



Penn Institute for Economic Research
Department of Economics
University of Pennsylvania
3718 Locust Walk
Philadelphia, PA 19104-6297
pier@econ.upenn.edu
<http://www.econ.upenn.edu/pier>

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“A Simple Search Model of Money with
Heterogeneous Agents and Partial Acceptability”

by

Andrei Shevchenko and Randall Wright

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A Simple Search Model of Money with Heterogeneous Agents and Partial Acceptability*

Andrei Shevchenko
Michigan State University

Randall Wright
University of Pennsylvania

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Abstract

Simple search models have equilibria where some agents accept money and others do not. We argue such equilibria should not be taken seriously – which is unfortunate if one wants a model with partial acceptability. We introduce heterogeneous agents and show partial acceptability arises naturally. There can be multiple equilibria with different degrees of acceptability. Given the type of heterogeneity we allow, the model is still simple: equilibria reduce to fixed points in $[0, 1]$. We show that with other forms of heterogeneity, equilibria are generally fixed points in set space, and there exists no method to reduce this to a problem in \mathbb{R}^1 .

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1 Introduction

The simplest search-theoretic model of monetary exchange endogenizes the acceptability of money, in the sense that there is a pure-strategy equilibrium where money is accepted and another where it is not. When these equilibria coexist there is typically also a mixed-strategy equilibrium where agents accept money with probability $\pi \in (0, 1)$ – or, equivalently, an equilibrium where some agents accept it and others do not.¹ In the equilibrium with $\pi \in (0, 1)$ we say that money is *partially acceptable*, or the economy is *partially monetized*. While the mixed-strategy equilibrium has been used in several applications in the literature (e.g., Kiyotaki and Wright [1993] and Soller-Curtis and Waller [2000]), we argue here that such equilibria should not be taken seriously.

These equilibria arise simply because of the fact that when there are two pure-strategy equilibria generically there is a mixed-strategy equilibrium in between, but in the model they make little economic sense. For one thing, they are unstable in a naive but natural sense, and in an evolutionary sense (Wright 1999). For another, an equilibrium of this sort is really an artifact of the extreme assumption that both goods and money are indivisible, an assumption made for tractability and not for economic content. If either goods or money are divisible these mixed-strategy equilibria do not exist. Moreover, even if one were to take seriously the notion that goods and money

¹The models we have in mind are versions of Kiyotaki and Wright (1991,1993). See Ljungqvist and Sargent (2000) for a textbook treatment, or Rupert et al. (2000) for a survey that discusses the basic model and many extensions in detail.

are indivisible – or at least that there may be some other nonconvexities with similar effects – if we allow agents to trade lotteries then again the mixed-strategy equilibria do not exist (Berentsen, Molico and Wright [in press]).

These arguments seems problematic for the case where money is partially accepted because agents use mixed strategies.² This is unfortunate, since there are good reasons for wanting a model that does display partial acceptability. One is the fact that we seem to see it in the world: at an anecdotal level, one could claim, for instance, that close to national borders some stores accept foreign currency while other do not, or that developing countries and transition economies can become partially dollarized in the sense that some locals use foreign currency while other do not. Also, as a pedagogical device, an equilibrium with partial acceptability would be quite useful, because we could use it to analyze how the degree acceptability depends endogenously on various parameters.

This paper attempts to resolve the issue by introducing heterogeneous agents into the standard model, and showing that partial acceptability arises naturally because agents with different characteristics will differ in their perceived costs and benefits from using cash. Agents can differ here in terms of their utility of consumption, cost of production, storage cost, and rate of time preference. For any general distribution of these characteristics, we show how to construct a statistic for each agent i , call it ξ^i , as a function of his characteristics, such that i accepts money in equilibrium iff $\xi^i \leq \mu$

²This is not to suggest that mixed-strategy or asymmetric equilibria are uninteresting in monetary economics generally. For example, we consider Aiyagari and Wallace (1991) and Renero (1998) quite interesting.

where μ is the measure of agents who accept money. If F is the CDF of ξ , which is derived from the underlying distribution of characteristics, then an equilibrium is simply a solution to $\mu = F(\mu)$.

There are several reasons for thinking this is useful. First, it is easy to see how the acceptability of money μ responds to changes in model parameters, like the severity of search frictions of the double coincidence problem. Also, since F can generally have more than one fixed point, the model displays an economically interesting multiplicity: if agents believe money is accepted by a low fraction of the population they are not very inclined to accept it; but if they believe it will be accepted by a higher fraction they are more so inclined. That acceptability is endogenous and at least to some extent a self-fulfilling prophecy has been a main theme in the search literature for some time. Simple search models do not display this phenomena, however, except in the extreme where money is either accepted or not, unless one takes seriously the mixed-strategy equilibrium.

Hence, we think our model makes some compelling points that existing textbook versions cannot. On a technical note, we also think that our method for reducing equilibrium to a fixed point of F is a contribution. Suppose that agents believe that individual i will accept money iff $i \in S$ for some subset of the population I . If they play best responses to this belief there will be a set that actually does accept money. So an equilibrium is generally a fixed point in set space. In any equilibrium of our model $S = \{i | \xi^i \leq \mu\}$ and the problem reduces to finding a fixed point of F , which is a number and not a set. Moreover, our assumptions are in a sense necessary for this

result: with forms of heterogeneity other than those allowed here, we show *there does not exist* a variable ξ^i such that equilibria necessarily have the form that - contains every i with ξ^i below some threshold.³

2 The Model

Time is continuous and agents live forever. The set of agents \mathbb{A} has measure 1. There is a set of indivisible and perishable goods \mathbb{G} , and different agents produce and consume different goods in this set. Assume i produces $g^i \in \mathbb{G}$ and consumes goods in a subset $\mathbb{G}^i \subset \mathbb{G}$ where $g^i \notin \mathbb{G}^i$. Agents meet bilaterally according to an anonymous random matching process with Poisson arrival rate α . Suppose two agents i and j meet at random; then we assume $\text{prob}(g^i \in \mathbb{G}^j) = x$ and $\text{prob}(g^j \in \mathbb{G}^i | g^i \in \mathbb{G}^j) = y$. Hence, a double coincidence occurs with probability xy . Notice that agents are symmetric here in the sense that α , x and y do not depend on agents' names; we argue below that while this is not necessary, in principle, it is important for tractability.⁴

We do allow heterogeneity in other dimensions. First, $\forall i \in \mathbb{A}$, agent i derives utility $u_i > 0$ from consuming any good in \mathbb{G}^i and disutility $c_i < u_i$ from producing g^i . Also, i has a rate of time preference r_i and a storage cost

³There are a few other monetary search models with heterogeneous agents, including Wallace and Zhou (1997) and Boyarchenko (2000), who pursue various substantive applications. Our theme is different: the goal is to show that a very simple model with the right kind of heterogeneity can be used to illustrate in a robust way that acceptability is endogenous, that it can depend on parameters in interesting ways, and that there can be multiple equilibrium with different degrees of acceptability for natural economic reasons.

⁴Also, we mention that there is no reason why \mathbb{G}^i could not change over time here, so that agents are interested in consuming different goods at different dates, as long as we maintain the other assumptions made above.

γ_i for holding money, where money here is an indivisible object that agents cannot produce or consume but may help to facilitate trade. As is standard in the simplest search-based models, we assume that an individual can only store $m \in \{0, 1\}$ units of money. One can motivate the unit upper bound on money holdings by assuming that once i produces he cannot produce again until he consumes.⁵ In any case, the fraction $M \in (0, 1)$ of the population with money are called buyers and the remaining $1 - M$ are called sellers.

A given agent $i \in \mathbb{A}$ is then fully described by his vector of characteristics $v_i = (u_i, c_i, r_i, \gamma_i)$, with some arbitrary function $\Phi(v_i)$ describing the distribution of characteristics over \mathbb{A} . Let V_m^i denote the value function for agent i when he is holding $m \in \{0, 1\}$ units of money. Let m_i denote the probability that agent i has money in steady state, and let π_i denote the probability that i accepts money if offered it in exchange. Then we have the standard continuous-time dynamic programming equations:

$$r_i V_0^i = \int_{\mathbb{A}} \alpha x m_j \pi_i (-c_i + V_1^i - V_0^i) dj + \int_{\mathbb{A}} \alpha x y (1 - m_j) (u_i - c_i) dj \quad (1)$$

$$r_i V_1^i = \int_{\mathbb{A}} \alpha x (1 - m_j) \pi_j (u_i + V_0^i - V_1^i) dj - \gamma_i. \quad (2)$$

The first term in (1) is the rate at which i when he is a seller meets an agent j who likes g_i and has money, $\alpha x m_j$, times the gain from taking the money in trade with probability π_i , integrated over \mathbb{A} . The second term is the rate at which meets an agent j without money and they enjoy a double

⁵In steady state, anyone with a unit of money must have acquired it in exchange for his production good; he therefore cannot produce again to acquire a second unit of money until he consumes, but he cannot consume without spending his money.

coincidence, $\alpha xy(1 - m_j)$, times the gain from a barter trade, also integrated over \mathbb{A} . The first term in (2) is the rate at which i when he is a buyer meets an agent j who has money and produces a good in \mathbb{G}^i , αxm_j , times the probability j takes the money, π_j , times the gain from trade, also integrated over \mathbb{A} . The final term is the disutility cost to i of storing money.

Notice we are using the fact that whether i wants to trade with j depends on v_i but not v_j – that is, your payoff in a trade depends on your type but not your partner’s type.⁶ This means we can define

$$- = \{i \in \mathbb{A} | \pi_i = 1\}$$

to be the set of agents who accept money (from everyone who has it). The best response condition for π_i is

$$\pi_i = \begin{cases} 1 & \Delta_i > 0 \\ [0, 1] & \Delta_i = 0 \\ 0 & \Delta_i < 0 \end{cases}$$

where $\Delta_i = -c_i + V_1^i - V_0^i$. Hence, $i \in -$ if $\Delta_i > 0$ and $i \notin -$ if $\Delta_i < 0$. We now have

$$r_i V_0^i = \alpha x \pi_i (-c_i + V_1^i - V_0^i) \int m_j dj + \alpha xy (u_i - c_i) \int_{\mathbb{A}} (1 - m_j) dj \quad (3)$$

$$r_i V_1^i = \alpha x (u_i + V_0^i - V_1^i) \int (1 - m_j) dj - \gamma_i. \quad (4)$$

⁶This is not true in all models, of course. Consider a divisible goods version where the terms of trade are determined by bargaining, as in Shi (1995) or Trejos and Wright (1995). Given that you can expect a better deal when you buy from a low cost rather than a high cost producer, the gains from trade depend on who you meet and not only on your own type. See Boyarchenko (2000).

We are interested in stationary equilibria where V_m^i and m_i do not depend on time. The distribution of money holdings in this economy must satisfy the steady state conditions:

$$m_i \int (1 - m_j) dj = (1 - m_i) \int m_j dj \quad \forall i \in - .$$

Rearrange this as

$$m_i = \frac{\int m_j dj \forall i}{\int dj} = \frac{M}{\mu} \quad \forall i \in - , \quad (5)$$

where M is the total money supply and $\mu = \mu(-) = \int dj$ is the measure of the population that accepts money – or, equivalently, $\mu = E\pi_i$. Hence, according to (5), every $i \in -$ ends up holding money with the same probability in steady state.⁷

A special case of our setup is the standard model with homogenous agents. In this version of the model, π is the (mixed strategy) probability the representative agent accepts money. It is then easy to see that there will be some π^* such that: if $\pi < \pi^*$ the best response is $\pi = 0$; if $\pi = \pi^*$ the best response is $\pi = [0, 1]$; and if $\pi > \pi^*$ the best response is $\pi = 1$. We may or may not have $\pi^* \in (0, 1)$ here, depending on parameters. If $\pi^* \in (0, 1)$ there are three Nash equilibria: $\pi = 0$; $\pi = \pi^*$, and $\pi = 1$. The mixed-strategy equilibrium π^* displays partial acceptability. However, it is clearly not a robust outcome in the following naive but natural sense: if any positive measure of agents for some reason make a mistake and, e.g., accept money with probability $\pi^* + \varepsilon$, for any $\varepsilon > 0$, the best response jumps from $\pi = \pi^*$ to $\pi = 1$.

⁷This presumes $\mu \geq M$; if not, there will be more agents holding money than accept money in steady state, which means money is not valued and at least some agents would dispose on it.

One can also show the mixed-strategy equilibrium is unstable in the evolutionary sense, and that with divisible goods or divisible money, or even with indivisible goods and money but agents who can trade lotteries, it cannot exist (see the references in the Introduction). Moreover, one would like to ask how acceptability responds to parameter changes; but here, in the pure-strategy equilibria it simply does not, and in the mixed-strategy equilibrium it does respond but goes the “wrong way” (obviously because it is unstable). Finally, while it is true the model illustrates that acceptability is to some extent a self-fulfilling prophecy, in the sense that there can be multiple equilibria, there can never be multiple interior equilibria with different degrees of partial acceptability. For all these reasons we pursue the heterogeneous agent case.

To characterize the set of agents who accept money - in our model, we calculate

$$\Delta_i = \frac{\alpha x[\mu - M - y(1 - M)](u_i - c_i) - c_i r_i - \gamma_i}{r_i + \alpha x \mu}. \quad (6)$$

Hence, $i \in \mathcal{A}$ if and only if $\Delta_i \geq 0$ if and only if $\xi_i \leq \mu$, where⁸

$$\xi_i \equiv \frac{c_i r_i + \gamma_i}{\alpha x (u_i - c_i)} + M + y(1 - M) \quad (7)$$

is a statistic that depends on only on exogenous parameters and the vector of characteristics for i , $v_i = (u_i, c_i, r_i, \gamma_i)$. The distribution of ξ_i across agents, $F(\xi_i)$, can be derived from the underlying distribution of exogenous characteristics $\Phi(v_i)$.

⁸We assume that agents accept money if $\Delta_i = 0$ in what follows; little of interest hinges on this tie-breaking rule here, except that one does have to worry about cases where there is a positive mass of the population in this situation, as discussed in the next footnote.

Finally, we close the model by observing that since $\pi = \{i | \xi_i \leq \mu\}$ the measure of π in equilibrium is simply the fraction of agents with ξ_i below the threshold μ ; that is, $\mu = F(\mu)$. Any equilibrium is therefore a fixed point $\mu \in [0, 1]$ of F . Note that this depends on the threshold property of equilibria. We were able to construct a variable ξ_i from primitives such that any equilibrium has the property that $\pi_i = 1$ iff ξ_i is below some threshold. This property in turn depends critically on the type of heterogeneity one assumes. More generally, an equilibrium is a fixed point π in set space and, as we shall see below, for types of heterogeneity other than the type we allow there is generally no way to reduce things to a fixed point problem in \mathbb{R}^1 .

So far we have only shown that any equilibrium has the threshold property, which says nothing about existence. To this end, note the following. If $F(0) = 0$ then $\mu = 0$ is an equilibrium. If $F(0) > 0$ then there are two cases: $F(1) = 1$, which implies $\mu = 1$ is an equilibrium; and $F(1) < 1$, which implies there must exist an equilibrium $\mu \in (0, 1)$ even if F is not continuous for the following reason. As a distribution function F is increasing, and so when $F(0) > 0$ and $F(1) < 1$ it must cross the 45° line because, although it could jump over the 45° line from below F cannot jump down. More formally, existence here is a special case of the Tarsky Fixed Point Theorem, which says the following:

Theorem 1 (*Tarsky*) *Suppose $F : [0, 1]^n \rightarrow [0, 1]^n$ is non-decreasing – that is, $F(x') \geq F(x)$ whenever $x' \geq x$. Then $\exists x^* \in [0, 1]^n$ such that $x^* = F(x^*)$.*

See any standard reference on fixed point theorems.⁹

Of course in monetary economies we usually want more: like, the existence of a *monetary* equilibrium, where $\mu > 0$. One way to get this is to find conditions that rule out the nonmonetary equilibrium – i.e. that imply $F(0) > 0$. This is *not* possible when $\gamma_i \geq 0 \forall i \in \mathbb{A}$: in this case, (7) implies $\xi_i > 0 \forall i \in \mathbb{A}$, and therefore $F(0) = 0$. Naturally, in this case, if agents believe $\pi = \emptyset$ then it is an equilibrium for no one to take money. However, we can set $\gamma_i < 0$ – a negative storage cost corresponding to money paying a positive dividend. Notice that $\xi_i < 0$ iff

$$-\gamma_i > r_i c_i + \alpha x[M + y(1 - M)](u_i - c_i). \quad (8)$$

For any agent i such that (8) holds, $\pi_i = 1$ is a dominant strategy. There is only one more detail to consider. If $-\gamma_i$ is too large, an agent with money may not be willing to part with it. To be sure that he is willing we need to check $u_i + V_0^i - V_1^i \geq 0$, which holds iff

$$-\gamma_i \leq r_i u_i + \alpha x[M + y(1 - M)](u_i - c_i). \quad (9)$$

We can impose (9) and still satisfy (8) as long as $u_i > c_i$. Hence, we can always assume the set

$$\mathbb{A}^0 = \{i \in \mathbb{A} | (9) \text{ and } (8) \text{ hold}\} \quad (10)$$

⁹Although we do not need continuity for existence, interesting things can happen when F is not continuous. For example, suppose F jumps at $\bar{\mu}$ from $F_L < \bar{\mu}$ to $F_R > \bar{\mu}$. There still exists a fixed point $\mu \neq \bar{\mu}$ by the above argument, but in addition we can construct equilibrium around $\bar{\mu}$ as follows. Every agent with $\xi_i < \bar{\mu}$ sets $\pi_i = 1$, every agent with $\xi_i > \bar{\mu}$ sets $\pi_i = 0$, and the mass of agents with $\xi_i = \bar{\mu}$ use a mixed strategy where $\pi_i = 1$ with probability π and $\pi_i = 0$ with probability $1 - \pi$, where π is determined so that $\Delta_i = 0$ for $\xi_i = \bar{\mu}$. Of course, this is just the method for constructing mixed-strategy equilibria in a model with homogeneous agents.

has positive measure, which implies $F(0) > 0$, and therefore the equilibrium that we know exists must be a monetary equilibrium.

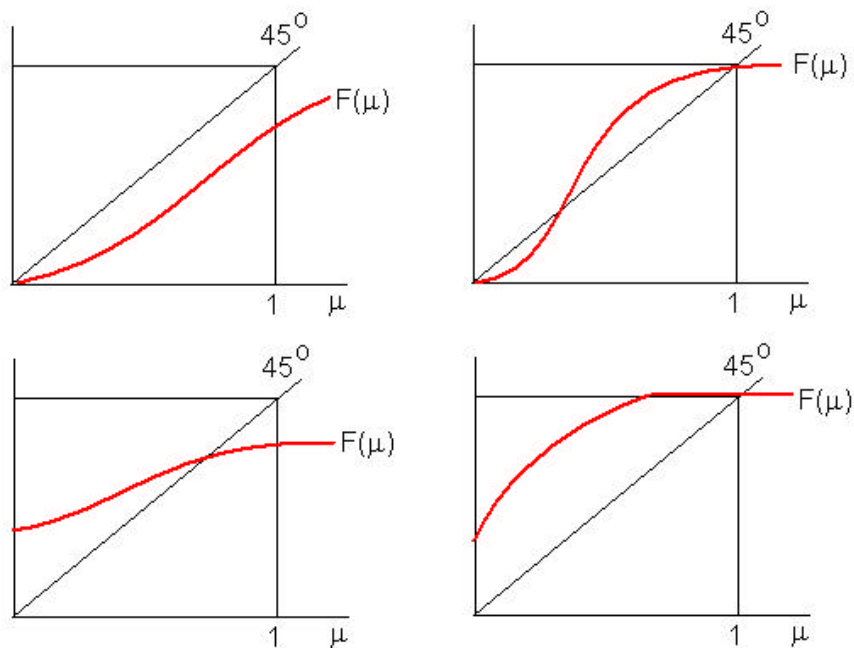


Figure 1: Some Possible Outcomes

It is clear that we can easily have multiple equilibria in the model. Figure 1 shows several possible outcomes. Three of the panels depict a unique fixed point: a nonmonetary equilibrium $\mu = 0$; a fully monetized equilibrium $\mu = 1$; and a partially monetized equilibrium $\mu \in (0, 1)$. The other panel depicts a case of multiple equilibria, one each of these three types. Obviously, we can also have multiple partially monetized equilibria with different degrees of acceptability in this model – something the model with homogeneous agents cannot deliver. The intuition is standard: the net benefit to accepting money

Δ_i is increasing in μ , because the greater the degree to which the economy is monetized the easier it is to find a seller who takes cash.

Just like we can assume \mathbb{A}^0 has positive measure to guarantee $F(0) > 0$, we can also assume the set \mathbb{A}^1 has positive measure to guarantee $F(1) < 1$, where $\mathbb{A}^1 = \{i \in \mathbb{A} | \gamma_i > (1 - M)(1 - y)\alpha x(u_i - c_i) - r_i c_i\}$. In this case, in any equilibrium, money must be partially acceptable, $\mu \in (0, 1)$. For the sake of illustration, if we suppose there is a unique such equilibrium, as in the lower left panel of Figure 1, the model is easily used to perform natural comparative static exercises. For example, increasing M or y or decreasing αx all shift F down and lead to a fall in equilibrium acceptability μ . One can do fancier things, like changing the distribution of any parameter in the vector of individual characteristics v_i , but the point should be clear: the model not only allows one to do comparative statics, it gives very reasonable answers.

We now argue that the type of heterogeneity we consider is in some sense the most general that works. The key feature is that our vector $v_i = (u_i, c_i, r_i, \gamma_i)$ depends on i but not on other types. This is a special case since we could have also assumed, for example, that the utility of consumption depends on the identities of the consumer i and the producer j , say u_{ij} . The same thing is true for the cost c_{ij} . Also, the arrival rate α_{ij} could index the rate at which type i meets type j ; indeed matching technologies like this have been used in the literature on international currency going back to Matsuyama, Kiyotaki and Matsui (1993). Additionally, the single- and double-coincidence probabilities could depend on both agents in a meeting,

x_{ij} and y_{ij} .

While these types of heterogeneity are certainly not without interest, in their presence the model is much less tractable. The reason is that we lose the threshold property of equilibrium: it is no longer the case that we can construct a statistic ξ_i such that all equilibria have property that the set of agents who accept money is equal to the set with ξ_i below some threshold. Without this property there is much less structure on the possible outcomes. If agents believe $\pi_j = 1 \forall j \in -$ where $- \subset \mathbb{A}$ is an arbitrary set, then each individual i will choose a best response π_i , which generates a set $- ' = T(-) = \{i \in \mathbb{A} | \pi_i = 1 \text{ is a best response given } - \}$. An equilibrium is a fixed point in set space, $- = T(-)$, which is of course a much more complicated object than what we have above.

To prove the point it suffices to consider an example. Let $\mathbb{A} = [0, 1]$, and partition agents into of three groups: $\mathbb{A}_1 = [0, 1/3)$, $\mathbb{A}_2 = [1/3, 2/3)$, and $\mathbb{A}_3 = [2/3, 1]$. For simplicity let $u_i = u$, $c_i = c$, and $r_i = r \forall i$, but let the storage cost γ_i differ across agents. Say for example that γ_i is monotonically increasing in i . Now assume an additional form heterogeneity exists in that α_{ij} differs across i and j . In particular, suppose

$$\alpha_{ij} = \begin{cases} \bar{\alpha} & \text{if } i, j \in \mathbb{A}_k \\ \underline{\alpha} & \text{otherwise} \end{cases}$$

where $\underline{\alpha} \ll \bar{\alpha}$. This simply says that two agents are much more likely to meet if they belong to the same subset \mathbb{A}_k than if they belong to different subsets. To illustrate the point, assume $\underline{\alpha} \approx 0$. Then the economy is really three sub-economies that do not interact.

These three sub-economies are each like our base model, and hence have the same type of possible equilibria. Suppose parameters are such that the situation for each sub-economy looks like the panel in Figure 1 with three equilibria, $\mu = 0$, $\mu = 1$, and $\mu = \mu^* \in (0, 1)$. We can assign each subeconomy a different equilibrium in many possible ways. One natural possibility is the following: $\pi_i = 1 \forall i \in \mathbb{A}_1$; $\pi_i = 1$ iff γ_i is below the relevant threshold γ^* $\forall i \in \mathbb{A}_2$; and $\pi_i = 0 \forall i \in \mathbb{A}_1$. In this case it is true that $\pi = \{i \in \mathbb{A} | \gamma_i < \gamma^*\}$, so that agents with lower storage costs are more likely to accept money and one can say that a threshold result obtains. But we could also do the opposite and set $\pi_i = 0 \forall i \in \mathbb{A}_1$; $\pi_i = 1$ iff γ_i is below $\gamma^* \forall i \in \mathbb{A}_2$; and $\pi_i = 1 \forall i \in \mathbb{A}_1$. Or we could set $\pi_i = 1 \forall i \in \mathbb{A}_1$; $\pi_i = 0 \forall i \in \mathbb{A}_2$; and $\pi_i = 1 \forall i \in \mathbb{A}_1$.

There is clearly no way to rank agents in this example according to some number ξ_i in such a way that all equilibria have the property that $\pi_i = 1$ iff ξ_i is below a threshold. Hence, an equilibrium generally will be a fixed point in set space $\pi = T(\pi)$, as described above. While the example perhaps appears special because of the extreme assumption $\underline{\alpha} \approx 0$, the point is nevertheless general (similar results can be derived in examples where $\underline{\alpha} > 0$). Again, we think this is interesting, but the goal here was to construct a tractable model.

3 Conclusion

In this paper we have attempted to construct a simple model of money that displays robust equilibria with different degrees of acceptability. To do this

we have extended the textbook search model, with indivisible goods and money, by introducing various types of heterogeneity. In principle, with heterogeneous agents the problem of finding an equilibrium is equivalent to a fixed point problem in set space. With our form of heterogeneity it reduces to a fixed point problem in $[0, 1]$. Although simple, the model achieves what we wanted: the acceptability of money is endogenous and depends on parameters in economically interesting ways, and there can be multiple equilibrium with different degrees of monetization. We think that this version should replace the standard model with homogeneous agents as the textbook model.

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