 6.1, we obtain more specific results for the case where f is uniform.

Let us first note that under $D(q) = 2 - q$, one has $R(q) = q(2 - q) = 2q - q^2$, so that R is maximized at $q = 1$, with $R(1) = 1$, which is our working normalization.

Equation (1) translates into $2 > c > (1-\delta)k$. Now q_k is the maximizer of $2q - q^2 - (1-\delta)kq$ so $q_k = [2 - (1-\delta)k]/2$. Also, q_c is the maximizer of $2q - q^2 - cq$ so $q_c = (2 - c)/2$. Then $\pi_k \equiv R(q_k) - (1-\delta)kq_k = [2 - (1-\delta)k]^2/4$ and $\pi_c \equiv R(q_c) - cq_c = (2-c)^2/4$. With respect to the terminal values, we obtain:

If $X + y > A$

$$H_1(X, y, A) = \begin{cases} 0 & \text{if } X \geq A \\ \frac{\delta}{1-\delta}\frac{2(A-X)}{A} & \text{if } A > X \geq q_k A \\ \delta k & \text{if } q_k A > X, \end{cases}$$

and if $X + y < A$

$$H_1(X, y, A) = \begin{cases} \frac{2(A-X-y)}{A} + \frac{\delta}{1-\delta}\frac{2(A-X)}{A} & \text{if } A > X \geq q_k A \\ \frac{2(A-X-y)}{A} + \delta k & \text{if } q_k A > X. \end{cases}$$

We also have

$$H_2(X, y, A) = \begin{cases} \frac{2(A-X-y)}{A} & \text{if } X + y \leq A \\ 0 & \text{if } X + y > A. \end{cases} \tag{A14}$$

In addition, equation (19) becomes

$$k[1 - (1 - s)\delta] - c = s\,\frac{\delta}{1 - \delta}\,\frac{2(A - X - x)}{A}, \tag{A15}$$

or, solving for x, we obtain

$$x = \frac{2s\delta(A - X) + [c - k + \delta k(1 - s)]\,(1 - \delta)\,A}{2s\delta}. \tag{A16}$$

Further, equation (20) becomes

$$k[1 - (1 - s)\delta] = sH_1(X + x, 0, A) +$$

$$2(1 - s)\left\{[1 - F(X + x - A)] - (X + x)\int\limits_{X+x-A}^{\infty} \frac{f(a)}{A + a}da\right\}. \tag{A17}$$

16.1 Solution with uniform f

In addition to linear demand, suppose now that f is uniform on some interval $[0, \bar{a}]$. Then $f(a) = 1/\bar{a}$ and $1 - F(a) = (\bar{a} - a)/\bar{a}$. In this case, equation (17) becomes

$$c = sH_2(X + x, y, A) +$$

$$\frac{2(1 - s)}{\bar{a}}\{[\bar{a} - (X + x + y - A)] + (X + x + y)\ln\frac{X + x + y}{A + \bar{a}}], \tag{A18}$$

where, from (A14), we have

$$H_2(X+x,y,A)=\begin{cases}\frac{2(A-X-x-y)}{A} & \text{if } X+x+y\leq A\\ 0 & \text{if } X+x+y>A.\end{cases} \tag{A19}$$

In addition, equation (A17) becomes

$$k[1-(1-s)\delta]=sH_1(X+x,0,A)+$$
$$\frac{2(1-s)}{\bar{a}}\{[\bar{a}-(X+x-A)]+(X+x)\ln\frac{X+x}{A+\bar{a}}], \tag{A20}$$

where

$$H_1(X+x,0,A)=\begin{cases}0 & \text{if } X+x\geq A\\ \frac{1}{1-\delta}\frac{2(A-X-x)}{A} & \text{if } A>X+x\geq q_kA\\ \frac{2(A-X-x)}{A}+\delta k & \text{if } q_kA>X+x.\end{cases} \tag{A21}$$

To find the range of parameters for which one has a corner or an interior solution we proceed as follows. The solution to (A20) is $x^o(0)$. We have an interior solution if

$$k[1-(1-s)\delta]-c>s[H_1(X+x^o(0),0,A)-H_2(X+x^o(0),0,A)] \tag{A22}$$

and a corner solution otherwise, where

$$H_1(X+x^o(0),0,A)-H_2(X+x^o(0),0,A)=\begin{cases}0 & \text{if } X+x^o(0)\geq A\\ \frac{\delta}{1-\delta}\frac{2[A-X-x^o(0)]}{A} & \text{if } A>X+x^o(0)\geq Aq_k\\ \delta k & \text{if } Aq_k>X+x^o(0).\end{cases}$$

Based on all these calculations, the specialized form of Proposition 5 in the paper is stated as follows.

Proposition 5A. *Suppose that $D(q)=2-q$ and that f is uniform on $[0,\bar{a}]$. In the post-entry stage and as long as demand keeps growing: (i) FDIs are positive in each and every period, and are found as follows. (ii) If (A22) holds, the optimal x is given by (A16) and is such that $(X+x)/A$ is constant; this x is then substituted into (A18) to find the optimal y. (iii) If (A22) does not hold, then $y=0$ and the optimal x solves (A20).*

Further, we rewrite (12) for the case of this example. In the *pre-entry* stage, the optimal export levels can be found by solving

$$c = sG_1(y, A) + \frac{2(1-s)}{\bar{a}}[(A+\bar{a}-y) + y\ln(\frac{y}{A+\bar{a}})], \tag{A23}$$

where

$$G_1(y, A) = \begin{cases} 0, & y \geq A \\ 2(A-y)/A, & y < A. \end{cases}$$

Finally, the threshold A^* for initial entry into the market is calculated as follows. We work with equation (10). There are two cases to consider, depending on whether the post-entry solution at A^* is such that one has a corner solution $x^o(0)$ or an interior one. Suppose first there is a corner solution. Then $x^*(A)$ is calculated from (A20) while $\bar{y}(A)$ is calculated from (A23). Substituting these values into (10), we obtain one equation in one unknown, A. Note also that equation (10), in the case examined here, becomes (writing $x^o(0)$ for $x^*(A)$ and $\bar{y}$ for $\bar{y}(A)$):

$$\begin{aligned}[1-(1-s)\delta]e = [(1-s)\delta-1]kx^o(0) + c\bar{y} + s[H(x^o(0), 0, A) - G(\bar{y}, A)] + \\ \frac{(1-s)[x^o(0)^2 - \bar{y}^2]}{2\bar{a}} + (1-s)\frac{x^o(0)}{\bar{a}}[2(A+\bar{a}-x^o(0)) + x^o(0)\ln(\frac{x^o(0)}{A+\bar{a}})] - \\ (1-s)\frac{\bar{y}}{\bar{a}}[2(A+\bar{a}-\bar{y}) + \bar{y}\ln(\frac{\bar{y}}{A+\bar{a}})].\end{aligned} \tag{A24}$$

Suppose now that we have an interior solution. Then $x^*(A)$ is calculated from (A16), $y^*(A)$ is calculated from (A18), and $\bar{y}(A) = \bar{y}$ is calculated from (A23). Substituting these values into (10), we have one equation in one unknown, A. Note that, in this case, we have $x^*(A) + y^*(A) = \bar{y}(A)$ (the proof of this claim can be found as part of the proof of Proposition 1, Section 1 of this Appendix). Then, in this case, equation (10) can be simplified as

$$[1-(1-s)\delta]e = [(1-s)\delta-1]kx^* + c(\bar{y}-y^*) + s[H(x^*, y^*, A) - G(\bar{y}, A)]. \tag{A25}$$

Depending on whether at the critical value A^* the investment path involves a corner or an interior solution, either (A24) or (A25) holds and can be solved uniquely to give the value A^*.

16.2 Numerical results

Section 9 of the published article discusses how one calculates the solution of the example with linear demand and uniform f presented above for particular parameter values and a particular realization of the A sequence. The published article also provides a graphical illustration of the calculated solutions. For completeness, we provide here, in a Table, the solution for one of the cases. This Table corresponds to Figure 5 in the published article.

A	s	x	y	$X+x$
0.5775	0.1000	0	0.2998	0
1.4821	0.1000	0	0.7951	0
2.3011	0.1000	0	1.3047	0
3.1644	0.1000	0	1.8812	0
3.2873	0.1000	0	1.9658	0
4.1165	0.1000	0	2.5496	0
4.5182	0.1000	3.0090	0	3.0090
4.8528	0.1000	0.2439	0	3.2529
5.4265	0.1000	0.4231	0	3.6760
5.5333	0.1000	0.0793	0	3.7553
5.7757	0.1000	0.1809	0	3.9362
6.2255	0.1000	0.3380	0	4.2741
6.7953	0.1000	0.4324	0	4.7065
7.4506	0.1296	0.3940	0	5.1005
7.6841	0.1452	0.1231	0	5.2236
7.8683	0.1574	0.0973	0	5.3208
8.7474	0.2160	0.1779	0.4359	5.4987
9.0131	0.2337	0.0792	0.5405	5.5780
9.6023	0.2729	0.2005	0.7439	5.7785
10.4892	0.3000	0.4373	0.9595	6.2158
10.8881	0.3000	0.2364	1.0383	6.4522
11.6944	0.3000	0.4778	1.2018	6.9300
12.5523	0.3000	0.5084	1.3812	7.4384
13.0291	0.3000	0.2825	1.4832	7.7209
13.1891	0.3000	0.0948	1.5177	7.8157

TABLE 1: Example with $D(q) = 2 - q$, $e = 12$, $c = 1.5$, $k = 10, \delta = 0.9$, f uniform on $[0, 1]$ and increasing s: $s = 0.1$ for $A \in [0, 7)$, $s = 0.0666A - 0.3666$ for $A \in [7, 10)$, and $s = 0.3$ for $A \in [10, \infty)$.

References

[1] Robert E. Baldwin. Trade policies in developed countries. In *Handbook of International Economics.* Volume 1. Edited by R.W. Jones and P.B. Kenen. Amsterdam: North Holland, 1984.

[2] Dirk Bergemann and Juuso Välimäki. Learning and strategic pricing. *Econometrica*, 64:1125-1149, 1996.

[3] Bruce A. Blonigen. In search of substitution between foreign production and exports. *Journal of International Economics*, 53:81-104, 2001.

[4] S. Lael Brainard. An empirical assessment of the proximity-concentration trade-off between multinational sales and trade. *American Economic Review,* 87: 520-544, 1997.

[5] Peter J. Buckley and Mark Casson. The optimal timing of a foreign direct investment. *Economic Journal*, 91:75-87, 1981.

[6] Richard E. Caves. International corporations: the industrial economics of foreign investment. *Economica*, 38:1-27, 1971.

[7] Kimberly A. Clausing. Does multinational activity displace trade? *Economic Inquiry*, 38:190-205, 2000.

[8] Avinash K. Dixit. The role of investment in entry deterrence. *Economic Journal*, 90:95-106, 1980.

[9] Avinash K. Dixit and Robert S. Pindyck. *Investment Under Uncertainty*. Princeton University Press, 1994.

[10] John H. Dunning. Trade, location of economic activity and MNE: A search for an eclectic approach. In B. Ohlin, P.O. Hesselborn, and P.M. Wijkman, editors, *International Allocation of Economic Activity*, pp. 395-418. Macmillan, 1977.

[11] John H. Dunning. *International Production and the Multinational Enterprise.* George Allen and Unwin, 1981.

[12] Wilfred J. Ethier. The multinational firm. *Quarterly Journal of Economics*, 101:805-833, 1986.

[13] Keith Head and John Ries. Overseas investment and firm exports. *Review of International Economics*, 9:108-122, 2001.

[14] Thomas Horst. The theory of the multinational firm: optimal behavior under different tariff and tax rates. *Journal of Political Economy*, 79:75-87, 1971.

[15] Ignatius J. Horstmann and James R. Markusen. Licensing versus direct investment: a model of internalization by the multinational enterprise. *Canadian Journal of Economics*, 20:464-481, 1987.

[16] Ignatius J. Horstmann and James R. Markusen. Strategic investments and the development of multinationals. *International Economic Review*, 28:109-121, 1987.

[17] Ignatius J. Horstmann and James R. Markusen. Exploring new markets: direct investments, contractual relations and the multinational enterprise. *International Economic Review*, 37:1-19, 1996.

[18] INSEAD Euro-Asia Centre. China's beer wars. 1998. Case 05/98-4755.

[19] Bruce Kogut and Nalin Kulatilaka. Operating flexibility, global manufacturing, and the option value of a multinational network. *Management Science*, 40:123-139, 1994.

[20] Robert E. Lipsey and Merle Yahr Weiss. Foreign production and exports of individual firms. *Review of Economics and Statistics*, 66:304-307, 1984.

[21] Giovanni Maggi. Endogenous leadership in a new market. *Rand Journal of Economics,* 27:641-659, 1996.

[22] James R. Markusen. The boundaries of multinational enterprises and the theory of international trade. *Journal of Economic Perspectives*, 9:169-189, 1995.

[23] James R. Markusen and Anthony J. Venables. Multinational firms and the new trade theory. *Journal of International Economics*, 46:183-203, 1998.

[24] S. Nicholas, W. Purcell, D. Merritt, and A. Whitwell. Foreign direct investment in Australia in the 1990's. 1994. University of Melbourne working paper.

[25] Rafael Rob. Learning and capacity expansion under demand uncertainty. *Review of Economic Studies*, 58:655-675, 1991.

[26] Kamal Saggi. Optimal timing of FDI under demand uncertainty. In Jean-Louis Mucchielli, Peter J. Buckley, and Victor V. Cordell, editors, *Globalization and Regionalization: Strategies, Policies, and Economic Environments.* The Haworth Press, 1998.

[27] Frank Rafael Smets. Essays on Foreign Direct Investment. Ph.D. dissertation, Yale University, 1993.

[28] Alasdair Smith. Strategic investment, multinational corporations and trade policy. *European Economic Review*, 31:89-96, 1987.

[29] Deborah L. Swenson. Foreign investment and the mediation of trade flows. UC Davis working paper, 1999.

[30] Anthony J. Venables. Equilibrium locations in vertically linked industries. *International Economic Review*, 37:341-359, 1996.

[31] Anthony J. Venables. Fragmentation and multinational production. *European Economic Review*, 43:935-945, 1999.

[32] Raymond Vernon. International investment and international trade in the product cycle. *Quarterly Journal of Economics*, 80:190-207, 1966.

[33] Nikolaos Vettas. Demand and supply in new markets: diffusion with bilateral learning. *Rand Journal of Economics*, 29:215-233, 1998.

[34] Hideki Yamawaki. Exports and foreign distributional activities: evidence on Japanese firms in the United States. *Review of Economics and Statistics*, 73:294-300, 1991.