Penn Institute for Economic Research
Department of Economics
University of Pennsylvania
3718 Locust Walk
Philadelphia, PA 19104-6297
pier@econ.upenn.edu
http://www.econ.upenn.edu/pier

PIER Working Paper 02-041

"Search-Theoretic Models of the Labor Market: A Survey"

by

Richard Rogerson and Randall Wright

http://ssrn.com/abstract_id=341660

Search-Theoretic Models of the Labor Market: A Survey

Richard Rogerson
University of Pennsylvania

Randall Wright
University of Pennsylvania

1 Introduction

This essay provides a survey of various models that use search theory to analyze labor markets. By search theory, we mean a framework in which trading *frictions* are modeled explicitly. In contrast with standard general equilibrium theory, search-theoretic models are based on the recognition that information about what is available in the market is not costless or easy to obtain, and because of this it can take time and other resources to locate agents with whom one can trade, to find the goods one wants to buy at favorable prices, to establish a suitable employment or other relationship, and so on. While general equilibrium theory has been applied successfully to environments that are both dynamic and stochastic, in search theory the dynamics and uncertainty explicitly affect the *trading process*. It is this aspect of the framework that makes it especially useful for thinking about many phenomena, including many labor market issues.

It is instructive to consider an example. Perhaps the most basic application of search theory concerns a worker looking for a job, when there is a known distribution of wages offered by different employers, but he does not know which employer is offering which wage. The worker therefore has to search, or sample, from this distribution. The key economic question here is how many employers should he contact before accepting a job? If sampling takes place at a fixed rate over time, his optimal search strategy is straightforward: there is a reservation wage w_R, defined by a simple condition, such that he should accept the first offer above w_R. This value depends on many aspects of the problem, including properties of the wage distribution, income from unemployment insurance, the frequency at which he gets to sample offers, and so on. His strategy implies the number of firms contacted, his duration of unemployment, and the wage he eventually accepts are random variables, all depending on many features of the problem. Hence, even in this simple setting, the theory already makes interesting predictions about individual unemployment experiences and, when aggregated, about the economy-wide unemployment rate and wages.

Similar predictions do not follow easily from standard competitive equilibrium theory. In the basic textbook model agents face a budget set and are allowed to choose any point in this set to maximize utility. This choice generally includes decisions about how much labor to supply at a given wage. Of course, the wage can vary over time and across states of the world, since all date- and state-contingent commodities, including labor, are different goods and hence generally have different date- and state-contingent prices, but this is all known *ex ante*. While there could be unemployment in the sense that some agents may supply zero labor, this is far less interesting than an explanation of unemployment based explicitly on frictions in the trading process. Also, there is no distribution of wages across workers, or across jobs, of a given type at a given date and state – the law of one price precludes this in any frictionless model. Search unemployment and a nondegenerate distribution of wages simply do not come up in the standard competitive paradigm; they are the bread and butter of search theory.

The above description of the labor market is too simplistic to dwell on for long, because in a sense it raises more issues than it resolves. First, where does the distribution of wage offers come from? Presumably wages and, more generally, all prices should be endogenous and chosen by some agents in the model. One thing competitive theory gets right with supply and demand is that prices are determined endogenously. Then again, one thing it gets wrong is that they are not chosen by anyone in the model, but by something outside the model – the auctioneer. The auctioneer is a very convenient device for solving or at least getting around the problem of price formation, but presumably this cannot be the last word on the problem. Search models not only allow us to discuss ways in which wages and other prices are determined by agents in the model, they allow us to study a wide variety of alternative ways of endogenizing prices, including bilateral bargaining, *ex ante* wage posting by employers, and other mechanisms.[1]

[1]A related point is that search theory allows us to make predictions about outcomes, such as unemployment, for any wage and price setting institution, including fixed prices,

Another issue with the above description of a labor market is that it is not necessarily trivial to go from a model of an individual unemployment spell to the economy-wide unemployment rate, because there are things that the individual takes as given that we need to determine in equilibrium. Consider the frequency with which the worker samples wage offers. First, it could depend on his search intensity. More importantly, in equilibrium, this rate even for a given intensity will generally depend on the number of firms posting vacancies, the number of workers competing for these vacancies, the intensity with which others are searching, and the acceptance or hiring decisions of those on the other side of the market. Clearly, these types of interactions need to be modeled *strategically*: individuals need to know the numbers of agents in the market as well as certain aspects of their behavior, and not just the prices that make up standard budget equations. The framework presented here allows us to make precise the ways in which these strategic interactions manifest themselves, in a tractable way.

A third issue that comes up is, what happens after a worker accepts a job? In the single-agent problem we can easily assume there are random terminations, say exogenous layoffs, which like the offer arrival rate could be taken parametrically by the individual. More generally, however, ending an employment relationship should be endogenous and can depend on, for example, what one learns about the quality of the relationship after it begins, how the productivity of the relationship evolves, and the outside opportunities with which the two parties come in contact. The search-theoretic approach is ideally suited to analyze partnership dissolution and formation. Of course, this is relevant in a wide variety of contexts, but the labor market is certainly one area in which the creation and destruction of long-term (employment) relationships is absolutely central. For example, we examine the extent to which these outcomes are efficient.

In what follows we will present the single-agent search problem and discuss all of the equilibrium issues raised in these introductory remarks, includ-

while standard competitive theory says nothing about what happens at any prices except equilibrium (market clearing) prices. In the language of game theory, search models specify what happens off the equilibrium path. Now, one can analyze standard markets with fixed wages or prices – as one does, e.g., after introducing minimum wage laws or rent control – if we also introduce some rationing scheme to determine what solves the allocation problem when the price mechanism cannot. Any such scheme is bound to be *ad hoc* in the context of a competitive equilibrium model. By contrast, in search theory rationing is done explicitly by time – indeed, this is what search theory is all about.

ing endogenizing the wage distribution, determining offer arrival rates, job destruction rates, and so on. Where appropriate we also compare the predictions with those of standard competitive theory. However, we emphasize at the outset that while search models are richer along some dimensions than standard supply and demand models, they are not necessarily inconsistent. One often hears (and teaches) that the market clearing allocations and prices of standard theory are meant to be approximations to a real world that is actually full of frictions of one form or another, but models that abstract from these frictions may well be accurate or useful approximations depending on the nature of the question one at hand. While ultimately it may be an empirical question, there is also value in studying this theoretically. Once we write down a model with explicit frictions, we can let these frictions become small and see the extent to which the limiting case resembles the competitive outcome.

Before getting into the paper, we should emphasize that search theory constitutes a huge, virtually overwhelming, branch of economics. In addition to labor markets, it has been applied in a great many areas in both micro and macroeconomics, including monetary theory, industrial organization, growth, public finance, and the economics of the marriage market, to name a few, all of which we must neglect lest this survey ends up unmanageable.[2] Search has been used in much fairly technical theoretical work, and has been a genuine workhorse in empirical economics, but we can neither delve into pure theory nor can we pay appropriate attention to all of the econometric issues and empirical results here.[3] Our focus is almost exclusively on using search models to help us organize our thinking about labor markets. The discussion is meant to be relatively rigorous, but we want to emphasize applications and issues as much as we can, without going into actual quantitative analysis, which is beyond the scope of the paper.

Moreover, as compared to some other branches of economics, search the-

[2] A few examples in monetary theory include Kiyotaki-Wright (1989, 1993), Shi (1995) and Trejos-Wright (1995); examples in the marriage market include Mortensen (1988), Burdett-Coles (1997, 1999) and Shimer-Smith (2000a); examples in industrial organization include Jovanovic (1982), Jovanovic-MacDonald (1994), Jovanovic-Rob (1989), Klemperer (19xx), and Fishman-Rob (2000).

[3] Two examples of theoretical work are Rubinstein-Wolinsky (1987) and Gale (1987), which are particularly worth mentioning because they are directly concerned with the question of whether models with frictions converge to competitive outcomes as the frictions vanish. See Devine-Kiefer (1991) and Wolpin (1995) for surveys of the empirical applications.

ory is relatively young, and so there does not exist anything close to a definitive treatise or standard textbook to which one can appeal for basic notation, techniques, or results. Therefore we will take time to develop somewhat carefully a few of these things in a way that will be useful in a variety of contexts. The approach throughout the applications will be to start with simple models and gradually add features that seem interesting in the context of data, policy, or other substantive issues. The different models are meant to be different applications of the same set of tools – really, of the same model. Our goal is that, with a little work, the reader should feel at the end of this article like search theory is a general and broadly applicable paradigm for organizing our thinking about labor markets in particular, and about economics generally, and our hope is that this will stimulate even more learning and research in this fertile area.

The rest of the article is organized as follows. Section 2 presents the basic single-agent search problem in a variety of different forms, with an eye especially towards the extensions that are most relevant for the equilibrium analysis to follow. Section 3 focuses on the endogenous determination of the wage distribution. Section 4 presents some simple equilibrium models of the labor market, and introduces some machinery that is common in such models, including the notion of a matching technology, as well as some basic bargaining theory. Section 5 discusses a particular version of the paradigm that is designed explicitly to analyze job creation and destruction. The final section concludes.[4]

2 Job Search

It makes sense to study *decision theory* – i.e., the problem of an individual agent, taking market conditions as given– before analyzing equilibrium models in search theory, just as, for example, we usually study consumer theory, before analyzing market equilibrium. So we begin with the problem of a single worker looking for a job. While the economics (and label) of search theory begins with Stigler (1961), his formulation was not really dynamic: he considered a buyer who chooses the number of price quotations, before the search process begins, to minimize expected price plus sampling cost. McCall

[4] Earlier surveys of search theory include Lipman-McCall (1976a) and Mortensen (1986). Mortensen-Pissarides (1999a,b) provide recent updates. See also Ljungqvist-Sargent (2000).

(1970), Mortensen (1970) and Gronau (1971) seem to be the first genuinely dynamic search models in the economics literature, and the presentation here will actually be very similar in many respects to those models.

2.1 The Basic Model

Consider a worker seeking to maximize

$$E\sum_{t=0}^{\infty}\beta^t y_t, \tag{1}$$

where $\beta \in (0,1)$ is the discount factor, y_t is income at t, and E denotes the expectation. Income is given by $y = w$ if employed at wage w and $y = b$ if unemployed (hours worked are fixed to unity for now, but this will be relaxed below). Although we refer to w as the wage, more generally it could be interpreted as capturing some general measure of the desirability of the job, which could depend on things like location, prestige, etc. Similarly, although we will refer to $b > 0$ as unemployment insurance, it can also be interpreted as including the value of leisure and home production, net of any cost of search.[5]

The individual chooses a policy indicating whether to accept any given job offer. We begin with the case where an unemployed individual samples one iid observation each period from a known distribution of (nonnegative) wage offers, characterized by the cdf $F(\bar{w}) = prob(w \leq \bar{w})$. If an offer is rejected the agent remains unemployed that period. Previously rejected offers cannot be recalled, although this assumption is actually not restrictive because the

[5] As is standard, we have formulated the problem as though the agent is interested in maximizing expected discounted income, as opposed to utility. If utility is linear and the market interest rate r satisfies $\frac{1}{1+r} = \beta$ then these are equivalent. However, the analysis can be intepreted more generally. If we assume risk aversion and *complete markets*, the worker can maximize expected utility by first maximizing income, which is the problem on which we focus, and then smoothing his consumption through markets. Another interpretation is that there are *no markets* for transferring income across time and states, so he must consume his income at each date, in which case one can simply reinterpret y_t as current utility. The case of a risk averse agent facing some but not complete markets is more difficult. Early theoretical analyses are Danforth (1979) and Hall-Lippman-McCall (1979); a recent version is Browning-Crossley-Smith (1999). Numerical analyses when risk averse consumers can save but cannot insure against income risk include Valdivia (1997) and Costain (1997).

problem is stationary – an offer that is not acceptable today will not be acceptable tomorrow. For now we assume that if an offer is accepted the worker keeps the job forever. Hence, $W(w) = w/(1-\beta)$ is the payoff to accepting an offer (W for the value of *working*), and $U = b + \beta E \max[W(w), U]$ is the payoff to rejecting and sampling again next period (U for the value of being *unemployed*), where E is the expectations operator.

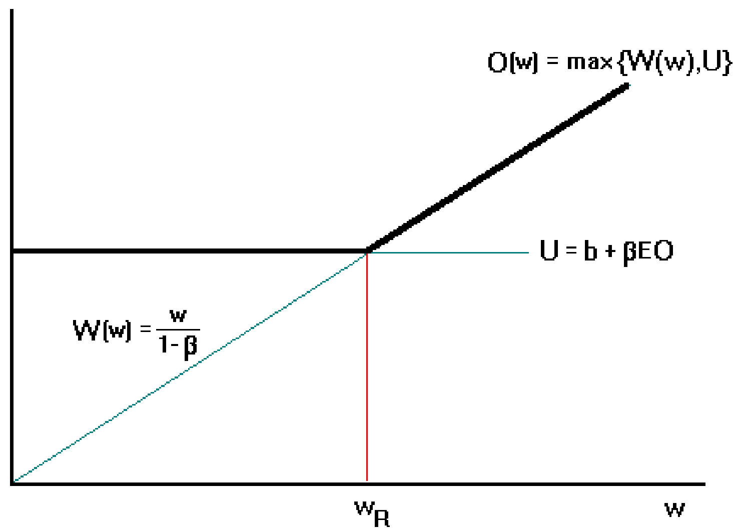

Figure 1: The Value Functions and Reservation Wage

The value of having offer w in hand, let's call it $O(w)$, satisfies the following Bellman's equation:

$$O(w) = \max[W(w), U] = \max\left\{\frac{w}{1-\beta}, b + \beta EO\right\}. \tag{2}$$

Since $W(w)$ is increasing, there is a unique w_R satisfying $W(w_R) = U$, called the *reservation wage*, with the property that the agent should reject $w < w_R$ and accept $w > w_R$. See Figure 1. Rearranging $W(w_R) = U$ implies $w_R = (1-\beta)b + (1-\beta)\beta EO$, which expresses w_R in terms of the unknown function

O. To eliminate EO, first use $U = w_R/(1-\beta)$ to rewrite (2) as

$$O(w) = \begin{cases} \frac{w}{1-\beta} & \text{for } w \geq w_R \\ \frac{w_R}{1-\beta} & \text{for } w < w_R \end{cases}$$

and integrate to get $EO = E\max(w, w_R)/(1-\beta)$. Substitution into $w_R = (1-\beta)b + (1-\beta)\beta EO$ yields

$$w_R = T(w_R) \equiv (1-\beta)b + \beta \int_0^\infty \max(w, w_R)dF(w). \tag{3}$$

The mapping T defined in (3) is easily shown to be a *contraction* (see Stokey-Lucas-Prescott 1989, for details), and so the contraction mapping theorem implies there exists a unique solution to $w_R = T(w_R)$ and, as shown in Figure 2, the sequence $w_{N+1} = T(w_N)$ converges to w_R as $N \to \infty$ starting from any initial value of w_0.[6]

A different expression for w_R is derived as follows. First, rewrite $U = b + \beta EO$ as

$$(1-\beta)U = b + \beta \int_{w_R}^\infty [W(w) - U]dF(w). \tag{4}$$

Then use $(1-\beta)U = w_R$ and $W(w) - U = (w - w_R)/(1-\beta)$ to get

$$w_R = b + \frac{\beta}{1-\beta} \int_{w_R}^\infty (w - w_R)dF(w). \tag{5}$$

This equates the utility per period from accepting w_R to the utility from rejecting, which is b plus the expected discounted improvement in next period's offer, and is the form often seen in the literature. Sometimes one also integrates by parts to arrive at

$$w_R = b + \frac{\beta}{1-\beta} \int_{w_R}^\infty [1 - F(w)]dw. \tag{6}$$

[6] As is standard in dynamic programming, w_N can be interpreted as the reservation wage from a finite horizon search problem, when there are N periods remaining, if we start at $w_0 = b$ (which is the the reservation wage for the one-shot problem). Hence, the infinite horizon problem in the text can be considered an approximation to a long finite problem, and in fact the approximation is very good for reasonable parameter values. Note that $w_R > b$ and that w_N converges monotonically; hence, the reservation wage increases (the worker gets more demanding) as the horizon gets longer.

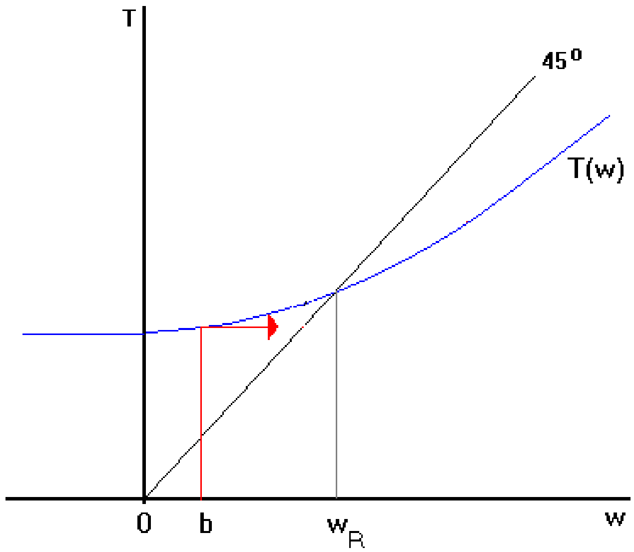

Figure 2: The T Mapping and Reservaion Wage

To get from (4) to (5) we solved for $W(w) - U$. In more complicated models, this may not be possible, so we need the following trick: first integrate (4) by parts to write

$$w_R = b + \beta \int_{w_R}^{\infty} W'(w)[1 - F(w)]dw,$$

and then insert $W'(w) = 1/(1-\beta)$ to get (6). In any case, versions of (6) will be encountered frequently below.

This very simple model already makes predictions about individual unemployment spells. The probability of getting a job each period, called the *hazard rate*, is given by $H = 1 - F(w_R)$.[7] The probability of being unemployed for exactly d periods is $(1-H)^{d-1}H$, and the average duration of

[7]Notice that H does not change over time here, although it increases over time in the finite horizon version outlined above since the reservation wage falls over time. The data suggest that actual hazard rates may decline with duration, but this is probably not due to a finite horizon being relevant for a typical worker. Indeed this pattern may simply be due to unobserved heterogeneity, i.e., it may not be that the hazard declines with duration

unemployment is

$$D = \sum_{d=1}^{\infty} d(1-H)^{d-1}H = \frac{1}{H}.$$

Notice, for example, that an increase in b – say, a more generous unemployment insurance policy – raises D because it raises w_R. It should be obvious that even though he is unemployed longer after b increases, the individual is not worse off. One of the first things that search theory teaches us is that people, at least to some degree, have a choice about how long to be unemployed. If b increases, the worker chooses to increase w_R, which increases D and the average accepted wage, say $w_A = E[w \mid w \geq w_R]$. Of course, not everything that leads to an increase in D makes him better off.[8]

2.2 Simple Extensions

In the above model the agent always receives one offer while he is unemployed. Actually, this subsumes the possibility of receiving no offers with positive probability, since $w = 0$ can be one of the offers, and of receiving more than one, since F can be reinterpreted as the distribution of the best offer in the period. Nevertheless, it is of interest to generalize things by explicitly allowing a random number n of offers each period. In the context of this example we will also show how one can move neatly from discrete to continuous time (which is useful because in some contexts continuous or discrete time may be more convenient, so it is good to know both). We will also show how this extension leads naturally to endogenous search intensity.

Following the presentation in Mortensen (1986), suppose the length of each period is given by Δ, and write the discount factor as $\beta = \frac{1}{1+r\Delta}$. Let $a(n, \Delta)$ be the probability of n offers, and $G(w, n, \Delta)$ the distribution of the

for a given worker, but that the workers with the longest spells of unemployment have the lowest hazard rates (see, e.g., Wolpin [1995] for a discussion).

[8] For example, consider changes in F. An increase (decrease) in all wages, either proportionately or by a constant, will increase (decrease) w_R and expected utility for a searcher, U. A mean preserving spread in F also increases w_R and U. Changes in F have implications for the average wage w_A, too, and sometimes counter-intuitive results can obtain. For instance, increasing all wages can actually reduce w_A. However, this and many other counter-intuitive results can be ruled out by assuming *log-concavity* – i.e., that $\log F$ is concave – an assumption first used in search theory by Burdett (1981). See also Mortensen (1986).

best of the lot, in a period. The generalization of (4) is

$$\frac{r\Delta}{1+r\Delta}U = b + \frac{1}{1+r\Delta}\sum_{n=1}^{\infty} a(n,\Delta)\int_{w_R}^{\infty}[W(w)-U]dG(w,n,\Delta). \qquad (7)$$

It is natural and convenient to assume offers arrive according to a Poisson process with parameter α: that is, $a(1,\Delta) = \alpha\Delta + o(\Delta)$, and $\sum_{n=2}^{\infty} a(n,\Delta) = o(\Delta)$, where $o(\Delta)$ is a function with the property that $o(\Delta)/\Delta \to 0$ as $\Delta \to 0$.[9] Inserting this into (7), rearranging, and taking the limit as $\Delta \to 0$, we have the flow value to being unemployed:

$$rU = b + \alpha\int_{w_R}^{\infty}[W(w)-U]dF(w) \qquad (8)$$

(an alternative derivation of this equation that some people may find easier follows from material presented below). The flow value of being employed at any w is $rW(w) = w$, and, in particular, $U = W(w_R) = w_R/r$. Hence, (8) implies

$$w_R = b + \frac{\alpha}{r}\int_{w_R}^{\infty}(w-w_R)dF(w). \qquad (9)$$

This, or the equation that results from integrating (9) by parts, is the continuous time reservation wage equation.[10] The hazard rate is now $H = \alpha[1-F(w_R)]$, the product of the exogenous arrival rate α, and the endogenous probability the offer is accepted $1-F(w_R)$. Our next goal is to make the first component endogenous, by allowing the individual to choose search intensity $s \in [0,1]$. Assume $\alpha = \alpha(s)$, where $\alpha' > 0$ and $\alpha'' < 0$, and let net unemployment income be $b - g(s)$, where g is the disutility of search effort and satisfies $g' > 0$ and $g'' > 0$. Note that one could normalize either α

[9] This is equivalent to assuming that $a(n,\Delta) = (\alpha\Delta)^n e^{-\alpha\Delta}/n!$, which is the Poisson density with parameter $\alpha\Delta$ (see any text on stochastic processes). Note that the random time until the next arrival, say τ, has a distribution function given by

$$\Phi(t) = pr(\tau \le t) = 1 - pr(\tau > t) = 1 - a(0,t).$$

So the time until the next arrival is an exponential random variable with distribution function $\Phi(t) = 1 - e^{-\alpha t}$. A feature of a Poisson process is that the probability distribution of τ is constant (independent of history, or memoryless); in particular, the expected time until the next arrival is always $1/\alpha$.

[10] Calling this the continuous time reservation wage is a bit of a misnomer, since exactly that same equation can be dervied in discrete time by setting $\Delta = 1$ in (7) and simply assuming directly $a(1,\Delta) = \alpha$ and $a(n,\Delta) = 0$ for $n > 1$.

or g to be linear, with no loss in generality, by measuring units appropriately. Also, we can guarantee an interior solution by assuming $g'(0) = 0$ and $g'(s) \to \infty$ as $s \to 1$.

Endogenizing intensity is interesting because even if w_R were such that agents accept all offers (which could be the optimal solution for some parameter specifications), we can still say that economic decisions are helping to determine unemployment experiences. That is, whether it is job acceptance or search intensity, unemployment becomes at least partly a choice. We emphasize that this does *not* mean individuals enjoy unemployment, any more than poor people enjoy consuming relatively little, but only that we are bringing it into the realm of choice theory (see Lucas 1978 for a similar point).

With endogenous intensity the flow value of being unemployed in (8) becomes

$$rU = \max_s \left\{ b - g(s) + \alpha(s) \int_{w_R}^{\infty} [W(w) - U] dF(w) \right\}. \tag{10}$$

The reservation wage w_R still satisfies (9), but now we also have the first order condition for s

$$\alpha'(s) \int_{w_R}^{\infty} (w - w_R) dF(w) \leq r g'(s), \quad = \text{if } s > 0, \tag{11}$$

which we get by differentiating (10) with respect to s and inserting $W(w) - U = (w - w_R)/r$. An optimal search strategy (w_R, s) is fully characterized by the two conditions (9) and (11). If there is no cost to search, $g(s) = 0$, individuals would search at the maximum feasible intensity, $s = 1$, and we are back to the base model. Implications of endogenous intensity will be discussed at several points below.

2.3 Turnover

Thus far we have assumed that once a worker obtains a job it is permanent. Sadly, but realistically, not all relationships last forever. We now allow for the possibility that jobs end stochastically, for some exogenous reason. In particular, let us return temporarily to the case of exogenous search intensity, and suppose that exogenous layoffs arrive in continuous time according to a Poisson process with parameter λ. We endogenize this in various ways below, but for now λ is a constant.[11] Also, because it is useful for some applications, we include the possibility that the individual may die or otherwise permanently exit the market according to an independent Poisson process with parameter δ, after which he gets 0 utility forever.

Generalizing the above arguments, we have the flow Bellman equations[12]

$$\begin{aligned}(r+\delta)U &= b+\alpha\int_{w_R}^{\infty}[W(w)-U]dF(w) & (12)\\ (r+\delta)W(w) &= w+\lambda[U-W(w)]. & (13)\end{aligned}$$

Again, the reservation wage satisfies $W(w_R)=U$, and the methods leading to (6) now yield

$$w_R = b+\frac{\alpha}{r+\delta+\lambda}\int_{w_R}^{\infty}[1-F(w)]dw. \tag{14}$$

Notice that λ and δ affect w_R only by changing the effective discount rate from r to $r+\lambda+\delta$. In any case, an individual in this model now goes through repeated spells of employment and unemployment over his lifetime. For example, if he lived forever ($\delta=0$), the fraction of time he spends unemployed is calculated to be $u=\lambda/(\lambda+H)$, where again $H=\alpha[1-F(w_R)]$.

In principle, models like this seem useful for organizing micro data on individual employment/unemployment histories and wages. However, although the above model does generate turnover, it does so only by exogenous lay-offs or death. It is clearly of interest to understand what factors determine

[11] It is interesting to allow λ as well as w to vary across jobs, since then the reservation strategy needs to be defined in terms of the pair (w,λ) (see Burdett-Mortensen 1980 and Wright 1987).

[12] An alternative (and quick) way to derive flow Bellman equations is as follows. Let time proceed in discrete periods of length Δ and write

$$U=\frac{1-\delta\Delta}{1+r\Delta}\left\{b\Delta+\alpha\Delta E\max[W(w),U]+o(\Delta)\right\},$$

where the term $1-\delta\Delta$ is the probability of surviving until the next period, and $o(\Delta)$ captures the probability of more than one Poisson arrival in a period, so that $o(\Delta)/\Delta\to 0$ as $\Delta\to 0$. Rearranging, we have

$$(r+\delta)\,\Delta U=(1-\delta\Delta)\left\{b\Delta+\alpha\Delta E\max[W(w)-U,0]+o(\Delta)\right\}$$

Dividing by Δ and taking the limit yields (12). Note again that we can get exactly the same results in discrete time simply by setting $\Delta=1$ and assuming $o(\Delta)=0$.

turnover more deeply, and this requires allowing for an endogenous component. We review three extensions that introduce potentially important factors for determining the rate of turnover: on-the-job search, on-the-job wage changes, and on-the-job learning. For simplicity, we set $\delta = 0$ here (or, as seen above, we can simply interpret it as included in r).

In reality, much turnover is accounted for by job to job transitions with no intervening period of unemployment. The on-the-job search model of Burdett (1978) allows us to produce these types of transitions. In addition, this model also explains how tenure at a particular job is correlated with other variables, such as the wage and quit rate. Following the presentation in Mortensen and Neuman (1984), suppose new offers arrive in continuous time at rate α_0 while unemployed and α_1 while employed. Each offer is a random draw from the same distribution F. For now, search intensity and hence the arrival rates α_0 and α_1 are exogenous. The strategy of an employed worker is obvious: accept any offer greater than your current wage (since there is no cost here to changing jobs; see Hey and McKenna 1979 for a model where there is). What needs to be determined is the strategy of an unemployed worker.

The methods leading to (12) and (13) now imply the flow Bellman equations

$$\begin{aligned} rU &= b + \alpha_0 \int_{w_R}^{\infty} [W(w) - U] dF(w) & (15) \\ rW(w) &= w + \alpha_1 \int_{w}^{\infty} [W(w') - W(w)] dF(w') + \lambda [U - W(w)]. & (16) \end{aligned}$$

Evaluating (16) at $w = w_R$ and combing it with (15), we have

$$w_R = b + (\alpha_0 - \alpha_1) \int_{w_R}^{\infty} [W(w') - W(w_R)] dF(w'). \tag{17}$$

Observe that w_R is greater or less than b as α_0 is greater or less than α_1. Thus, for example, when offers arrive more frequently while employed the individual will accept some offers strictly less than b. In any case, (17) is not the final answer since as above we have to eliminate the endogenous W. The usual technique of integrating (17) by parts and inserting $W'(w)$, which we get by differentiating (16), leads to the generalized reservation wage equation[13]

$$w_R = b + (\alpha_0 - \alpha_1) \int_{w_R}^{\infty} \left[\frac{1 - F(w)}{r + \lambda + \alpha_1 [1 - F(w)]} \right] dw. \tag{18}$$

[13] We can also make search intensity endogenous with on-the-job search. For example, let the arrival rates be $\alpha_0 = \alpha(s_0)$ and $\alpha_1 = \alpha(s_1)$, where s_0 and s_1 are the resources devoted

Another factor which may influence job turnover is that the wage associated with a particular job may change at some point, even though outside wage opportunities are unchanged. To formalize this we suppose that according to a Poisson process with parameter γ the wage changes from w to a new draw w' from the (conditional) distribution $F(w' \mid w)$. Ignoring on-the-job search and layoffs, for the sake of illustration, the flow value of employment becomes

$$rW(w) = w + \gamma \int_0^\infty \max[W(w') - W(w), U - W(w)] dF(w' \mid w). \qquad (19)$$

Notice that when w changes to w' the worker has two options: accept w' or quit to unemployment (in contrast to the on-the-job search model, in which he can reject w' in favor of the current wage).

A natural specification is that $F(w' \mid w_2)$ first order stochastically dominates $F(w' \mid w_1)$ whenever $w_2 > w_1$. Then $W(w)$ is increasing, and hence there is a single reservation wage w_R for employed and unemployed agents: if $w < w_R$ then reject (quit) and if $w > w_R$ then accept (stay) while unemployed (employed). When employed at w, if the wage falls to $w' < w_R$ the worker quits to unemployment. Stochastic dominance implies that the quit rate is decreasing in w. If we take the limiting case where $F(w' \mid w) = F(w)$ (independence), then the usual techniques easily generate

$$w_R = b + \frac{\alpha - \gamma}{r + \gamma} \int_{w_R}^\infty (w - w_R) dF(w).$$

Notice that $\gamma > \alpha$ implies then agent would accept an offer $w < b$.

Another important feature with implications for turnover is learning about the quality of the relationship while matched. Jovanovic (1979a) introduced this feature to rationalize some of the same observations that the on-the-job search model was designed to explain.[14] He considers the case where workers have to learn over time about how good they are at any job based on

---

to search while unemployed and employed, and with no loss in generality normalize the costs to be linear, say $k_0 s_0$ and $k_1 s_1$. One can show that if $k_0 < k_1$ then more search effort will be made by the unemployed than employed workers. Also, s_1 is decreasing in w, so workers with higher wages search less, and at a wage sufficiently high s_1 goes to 0.

[14] See also Wilde (1979). Jovanovic (1979b) did not have learning, but has human capital involve on the job. In any case, the kind of learning we are considering here is to be distinguished from learning about the distribution F while searching, which is also interesting; Burdett-Vishwanath (1998a) provide an example and further references to this

productivity observations that are a function of both true productivity and random shocks. Here we present the very simple discrete-time version in Wright (1986), where all learning takes place in one period. Assume an offer is a signal ω, where ω is drawn from the distribution function $G(\omega)$, depending on both the true wage w and some noise; e.g., we could have $\omega = zw$ where z is random. Assume $F(w \mid \omega_2)$ first order stochastically dominates $F(w \mid \omega_1)$ whenever $\omega_2 > \omega_1$.

In discrete time, the value of search is now

$$U = b + \beta \int_0^\infty \max\{E[W(w) \mid \omega], U\} dG(\omega).$$

The value of employment at a known w is given by

$$W(w) = w + \beta \max[W(w), U],$$

since one period after accepting an offer the true value of w is observed and the worker decides whether to stay or quit. If he quits the payoff is U, assuming he must wait one period for the next offer. Since $W(w)$ is increasing, stochastic dominance implies $E[W(w) \mid \omega]$ is increasing in ω. Hence, there is a reservation signal ω_R such that offers should be accepted if $\omega \geq \omega_R$. Once w is revealed, the worker stays if $w \geq w_R$. Notice workers can be confused into accepting jobs they would prefer to reject, and vice-versa. The two possibilities are not equally likely, however, since the situation is not symmetric.[15]

More general learning models, like Jovanovic (1979a), have the implication that reservation signals will increase with tenure. This is because at the beginning of an employment spell there is a lot of uncertainty, so a low value of the signal is not necessarily a bad sign. Similarly, a good signal is also not so informative, but since the worker can always quit, again the situation is not symmetric. Therefore, the more that is known about a situation the

literature. Note that with learning about F it is not even guaranteed that a reservation wage policy will be optimal. Suppose, for example, that we know that either: a) $w = w_0$ with prob 1; or b) $w = w_1$ with prob π and $w = w_2$ with prob $1 - \pi$; where $w_2 > w_1 > w_0$ and π is small. It should be clear that it may be optimal to accept w_0 but not w_1. Rothschild (1974) presents conditions that guarantee a reservation wage policy *is* optimal with learning.

[15] Indeed, this asymmetry will follow from any model where the employed can quit easier than the unemployed can recall a previously rejected offer or generate a new offer, as seems reasonable.

more demanding one tends to be. Also, individuals with a long tenure at a job have already learned a lot and so they are less likely to quit. Furthermore, given that they are still there, they are more likely to be earning higher wages. Hence, this model also predicts quit rates fall and wages rise with tenure.

2.4 Discussion

There are many applications of the above problems, several of which we discuss below. There are also many other extensions of the framework that we do not have time to review in detail, including many interesting dynamic applications. For example, one can consider the case where offers are not iid[16] Or, one can study the case where unemployment insurance varies over time (Burdett 1979, Mortensen 1977, Albrecht-Vroman 2000, Coles-Masters 2000). Also, there are extensions of the basic search method, such as the case of so-called systematic search, where you first look at the locations that are best according to your prior, and if those do not pan out then you proceed to less favorable locations, typically lowering your reservation wage along the way (Salop 1973).

While these and many other extensions of the single-agent scenario are interesting, at this point we want to move on to equilibrium models. To explain why, first note that the above results always had an appeal for many in both micro and macroeconomics; e.g., for the former there are explicit predictions about the duration of unemployment spells, for the latter there is the foundation of what Friedman called the natural rate of unemployment. Also, there are predictions about how certain policies, such as changes in unemployment insurance, would affect the nature of individual unemployment experiences and the aggregate rate. While these things are appealing, there are some issues that need to be addressed.

[16] Lippman-McCall (1976b), Jovanovic (1987) and Lippman-Mamer (1989) let F change over time. For example, suppose there are two distributions, F_1 and F_2, where F_2 first order stochastically dominates F_1, that shift over time according to a stationary Markov process: π_{ij} is the probability of F_j next period given F_i this period. Assume $\pi_{22} > \pi_{12}$ (persistence). Even ignoring on-the-job search or learning, quits may occur because an offer that was acceptable given F_i may not be acceptable once we switch to F_j. One can show that for each i there is a reservation wage R_i, with $R_2 \geq R_1$. Quits occur when F_1 switches to F_2 if you happen to be employed at $w \in (R_1, R_2)$. Thus, if we interpret shifts in F in the natural way, this model implies quits are pro-cyclical.

One thing to say is that the single-agent model is not general equilibrium; indeed, the early literature was once caricatured by Rothschild (1973) as *partial-partial* equilibrium theory, because it not only considers only one market (the labor market), it only considers one side of that market (workers). This is somewhat beside the point, since, as is often the case, a clever economist can technically recast the same model using general equilibrium language without changing anything of substance. Consider a group of workers searching for jobs in "nature" – say, looking for good fishing spots – and productivity at different locations is simply a feature of the physical environment, captured by $F(w)$. It is easy to formulate general equilibrium versions of such an environment that are hard to distinguish form the decision theory analyzed above, with a steady state (natural) rate of unemployment given by $u = \lambda/(\lambda + H)$.[17]

Clearly, the issue is not general equilibrium *per se*. Rather, for analyzing some issues, a model that assumes certain things are fixed cannot address whether these things represent channels that may be important. Consider the case of an increase in unemployment benefits. The above analysis tells us that this will impact on worker's behavior through the reservation wage, and more generally also through search intensity. This implicitly assumes that the distribution of wages does not change, the offer arrival rate as a function of search intensity does not change, and the layoff rate does not change. Economic intuition suggests that there could be significant effects along each of these margins, and to analyze this we need to incorporate them into the analysis. The main step that needs to be taken in order to do this is to add agents on the other side of the market – firms. We proceed to do this in the next section, where we focus primarily on wages, and in the sections that follow, where we focus more on arrival and layoff rates.

3 Wage Distributions

As we said, while it is not *logically* wrong to think of $F(w)$ as exogenous, for many applications it is of interest to have wages determined inside the model. Because a key element of the original search theory is that a given worker faces a distribution of wages for their labor services, a natural issue that emerged was to understand under what conditions a market for identical

[17] An example in the spirit of this interpretation is Ljunqvist and Sargent (1998).

workers could generate a nondegenerate wage distribution in equilibrium.[18] There were two senses in which this issue was of interest. First, since the basic model suggested that search was only relevant if the wage distribution was nondegenerate, economists who thought search was important had to also develop of theory of wage dispersion. Second, for those economists who took wage dispersion for identical workers (or price dispersion for a homogeneous good) as a fact of life, the hope was that search frictions would generate equilibria in which the law of one price does not hold.

Diamond (1971) was one of the first to address this question (for product markets, but the issues are similar in the labor market). Although his model does *not* generate wage dispersion, it is useful to consider his analysis, both because it offers insight into the issues and because it shaped much of the subsequent analysis.. Consider a model with a large number of homogeneous workers and firms, where each worker solves a simple search problem like the one in the previous section, i.e., they randomly sample one offer each period, have unemployment income equal to b, die at rate δ, etc. Each firm has a constant returns to scale technology with labor as the only input, with marginal product $p > b$. Assume that wages are set by firms in a *wage-posting game*: at the beginning of each period a firm commits to a wage, given the wages chosen by the others, to maximize expected profits. Let F be the cdf for wages. A Nash equilibrium F requires that every wage posted with positive probability earns the same profit, and no other wage earns any greater profit.

Diamond's finding is rather striking: there is a unique equilibrium, and in this equilibrium all employers set the same wage, equal to the value of unemployment income: $w = b$. The proof is simple. Given workers are homogeneous, for any cdf F they all choose the same reservation wage w_R. Clearly, no firm will post $w < w_R$ as this would mean they hire no one, and no firm will post $w > w_R$ as they can hire every worker they contact at $w = w_R$. To see why it turns out that $w = b$ in equilibrium, assume all firms are posting $w > b$, and consider the situation of an individual firm. If they deviate and offer a wage that is slightly less than w they will still hire every worker they meet (i.e., workers' reservation wage will also fall infinitesimally since the loss in wages is less than the cost of waiting another period for an

[18] Obviously the most basic competitive model can generate wage dispersion for workers who vary in productivity; what is relevant in this context is wage dispersion across offers for a given worker type.

offer of w). Because this argument remains valid as long as $w > b$, the unique equilibrium wage must be equal to b. Indeed, this is still true if we relax the assumption of homogeneous firms by assuming marginal products differ.

The conclusion seemed rather severe: even if productivities differ across firms, search frictions do not yield a nondegenerate distribution of wages if workers are homogeneous. Not only could search theory not help rationalize the wage or price dispersion found in the real world, it even seemed on shaky ground logically, since in the basic model it is the presence of wage dispersion that motivates search in the first place. Many researchers subsequently developed models in which equilibrium wage or price distributions are nondegenerate, including Butters (1977), Reinganum (1979), MacMinn (1980), Burdett-Judd (1983), Robb (1985), Albrecht-Axel (1984), and Burdett-Mortensen (1998). We review several alternatives in this section.

Before proceeding, we highlight some features of the Diamond model that turn out to be important. First, the exact nature of the search friction matters; for example, it can matter if searchers sometimes receive two offers at once. Second, even if workers have identical productivities, it turns out that other types of heterogeneity matter, as we will see below. A third feature is the assumption that firms post wages, as opposed to, say, determining wages through bargaining, as we will discuss at length later. Lastly, we think it is interesting to note how this literature has evolved: while early efforts were devoted to trying to find conditions in which there was *some* wage dispersion in equilibrium, more recently there has been much success using these models to account for various aspects of the actual wage distributions in the data.

3.1 Worker Heterogeneity

To motivate the various models that follow, ask yourself this: why might one expect to find wage dispersion? The answer is that search frictions produce a natural trade-off: while posting a higher wage results in lower profit per worker, it will potentially increase the rate at which workers can be hired, since more workers will be willing to accept the job at a higher wage. It turns out that in the Diamond model this trade-off is actually non-existent, since in equilibrium if you increase your wage relative to other firms there is no increase in the rate at which you can hire when all workers have the same reservation wage. In view of this, a natural approach to generating wage dispersion is to allow for heterogeneity in some dimension that will generate heterogeneity in reservation wages.

The Albrecht-Axel model assumes some workers have unemployment income b_1 and others have $b_2 > b_1$. This implies that for a given wage distribution F there will be two different reservation wages, say w_1 and $w_2 > w_1$. The obvious generalization of Diamond's result implies that no firm will post a wage other than w_1 or w_2. But, it seems possible that some firms could post w_1 and others w_2, as long as these imply equal profit. This seems possible, in principle, since they can set $w = w_1$ and hire only b_1 workers, or set $w = w_2$ and hire all workers. The former implies a higher profit per worker but a lower arrival rate of workers, whereas the latter implies a higher arrival rate of workers but lower profit per worker.[19]

Suppose there are large numbers of firms and workers, and normalize the measure of firms to be 1, and let the measure of workers be $L = L_1 + L_2$ where L_j is the measure with unemployment income b_j. As we said, given any distribution F, all type 1 workers choose reservation wage w_1 and all type 2 workers choose reservation wage $w_2 > w_1$, and in equilibrium all firms post either w_1 or w_2. Let σ be the fraction of firms posting w_2, to be determined endogenously below. Given that at most two wages are posted, F is completely summarized by w_1, w_2 and σ. Note, however, that F is the distribution of wages across firms, which generally differs from the distribution of wages across workers, as we will see.

As in the previous section, let the rate at which workers contact firms be α and the separation rate be λ, both of which are exogenous for now. Since firms have a constant returns to scale production technology with marginal product $p > b_2$, they will employ every worker that accepts their wage. Following the logic of the Diamond model, one can show that in equilibrium the highest wage posted satisfies $w_2 = b_2$. To determine w_1, note that b_1 workers accept both $w = w_1$ and $w = w_2$, and their value functions satisfy

$$\begin{aligned}
rU_1 &= b_1 + \alpha\sigma\left[W_1(w_2) - U_1\right] \\
rW_1(w_1) &= w_1 \\
rW_1(w_2) &= w_2 + \lambda\left[U_1 - W_1(w_2)\right].
\end{aligned}$$

Although they may look slightly different, these are merely special cases

[19] Albrect-Axel (1984) do not actually assume all firms earn the same profit, but rather that productivity p is distributed in some way across firms, and look for a cutoff p^* such that firms with $p < p^*$ pay $w = w_1$ and firms with $p > p^*$ pay $w = w_2$. On this dimension, the model here is actually more similar to the consumer search models in Diamond (1987) or Curtis and Wright (2001).

of what we saw in the previous section that apply when F is a two-point distribution.

Note that although b_1 workers accept $w = w_1$ they get no capital gain from doing so, since w_1 is their reservation wage, and similarly they suffer no capital loss when laid off from w_1. Using $W_1(w_1) = U_1$ and $w_2 = b_2$, we find

$$w_1 = \frac{(r+\lambda)b_1 + \alpha\sigma b_2}{r+\lambda+\alpha\sigma}. \tag{20}$$

We now know w_1 and w_2 as functions of the underlying parameters and σ, say $w_j = w_j(\sigma)$. In steady state, the unemployment rates for type 1 and 2 are given by $u_1 = \frac{\lambda}{\alpha+\lambda}$ and $u_2 = \frac{\lambda}{\alpha\sigma+\lambda}$, while total unemployment is $L_1u_1 + L_2u_2$. The employment rates can be written $e_1 = 1 - u_1$, $e_2 = 1 - u_2$, while the aggregate employment rate is $e = (L_1e_1 + L_2e_2)/L$.

The next step is to analyze firms, who are assumed to maximize steady state profit, $\Pi = n(p-w)$. To compute Π we first derive the steady state number of workers at a given firm posting w_j, denoted n_j. Since the total number of contacts between type j workers and firms per period is $\alpha L_j u_j$, this is also the arrival rate of type j workers for a given firm, since we have normalized the number of firms to 1. Hence, w_1 firms recruit workers at rate $\alpha L_1 u_1$ and w_2 firms recruit at rate αu. This is of course the key trade off: paying higher wages increases recruitment at the expense of profit per worker. Since a firm of type j loses λn_j workers per period, we have $n_1 = \alpha L_1 u_1/\lambda$ and $n_2 = \alpha u/\lambda$. Hence,

$$\begin{aligned}
\Pi_1 &= n_1(p-w_1) = \frac{\alpha L_1}{\alpha+\lambda}\left[p - \frac{(r+\lambda)b_1+\alpha\sigma b_2}{r+\lambda+\alpha\sigma}\right] \\
\Pi_2 &= n_2(p-w_2) = \left[\frac{\alpha L_1}{\alpha+\lambda} + \frac{\alpha L_2}{\alpha\sigma+\lambda}\right](p-b_2),
\end{aligned}$$

after inserting w_j as well as u_j.

The difference in profits is given by

$$\Pi_2 - \Pi_1 = \frac{\alpha u}{\lambda}(p-w_2) - \frac{\alpha L_1 u_1}{\lambda}(p-w_1), \tag{21}$$

which is proportional to

$$\begin{aligned}
T(\sigma) &= (r+\lambda+\alpha\sigma)\left\{(p-b_2)\left[\lambda L_1 + (\alpha+\lambda)L_2\right] - (p-b_1)\lambda L_1\right\} \\
&\quad -r\alpha\sigma L_1(b_2-b_1).
\end{aligned} \tag{22}$$

As we said, an equilibrium is a wage distribution such that every wage posted with positive probability earns the same profit, and no other wage earns any greater profit. Given that a fraction σ of the firms post $w_2(\sigma)$ and the rest post $w_1(\sigma)$, any equilibrium is completely characterized by a value of σ such that one of the following holds: $\sigma = 1$ and $T(1) > 0$; $\sigma = 0$ and $T(0) < 0$; or $0 < \sigma < 1$ and $T(\sigma) = 0$. In the first case, all firms post $w_2 = w_2(1) = b_2$; in the second all firms post $w_1 = w_1(0) = b_1$; and in the third case some firms post $w_2 = b_2$ while others post $w_1 \in (b_1, b_2)$, but both earn the same profit, $\Pi_1 = \Pi_2$.

One can show there always exists a unique equilibrium σ, and $0 < \sigma < 1$ if and only if $\underline{p} < p < \overline{p}$ where

$$\underline{p} = b_2 + \frac{\lambda L_1(b_2 - b_1)}{(\alpha + \lambda)L_2} \text{ and } \overline{p} = \underline{p} + \frac{r\alpha L_1(b_2 - b_1)}{(r + \alpha + \lambda)(\alpha + \lambda)L_2}.$$

When productivity is very low all firms pay $w_1 = b_1$, when it is very high all firms pay $w_2 = b_2$, and when it is intermediate we have wage dispersion. If so desired, one can solve $T(\sigma) = 0$ for σ and substitute to derive the explicit distribution of wages posted. The distribution of wages across workers is simple: e_j workers earn w_j. For $\sigma \in (0, 1)$, σ will be less than e_2 because high wage firms are larger. Of course, it is precisely the fact that high wage firms are larger that equates the profit from posting w_1 and w_2.

3.2 A Shirking/Crime Model

The above model can generate dispersion because firms that pay higher wages recruit at a faster rate. A different approach would be a model where firms that pay higher wages lose workers at a slower rate. The Diamond model did not possess this trade-off since all firms lost workers at the exogenous death rate δ. Here we present a model that can be interpreted as one of either *shirking*, as in some of the efficiency wage literature, or of *crime*.[20] Thus, any employed worker randomly comes across an opportunity to shirk, or to engage in some criminal activity, according to a Poisson process with arrival rate μ. The gross reward to this activity is denoted K. However, there is also a probability ν that he gets caught. In general, if a worker is caught he

[20] See Shapiro-Stiglitz (1984) or Weiss (1980) for a discussion of efficiency wage models in terms of shirking. The presentation here is more along the lines of the model of crime in Burdett-Lagos-Wright (2000).

is put in jail for a while, and then released back into the pool of unemployed workers.

Although there are good reasons to include jail time in the model, for simplicity, we assume here that jail time is zero – i.e., when workers are caught they lose their job, but are back on the streets looking for another job right away. What matters for the argument is simply that the employment relationship is terminated. Also, to show how things differ from Albrecht-Axel, we assume that workers are homogeneous. As in Diamond's model this implies that there is a common reservation wage w_R, and firms can hire any worker they contact by posting $w = w_R$. However, a plausible alternative now is to pay something above w_R to induce the worker to refrain form the activity in question (shirking, crime, or whatever one wants to call it). Firms may find this profitable since, after all, they suffer a capital loss when a worker leaves.[21]

Let $w_C > w_R$ denote the *critical wage* at which a worker would refrain from the activity in question rather than risk losing his job, defined by $K + \nu[U - W(w_C)] = 0$. Clearly, in equilibrium no firm would post any wage other than w_R or w_C, and let σ be the fraction posting the higher wage. Bellman's equations for a worker are

$$\begin{aligned} rU &= b + \alpha\sigma[W(w_C) - U] \\ rW(w_R) &= w_R + \mu K \\ rW(w_C) &= w_C + \lambda[W(w_C) - U]. \end{aligned}$$

Notice that although workers accept w_R, they get no capital gain from doing so since $W(w_R) = U$; likewise, they suffer no capital loss from losing the job. Using $K + \nu[U - W(w_C)] = 0$ and $W(w_R) = U$, we have

$$\begin{aligned} w_C &= b + (r + \lambda + \alpha\sigma)K/\nu \\ w_R &= b - \mu K + \alpha\sigma K/\nu. \end{aligned}$$

The steady state distribution of wages across firms is characterized by w_R, w_C and σ, although once again this differs from the steady state distribution of wages across workers, since low wage firms lose their workers more

[21] One reason that jail time is interesting in the model is the following. In the basic efficiency wage model, the firm is supposed to punish a worker by laying him off when he is caught shirking, but this is not really in the firm's interest – what is the point of getting rid of a worker, only to search for another who will behave exactly the same? If an outside authority like the criminal justice system exogenously takes the worker out of the job this issue does not come up.

frequently and hence are smaller. That is, all firms recruit at the same rate $\alpha L u$, but firms paying w_R lose workers at rate $\lambda + \mu\nu$ while firms paying w_C lose workers only at rate λ. Hence, $n_R = \alpha L u/(\lambda + \mu\nu)$ and $n_C = \alpha L u/\lambda$. It follows that steady-state profits for the two types of firms are given by:

$$\begin{aligned} \Pi_R &= n_R(p - w_R) = \frac{\alpha L u}{\lambda + \mu\nu}\left[p - b + \mu K - \alpha\sigma K/\nu\right] \\ \Pi_C &= n_C(p - w_C) = \frac{\alpha L u}{\lambda}\left[p - b - (r + \lambda + \alpha\sigma)K/\nu\right]. \end{aligned}$$

One can now show that $\Pi_C - \Pi_R$ is proportional to

$$T(\sigma) = \mu\nu(p - b) - \lambda(r + \lambda)K/\nu - \mu(r + 2\lambda)K - \mu\alpha K\sigma. \tag{23}$$

As in the Albrecht-Axel model, it is a matter of algebra to show that there is a unique equilibrium, and $0 < \sigma < 1$ if and only if p is in some region as determined by solving $T(0) = 0$ and $T(1) = 0$. For very low (high) p no firm (every firm) finds it profitable to induce workers to refrain from bad behavior, and so the unique wage is w_R (w_C); for intermediate p we have wage dispersion.

Whereas Albrecht-Axel assumes intrinsically heterogeneous workers, the crime model assumes that in any period there will be *ex post* differences across individuals (some will have a crime opportunity and others will not; some will earn w_R and some earn w_C). This suggests an alternative version of Albrecht and Axel with ex-ante homogenous workers, but where unemployment income, or more generally, nonmarket opportunities, for each individual evolves according to a two-state Markov process, taking on the values b_1 and b_2. Without going through the details, one should be able to see that a firm can offer a low wage and attract searchers with a low current value of b, but these workers will leave if their value of b increases. This of course assumes that the wage is fixed – but that is a basic assumption to all of the models considered here.

3.3 On-the-Job Search

In the two models presented above firms can pay higher wages to increase the inflow or reduce the outflow of workers. The Burdett-Mortensen (1998) model has both, but through a different mechanism: on-the-job search. In their basic model, workers are homogenous and there are no opportunities for shirking or criminal activity, but employed workers continue to sample new offers and leave whenever they get an offer above their current wage. As in the previous section, let the offer arrival rates be α_0 and α_1 while unemployed and employed respectively, and assume every offer is a random draw from $F(w)$.[22] Since all unemployed workers are identical they have a common reservation wage, w_R. Clearly no firm posts w below this common reservation wage w_R, so all jobs are accepted by unemployed workers, and the unemployment rate is $u = \lambda/(\lambda + \alpha_0)$. For ease of presentation, we begin with the special case $\alpha_0 = \alpha_1 = \alpha$, which implies $w_R = b$ by (18), and return to the general case later.

The analysis of this model is more intricate than those considered above, since the worker inflow rate for a firm posting wage w now depends upon the distribution of wages paid across workers (since they can attract any worker they contact whose wage is below w). The first thing we need to do is to compute the steady state distribution of wages across workers, $G(w)$, given a distribution of wages across firms, $F(w)$. As above, these differ because firms posting different wages employ different numbers of workers. It is a matter of simple analysis to show[23]

$$G(w) = \frac{\lambda F(w)}{\lambda + \alpha [1 - F(w)]}. \tag{24}$$

We now describe the problem of an individual firm, focusing on the case $r \to 0$ for simplicity (see Coles [1997] for the case where $r > 0$). Steady state profit for a firm posting w is $\Pi(w) = n(w)(p - w)$, where $n(w)$ is its steady state number of workers. To compute $n(w)$, simply note that the number of workers employed at a firm paying w must equal the number of workers earning w divided by the number of firms paying w. Assuming differentiability, which we will verify below, this means $n(w) = G'(w)(1 -$

[22]It can make a difference in this model if, instead of matching with firms at random, one assumes balanced matching in the sense of Burdett-Vishnawath (1988b) – i.e., if you are more likely to get an offer from a larger firm than a smaller firm; see Robin-Roux (1998).

[23]Consider the following argument. Given any w, the number of workers employed at a wage no greater than w is $G(w)(1-u)$. This increases over time at rate $\alpha u F(w)$, the rate at which unemployed workers contact a firm paying less than w, and decreases over time at rate $\lambda G(w)(1-u) + G(w)(1-u)\alpha[1 - F(w)]$, the rate at which workers employed at less than w are terminated for exogenous reasons plus the rate at which they move to firms paying more than w. Equating these flows and inserting $u = \lambda/(\lambda + \alpha)$ implies (24).

$u)/F'(w)$. Therefore

$$\Pi(w) = \frac{G'(w)}{F'(w)}(1-u)(p-w) = \frac{\alpha\lambda(p-w)}{\{\lambda + \alpha[1-F(w)]\}^2}, \tag{25}$$

after inserting u and G', the latter of which we get from differentiating (24).

Again, equilibrium requires that all wages paid yield the same profit, which is at least as large as the profit from posting any other wage. What could an equilibrium F possibly look like? We argued above that no firm posts $w < w_R = b$. Also, clearly no firm posts $w > p$, since this implies $\Pi < 0$. Some other features of any equilibrium are the following: (1) F contains no mass points; (2) some firm pays exactly $w = b$; ; and (3) there can be no gaps on the support of F.[24] Summarizing, the support of F is $[b, \bar{w}]$ for some upper bound $\bar{w} < p$, and there are no gaps or mass points on the support.

The key next step is to use the fact that firms earn equal profits from all wages paid, including the lowest wage b: thus, $\Pi(w) = \Pi(b)$ for every $w \in [b, \bar{w}]$. Since $F(b) = 0$, we have $\Pi(b) = \alpha\lambda(p-b)/(\alpha+\lambda)^2$. Equating this to the expression for $\Pi(w)$ in (25), we have an equation in $F(w)$ that can be solved to yield

$$F(w) = \frac{\lambda+\alpha}{\alpha}\left(1 - \sqrt{\frac{p-w}{p-b}}\right). \tag{26}$$

This is the form F must take in equilibrium: it is the unique wage distribution that implies equal profit for all wages paid. To complete the description of F it only remains to find $\bar{w}$, which we easily get from solving $F(\bar{w}) = 1$.

In words, the outcome is as follows. First, all unemployed workers accept the first offer they receive since all offers are above $w_R = b$. They move up the wage distribution each time a better offer comes along, but also return to unemployment periodically, due to exogenous layoffs. There is a nondegenerate distribution of wages offered by firms, F, and of wages earned by

[24] To show (1) suppose there were a mass point at w. Then, if firm offers $w + \varepsilon$ instead of w it can increase its inflow of workers by a discrete amount for any $\varepsilon > 0$, whereas the decrease in profit per worker goes to 0 as ε goes to 0. To show (2), suppose the lowest wage paid is $w' > b$. Then any firm paying w' can increase profit by paying $w = b$, since it still attracts and loses the same number of workers (given there are no mass points), which means $n(b) = n(w')$. Hence, the lowest wage paid is exactly b. To show (3), suppose there is an non-empty interval $[w', w'']$, with $w' > b$ and some firm paying w'' but no firm paying $w \in [w', w'']$. Then the firm paying w'' can make strictly greater profit by paying $w'' - \varepsilon$ for some $\varepsilon > 0$.

workers, G, which are different since different wages imply different numbers of workers: firms paying higher wages attract more workers from other firms and lose fewer workers to other firms. Hence, high wage firms are larger in equilibrium, although all firms earn the same profit.

The model has many interesting extensions. First, it is not much harder to solve with $\alpha_0 \neq \alpha_1$. The result is

$$F(w) = \frac{\lambda + \alpha_1}{\alpha_1}\left(1 - \sqrt{\frac{p-w}{p-w_R}}\right), \tag{27}$$

where now w_R is endogenous (in the case $\alpha_0 = \alpha_1$ we knew $w_R = b$). To determine w_R, one needs to solve (18), which can be done *explicitly* given the functional form in (27). The answer is

$$w_R = \frac{(\lambda + \alpha_1)^2 b + (\alpha_0 - \alpha_1)\alpha_1 p}{(\lambda + \alpha_1) + (\alpha_0 - \alpha_1)\alpha_1}. \tag{28}$$

To highlight one reason why this generalization is interesting, consider the limit as either $\alpha_1 \to 0$ or $\lambda \to \infty$. To see what happens, solve $F(\bar{w}) = 1$ explicitly for

$$\bar{w} = p - \left(\frac{\lambda}{\lambda + \alpha_1}\right)^2 (p - w_R).$$

In the limit, the highest wage offered $\bar{w}$ is equal to w_R, which is also the lowest wage offered since no firm ever offers $w < w_R$. Hence, there is a single wage, $w = w_R$. Moreover, (28) implies that in the limit $w_R = b$, and so all employers offer workers their value of unemployment income. The Diamond solution emerges as a special case.

Another interesting result comes from taking the limit of

$$G(w) = \frac{\lambda\left(1 - \sqrt{\frac{p-w}{p-w_R}}\right)}{\alpha_1\sqrt{\frac{p-w}{p-w_R}}}$$

as $\alpha_1 \to \infty$. The result implies $\bar{w} = p$ and $G(w) = 0$ for all $w < p$. Hence, in the limit all workers earn exactly $w = p$. Moreover, as $\alpha_0 \to \infty$, the unemployment rate u becomes 0. Hence, something that resembles the competitive solution emerges – i.e., all workers always earning their marginal product – can be thought of as the limiting case as the search frictions vanish, in the sense that both α_0 and α_1 get large. Thus, the model generates both the competitive outcome and the Diamond solution as special cases.

One can also let firms be heterogenous with respect to productivity p in the model. Given a finite number of firm types, there is a distribution of wages paid by each type, and all firms with productivity p_2 pay a greater wage than all firms with productivity $p_1 < p_2$. Thus, higher productivity firms necessarily end up larger. This is an important extension because with constant p the wage distribution given by (26) has an increasing density, which is not what one sees in the data. With heterogenous firms, however, F can have a decreasing density, even if the underlying distribution of p does not. Additionally, as shown in van den Berg (2000), with heterogeneous firms there can be multiple equilibria (see below for a simple example based on the same economics).

3.4 Other Issues

We have reviewed three models that generate endogenous wage dispersion, and of course, one can combine them. For example, we can have on-the-job search and workers who differ with respect to b, integrating Burdett-Mortensen with Albrecht-Axel. This is actually important, for the following reasons. The on-the-job search model can do a good job of accounting for the empirical distribution of wages, at least once we allow heterogeneous firms, but it does less well in accounting for individual employment histories. Especially problematic is observed negative duration dependence (i.e., hazard rates that decrease with the length of unemployment spells). In contrast, the Albrecht-Axel model does a better job of accounting for negative duration dependence since the unemployment pool has workers with different outflow rates, but does less well in accounting for the wage data. Models that combine on-the-job search and worker heterogeneity have greater scope to account for both wages and unemployment data (see Bontemps, Robin and van den Berg 1999).

One can also integrate the on-the-job search and crime/shirking frameworks. In the resulting model, an endogenous fraction σ of firms pay $w \in [w_C, \bar{w}]$, and the remaining $1 - \sigma$ pay $w \in [w_R, \hat{w}]$, where w_R is the reservation wage and w_C the critical wage that dissuades criminal activity. The distributions on $[w_R, \hat{w}]$ and $[w_C, \bar{w}]$ have no gaps or mass points, and one can solve for their closed forms as a function of σ. However, there is a gap between $\hat{w}$ and w_C (since by increasing w from $w_C - \varepsilon$ to w you can generate a discrete drop in the rate at which workers flow out, increasing profit). We can have $\sigma = 0$ or 1 in equilibrium, in which case things look a lot like the basic Burdett-Mortensen model, but we can also have $0 < \sigma < 1$. Indeed, the model can have multiple equilibrium values of σ, although only when criminals are actually sent to jail, and not just to unemployment.[25]

We also want to mention that a slightly different version of any of the above models can be formulated, which gives very similar results but is based on a different vision of employers. Rather than having a fixed number of firms that meet workers at some constant rate and all hire as many as they can get, suppose that to attract workers firms have to post vacancies, which is costly. Each vacancy can be filled by at most 1 worker. We assume that each employer can post only one vacancy, but allow entry by firms.[26] Although we will go into models with entry in much more detail below, it is worth introducing the idea here to show how the endogenous wage distribution models can be recast in an alternative form. Mortensen (2000) does so for the Burdett-Mortensen model, and here we will do it for the Albrecht-Axel model.

The basic setup is the same as Albrecht-Axel. Thus, any equilibrium wage distribution has a fraction σ of vacancies posting $w_2 = b_2$ and the remaining $1-\sigma$ posting w_1 as given by (20), and the unemployment rates are $u_1 = \frac{\lambda}{\alpha+\lambda}$ and $u_2 = \frac{\lambda}{\alpha\sigma+\lambda}$. However, here we need to be more careful with the arrival rates. As will be discussed below, to determine arrival rates in general one can assume a matching function that maps the number of searching workers and firms, u and v, into the total number of meetings, $m = m(u, v)$, but for now consider the special case $m(u, v) = A \min\{u, v\}$. Assume that firms enter, or post vacancies, as long as expected profit exceeds the fixed cost of entry, k. Clearly, we will have $v \geq L$ as long as the cost of entry is not too high, since firms make positive profit when $v = L$. Given $v \geq L$ we know $v \geq u$; so the arrival rate for workers is the fixed constant $\alpha = A$, and the arrival rate for firms is $\alpha u/v$. The rate at which a firm meets type j workers

[25] See Burdett-Lagos-Wright (2000). Intuitively, multiple equilibria can arise as follows: Suppose more firms are paying above w_C. This makes the value of search higher, so workers are more reluctant to commit a crime because there is more to lose from spending time in jail. This makes it cheaper to pay above w_C, and hence we can have multiple equilibria. If, however, workers do not have to spend any time in jail, but simply become unemployed when caught, this cannot happen.

[26] Actually, it is equivalent to assume that a fixed number of employers can each post as many vacancies as they like, since only the total number of vacancies will be determined here. What is important is that posting vacancies is required to generate meetings and that this activity is costly.

is therefore $\frac{\alpha}{v}L_j u_j$.

Let V_j and J_j be the value functions for a given firm searching for a worker and matched with a worker (V for the value of a *vacancy* and J for the value of a filled *job*), given it posts w_j. Since a firm posting w_1 hires only type 1 workers,

$$\begin{aligned} rV_1 &= \frac{\alpha}{v}L_1 u_1 (J_1 - V_1) \\ rJ_1 &= p - w_1 + \lambda(J_1 - V_1). \end{aligned}$$

The entry condition says that if any firms paying w_1 enter at all then we must have $V_1 = k$. Inserting this and solving we get

$$rV_1 = rk = \frac{\alpha}{v}L_1 u_1 \frac{p - w_1 - rk}{r + \lambda}.$$

Repeating the exercise for any firm posting w_2, if firms enter at w_2, we must have

$$rV_2 = rk = \frac{\alpha}{v}u\frac{p - w_2 - rk}{r + \lambda}.$$

Entry by both types implies $rV_1 = rk = rV_2$, or

$$T(\sigma) = u(p - w_2) - L_1 u_1 (p - w_1) - L_2 u_2 rk = 0. \tag{29}$$

Comparing this with (22) from Albrecht-Axel, we see that the equilibrium function in the two models reduces to exactly the same thing for either small r or k. Hence, for small r or k the equilibrium is the same, except for the interpretation: now posting a high wage does not mean the firm becomes larger, but that it recruits faster.

To close this section, we reiterate that every model discussed involves wage posting by firms, and the firm agrees to employ every worker it contacts at that wage independent of their characteristics. If the firm could condition wage offers on worker type it is simple to get wage dispersion. For example, in the Albrecht-Axel environment it is an equilibrium for firms to pay $w_j = b_j$ to any type j worker who shows up (a generalized version of the Diamond result). One way to think of firms posting wages conditioned on worker type is as an extreme bargaining assumption where firms get to make take-it-or-leave-it offers to anyone who shows up. If we go to the other extreme and give workers all the bargaining power then the equilibrium has a single wage, $w = p$. In principle, given heterogeneity, any bargaining solution intermediate between take-it-or-leave-it offers by the firm and take-it-or-leave-it offers by the worker can generate wage dispersion, as we will see later.[27]

4 Two-Sided Search

The preceding section focused on the issue of how to endogenously generate an equilibrium with wage dispersion. This necessitated introducing firms into the analysis. Any such model is an example of a *two-sided* search model, since it considers behavior on both sides of a match. While the wage dispersion literature tended to take the Diamond model as its starting point, this formulation is really just one of many possibilities. Some of the key choices that one makes in writing down any two-sided search problem concern what determines the rate at which meetings occur, how the output of a match is determined, and how the parties decide on compensation. In the Diamond model, meeting rates were taken to be exogenous, the output of a match was a homogeneous good, and compensation was determined through wage posting. Each choice is one of several alternatives available to a modeler, and generally the best option depends upon the issue. In this section we provide an overview of several of these options.

4.1 Nontransferable Utility

Usually economists model employment relationships under the assumption of transferable utility: the worker and firm together produce some output which is to be somehow divided between the parties. However, it is also clear that there can be aspects of employment relationships that may not fit this description all that well, such as how one gets along with one's boss, where

[27] If we assume firm heterogeneity with respect to p then it is easy to generate wage dispersion if workers make take-it-or-leave-it offers, subject to $w \leq p_j$ (since a firm could always reject). Of course, there may be no $w \leq p_j$ at which the worker prefers employment to continued search. Call a firm *active* if there is positive probability a worker will become employed there. Let the fraction σ of firms have $p = p_2$ and $1 - \sigma$ have $p = p_1 < p_2$. Assume workers sample randomly from the set of *active* firms at rate α. One can show there is an equilibrium where all firms are active (so some workers get $w = p_1$ while others get $w = p_2$) iff $p_2 \leq p_A = \frac{r+\lambda+\alpha\sigma}{\alpha\sigma} p_1 - \frac{r+\lambda}{\alpha\sigma}$. There is also an equilibrium with only p_2 firms active iff $p_2 \geq p_B = \frac{r+\lambda+\alpha}{\alpha} p_1 - \frac{r+\lambda}{\alpha}$. These equilibria coexist in the nonempty region where $p_B < p_2 < p_A$, along with an equilibrium in which a fraction of p_1 firms are active. This multiplicity is in the same spirit as van den Berg (2000).

the job is located, and so on. Therefore some models of the labor market explore the implications of nontransferable utility. Moreover, very many of the search-based models of the marriage market also assume that utility is nontransferable. Hence, we begin here with a model in which the output from a match is entirely nontransferable. We later discuss the case in which there is a mix of transferable and non-transferable components.

Consider an economy with a large group of workers of measure L_w and a large group of employers with measure L_e. Each unmatched employer is searching for a single unemployed worker, and vice-versa. For simplicity, assume for now that $L_e = L_w = 1$. Then, given a worker can only match with one employer and vice-versa, the number of unemployed workers u is always equal to the number of firms with a vacancy. Also, assume for now that all individuals are *ex ante* identical. In particular, workers all produce output y if employed, but each worker and employer have idiosyncratic tastes concerning who they are matched with. So, although there is no such thing as an objectively better worker or employer, any individual may prefer one match over another.

As in Burdett-Wright (1998), we formalize this by assuming that in a random meeting, the payoffs to the worker and employer are given by z_w and z_e respectively, where z_w and z_e are (independent) iid draws from distributions with cdf F_j. Assume for now that the wage rate is exogenously given as w for all matches, and that this element is incorporated into z_j (that is, payoffs depend on the wage, but can also depend on other non-wage characteristics). Assume that the meeting rate is given by α_0 for both types of agents, and that each agent of type j dies at rate δ_j, at which point he is replaced by another agent exactly the same who starts life unmatched. There is also an exogenous layoff rate for matches given by λ_0. Agents cannot search while matched (see Webb [1999a,b] for extensions that relax some of these assumptions).

A key feature of this set-up is that when two agents meet their evaluations of the match are not perfectly correlated, i.e., the match may be good for one of the parties but not for the other. But, by assumption, there is nothing that the former can do to affect the latter's evaluation; this is what it means for utility to be nontransferable. Of course, for a relationship to be consummated both parties must agree. Because a given agent may not be acceptable to every agent they meet, effective offer arrival rates, denoted by α_w and α_e, are not necessarily the same as α_0, and need to be determined endogenously.

First note that every agent of type j, given α_j and δ_j, faces a standard search problem just like in Section 2, which is solved by a reservation utility level w_j satisfying a version of (14). Although we emphasize that w_j is a reservation *utility level* here, rather than a reservation wage, the analytics are the same. For workers, e.g., we have

$$w_w = b_w + \frac{\alpha_w}{r + \delta_w + \lambda_w} \int_{w_w}^{\infty} [1 - F_w(z_w)]\, dz_w, \tag{30}$$

where α_w and λ_w are the arrival and layoff rates from the worker's perspective. A symmetric equation holds for firms, giving their reservation utility level w_e.

In equilibrium, as we said, not all contacts result in an offer. For workers, we have $\alpha_w = \alpha_0[1 - F_e(w_e)]$, since to get an offer they need a contact and the employer must be willing to hire them, which requires $z_e \geq w_e$. Also, even if λ_0 is exogenous, agents still have to worry about death rates of agents on the other side of the market. For a worker, e.g., $\lambda_w = \lambda_0 + \delta_e$. Substituting α_w and λ_w into (30) we have

$$w_w = b_w + \frac{\alpha_0[1 - F_e(w_e)]}{r + \delta_w + \delta_e + \lambda_0} \int_{w_w}^{\infty} [1 - F_w(z_w)]\, dz_w.$$

This implies a relation $w_w = \rho_w(w_e)$, which can be thought of as the *best response function* of workers to the strategy of firms.[28] Symmetrically, we have $w_e = \rho_e(w_w)$. A (steady state) equilibrium is given by an intersection of the two best response functions in (w_w, w_e) space.

One can show that a steady state equilibrium exists, and that it is unique if F_j satisfies a log-concavity property (see Burdett-Wright 1998 for proofs of these assertions). However, without log-concavity, steady state equilibrium is *not* generally unique. The intuition is as follows. Suppose one side of the market, say workers, are very demanding about the kinds of offers they accept (they set w_w very high). Then on the other side, firms get very few offers (α_e is very low), and so they cannot afford to be too demanding (they set w_e low). This means that the workers get plenty of offers (α_w is very high), which justifies being demanding. So, a high w_w and a low w_e could be a self-fulfilling prophecy; but so could the opposite.

[28] It looks as though the dependence of w_w on w_e occurs only through the arrival rate $\alpha_w = \alpha_0[1 - F_e(w_e)]$, but this is not true in general. For instance, suppose z can change during the relationship at rate γ, as in the problem described by (19); then $\lambda_w = \lambda_0 + \delta_e + \gamma F_e(w_e)$. Hence, generally *both* the arrival and layoff rates for one side depend on the strategies of agents on the other side of the market.

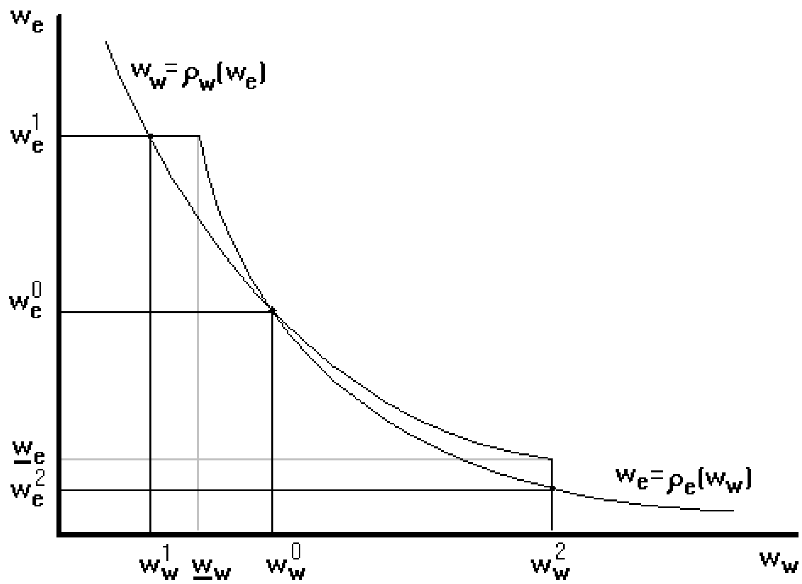

Figure 3: Multiple Equilibria with Nontransferable Utility

An example is shown in Figure 3, drawn for the case in which z_j has a Pareto distribution above some lower bound $\underline{w}_j$ (which, we note, is not log concave). There is a steady state equilibrium at (w_w^1, w_e^1) where firms are very demanding and workers actually accept all offers, at (w_w^2, w_e^2) where the opposite holds, and at the intermediate point (w_w^0, w_e^0). The steady state equilibrium unemployment (equals vacancy) rate in a given equilibrium is

$$u^* = \frac{\lambda_0 + \delta_e + \delta_w}{\lambda_0 + \delta_e + \delta_w + H},$$

where now the hazard rate is $H = \alpha_0[1 - F_w(w_w)][1 - F_e(w_e)]$. Generally, u^* differs across steady state equilibria. Hence, the natural rate of unemployment is not particularly natural here, in the sense that it depends on which equilibrium the economy ends up in.[29]

[29] Although we focus almost exclusively on steady states in this survey, in many of the models dynamics are straightforward. In particular, suppose the arrival rate α_0 does not depend on the unemployment rate – which it will not under plausible assumptions

Even when there is a unique equilibrium, this model has many interesting implications. A neat example is Masters (1999), who considers in some sense a blend of transferable and non-transferable utility. In particular, he takes the model as described above but additionally assumes, as in the previous section, that employers play a wage-posting game to determine w. Masters shows that, under a log-concavity assumption, there is a unique symmetric (single-wage) equilibrium in the wage-posting game, w^* (it is not known if there also exist other equilibria with wage dispersion in his set up). Given w^*, some meetings result in a match being consummated while others do not, depending on the random nonwage characteristics. A key result is that w^* is less than the value that minimizes unemployment. Hence, an increase in the legislated minimum wage leads to a reduction in unemployment, for simple and natural reasons.[30]

4.2 Transferable Utility

With nontransferable utility, there is no scope for any payments between the parties in a match. In contrast with the above model, many applications to the labor market assume that utility is *perfectly transferable*. In general this implies that when a worker and employer meet and decide to form a match they also need to decide what payments will accrue to each of the parties. Or, put somewhat differently, one needs to decide how wages will be determined. We have already seen one mechanism for doing this, the wage-posting games analyzed in Section 3, and we have alluded to other alternatives, such as the extreme form of bargaining where either workers or firms make take-it-or-leave-it offers. Here we discuss a very popular alternative, which is to assume a fairly general form of bargaining.

discussed below. Then if we start at $u \neq u^*$ it is an equilibrium for agents to set w_j to its steady state value for all t, and u_t will converge to u^*. When there is a unique steady state, this is typically the only equilibrium; however, when there are multiple steady states there can be other equilibria where w_j changes over time. In some models, the dynamics are very complicated; see Mortensen (1988, 1999).

[30] The intuition is similar to that coming from a standard textbook model of labor supply and demand: if firms have monopsony power they set wages too low, and efficiency can be increased by minimum wage legislation. However, the textbook model of supply and demand does not generate unemployment in the sense that individuals are searching for a job, nor does it make any predictions about the flows of workers between employment and unemployment. Hence, this model presents a richer picture of the issue than does the standard textbook treatment.

Consider a meeting between a worker and a firm. They receive a draw y from a cdf F that represents the output they would produce in each period that they remain together. Let w be the wage payment to the worker that is the solution to the bargaining problem, so that the employer receives $\pi = y - w$. Restricting attention to steady states, let U and $W(w)$ denote the worker's value functions, and V and $J(\pi)$ denote the employer's value functions, where $J(\pi)$ is the value of being matched when the profit flow is given by π. Then $W(w) \geq U$ and $J(\pi) \geq V$ are necessary for a relationship to be consummated. The techniques described above imply this is equivalent to $w \geq w_R$ and $\pi \geq \pi_R$, where w_R is the reservation wage for the worker and π_R the reservation profit for the firm. We will now describe conditions under which these two conditions reduce to the single condition $y \geq y_R$, which means workers and firms always agree on whether to form and maintain a relationship. Note that this was not the case in the nontransferable utility model, where we can have $w > w_R$ and $\pi < \pi_R$, or vice-versa, and the match will not be consummated.

Since the bargaining problem confronting a pair who meet constitutes a bilateral monopoly situation, there is no single correct way of approaching the issue. However, much of the literature adopts the generalized Nash solution. In words, the generalized Nash bargaining solution maximizes the payoff minus the threat point of one agent, raised to some power θ, times the payoff minus the threat point of the other agent, raised to $1-\theta$, where θ is called the bargaining power of the first agent. The threat points and bargaining power are primitives in this approach. To understand this, note that Nash (1950) did not actually analyze the bargaining *process* at all, but took as given a small number of reasonable axioms concerning the outcome and showed that his solution is the unique outcome satisfying these axioms.[31]

The Nash solution, while elegant and very practical, raises several questions about the process of bargaining (how do the parties actually reach the suggested outcome?), and about the threat points and bargaining power. However, one can provide a game-theoretic description of the bargaining process, of the sort studied by Rubinstein (1982), that has a unique sub-

[31] The axioms include Pareto efficiency, symmetry, and two others that are somewhat more technical in nature. He shows that there is a unique outcome satisfying these four axioms, and it is the solution given in the text with $\theta = 1/2$. Relaxing the symmetry axiom implies that for any bargaining power parameter θ the solution satisfies the other axioms, which is the generalized Nash solution. See Osborne-Rubenstein (1990), e.g., for further discussion and proofs of the assertions in this subsection.

game perfect equilibrium that approximates the Nash solution. That is, as the time between counteroffers in the negotiations becomes small, the equilibrium outcome converges to that predicted by the Nash solution, for a particular choice of the threat points and bargaining power, depending on details of the underlying game. For example, suppose each agent has a given probability of making an offer (as opposing to responding to the other agent's offer) in each round of bargaining; then, other things being equal, his bargaining power equals that probability.

Of course, this only pushes the parameter θ one level back – where do these probabilities come from? From one perspective this indeterminacy may seem troubling. From another, it may be viewed as a virtue. In fact, the nature of bargaining may differ across industries, countries, or situations, and allowing the parameter θ to vary is one way to try and capture these differences. We can use this framework to address the implications of different assumptions about bargaining power on variables like the wage, unemployment, and so on.

To pursue things further it is instructive to consider the following case. Given value functions U and $W(w)$ for the worker and V and $J(\pi)$ for the firm, the generalized Nash solution solves

$$w = \arg\max\, [W(w) - U]^{\theta}[J(y - w) - V]^{1-\theta}, \tag{31}$$

where here the threat points are taken to be the values of being unmatched.[32] The solution to the maximization problem satisfies

$$\theta[J(y - w) - V]W'(w) = (1 - \theta)[W(w) - U]J'(y - w), \tag{32}$$

[32] This is a natural but the not the only possible way to specify the threat points. In terms of the underlying bargaining game, one can show the following. Suppose that if one player rejects the other's offer, during the period until the next offer is proposed, agents are allowed to continue searching, and if they meet someone else then they abandon the agent with whom they were previously negotiating. The equilibrium of this game is the Nash solution with threat points given by the value functions of search, as in the text. Alternatively, if the game is such that agents are not allowed to continue searching between offers, then the equilibrium is the Nash solution with threat points equal to 0. As there is no presumption that one game is necessarily a better description of actual bargaining than the other, the choice depends on context and convenience. A final issue is that one sometimes has to worry about outside options. For example, in the case where the threat points are 0 because agents cannot meet anyone else between offers, we still want to allow them to abandon (permanently) the agent with whom they are negotiating to go back on the market. This adds contraints to the problem of the form $W(w) \geq U$ and $J(\pi) \geq V$. These constraints will not bind in any applications studied here.

which in principle can be solved for w.

At this stage we need to be more explicit about how payoffs in the relationship depends on w. For instance, suppose there is some exogenous break up rate λ and all agents live forever. Then the value functions satisfy

$$\begin{aligned} rW(w) &= w + \lambda[U - W(w)] \\ rJ(\pi) &= \pi + \lambda[V - J(\pi)]. \end{aligned}$$

Hence, $W'(w) = J'(\pi) = \frac{1}{r+\lambda}$. Inserting these into (32), and rearranging, we have

$$W(w) = U + \theta[J(y-w) - V + W(w) - U].$$

This says that in terms of total lifetime expected utility (which is what W measures), the worker receives his threat point U plus a share of the *surplus*, $S = J(y-w) - V + W(w) - U$, where S measures the total utility available in the match over what the pair can earn by abandoning each other and continuing to search.

In other words, in this case, the Nash solution is to *split the surplus* according to θ. It is important to point out, however, that splitting the surplus is *not* generally the same as the Nash solution, which is formally given by (31), except under special circumstances (like linear utility) that happen to be satisfied here. In any case, note that if we write $W(w) - U = \frac{w-w_R}{r+\lambda}$ and $J(\pi) - V = \frac{\pi-\pi_R}{r+\lambda}$, we can rewrite (31) as

$$w = \arg\max\ [w - w_R]^\theta [y - w - \pi_R]^{1-\theta}. \tag{33}$$

The solution is

$$w = w_R + \theta(y - \pi_R - w_R).$$

Hence, one could say that the Nash solution also splits the surplus in terms of the current flow utility. More importantly, notice that $w \geq w_R$ if and only if $y \geq y_R = \pi_R + w_R$. Similarly, $\pi \geq \pi_R$ if and only if $y \geq y_R$. Hence, both the worker and the firm agree to consummate the relationship if and only if $y \geq y_R$, as claimed above.

An implication of this is that we can think of an equilibrium in two equivalent ways. First, we can find the two reservation values (w_R, π_R). For example, for the worker, standard techniques yield

$$w_R = b_w + \frac{\alpha_w}{r+\lambda}\int_{y_R}^{\infty}(w - w_R)dF(y) = b_w + \frac{\alpha_0}{r+\lambda}\int_{y_R}^{\infty}\theta(y - \pi_R - w_R)dF(y)$$

after inserting $w = w_R + \theta(y - \pi_R - w_R)$ and $\alpha_w = \alpha_0$. The latter condition, that α_w equals the exogenous contact rate, holds because every time the worker wants to match the firm also wants to match (i.e., $w \geq w_R$ if and only if $\pi \geq \pi_R$ if and only if $y \geq y_R$). Integrating by parts, we have

$$w_R = b_w + \frac{\alpha_0 \theta}{r + \lambda} \int_{y_R}^{\infty} [1 - F(y)] dy. \tag{34}$$

Symmetrically, for firms we have

$$\pi_R = b_e + \frac{\alpha_0 (1 - \theta)}{r + \lambda} \int_{y_R}^{\infty} [1 - F(y)] dy, \tag{35}$$

where b_e is their flow payoff to search.

A solution (w_R, π_R) to (34) and (35) is an equilibrium, from which we can compute wages, profits, unemployment, etc. This method is analogous to what we did in the nontransferable utility model. However, here we can alternatively add (34) and (35) to get one equation in y_R:

$$y_R = b_w + b_e + \frac{\alpha_0}{r + \lambda} \int_{y_R}^{\infty} [1 - F(z)] dz. \tag{36}$$

It is easy to see that (35) has a unique solution. Hence, there is a unique equilibrium value for y_R, and for the unemployment rate $u = \frac{\lambda}{\lambda + H}$, where $H = 1 - F(y_R)$ here. A key point (that will be very useful in the next section) is that in transferable utility models with Nash bargaining, for every match the worker wants to accept we have $y \geq y_R$, and so the firm necessarily agrees.[33]

4.3 The Meeting Technology

In all of the models considered thus far we have assumed that meeting or contact rates are exogenous (although this is not to say that offer rates are exogenous, as the nontransferable utility model illustrates). We now want to discuss a more general approach, in the guise of the *matching function.* Suppose at a given point in time there are v vacancies posted and u unemployed

[33] These results contrast sharply with the nontransferable utility model, where there can be multiple equilibria. It remains true, however, that under the same conditions as before, it is still an equilibrium for agents to use their steady state reservation strategies for any intitial unemployment rate. That is, along the transition path as u converges to u^* the arrival rates are constant and so y_R is constant.

workers searching. Then the matching function gives the number of contacts between firms and workers as $m = m(u, v)$ (we add search intensity below). This relationship is at least for now an exogenous technological specification, like a production function. Assuming all workers are the same and all firms are the same, the arrival rates for unemployed workers and employers with vacant jobs are given by

$$\alpha_w = \frac{m(v, u)}{u} \text{ and } \alpha_e = \frac{m(v, u)}{v}. \tag{37}$$

We assume that the function m is non-negative, increasing in both arguments, and concave. It is often assumed that $m \leq \min\{u, v\}$ (especially in discrete time models). It is also sometimes convenient to assume the meeting technology displays constant returns to scale, i.e., $\chi m(u, v) = m(\chi u, \chi v)$. A key implication of constant returns is that α_w and α_e are functions only of the ratio v/u, which is often referred to as a measure of "market tightness" in the literature. Constant returns implies that once we know α_w we know α_e, and vice-versa; this will be very useful below.

Given arbitrary but fixed numbers L_w of workers and L_e of employers, and given that each employment relationship involves one of each, the numbers of unemployed workers and vacant positions are related by the identity $L_w - u = L_e - v$. This means we can write $m = m(u, L_e - L_w + u)$. For example, if $L_w = L_e$, we can write the matching technology as $M = m(u, u) = M(u)$. Then the rate at which the representative agent contacts someone is $\alpha_0 = \alpha_0(u) = M(u)/u$. Constant returns to scale in $M(u)$ implies $\alpha_0' = 0$, increasing returns implies $\alpha_0' > 0$, and so on. Implicitly, we were assuming constant returns in the models analyzed previously when we wrote the contact rate α_0 as a constant, say, in condition (36). Indeed, with increasing returns there can be more than one solution to (36): intuitively, a higher w_R leads to a higher α_0 which yields a higher w_R. However, even with increasing returns, it is good to know that log-concavity implies a unique solution (Burdett-Wright 1998).

The general idea of a matching technology is meant to represent the fact that it is difficult for searching workers and firms to get together. There are many ways in which one could choose to model this more deeply.[34] As developed by Diamond (1981, 1982a,b), Mortensen (1982a,b), Pissarides (1985,1990), and others, the matching function approach allows us to be

[34] Examples include Montgomery (1991), Peters (1984,1991), Burdett-Shi-Wright (2000), and Lagos (2000).

somewhat agnostic about the actual mechanics of the search process and view $m(u,v)$ as an economic primitive, as we said, somewhat like a production function. Just as the standard production function maps inputs, say labor and capital, into output, the matching function maps search by workers and recruitment by firms into meetings. Of course, there are advantages and disadvantages to this approach.

On the one hand, assuming a meeting technology m is a flexible way to incorporate what seem like generic features of any reasonable search process, e.g., the fact that more search on either side of the market implies more meetings. How effective is extra search effort in generating more meetings, how important is search by workers or recruitment by firms, and so on, are questions that may best be viewed as empirical issues, and by not restricting m to correspond to any particular specification of the search process we are not limiting our ability to capture reality. On the other hand, assuming an m obviously makes the matching process somewhat of a black box. For instance, in some more complicated situations, such as heterogeneous workers, it is less clear what one should assume about the implications for meetings if we do not know the underlying mechanics. Nonetheless, the meeting technology has proven to be a simple and useful way to proceed, and we will use it extensively below.[35]

5 Job and Worker Flows

Traditionally, macroeconomists have focused much attention on the *level* of employment and unemployment. However, a growing empirical literature has documented the large *flows* of workers and jobs. The flow exhibit interesting patterns over the business cycle and across countries (see the survey by Davis-Haltiwanger [2000]). Many researchers believe that a promising route for understanding the labor market is to analyze these underlying flows in more detail. Search models, in general, are obviously well-designed to address the issue. In this section we pursue a class of models used to account for job and worker flows, emanating from the work of Pissarides (1985, 1990).

[35] We point out that there is an interesting alternative to the specification in the text, which is to assume the number of meetings depends on new entry of unmatched workers and firms, rather than the stocks of existing unemployed workers and vacancies; see Coles-Smith (1996,1998). See Petrongolo-Pissarides (2000) for an extensive summary of the empirical literature on matching functions more generally.

In contrast to the literature on wage distributions, with exogenous contact rates and wages determined in a posting game, the models reviewed here tend to assume contact rates are determined endogenously through a matching function and wages are determined by bargaining. What distinguishes the various models we present is that each stresses a different margin of adjustment. The margins that are most relevant of course depend on the issue one wants to address. The margins that we consider below are: entry (or, recruitment effort) by firms; search intensity by workers; the decision to consummate a match in a given meeting; the decision to terminate ongoing matches; and the choice of hours. Though one could write down a unified model with all of these features, we think it facilitates understanding to consider each in isolation, as we have done throughout this survey paper. Still, we emphasize the common features of the models.

5.1 The Basic Model

We begin with a benchmark model, corresponding to the one in Pissarides (1985, 1990) (see Howitt-McAfee [1987] and Howitt [1988] for some related work). The key feature is the ability of firms to create jobs, by posting vacancies and searching for workers. Let v denote the number of vacancies and k the cost of posting a vacancy per period (this is a flow cost, but one can add an initial fixed cost to creating a vacancy without affecting the main results). It does not matter for most of what we do here whether one thinks of a given number of firms, each deciding how many vacancies to post, or of each firm being allowed to post only one vacancy and the number of firms as the endogenous variable, since we will impose a free entry condition that drives the value of the marginal vacancy to 0. For most of what follows we adopt the former interpretation. Also, for now, all matches produce the same output y per period.

There is a number of homogeneous workers, fixed at $L_w = 1$. Unmatched workers search costlessly while matched workers are unable to search. The steady-state unemployment rate is given by $u = \frac{\lambda}{\lambda+\alpha_w}$, where now $\alpha_w = m(u,v)/u$ and m is the meeting technology as in the previous section. As discussed above, assuming constant returns, once we know α_w we also know α_e since both are functions of the ratio u/v. The value functions of an unfilled vacancy, a filled job, an unemployed worker, and an employed worker are V,

J, U, and W.[36] We assume transferable utility, and wages will be determined by the generalized Nash bargaining solution

$$W = U + \theta[W + J - U - V] = U + \theta S, \tag{38}$$

where the surplus S is given by

$$S = J + W - U - V. \tag{39}$$

The most interesting part of this model is the decision by firms to post vacancies. In discrete time, the value of posting a (single) vacancy is

$$V = \beta\alpha_e J - k,$$

since each vacancy v posted today costs k and generates a filled job next period with probability α_e. This implies the equilibrium free entry condition

$$\beta\alpha_e J \leq k, \quad = \text{ if } v > 0. \tag{40}$$

We will focus on outcomes with $v > 0$, and give conditions below to guarantee this is the case, so that (40) holds with equality. It is immediate that in equilibrium free entry drives the value of a vacancy to 0, and so we need not keep track of V in what follows.

The rest of the model is standard. The discrete time Bellman equation for J is

$$J = y - w + \beta(1 - \lambda)J, \tag{41}$$

while the Bellman equations for workers are

$$\begin{aligned} W &= w + \beta(1 - \lambda)W + \beta\lambda U & (42) \\ U &= b + \beta\alpha_w W + \beta(1 - \alpha_w)U. & (43) \end{aligned}$$

Formally, an equilibrium includes the value functions (J, W, U), the wage w, and the unemployment and vacancy rates (u, v) satisfying the Bellman equations, the bargaining solution, and the free entry and steady state conditions.

[36] Note that J represents the present discounted value for a firm from an existing job, and not from all jobs he may have currently filled, under the interpretation that a given firm can post as many vacancies as it likes.

The arrival rates α_w and α_e are implicitly part of the definition, too, but they are known functions of u/v.[37]

One approach to solving the model involves trying to find the steady state wage. Start with some arbitrary w_0, solve (41) for J, and then use this in (40) to solve for α_e and, by constant returns, also for α_w. This allows us to determine W and U. This value of w_0 is an equilibrium wage if the implied values for J, W, and U are such that the bargaining condition (38) holds. Put differently, substituting the values of J, W, and U in terms of w into (38) gives an equation that implicitly defines the equilibrium wage rate. While this works, we instead adopt a method that allows us to bypass w. Although in the base model the two methods are equally simple, in more complicated versions our method can be easier.

The idea is to work directly with the surplus S given in (39). The first step is to find an expression for S in terms of the primitives of the model and values that are determined outside of a given match (i.e., α_w and α_e). Direct substitution of J and W into (39) yields

$$S = y + \beta(1-\lambda)S - (1-\beta)U \tag{44}$$

Now use $(1-\beta)U = b + \beta\alpha_w\theta S$ (from the value function for U and the bargaining condition) to write

$$S = y - b + \beta(1-\lambda)S - \beta\alpha_w\theta S \tag{45}$$

Note that w does not enter this expression. Intuitively, in the context of a match, w is simply a transfer from one party to another and does not affect the total surplus.

The next step is to obtain expressions that characterize optimal choices for each of the decisions that get made outside the context of a match, given the value of S. In this particular model the only such decision concerns posting vacancies. Using the free entry condition and the fact that bargaining implies $J = (1-\theta)S$, we have

$$k = \beta\alpha_e(1-\theta)S. \tag{46}$$

[37] Although we focus on steady states, this is another example of a model (like the ones in the previous section) for which the dynamics are straightforward. Basically, the key observation is that the free entry condition pins down α_e and therefore α_w. So, given any initial unemployment rate, it is an equilibrium for vacancies to adjust so that u/v jumps to the steady state level, which implies all other variables are constant along the path as u and v converge to their steady state levels.

The two equations (45) and (46) completely characterize the equilibrium. This method is quite general: as we will see below, the set of conditions that characterize equilibrium will always take on the same form, with one equation that defines S in terms of primitives and values that are chosen outside of a match, and one equation for each endogenous decision variable that gets made outside the context of a match as a function of S.

Further description of equilibrium now amounts to manipulation of (45) and (46). In this case it is easy to combine them to get

$$(y-b)\beta\alpha_e(1-\theta) = k[1-\beta(1-\lambda)+\beta\alpha_w\theta]. \tag{47}$$

Recall that constant returns in the matching function implies that both α_e and α_w depend only on the ratio v/u. It follows that (47) is one equation in the one unknown, v/u. It is easy to see that this equation has at most one solution, since the left hand side is decreasing while the right hand side is increasing in v/u. Standard conditions guarantee that a positive solution exists.[38]

Hence, one can show that there exists a unique equilibrium value for v/u. >From this we know the arrival rates and the unemployment rate $u = \frac{\lambda}{\alpha_w+\lambda}$. It is now straightforward to determine w: simply solve for S from either (46) or (45) and rearrange (41) to yield

$$w = y - [1-\beta(1-\lambda)]\,(1-\theta)S. \tag{48}$$

A number of comparative statics results follow easily from (47). For instance, one can show that v/u and α_w are decreasing in k, θ, or λ , and increasing in $y-b$ or β. Also, w is increasing in $y-b$, β and θ and decreasing in k and λ.

It is of interest to compare this model to the basic job search problem with exogenous layoffs. Both models yield predictions about the equilibrium unemployment rates and worker flows, and in both the flow from employment to unemployment is exogenous. Hence, in both cases the models are effectively concerned only with the flow of workers from unemployment to employment. However, whereas the simple job search problem determined this flow entirely from the worker's perspective, this model determines it entirely from the

[38] One assumes two things: first, we have the standard Inada-type conditions, $\alpha_w \to 0$ and $\alpha_e \to 1$ as $v/u \to 0$, and $\alpha_w \to 1$ and $\alpha_e \to 0$ as $v/u \to \infty$; and second, $k < \frac{\beta(1-\theta)(y-b)}{1-\beta(1-\lambda)}$. The latter condition effectively states that if a firm could obtain a worker with probability one next period it would find it profitable to post a vacancy.

firm's perspective, in the sense that workers are completely passive. A further distinction is that this model endogenously determines the equilibrium wage (which happens to be constant across matches in this case). In each of the extensions considered below we will introduce elements which give rise to more interesting worker behavior.

5.2 Search Intensity

As we said, in the basic model the only interesting decision is that of firms to post vacancies – workers are passive in the sense that they search at some fixed intensity and accept any offer they get. The first extension we consider is to allow workers to choose search intensity $s \in [0,1]$. As in the decision-theoretic model of Section 2, we assume a disutility cost $g(s)$ of search per period, where $g(0) = 0$, $g' > 0$ and $g'' > 0$, and to ensure an interior solution we also assume $g'(0) = 0$ and $g'(s) \to \infty$ as $s \to 1$. However, given we are now in an equilibrium setting, we need to be more careful about the way intensity affects arrival rates.

To this end, write total search effort by unemployed workers as $u\bar{s}$, where $\bar{s}$ is average intensity.[39] Then the matching function will be $m = m(\bar{s}u, v)$, which as before is assumed to exhibit constant returns to scale in its two arguments. The probability that a given worker contacts a firm depends now not only on the total amount of search on each side of the market, but also on the level of his own effort relative to $\bar{s}$:

$$\alpha_w = \frac{m(\bar{s}u, v)}{u}\frac{s}{\bar{s}}.$$

The first term m/u is the average meeting rate for workers, while the second term scales this rate based on relative intensity.

A steady-state equilibrium now includes all the objects listed in the previous subsection, (J, W, U), w, and (u, v), plus the new variable s, satisfying all the conditions given earlier plus the new requirement that $s = \bar{s}$ solves

$$U = \max_s \left\{ b - g(s) + \beta \left[\alpha_w W + (1 - \alpha_w) \right] U \right\},$$

where we note that individuals take $\bar{s}$, u and v as given when choosing their value of s. Again, the arrival rates are implicitly part of the definition of

[39] We will focus here on symmetric equilibria where all unemployed workers search with the same intensity; given g is strictly convex, however, this is not restrictive.

an equilibrium, and can be recovered from the other variables. Note that while α_e is only a function of the ratio $v/\bar{s}u$, this is not true of α_w, although α_w/s does depend only on this ratio. In any case, as in the typical model with search intensity chosen on both sides of the market, there is always a degenerate equilibrium, since if one side of the market does not search then no one on the other side will search either. We focus on nondegenerate outcomes from now on.

Analyzing a nondegenerate steady state equilibrium proceeds very much as before, using the surplus $S = J + V - U$. We can again derive

$$\begin{aligned} S &= y - b + \beta(1-\lambda)S - [-g(\bar{s}) + \beta\alpha_w\theta S] \qquad (49) \\ k &= \beta\alpha_e(1-\theta)S, \qquad (50) \end{aligned}$$

but there is now an additional equation that gives the first order condition for search intensity, which when evaluated at $s = \bar{s}$ implies

$$g'(\bar{s}) = \beta\alpha_w\theta S/\bar{s}. \qquad (51)$$

The first equation defines S in terms of primitives and decisions taken outside of a match, in this case $\bar{s}$ and α_w. The second and third equations characterize optimal decisions in terms of primitives and S, those decisions being the number of vacancies to post and search effort by workers.

Establishing the existence of a unique steady state is straightforward. Combine (51) and (49) to obtain

$$S = y - b + \beta(1-\lambda)S - [-g(\bar{s}) + \bar{s}g'(\bar{s})]\,. \qquad (52)$$

Properties of the function g imply the term is square brackets is increasing in $\bar{s}$, and so (52) implies a negative relationship between $\bar{s}$ and S. We now use this to derive two relationships between $v/\bar{s}u$ and $\bar{s}$, one upward sloping and one downward sloping. First, if $\bar{s}$ and S are negatively related then equation (50) implies a negative relationship between $v/\bar{s}u$ and $\bar{s}$. Following Mortensen and Pissarides (1994) we call this the job creation, or JC, curve, since it gives the optimal level of $v/\bar{s}u$ given the search intensity of workers. Second, the negative relationship between $\bar{s}$ and S, when substituted into (51) implies a positive relationship between $v/\bar{s}u$ and $\bar{s}$. We call this the search intensity, or SI, curve, since it gives the optimal choice of search intensity taking the value of $v/\bar{s}u$ as given. See Figure 4. The intersection of JC and SI gives the unique nondegenerate equilibrium. Given our earlier assumptions it is straightforward to show that these two curves do in fact intersect. From $v/\bar{s}u$ we know $m(\bar{s}u, v)/\bar{s}u$, and hence α_w, u and all of the other endogenous variables.

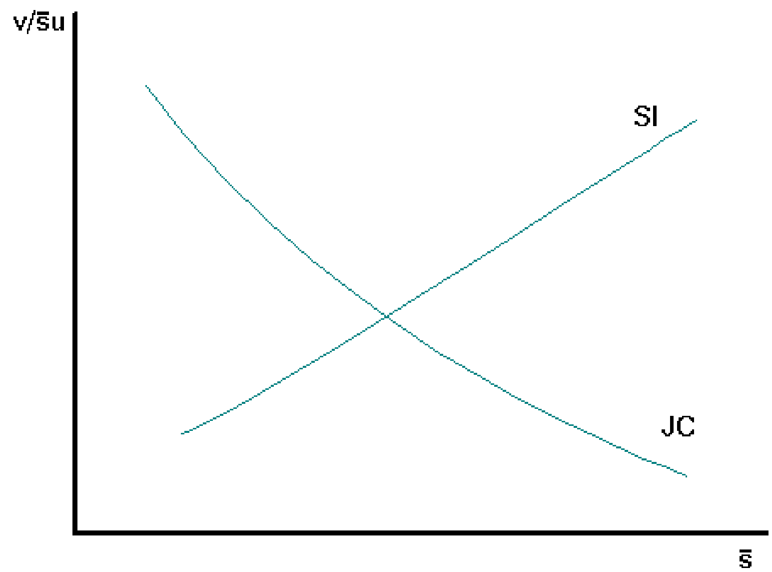

Figure 4: Job Creation and Search Intensity

5.3 Match-Specific Job Creation

In the above models there is unemployment in steady state because it takes time for workers to contact firms and vice-versa, but every contact leads to a match being formed, and the wage is the same in every job. This seems special, as compared to some of the models we have outlined earlier (e.g., it corresponds to agents sampling from degenerate distributions). For many applications, including policy analysis, changes in the set of contacts that lead to jobs being consummated may be of first-order importance. Indeed, the first policy example we discussed in Section 2 concerned an increase in b, which by increasing the reservation wage increased the length of the average unemployment spell and the expected wage. Here we extend the basic model to incorporate this margin. While there are many ways it could be done, we assume that when a worker and a firm meet they draw a match specific productivity y from the cdf F.[40]

The value functions for matched firms and workers are now written $J(y)$ and $W(y)$ respectively, and the equilibrium wage $w(y)$, since all of these may depend on idiosyncratic productivity. In principle, we have to determine the conditions when both firms and workers want to consummate a match. However, as should be clear from the previous section, given the bargaining solution, workers and firms will always agree about whether to form an employment relationship: they consummate a match if and only if $y \geq y_R$, where y_R is to be determined below. The argument is exactly the same as the discussion of (36), even though this model may appear slightly different due to the more general matching technology and free entry condition (basically, these additional complications do not affect, or are taken as given in, the bilateral decision of a pair deciding on an individual match). For simplicity, we return to the case where worker search effort is fixed.

An equilibrium is now described by all the same objects as above, although now J, W, etc. are indexed by y, plus there is a new variable, the reservation productivity level y_R. The equilibrium will also determine the distribution of productivity across existing relationships, or equivalently, given the wage function $w(y)$, the wage distribution $G(w)$. Only a minor modification of previous methods is required. We now have

$$
\begin{aligned}
J(y) &= \max\{y - w(y) + \beta(1-\lambda)J(y), 0\} \\
W(y) &= \max\{w(y) + \beta[(1-\lambda)W(y) + \lambda U], U\}.
\end{aligned}
$$

The fact that matches are consummated if and only if $y \geq y_R$ means that we can add these equations inside the max operator to define the surplus function for all acceptable matches,

$$S(y) = J(y) + W(y) - U.$$

Straightforward manipulation gives

$$S(y) = \max\{y - b + \beta(1-\lambda)S(y) - \beta\alpha_w\theta ES(y), 0\}. \tag{53}$$

The free entry condition is

$$k = \beta\alpha_e(1-\theta)ES(y), \tag{54}$$

[40] Productivity is assumed to be observed by both parties at the time of meeting, before the match is consummated, and is constant over the duration of the match.

where the expectation is with respect to F. As before, (53) defines $S(y)$ in terms of primitives and values that are determined outside the match, and (54) characterizes the optimal choice of vacancies in terms of primitives and S. However, there is a slight difference between this case and the two cases considered earlier. Now the first equation not only defines $S(y)$ but also implicitly defines y_R. The reason is that the decision to form a match is a decision that gets made after contact between the worker and the firm has occurred. So, while this is a model in which there are two decisions, one of them is taken outside and one is taken inside the context of a match. In cases such as these, those decisions that get taken inside the match have their conditions imbedded in the implicit definition of the surplus function $S(y)$.

For a given value of α_w, (53) determines y_R. Furthermore, for $y \geq y_R$ the function $S(y)$ is linear with slope equal to $\frac{1}{1-\beta(1-\lambda)}$. Given this, there are different ways to characterize equilibrium. Consistent with what we did above, one can show the relevant equations imply two relationships between v/u and y_R, one upward sloping and one downward sloping. To obtain the first relationship, note that y_R is increasing in α_w. We call this the match formation, or MF equation, since it gives the optimal match formation decision taking as given the ratio v/u. To obtain the second, use (53) to solve for $ES(y)$ and note that it is a decreasing value of α_w. Since each value of α_w is associated with a value of y_R, it follows that there is implicitly a negative relationship as well between $ES(y)$ and y_R. Substituting this into equation (54) yields a negative relationship between y_R and v/u. This is the JC curve, as above. The intersection of the two curves gives the unique equilibrium. As before, in order to establish that there is an intersection of these two curves we need to assure that expected match productivity is sufficiently high to make it worthwhile for a firm to post a vacancy when there is probability 1 of meeting a worker.

One can also recover the equilibrium wage function $w(y)$ as before by using the value function for $J(y)$ and noting that $J(y) = (1-\theta)S(y)$ and that $S(y_R) = 0$. It follows that wages are equal to some value w_R if $y = y_R$ and increase linearly in y with slope $\frac{\theta}{1-\beta(1-\lambda)}$. The distribution of productivity here is simply $F(y)$ truncated at y_R, and the equilibrium wage distribution G is then that implied by the productivity distribution and $w(y)$.[41] The model

[41] Obviously, $G(w)$ is generally nondegenerate. This is an example of what we had in mind earlier when we said that in bargaining (as opposed to wage-posting) models, wage dispersion emerges directly from heterogeneity.

makes predictions about arrival rates, reservation productivity, wages, and so on. Comparative statics results follow easily. For example, as in the previous case, increasing k, λ, θ, or b will lead to a decrease in v/u while an increase in β will lead to an increase in v/u. Also, increasing k or decreasing b lowers y_R.

5.4 Job Destruction

So far in this section job creation is endogenous but job destruction is exogenous. That is, we have simply assumed that with probability λ matches terminate for reasons outside the model. For many applications of interest, changes in the rate at which matches break up are potentially important. We know from the discussion throughout this survey that there are several ways to endogenize breakups, including on-the-job search and learning. Here we consider the Mortensen-Pissarides (1994) model, which incorporates on-the-job productivity changes, as we did for the single-agent model in the discussion surrounding (19). This extension is particularly significant because it yields an equilibrium model in which both the flows into and out of unemployment are endogenous. Given that these flows vary a lot across countries, the model allows one to begin thinking formally about the various factors that may account for these differences.

Let y be the current output produced by a worker-firm pair, and assume this evolves stochastically according to $F(y' \mid y) = prob(y_{t+1} \leq y' \mid y_t = y)$, where $y \in [0, \bar{y}]$ for some $\bar{y}$. Realizations of y are iid across matches. We assume $F(y' \mid y_2)$ first order stochastically dominates $F(y' \mid y_1)$ whenever $y_2 > y_1$; as in the single-agent model, this implies the expected value of having a job will be increasing in current productivity. Agents observe current productivity before they decide whether to continue or terminate a match, and if the match is terminated the worker is allowed to search during the period. It remains to specify the level of productivity in new matches. Rather than having new matches draw y at random, here we assume that all new matches begin with the same productivity level, y_0. Hence, job creation is kept simple here, since we have studied it before, and we can focus on the job destruction margin.[42]

[42] If a firm has a match with productivity y and it terminates, we assume the next worker it matches with also starts at y_0 and not the previous y. That is, the value y is entirely match specific and when the match breaks up this value is lost.

An equilibrium is defined as the natural extension of the previous models. In particular, the Bellman equations for matched agents are

$$\begin{aligned} J(y) &= \max\left\{Ey' - w(y) + \beta E J_e(y'), 0\right\} \\ W(y) &= \max\left\{w(y) + \beta E W(y'), U\right\}, \end{aligned}$$

where the expectations are conditional on the current y. As in the previous models with transferable utility, in which workers and firms always agreed on whether to consummate a match, exactly the same logic here implies they always agree on whether to end a match. Hence, the analysis of equilibrium follows the same steps as before.

Specifically, we have the following two equations

$$S(y) = \max\left\{y - b + \beta E S(y') - \beta\alpha_w\theta S(y_0), 0\right\} \tag{55}$$

$$k = \beta\alpha_e(1-\theta)S(y_0). \tag{56}$$

Solving for equilibrium amounts to solving these two equations for the reservation productivity y_R and the value of v/u. We will show that these two equations can be used to produce two relationships between these two variables, one of which is increasing and the other of which is decreasing. To get the first relationship, note that (55) implies y_R is increasing in α_w and hence in v/u. We call this relationship the job destruction curve since it gives the optimal decision for destroying a match given the value of v/u.

The second relationship is slightly more complicated. First, (55) implies $S(y)$ is decreasing in α_w, and so in particular letting $y = y_0$, $S(y_0)$ is a decreasing function of α_w. This induces a relationship between $S(y_0)$ and y_R, since each value of α_w implies values for these two objects, and this relationship between $S(y_0)$ and y_R is negative. Substituting this relationship into (56), one obtains a negative relationship between y_R and v/u. This is again the job creation curve seen in Figure **??**. The intersection of the job creation and job destruction curves gives the unique steady state equilibrium. As always, a sufficient condition that these two curves do indeed intersect is that the expected return to a firm is positive if they are assured of meeting a worker with probability one.

5.5 Hours

All of the models analyzed so far assume that hours or work effort is fixed – i.e., either the worker works or he does not. We now make two changes to the environment of the basic model. First, a worker derives flow utility $w - g(h)$ from a job paying income w when he works h hours, where $0 \leq h \leq 1$. Assume $g'(h) > 0$, $g''(h) > 0$, $g(0) = 0$, $g'(0) = 0$ and $g'(1) = \infty$. Second, output from a match is now $f(h)$, where $f'(h) > 0$, $f''(h) < 0$, $f(0) = 0$, and $f'(0) > 0$. For ease of presentation, we assume search intensity and productivity (both in and across jobs) are fixed here. An equilibrium is defined as the natural extension of what came before except h is now endogenous, which means we need one more equilibrium condition. There are two ways to introduce the new condition (or at least two ways to interpret it), and we discuss them in turn.

First, we could consider a formulation in which hours are jointly determined with wages in the bargaining process – i.e., in the Nash bargaining problem the maximization is with respect to (w, h). Then an argument very similar to the one behind (33) leads to

$$(w, h) = \arg\max\,[w - g(h) - w_R]^{\theta}[f(h) - w - \pi_R]^{1-\theta},$$

where w_R and π_R are the reservation wage and profit flows. It is immediate from the first order conditions from this problem that $g'(h) = f'(h)$. Hence, the marginal disutility of labor equals the marginal product. Given the value of h that satisfies this condition, the analysis proceeds as usual.

The other formulation is to assume that the wage payment is a function of the hours worked, $w(h)$, that both firms and workers take as given when they decide on h. Of course, in equilibrium these decisions must be consistent. To pursue this approach, write the Bellman equations for matched agents as

$$\begin{aligned} J &= \max_h \{f(h) - w(h) + \beta(1-\lambda)J\} \\ W(h) &= \max_h \{w(h) - g(h) + \beta[(1-\lambda)W + \lambda U]\}. \end{aligned}$$

Given that in equilibrium workers and firms choose the same h, we have

$$S = \max_h \{f(h) - g(h) + \beta(1-\lambda)S - \beta\alpha_w\theta S\}.$$

This condition also yields $g'(h) = f'(h)$.

This expression for the surplus together with the usual free entry condition have the same general structure as the equilibrium conditions in each of the above models. Again, once we know h from $g'(h) = f'(h)$, things reduce to the benchmark model with fixed hours. Of course, a change in any exogenous variable, such as b, now affects hours worked and indirectly this affects all of the other variables in equilibrium.

5.6 Extensions and Applications

To the basic model described above one can add many other things. Pissarides (1994) and Mortensen (1996) allow on-the-job search, and Pries (1998) and Pries-Rogerson (2000) add learning. Also of interest is a literature which incorporates the main features of the above models into versions of the stochastic growth model. Early examples include Andolfatto (1996) and Merz (1995), who do it for the basic Pissarides (1985) model. More recently several authors have done this for some of the extensions described above, including Merz (1999) and den Haan-Ramey-Watson (2000), who effectively do it for the Mortensen-Pissarides model. One issue that has received considerable attention in business cycle versions of these models is "cleansing" – the notion that in bad economic times it is low productivity matches that are destroyed. This is an implication of the Mortensen-Pissarides model. Caballero-Hammour (1994,1996) discuss this more extensively. More recently, Barlvey (1999a,b) argues that recessions are "sullying" rather than "cleansing" because they inhibit workers' ability to move up the job ladder.

There is also a literature that relaxes the assumption that workers and firms were identical *ex ante*. Acemoglu (1999, 2000) and Albrecht-Vroman (1999) consider workers that differ in skill and show how this can affect the types of jobs that firms create and the wage and unemployment levels for different skill groups. Mortensen-Pissarides (1999c) examine how various policies impact differently across skill groups. Shimer (1999) considers workers that are heterogeneous and explicitly allows for the possibility that multiple workers show up simultaneously to apply for a given vacancy, and studies how this can affect the unemployment dynamics of low skill workers. Another extension in the literature follows Lucas-Prescott (1974) in not relying on a reduced form matching function, and assumes wages are determined competitively.. Recent examples include Greenwood-MacDonald-Zhang (1996), Gomes-Greenwood-Rebelo (1999) and Alvarez-Veracierto (1999).

We alluded earlier to the literature that uses these models to understand different behavior of worker and job flows across countries and over the business cycle. Millard-Mortensen (1997) show how different policy regimes in the US and UK can account for their very different labor market outcomes. Delacroix (1999a,b) and Blanchard-Portugal (1999) study the effect of various employment protection policies in the model. Cole-Rogerson (1999) discuss how well the basic Mortensen-Pissarides model can account for the behavior of job creation and destruction over the business cycle. Shi-Wen

(1997) use a version of the model to understand the implications for optimal taxation. While we cannot do justice to all the work in the area, it should suffice to say that this represents an extremely active and productive area of current research into the dynamics of labor markets.

6 Efficiency

We now turn our attention to characterizing efficient allocations, and understanding the relationship between efficient allocations and decentralized outcomes, in equilibrium search models. The first thing to mention is that, given an economy with many agents, there are typically many allocations that satisfy Pareto efficiency. We focus on those that maximize the sum of equal weighted utility, or, equivalently, the present discounted value of output net of the disutility of working and cost posting vacancies. We will discuss mainly the models in the previous section. For this class of models a fair amount is known about efficient allocations and their relationship to decentralized equilibria, and a fairly simple yet general result summarizes this relationship, as was first demonstrated by Hosios (1990). In order to illustrate this result in as simple a context as possible we will begin with a one-shot version of the Pissarides model that served as our benchmark model, and then proceed to dynamic models.

6.1 One-Shot Models

Consider a model that looks just like the Pissarides model except that everything happens in one period. Specifically, assume that all workers are initially unmatched, so $u = 1$. Entrepreneurs decide how many vacancies v to post, each at cost k, and then matches form according to $m(1, v)$. Each match produces output y, but at a utility (opportunity) cost of b for each worker. Then the economy ends. In the decentralized version of this economy, wages are again determined by Nash bargaining, which implies $w = b + \theta(y - b)$, where θ is the worker's bargaining power. It is easy to see that in equilibrium vacancies are posted until

$$k = \alpha_e(1 - \theta)(y - b). \tag{57}$$

Now consider what a social planner would do in this economy. His optimization problem can be written

$$\max_v \{-kv + m(1,v)(y-b)\},$$

which yields the first order condition

$$k = m_v(1,v)(y-b). \tag{58}$$

Denote the social planner's optimal choice of vacancies by v^*. Comparing (57) and (58), we see that the two solutions for v coincide if and only if $\theta = \theta^*$ where

$$\theta^* = 1 - \frac{v^* m_v(1,v^*)}{m(1,v^*)}, \tag{59}$$

sometimes referred to as the Hosios (1990) condition. If $\theta > \theta^*$ ($\theta < \theta^*$) then the decentralized equilibrium will entail $v < v^*$ ($v > v^*$). This condition has a simple interpretation: the share of the surplus that accrues to the firm should equal the elasticity of the matching function with respect to vacancies. If m displays constant returns to scale then (59) is equivalent to

$$\theta^* = \frac{m_u(1,v^*)}{m(1,v^*)},$$

which says the share of the surplus going to workers should equal the elasticity of the matching function with respect to unemployment.

>From one perspective, the fact that there is some value of θ for which the decentralized equilibrium is efficient is not surprising. After all, for an allocation to be efficient, we simply need the right number of vacancies posted, and as θ varies so does the v. By choosing θ appropriately it is possible to get $v = v^*$. Another perspective on this comes from considering wage determination. Different choices of θ imply different wage rates, and we know that the number of vacancies is decreasing in w. So the result says that there is a value of the wage which generates $v = v^*$. Of course, the fact that the value of θ that achieves the efficient outcome has the interpretation offered above may be surprising, even if the existence of such a θ is not.

To illustrate that the result is actually more substantive than saying it is possible to target one variable with one free parameter, consider the same model except that the search intensity for workers is endogenous. Following standard arguments, the equilibrium values of v and s satisfy

$$k = \alpha_e(1-\theta)(y-b) \text{ and } g'(s) = \frac{\alpha_w}{s}\theta(y-b). \tag{60}$$

The social planner's problem is

$$\max_{v,s}\{-kv - g(s) + m(s,v)(y-b)\},$$

which yields the first order conditions

$$k = m_v(s,v)(y-b) \text{ and } g'(s) = m_s(s,v)(y-b). \tag{61}$$

Let v^* and s^* denote the solution to the planner's problem. Comparing (60) and (61), it is apparent that the solutions are equal if and only if

$$m(s^*,v^*)(1-\theta) = v^* m_v(s^*,v^*) \text{ and } m(s^*,v^*)\theta = s^* m_s(s^*,v^*). \tag{62}$$

As pointed out above, under constant returns, the two conditions in (62) both hold if $\theta = \theta^*$, as defined in (59), although the share of the surplus that now goes to workers must equal the elasticity of the matching function with respect to worker search intensity rather than simply unemployment. The result no longer seems so unsurprising: in this model there are two variables to be determined, s and v, but there is only one free parameter, θ. Moreover, both of these decisions take place outside the context of a match. It is instructive to consider things from the perspective of wages again. Varying the value of θ amounts to varying the wage in equilibrium. Since s and v both are endogenous, it is not clear a priori whether one can target both using only the wage as an instrument. It turns out that one can.

We could consider each of the extensions discussed earlier in this one-shot model, and we would find the same result in each case. Namely, the equilibrium allocation is efficient if and only if the share of the surplus going to workers (firms) is equal to the elasticity of the matching function with respect to worker search effort (firm search effort). Instead, we next turn our attention to the dynamic models analyzed earlier. Although the mechanics are slightly more complicated in these models than in one-shot models, it will turn out that we obtain exactly the same Hosios condition. Nonetheless, it is instructive to see how the result can be established in the dynamic settings, especially since the methods we use are different than those used by Hosios.

6.2 A Benchmark Dynamic Model

We begin with the benchmark Pissarides model. Note that here we take the dynamics into account, and formulate the social planner's problem recursively. The state variable is the measure of matched workers (the employment rate), e. The value function for the planner, $Y(e)$, satisfies the Bellman equation

$$Y(e) = \max_v \{(y-b)e - kv + \beta Y(e')\} \tag{63}$$

where $e' = (1-\lambda)e + m(1-e, v)$. One can easily show that $Y(u)$ is affine; i.e. $Y(u) = a_0 + a_1 e$ for some constants a_0 and a_1.[43] These constants have useful interpretations: a_0 is the value to the social planner of an unemployed worker, and a_1 is the added value having a worker matched rather than unemployed, or the *surplus* from matching a worker and firm.

The first order condition for the maximization problem in (63) is

$$k = \beta m_v(1-e, v) Y'(e'). \tag{64}$$

Since $Y(u) = a_0 + a_1 e$, the derivative Y' is constant, and it follows that the optimal choice of v has the property that $m_v(1-e, v)$ is also a constant, independent of e. Given $m(u, v)$ satisfies constant returns, it follows that the optimal policy has the property that the value of $v/(1-e)$ is constant. To determine this value, first differentiate (63) to obtain

$$Y' = -\frac{y-b}{1-\beta(1-\lambda)+\beta m_u(u,v)}. \tag{65}$$

Then substitute (65) into (64) to arrive at

$$k\left[1-\beta(1-\lambda)+\beta m_u(u,v)\right] = (y-b)\beta m_v(u,v) \tag{66}$$

This is one equation in the one unknown, v/u, and completely characterizes the planner's solution.

How does this compare with the equilibrium? Recall that the equilibrium outcome was parameterized by bargaining power θ: different values of θ lead to different steady state equilibrium values of v/u. The natural question to ask is whether there is some value of θ for which the equilibrium and social planner's problem yield the same outcome. Comparing equations (66) and (47), we see that the two solutions coincide if and only if

$$m_u(u,v) = \theta\alpha_w \text{ and } m_v(u,v) = (1-\theta)\alpha_e. \tag{67}$$

Again, with constant returns in m, the two conditions in (67) both hold if and only if $\theta = \theta^*$ where θ^* satisfies

$$\theta^* = m_u(1, \frac{v^*}{u})/m(1, \frac{v^*}{u}). \tag{68}$$

[43]The mapping defined by (63) is a contraction and takes affine functions into affine functions. Since the set of affine functions is closed, the result follows.

This of course is the same as the Hosios condition (59) from in the one-shot model.[44]

6.3 A General Model

We next consider the model with random match-specific productivity y, and show that exactly the same results obtain, although the derivation is more complex. To make the analysis less cumbersome, consider the case in which $y \in \{y_1, y_2 ... y_N\}$. The state variable for the planner is now distribution of existing matches, say $\mu = (\mu_1, \mu_2, ...\mu_N)$ where μ_i is the fraction of workers in matches with idiosyncratic productivity y_i. One value of y_i is also the productivity of new matches, y_0, but we do not need to specify which one; we simply let μ_0 be a vector which has a 1 in the argument corresponding to y_0 and zeroes elsewhere. Note that unemployment is $u = 1 - \sum_i X_i \mu_i$, where we now use X_i to denote the decision rule of the social planner with regard to match termination: $X_i = 1$ implies that matches of type i terminate and $X_i = 0$ implies they do not. Let $X = (X_1, X_2 ... X_N)$. Also, $X \cdot \mu$ will correspond to a vector whose i^{th} argument is equal to $X_i \mu_i$.

The Bellman equation for the planner's problem is now

$$Y(\mu) = \max_{X,v} \left\{ \sum_i (1 - X_i)(y_i - b)\mu_i - kv + \beta Y(\mu') \right\}, \tag{69}$$

where $\mu' = (1 - X) \cdot \mu + m(u, v)\mu_0$ is the distribution of match productivity next period. While (69) may seem forbidding at first glance, a few observations make it much more manageable. First, similar to what we did before, one can show that Y is affine: $Y(\mu) = a_0 + \sum_i a_i \mu_i$, and moreover, if $X_j = 1$ then $a_j = 0$. The constants have the same interpretation here as in the earlier case: a_0 is the value to the planner of an unemployed worker, and a_i is the added value of putting a worker in a match with productivity y_i. Also, as in the previous section, the fact that Y is affine implies that v/u will again be a constant.

This implies that the decision to continue or terminate a match for a given realization of y is independent of the distribution of existing matches, μ, since

[44] Although we focused on steady states in previous sections, the dynamics are simple in this model, and imply that v/u is constant along the entire equilibrium path (not just in steady state). Hence, we have really shown that (59) implies the entire equilibrium allocation is efficient, and not just the steady state allocation.

the alternative to continuing the match is one more unemployed worker, the value of which is a_0 and hence independent of the state. It follows that it is sufficient to consider the following two functional equations to characterize the social planner's problem

$$\begin{aligned} Y_1(y) &= \max\{y + \beta E Y_1(y'), Y_0\} && (70) \\ Y_0 &= \max_{v/u}\{b - k(v/u) + m(1, v/u) Y_1(y_0) + [1 - m(1, v/u)] Y_0\} && (71) \end{aligned}$$

The first equation is the value of a match with productivity y, given that the value of an unemployed worker is Y_0. The second is the value to the planner of an unemployed worker, given $Y_1(y)$. Note that in terms of the earlier analysis, $Y_1(y_i) = a_0 + a_i$, $i = 1, 2...n$, and $Y_0 = a_0$.

Define $S_m(y) = Y_1(y) - Y_0$, the surplus associated with a match. Then it is straightforward to derive[45]

$$\begin{aligned} k &= \beta m_v S_m(y_0) && (72) \\ S_m(y) &= \max\{y - b + \beta E S_m(y') - m_u(1, (v/u)^*) \beta S_m(y_0), 0\}. && (73) \end{aligned}$$

Now compare (72) and (73) with (53) and (54) from the equilibrium analysis, which we repeat here for convenience:

$$\begin{aligned} k &= \beta \alpha_e (1 - \theta) S(y_0) \\ S(y) &= \max\{y - b + \beta E S(y') - \beta \alpha_w \theta S(y_0), 0\}. \end{aligned}$$

[45] First, (70) implies

$$S_m(y) = \max\{y - b + \beta E S_m(y') - (1 - \beta) Y_0, 0\}$$

Consider (71) and let v^* be the optimal solution. Then

$$Y_0 = -k(v/u)^* + \beta\{m(1, (v/u)^*) Y_1(y_0) + [1 - m(1, (v/u)^*)] Y_0\},$$

which implies

$$(1 - \beta) Y_0 = -k(v/u)^* + \beta m(1, (v/u)^*) S_m(y_0).$$

The first order condition from (71) is (72). Substitute this into the expression for Y_0 to obtain

$$(1 - \beta) Y_0 = \beta[m(1, (v/u)^*) - m_v(1, (v/u)^*)(v/u)^*] S_m(y_0).$$

Constant returns implies that $m(1, (v/u)^*) - m_v(1, (v/u)^*)(v/u)^* = m_u(1, (v/u)^*)$. Finally, substitute back into the expression for $S_m(y)$ to obtain (73).

Once again, these two sets of conditions are equivalent if and only if $m_v(1, v^*) = (1-\theta)\alpha_e$. It follows that the planner's solution is identical to the equilibrium if and only if θ satisfies the Hosios condition.

6.4 Discussion

We have demonstrated that for a large class of models there is indeed a simple connection between efficient allocations and decentralized allocations, as described by the Hosios condition. However, it would be misleading to suggest that such a condition holds in "all" search and matching models. There are many well known extensions for which not only does the Hosios condition fail, but for which there is no value of the bargaining parameter θ such that the decentralized allocation is efficient. Indeed, a general understanding of the nature of efficient allocations in two-sided search models is still being developed.[46] Here we briefly review some of the issues and models.

An interesting case, studied in slightly different contexts by Coles-Smith (1993), Davis (1995) and Masters (1998) is the following. Suppose workers must decide on how much human capital to acquire, prior to searching for a job. Then, one can show that generically there is no θ that yields efficient allocations. Basically, the value of θ that provides the right incentive for investment in human capital is different from the value of that produces the right incentive for firms (whether firms are choosing the number of vacancies to post, the types of job to create, or investment in physical capital). In general, the inefficiencies can result when *ex post* bargaining does not provide the right incentives for *ex ante* investments – referred to as the holdup problem.[47]

Another case where there is no θ that yields the efficient outcome, analyzed by Smith (1999), is when firms have concave production functions and

[46] For example, recent work by Shimer-Smith (2000b) shows that in some models of heterogeneous agents the efficient outcome may not even be a steady state, but may cycle. Although this may sound surprising, the intuition is simple: when the pool of unmatched agents contains mostly high productivity types, e.g., it is efficient for agents to be very demanding and hold out for a high productivity partner; as the pool becomes depleted, however, they should lower their standards. By lowering their standards they reduce the fraction of low productivity types in the pool, and we are back to where we started.

[47] This is particularly problematic in search models, since the investments may have to be made before the worker and firm meet, which means they cannot avoid the problem by signing appropriate contracts. However, Acemoglu and Shimer (1998) show that in some search models the holdup problem can be avoided if we assume so-called directed search.

bargain with each worker individually. Smith (1999) shows that firms are inefficiently large, and there is no value of the sharing parameter that corrects this. And, when we do not have constant returns in the meeting technology, we cannot choose any θ to generate efficiency in the simplest model: e.g., it is apparent that (62) cannot hold if we do not have constant returns. Of course, it has been understood since Diamond (1981,1982a,b) that increasing returns to matching leads to inefficiencies. Basically, the idea is that when an agent decides to enter (or to increase search intensity) he takes into account his own cost and benefit but not that of others. A similar issue comes up in some on-the-job search models, such as Imai-Burdett-Wright (1999). In this context individuals take into account the costs and benefits of search that accrue to them and ignore the cost this imposes on their current partners, implying that there will be excessive on-the-job search.

One can also ask about the efficiency of the wage-posting equilibria in Section 3. In a general sense, wage-posting where firms must commit to pay the same wage no matter what are unlikely to yield efficient outcomes, since firms cannot adjust to circumstances. Generally this may lead to inefficient separations and may preclude matches being consummated even though the surplus is positive. For instance, in Albrecht-Axel, a planner would want all meetings to result in matches being consummated, but this does not happen in the equilibrium with wage dispersion. The basic Burdett-Mortensen model, on the other hand is a special case in which efficiency does result. In their model all meetings involving unemployed workers do result in matches being formed. All other meetings are irrelevant from the social planner's perspective, since productivity is the same in all matches. Note that an important feature of the Burdett-Mortensen model is that arrival rates are assumed to be constant – there is no issue of workers contacting firms at the efficient rate.[48]

[48] There is another approach that we do not really have time to go into– what Moen (1997) calls a *competitive search equilibrium.* The idea is that there are different markets in which firms can post vacancies, which each market corresponding to a different v/u ratio. He shows that the resulting equilibrium is efficient. See also Shimer (1995) and Mortensen-Wright (2000).

7 Conclusions

We have provided a review of various search-based models of labor markets. In contrast to standard textbook "supply and demand" models, search theory emphasizes the frictions inherent in the exchange process. As should be clear by now, there is no single or canonical specification of a search model. Specifications are distinguished along many dimensions, including such things as how wages are determined, how contact rates are determined, whether there is entry of agents, etc. The specification that is best suited for a particular application depends on the issue. At the same time, this survey should make also clear that there is indeed a common framework underlying all of these specifications, with a common set of tools and methods that are useful across a wide variety of specifications.

Moreover, despite the variety of models than one can consider, we believe there are two basic phenomena that lie at the heart of many important labor market issues and arise quite naturally in models that allow for trading frictions: wage dispersion and unemployment. These phenomena do not arise naturally out of textbook "supply and demand" models. Our focus has been to overview some of the many ways in which these phenomena can be studied using search theory. There is much more that can be and has been done with search models, and we have only scratched the surface. We hope the survey has at least conveyed some of the basic ideas from this interesting and useful approach.

References

[1] **Acemoglu, Daron,** "Changes in Unemployment and Wage Inequality: An Alternative Theory and Some Evidence," *American Economic Review,* December 1999, 89, 1259-78.

[2] **Acemoglu, Daron,** "Good Jobs and Bad Jobs," 2000, forthcoming, Journal of Labor Economics.

[3] **Acemoglu, Daron and Robert Shimer,** "Holdups and Efficiency with Search Frictions," *International Economic Review*, 1999, **40,** 827-850.

[4] **Albrecht, James and Bo Axell,** "An Equilibrium Model of Search Unemployment," *Journal of Political Economy,* 1984, 92, 824-40.

[5] **Albrecht, James and Susan Vroman,** "A Matching Model with Endogenous Skill Requirements," mimeo, Georgetown University, 2000a.

[6] **Albrecht, James and Susan Vroman,** "Wage Dispersion in a Job Search Model with Time-Varying Unemployment Benefits," mimeo, Georgetown University, 2000b.

[7] **Alvarez, Fernando and Marcelo Veracierto,** "Labor Market Policies in an Equilibrium Search Model," *NBER Macroeconomics Annual*, 1998, 265-304.

[8] **Andolfatto, David,** "Business Cycles and Labor Market Search," *American Economic Review,* 1996. 86, 112-32.

[9] **Barlvey, Gadi,** "The Sullying Effects of Recessions," mimeo, Northwestern University, 1999a.

[10] **Barlvey, Gadi,** "Is There a Cleansing Effect in Recessions? Evidence From Wage Data," mimeo, Northwestern University, 1999b.

[11] **Blanchard, Olivier J. and Peter A. Diamond,** "The Beveridge Curve," *Brookings Papers on Economic Activity*, 1989, 1, 1-60.

[12] **Blanchard, Olivier and Pedro Portugal,** "What Hides Behind an Unemployment Rate: Comparing Portuguese and U.S. Labor Markets," mimeo, MIT, 2000.

[13] **Bontemps, Christian, Jean-Marc Robin, and Gerard J. van den Berg,** "An Empirical Equilibrium Job Search Model with Search on the Job and Heterogeneous Workers and Firms," *International Economic Review,* 2000, 40, 1039-74.

[14] **Browning, Martin, Thomas Crossley and Eric Smith,**"Asset Accumulation and Short Term Employment," mimeo, University of Essex, 1999.

[15] **Burdett, Kenneth,** "A Theory of Employee Job Search and Quit Rates," *American Economic Review,* March 1978, 68, 212-20.

[16] **Burdett, Kenneth,** "Search, Leisure, and Individual Labor Supply," in *Studies in the Economics of Search,* S. A. Lippman and J. J. McCall, eds. Amsterdam: North-Holland, 1979.

[17] **Burdett, Kenneth,** "A Useful Restriction on the Offer Distribution in Job Search Models," in *Proceedings of a Symposium at the Industrial Institute for Economic and Social Research.* Stockholm, 1981.

[18] **Burdett, Kenneth and Melvyn G. Coles,** "Marriage and Class," *Quarterly Journal of Economics*, 1997, 112, 141-68.

[19] **Burdett, Kenneth and Melvyn G. Coles,** "Long-Term Partnership Formation: Marriage and Employment," *Economic Journal,* 1999, 109, 307-34.

[20] **Burdett, Kenneth, Ryoichi Imai and Randall Wright,** "Unstable Relationships," mimeo, University of Pennsylvania, 1999.

[21] **Burdett, Kenneth and Kenneth L. Judd,** "Equilibrium Price Distributions," *Econometrica,* 1983, 51, 955-70.

[22] **Burdett, Kenneth, Ricardo Lagos and Randall Wright,** "Inequality, Unemployment and Crime," mimeo, University of Pennsylvania, 2000.

[23] **Burdett, Kenneth and Dale T. Mortensen, "**Search, Layoffs, and Labor Market Equilibrium," *Journal of Political Economy,* 1980, 88, 652-72.

[24] **Burdett, Kenneth and Dale T. Mortensen,** "Wage Differentials, Employer Size, and Unemployment," *International Economic Review,* 1998, 39, 257-73.

[25] **Burdett, Kenneth and Tara Vishwanath,** "Declining Reservation Wages and Learning," *Review of Economic Studies,* 1988a, 5, 655-65.

[26] **Burdett, Kenneth and Tara Vishwanath,** "Balanced Matching and Labor Market Equilibrium," *Journal of Political Economy,* 1988b, 96, 1048-65.

[27] **Burdett, Kenneth and Randall Wright,** "Two-Sided Search with Non-Transferrable Utility," *Review of Economic Dynamics,* January 1998, 1, 220-45.

[28] **Burdett, Kenneth, Shouyong Shi and Randall Wright,** "Pricing with Frictions," mimeo*,* University of Pennsylvania, 2000.

[29] **Butters, Gerard R.,** "Equilibrium Distributions of Sales and Advertising Prices," *Review of Economic Studies,* 1977, 44, 465-91.

[30] **Caballero, Ricardo J and Mohamad L. Hammour,** "The Cleansing Effect of Recessions," *American Economic Review* 84:5 (1994), 1350-1368.

[31] **Caballero, Ricardo J and Mohamad L. Hammour,** "On the Timing and Efficiency of Creative Destruction," *Quarterly Journal of Economics,* 1996, 111, 805-52.

[32] **Cole, Harold L. and Richard Rogerson,** "Can the Mortensen-Pissarides Matching Model Match the Business Cycle Facts?" *International Economic Review,* 1999, 40, 933-60.

[33] **Coles, Melvyn G.** "Equilibrium Wage Dispersion, Firm Size and Growth." *Universitat Autonoma de Barcelona DEHA/IAE Discussion Paper,* 1997.

[34] **Coles, Melvyn G. and Adrian Masters,** "Duration Dependent UI Payments in a Model of Equilibrium Unemployment," mimeo, University of Essex, 2000.

[35] **Coles, Melvyn G. and Eric Smith,** "Cross Section Estimation of the Matching Function: Evidence from England and Wales," *Economica.* 1996, 63, 589-87.

[36] **Coles, Melvyn G. and Eric Smith,** "Marketplaces and Matching," *International Economic Review,* 1998, 39, 239-54.

[37] **Costain, James S.,** "Unemployment Insurance in a General Equilibrium Model of Job Search and Precautionary Savings." *Ph.D Dissertation.* University of Chicago, 1997.

[38] **Curtis, Elisabeth and Randall Wright,** "Price Setting, Price Dispersion, and the Value of Money – or, The Law of Two Prices," mimeo, University of Pennsylvania, 2001.

[39] **Danforth, John P.,** "On the Role of Consumption and Decreasing Absolute Risk Aversion in the Theory of Job Search," in Studies in the Economics of Search, S. Lippman and J. McCall (eds.) North-Holland, 1979.

[40] **Delacroix, Alain,** "Transitions into Unemployment and the Nature of Firing Costs," mimeo*,* Purdue University, 1999a.

[41] **Delacroix, Alain,** "Firing Costs, Minimum Wage and the Outside Option Principle," mimeo, Purdue University, 1999b.

[42] **den Haan, Wouter J., Garey Ramey and Joel Watson,** "Job Destruction and the Propagation of Shocks," *American Economic Review,* 2000, 90, 482-98.

[43] **Devine, Theresa J. and Nicholas M. Kiefer,** *Empirical Labor Economics: A Search Approach*, Oxford: Oxford University Press, 1991.

[44] **Diamond, Peter A.,** "A Model of Price Adjustment." *Journal of Economic Theory,* 1971, 3, 156-68.

[45] **Diamond, Peter A.,** "Mobility Costs, Frictional Unemployment, and Efficiency," *Journal of Political Economy,* 1981, 89, 798-812.

[46] **Diamond, Peter A.,** "Aggregate Demand Management in Search Equilibrium," *Journal of Political Economy*, 1982a, 90, 881-894.

[47] **Diamond, Peter A.,** "Wage Determination and Efficiency in Search Equilibrium," *Review of Economic Studies*, 1982b, 49, 217-227.

[48] **Diamond, Peter A.,** "Consumer Differences and Prices in a Search Model," *Quarterly Journal of Economics,* 1987, 102, 429-436.

[49] **Fishman, Arthur and Rafael Rob,** "Product Innovation by a Durable-Good Monopoly," *Rand Journal of Economics,* 2000, 31, 237-52.

[50] **Gale, Douglas,** "Limit Theorems for Models with Sequential Bargaining," *Journal of Economic Theory,* 1987, 43, 20-54.

[51] **Gomes, Joao, Jeremy Greenwood and Sergio Rebelo,** "Equilibrium Unemployment," mimeo, Northwestern University, 2000.

[52] **Greenwood, Jeremy, Glenn M. MacDonald and Guang-Jia Zhang,** "The Cyclical Behavior of Job Creation and Job Destruction: A Sectoral Model, " *Economic Theory,* 1996, 7, 95-112.

[53] **Gronau, Reuben,** "Information and Frictional Unemployment," *American Economic Review,* 1971, 61, 290-301.

[54] **Hall J., Steven A. Lippman and John J. McCall,** "Expected Utility Maximizing Job Search," in Studies in the Economics of Search, S. Lippman and J. McCall (eds.) North-Holland, 1979.

[55] **Hey, John D. and Chris J. McKenna,** "To Move or Not to Move?" *Economica,* 1979, 46, 175-85.

[56] **Hosios, Arthur J.,** "On the Efficiency of Matching and Related Models of Search and Unemployment," *Review of Economic Studies,* 1990, 57, 279-98.

[57] **Howitt, Peter,** "Business Cycles with Costly Search and Recruiting," *Quarterly Journal of Economics,* 1988, 103, 147-65.

[58] **Howitt, Peter and R. Preston McAfee,** "Costly Search and Recruiting," *International Economic Review,* 1987, 28, 89-107.

[59] **Jovanovic, Boyan,** "Job Matching and the Theory of Turnover," *Journal of Political Economy,* 1979a, 87, 972-90.

[60] **Jovanovic, Boyan,** "Firm-specific Capital and Turnover," *Journal of Political Economy,* 1979b, 87, 1246-60.

[61] **Jovanovic, Boyan,** "Selection and the Evolution of Industry," *Econometrica,* 1982, 50, 649-70.

[62] **Jovanovic, Boyan and Glenn M. MacDonald,** "Competitive Diffusion," *Journal of Political Economy,* 1994, 102, 24-52.

[63] **Jovanovic, Boyan and Rafael Rob,** "The Growth and Diffusion of Knowledge," *Review of Economic Studies,* October 1989, 56, 569-82.

[64] **Kiyotaki, Nobuhiro and Randall Wright,** "On Money as a Medium of Exchange," *Journal of Political Economy,* August 1989, 97, 927-54.

[65] **Kiyotaki, Nobuhiro and Randall Wright,** "A Search-Theoretic Approach to Monetary Economics," *American Economic Review,* March 1993, 83, 63-77.

[66] **Lagos, Ricardo,** "An Alternative Approach to Search Frictions," *Journal of Political Economy,* 2000, 108, 851-73.

[67] **Lippman, Steven A. and John J. McCall,** "The Economics of Job Search: a Survey," *Economic Inquiry,* 1976a, 14, 347-68.

[68] **Lippman, Steven A. and John J. McCall,** "Job Search in a Dynamic Economy," *Journal of Economic Theory,* 1976b, 12, 365-90.

[69] **Lippman, Steven A. and John W. Mamer,** "A Simple Search Model with Procyclical Quits," *Journal of Economic Dynamics & Control,* 1989, 13, 247-53.

[70] **Ljungqvist, Lars and Thomas J. Sargent,** "The European Unemployment Dilemma," *Journal of Political Economy,* 1998, 106, 514-50.

[71] **Ljunqvist Lars and Thomas J. Sargent,** *Recursive Macroeconomic Theory,* Cambridge: MIT Press, 2000.

[72] **Lucas, Robert E., Jr.,** "Unemployment Policy," *American Economic Review: Papers and Proceedings,* 1978, 68, 353-57.

[73] **Lucas, Robert E., Jr. and Edward C. Prescott,** "Equilibrium Search and Unemployment," *Journal of Economic Theory,* 1974, 7, 188-209.

[74] **MacMinn, Richard D.,** "Search and Market Equilibrium," *Journal of Political Economy,* 1980, 88, 308-27.

[75] **Masters, Adrian M.,** "Efficiency of Physical and Human Capital Investments a Model of Search and Bargaining," *International Economic Review,* 1998.

[76] **Masters, Adrian M.,** "Wage Posting in Two-Sided Search and the Minimum Wage," *International Economic Review,* 1999, 40, 809-26.

[77] **McCall, John J.,** "Economics of Information and Job Search," *Quarterly Journal of Economics,* 1970, 84, 113-26.

[78] **Merz, Monika,** "Search in the Labor Market and the Real Business Cycle," *Journal of Monetary Economics,* 1995, 36, 269-300.

[79] **Merz, Monika,** "Heterogeneous Job-Matches and the Cyclical Behavior of Labor Turnover," *Journal of Monetary Economics,* 1999, 43, 91-124.

[80] **Millard, Stephen P. and Dale T. Mortensen,** "The Unemployment and Welfare Effects of Labour Market Policy: A Comparison of the USA and the UK," in *Unemployment Policy: Government Options for the Labour Market.* D.J. Snower and G. de la Dehesa, eds. New York: Cambridge University Press, 1997.

[81] **Moen, Espen R.**, "Competitive Search Equilibrium," *Journal of Political Economy,* 1997, 105, 385-411.

[82] **Montgomery, James D.**, "Equilibrium Wage Dispersion and Interindustry Wage Differentials," *Quarterly Journal of Economics* 106, 1991, 163-179.

[83] **Mortensen, Dale T**., "A Theory of Wage and Employment Dynamics," in *Microeconomic Foundations of Employment and Inflation Theory*, E. S. Phelps, *et. al.* (eds), New York: W.W. Norton, 1970.

[84] **Mortensen, Dale T.**, "Unemployment Insurance and Job Search Decisions," *Industrial and Labor Relations Review,* 1977, 30, 505-17.

[85] **Mortensen, Dale T.,** "The Matching Process as a Noncooperative Bargaining Game," in *The Economics of Information and Uncertainty*, ed. John J. McCall, Chicago: University of Chicago Press for the National Bureau of Economic Research, 1982b.

[86] **Mortensen, Dale T.,** "Property Rights and Efficiency in Mating, Racing, and Related Games," *American Economic Review*, 1982b, 72, 968-979.

[87] **Mortensen, Dale T.,** "Job Search and Labor Market Analysis," in *Handbook in Labor Economics*, O. Ashenfelter and R. Layard (eds.), Amsterdam: North Holland, 1986.

[88] **Mortensen, Dale T.,** "Matching: Finding a Partner for Life or Otherwise," *American Journal of Sociology*, 1988, 94, S215-S240.

[89] **Mortensen, Dale T.,** "The Persistence and Indeterminacy of Unemployment in Search Equilibrium," *Scandinavian Journal of Economics*, 1989, 91, 347-370.

[90] **Mortensen, Dale T.,** "Equilibrium Unemployment with Wage Posting: Burdett-Mortensen Meets Pissarides," mimeo, Centre for Labour Market and Social Research, Aarhus, Denmark, 1998

[91] **Mortensen, Dale T.,** "Equilibrium Unemployment Dynamics," *International Economic Review,* 1999, 40, 889-914.

[92] **Mortensen, Dale T. and George. R. Neumann,** "Choice or Chance? A Structural Interpretation of Individual Labor Market Histories," in *Studies in Labor Market Dynamics,* G.R. Neumann and W. Westergaard-Neilsen (eds.), Heidelberg: Springer-Verlag, 1984.

[93] **Mortensen, Dale T. and Christopher A. Pissarides,** "Job Creation and Job Destruction in the Theory of Unemployment," *Review of Economic Studies*, 1994, 61, 397-415.

[94] **Mortensen, Dale T. and Christopher A. Pissarides,** "New Developments in Models of Search in the Labor Market," in *Handbook in Labor Economics*, O. Ashenfelter and D. Card (eds.), Amsterdam: North Holland, 1999a.

[95] **Mortensen, Dale T. and Christopher A. Pissarides,** "Job Reallocation, Employment Fluctuations, and Unemployment Differences," in M. Woodford and J. Taylor (eds.), *Handbook of Macroeconomics*, Amsterdam; North Holland, 1999b.

[96] **Mortensen, Dale T. and Christopher A. Pissarides,** "Unemployment Responses to 'Skill-Biased' Shocks: The Role of Labour Market Policy," *Economic Journal*, 1999c, 109, 242-65.

[97] **Mortensen, Dale T., and Randall Wright**, "Competitive Pricing and Efficiency in Search Equilibrium," mineo, University of Pennsylvania, 2000.

[98] **Nash, John,** "The Bargaining Problem," *Econometrica,* 1950, 18, 155-62.

[99] **Osborne, Martin J. and Ariel Rubinstein,** *Bargaining and Markets,* London: Academic Press, 1990.

[100] **Peters, Michael,** "Equilibrium with Capacity Constraints and Restricted Mobility," *Econometrica*, 1984, 52, 1117-1129.

[101] **Peters, Michael,** "Ex Ante Price Offers in Matching Games: Steady States," *Econometrica*, 1991, 59, 1425-1454.

[102] **Petrongolo, Barbara and Christopher A. Pissarides,** "Looking into the Black Box: A Survey of the Matching Function," mimeo, London School of Economics, 1999.

[103] **Pissarides, Christopher A.**, "Short-run Equilibrium Dynamics of Unemployment, Vacancies, and Real Wages," *American Economic Review,* 1985, 75, 676-90.

[104] **Pissarides, Christopher A.,** *Equilibrium Unemployment Theory*, Oxford: Basil Blackwell, 1990.

[105] **Pissarides, Christopher A.,** "Search Unemployment with On-the-Job Search," *Review of Economic Studies,* 1994, 61, 457-75.

[106] **Pries, Michael J.,** "Persistence of Employment Fluctuations: A Model of Recurring Job Loss," mimeo, Stanford University, 1998.

[107] **Pries, Michael J and Richard Rogerson,** "Labor Market Institutions and Hiring Policies," mimeo, University of Pennsylvania, 2000.

[108] **Reinganum, Jennifer F.,** "A Simple Equilibrium Model of Price Dispersion," *Journal of Political Economy,* 1979, 87, 851-8.

[109] **Rob, Rafael,** "Equilibrium Price Distributions," *Review of Economic Studies,* 1985, 52, 487-504.

[110] **Robin, Jean-Marc and Sebastien Roux,** "Equilibrium Search Decisions with Decreasing Returns to Scale and Hiring Costs: Theory and Estimation," mimeo, 1998.

[111] **Rothschild, Michael,** "Models of Market Organization with Imperfect Information: A Survey," *Journal of Political Economy,* 1973, 81, 1283-308.

[112] **Rubinstein, Ariel,** "Perfect Equilibrium in Bargaining Model," *Econometrica,* 1982, 50, 97-109.

[113] **Rubinstein, Ariel and Asher Wolinsky,** "Equilibrium in a Market with Sequential Bargaining," *Econometrica,* 1985, 53, 1133-50.

[114] **Salop, Steven C.,** "Wage Differentials in a Dynamic Theory of the Firm," *Journal of Economic Theory,* 1973, 6, 321-44.

[115] **Shapiro, Carl and Joseph E. Stiglitz,** "Equilibrium Unemployment as a Worker Discipline Device," *American Economic Review,* 1984, 74, 433-44.

[116] **Shi, Shouyong**, "Money and Prices: A Model of Search and Bargaining," *Journal of Economic Theory,* 1995, 67, 467-96.

[117] **Shi, Shouyong and Quan Wen,** "Labor Market Search and Capital Accumulation: Some Analytical Results," *Journal of Economic Dynamics and Control,* 1997, 21, 1747-76.

[118] **Shimer, Robert,** "Contracts in Frictional Labor Markets," mimeo, Princeton University, 1995.

[119] **Shimer, Robert,** "Job Auctions," mimeo, Princeton University, 1999.

[120] **Shimer, Robert and Lones Smith**, "Assortative Matching and Search," *Econometrica* 68:2 (2000a).

[121] **Shimer, Robert and Lones Smith,** "Matching, Search, and Heterogeneity," mimeo, Princeton University, 2000b.

[122] **Smith, Eric,** "Search, Concave Production and Optimal Firm Size," *Review of Economic Dynamics*, 1999, 2, 456-471.

[123] **Stigler, George,** "The Economics of Information," *Journal of Political Economy,* 1961, 69, 213-225.

[124] **Stokey, Nancy L. and Robert E. Lucas, Jr. with Edward C. Prescott,** *Recursive Methods in Economic Dynamics,* Cambridge and London: Harvard University Press, 1989.

[125] **Trejos, Alberto and Randall Wright,** "Search, Bargaining, Money and Prices," *Journal of Political Economy,* 1995, 103, 118-41.

[126] **Valdivia, Victor,** "Evaluating the Welfare Benefits of Unemployment Insurance," mimeo, 1995.

[127] **van den Berg, Gerard J.,** "Multiple Equilibria and Minimum Wages in Labor Markets with Informational Frictions and Heterogeneous Production Technologies," mimeo, Erasmus University, 2000.

[128] **Webb, Tracy,** "Marriage and Divorce," mimeo, University of Bristol, 1999a.

[129] **Webb, Tracy,** "Equilibrium Quits and Layoffs in a Model with On-the-Job Search," mimeo, University of Bristol, 1999b.

[130] **Weiss, Andrew,** "Job Queues and Layoffs in Labor Markets with Flexible Wages," *Journal of Political Economy,* 1980, 88, 526-38.

[131] **Wilde, Louis L.,** "An Information-Theoretic Approach to Job Quits," in *Studies in the Economics of Search,* S.A. Lippman and J.J. McCall (eds.), Amsterdam: North-Holland, 1979.

[132] **Wolpin, Ken. I.,** *Empirical Methods for the Study of Labor Force Dynamics,* Luxembourg: Harwood Academic Publishers, 1995.

[133] **Wright, Randall,** "Search, Layoffs, and Reservation Wages," *Journal of Labor Economics,* 1987, 5, 354-365.

[134] **Wright, Randall,** "Job Search and Cyclical Unemployment," *Journal of Political Economy,* 1986, 94, 38-55.