Penn Institute for Economic Research
Department of Economics
University of Pennsylvania
3718 Locust Walk
Philadelphia, PA 19104-6297
pier@econ.upenn.edu
http://www.econ.upenn.edu/pier

PIER Working Paper 02-039

"Price Setting, Price Dispersion, and the Value of Money – or The Law of Two Prices"

by

Elisabeth Curtis and Randall Wright

http://ssrn.com/abstract_id=339200

Price Setting, Price Dispersion, and the Value of Money – or, The Law of Two Prices*

Elisabeth Curtis
Drexel University

Randall Wright
University of Pennsylvania

August 2, 2001

Abstract

We study models that combine search, monetary exchange, price posting by sellers, and buyers with preferences that differ across random meetings – say, because sellers in different meetings produce different varieties of the same good. We show how these features interact to influence the price level (i.e., the value of money) and price dispersion. First, price-posting equilibria exist with valued fiat currency, which is not true in the standard model. Second, although both are possible, price dispersion is more common than a single price. Third, perhaps surprisingly, we prove generically there cannot be more than two prices in equilibrium.

JEL Nos. C78, D83, E31
Keywords: search, money, price posting, price dispersion

*We thank Alan Head, Adrian Masters and Robert Shimer for very useful comments and discussions, as well as the NSF for financial support. Address correspondence to Randall Wright, Department of Economics, University of Pennsylvania, 3718 Locust Walk, Philadelphia PA 19104.

1 Introduction

This paper studies search-based models of exchange in which sellers post prices. A key aspect of our specification is that a buyer's preferences differ across random meetings with sellers. That is, even after you locate a seller with the good you want, you may get higher or lower utility from any given quantity of that good – perhaps because different sellers produce different varieties of the good, or perhaps simply because you may be more or less "hungry" at different points in time. We use the model to analyze the price level (i.e., the value of money) and price dispersion. First, we show that there exist price-posting equilibria with valued fiat money in this framework, something that is *not* true in standard models. Second, although both are possible, equilibria with price dispersion are more common than equilibria with a single price, in the sense that the former exist for a strict subset of the parameters for which the latter exist. Third, perhaps surprisingly, we prove that generically there are *at most two* prices in equilibrium.

To put things into perspective it is useful to start with the contribution of Diamond (1971). Like ours, Diamond's framework is search model with price posting. Given homogeneous buyers he shows there is a unique equilibrium price distribution and it is degenerate – everyone sets the same price. Moreover, this common price extracts all gains from trade from buyers. Now, Diamond's model does not have anything to do with *money* in the sense that a monetary economist would use the term, but the logic goes through in exactly the same way in the standard search-based model of fiat currency with endogenous prices, such as Shi (1995) or Trejos and Wright (1995). That is, if sellers set prices in this model, there is a unique equilibrium, it involves all sellers charging the same price, and that price extracts all gains from trade from buyers. This result is striking in a monetary economy for the following reason: if sellers capture all gains from trade, money is not useful, and so equilibria with valued fiat currency cannot exist. That is, the unique equilibrium is the nonmonetary equilibrium.

From the literature research on price dispersion, in non-monetary models, it is known that making searchers heterogeneous in some ways may overturn Diamond's (1971) results. For example, in the context of the labor market, Albrecht and Axell (1984) show that if workers differ in their intrinsic values of leisure then there can exist equilibrium where different firms post different wages. Diamond (1987) does something similar in a consumer search model. Such tricks do *not* work in monetary economies: if some agents enjoy permanently higher utility from consumption then monetary equilibria still unravel. However, our way of introducing meeting-specific differences in utility does allow us to construct equilibria with valued fiat currency. Intuitively, in our framework sellers face a trade-off between selling sooner and realizing greater profits per sale, and as long as they put some weight on the former consideration buyers can derive a strictly positive surplus in some meetings, which means that money can be valued as a medium of exchange.[1]

We characterize the set of parameters for which monetary equilibria exist, and show that these equilibria are similar in some respects to those in other models but also have some unusual properties. Moreover, once monetary equilibria exist, the trade-off facing sellers makes it plausible that there may be more than one price, the way there can be different wages in Albrecht and Axell (1984) or prices in Diamond (1987). For a simple version of the model where the preference parameter can take on $K = 2$ different values across meetings, we characterize the set of equilibria and determine when there will be one price or two (with $K = 2$ it is immediate that there can never be more than two prices for the reason explained in the next paragraph). We find that a single-price equilibrium exists on a strict subset of the parameters for which there exists a two-price equilibrium. Hence, price dispersion is not only possible it is more typical than a single price.

The basic logic behind Diamond's original result is that, in any candidate equilibrium, all buyers have a common reservation price and no seller wants to post anything other than that price. In general, when the preference parameter can take on K different values across meetings, there are K distinct

[1] Sellers ostensibly face the same tradeoff in a version of the model where agents have permanently different preferences, but it cannot lead to valued fiat money. The reason will be explained in more detail below, but the basic idea is as follows: in any candidate monetary equilibria some agents will get zero surplus, so they drop out, and in the end we are back to the case with homogeneous agents, where the trade-off no longer exists and money cannot be valued.

reservation prices and no seller wants to post anything other than one of them – which is why there can never be more than two prices posted when $K = 2$. However, we prove the strong and perhaps surprising result that generically there will never be more than 2 prices actually posted for *any* K. We think this is interesting for the following reason. Standard arbitrage considerations in frictionless models imply there cannot be more than one price for a given good. These arbitrage considerations do not apply in search-based models, so that the law of one price generally does not hold, but what we find is that a slightly weaker law of two prices does.[2]

The rest of the paper is organized as follows. Section 2 describes the basic assumptions concerning search, money, pricing, and the structure of preferences. Section 3 studies the case of $K = 2$. For this case we characterize the set of equilibria, derive welfare and comparative static results, and work out an explicit example. Section 4 studies the case of general K, and proves the law of two prices. Additionally, we sketch the case $K = 3$ in order to show that two prices are more common than one, and that the two prices could be either the two highest reservation prices, the two lowest, or the lowest and the highest. Section 5 discusses some alternative assumptions in order to sort out what is most important for the results, and shows that some

[2] While the proof is actually very simple, discussion of the economic intuition as well as the critical assumptions behind this result seem best postponed until some more details of the model are at hand. However, we want to be up front about one assumption in particular: the differences in preferences here are idiosyncratic to meetings and not intrinsic characteristics of individuals. We explain how this matters in detail below.

results, including the law of two prices, also hold in nonmonetary models. Section 6 concludes.

2 The Basic Framework

There is a $[0,1]$ continuum of infinite-lived agents who discount at rate r. Agents trade in a bilateral random-matching process with Poisson arrival rate α. So that there is potentially a role for a medium of exchange, we assume there are N goods and N types of agents, where type n produces only good n and consumes only good $n+1$ (mod N). For $N > 2$ the probability of a double coincidence of wants in any meeting is 0, and the probability of a single coincidence is $x = 1/N$. A key assumption is that not all single coincidence meetings are created equal: when a type n agent meets a random type $n+1$ agent, the former derives utility from q units of the latter's output given by $U = \delta_k q$ with probability λ_k, $k = 1, 2, \ldots K$. One way to motivate this is to assume each agent produces one of K varieties of his good, and different customers prefer different varieties.[3] The cost for any agent (in disutility) from producing q units of his production good is $c(q)$, where $c'(q) > 0$ and $c''(q) > 0$ for all $q > 0$, $c'(0) = c(0) = 0$, and $c(\hat{q}) = \hat{q}$ for some $\hat{q} > 0$.

[3] One interpretation of this story is that it combines the specification for prefernces over goods in Kiyotaki and Wright (1989) with the specification for preferences over variety in Kiyotaki and Wright (1991). Alternatively, for our purposes it is equvialent to simply assume type n goods all come in one variety, but in any given meeting the buyer has a random utility shock.

As is standard in this type of model, exchange must be quid pro quo, and if we assume that consumption goods are storable only by their producers then all trade must use money. Money consists of storable objects, referred to here as *dollars*, that no one can consume or produce. A fraction M of the population are each initially endowed with one dollar. For simplicity, as in much of the literature, we assume that dollars are indivisible and that each agent has a storage capacity of one dollar. Hence, every exchange involves 1 dollar being traded for some amount of output to be determined below. This means that it is always the case that the fraction M of the population, called *buyers*, hold 1 dollar each, while the fraction $1-M$, called *sellers*, are without money. In each trade money will change hands, so that the buyer becomes a seller and vice-versa, but in the aggregate there are always M buyers and $1-M$ sellers.

These assumptions are essentially identical to those in the base model of Shi (1995) or Trejos and Wright (1995), except for the random utility generated in each single coincidence meeting (one could say the standard model is a special case where all type n goods come in the same variety). However, rather than having agents bargain after they meet, we assume that sellers post prices ex ante. That is, each agent without money picks a q and stands ready to trade q units of his output in exchange for a dollar, for an implied price of $p = 1/q$ (one might prefer to say they post quantities, but this is the same as posting prices here). One interpretation is that each seller sets a vending machine with the capacity to produce q units of his output. When a buyer encounters the machine, he can either put in his dollar and receive q, at which point he consumes and becomes a seller, or walk away and continue as a buyer.

Previous models in the related monetary literature typically assume q is determined after agents meet according to some bargaining rule, such as the generalized Nash solution where the buyer has bargaining power β. Given δ is nonrandom, $\beta = 0$ is equivalent to ex ante price posting by the seller. As we show below, in the standard model $\beta = 0$ implies monetary equilibria do not exist. Once we introduce the random element in each single coincidence meeting, δ, and given q cannot depend on δ (e.g., the vending machine cannot distinguish buyers' tastes) sellers face a trade-off between the probability of a sale and profit per sale. If they put enough weight on the former they may set q so that monetary equilibria will exist. We want to know when this is the case, and to characterize the nature of price dispersion that arises from the trade-off.[4]

[4]Although most related work assumes bargaining, there are exceptions. For example, Jafarey and Masters (2000) also consider price posting in a model that is in some respects similar to ours, but make assumptions to guarantee that the is a unique price in equilibrium (see below for details). Green and Zhou (1998) and Zhou (1999) also have price posting, and sellers face a similar trade-off in terms of profit per sale versus the probability of a sale, except in their model it is because buyers in different meetings may have different amounts of money rather different preferences. In any case, they also only discuss single-price equilibria. Also note that price dispersion is more easily generated if we assume bargaining rather than posting, as long as agents are heterogeneous with respect to some characteristic such as their money holdings or preferences; see Molico (1998), Camera and Corbae (1999), Wallace and Zhou (1997), Boyarchenko (2000) or Kudoh (2000). Our goal has precedence in other search applications, including models of the labor market,

Generally, a steady state equilibrium is described by a distribution, say $F(q) = pr(q_j \leq q)$, such that each seller j sets q_j to maximize expected utility given the value of q set by every other agent, and also given that buyers use utility maximizing search strategies. Let V_m and V_0 be the payoffs or value functions of buyers and sellers, respectively, given $F(q)$. Although there always exists a nonmonetary equilibria where $V_0 = V_m = 0$, we focus here on monetary equilibria, where $V_m > V_0 > 0$. As we said, we are interested in characterizing the equilibrium distribution F, especially in finding out when it exhibits price dispersion and what is the nature of that price dispersion. Also we want to study how prices, quantities, and payoffs depend on the underlying parameters and on the kind of equilibrium we are in.

3 A Simple Model

In order to develop some basic insights, in this section we study the case of $K = 2$; that is, in any meeting with a seller (or, his vending machine) a buyer realizes $U = \delta_1 q$ with probability λ_1 and $U = \delta_2 q$ with probability $\lambda_2 = 1 - \lambda_1$. To reduce notation, here we normalize $\delta_2 = 1$ and write $\delta_1 = \delta \in (0, 1)$ as well as $\lambda_1 = \lambda \in (0, 1)$. In the event the buyer realizes $\delta_2 = 1$ we say he likes the sellers good a lot, and in the event he realizes $\delta_1 = \delta$ we say he likes it only a little.

---

where the emphasis is placed on deriving endogenous wage distributions with posting (see Albrecht and Axell 1984, Burdett and Mortensen 1998, Albrecht Vroman 2000, e.g.).

The continuous time Bellman equation for a buyer is

$$
\begin{aligned}
rV_m &= (1-M)\lambda \int \max\{q+V_0-V_m, 0\}\, dF(q) \\
&\quad +(1-M)(1-\lambda) \int \max\{\delta q+V_0-V_m, 0\}\, dF(q),
\end{aligned} \tag{1}
$$

where we have normalized time with no loss in generality so that $\alpha x = 1$. In words, (1) sets the flow return to having a dollar equal to the rate at which you locate goods you like a lot, $(1-M)\lambda$, times the expected net gain from trading or not depending on the posted q, plus the rate at which you locate goods you like a little, $(1-M)(1-\lambda)$, times the expected net gain from trading or not again depending on q. Clearly, for any distribution $F(q)$ a buyer's decision about whether to accept or reject q will have a conditional reservation property: if you meet a seller posting q and like his good a lot, accept iff $q \geq q_L$ where $q_L = V_m - V_0$; and if you meet a seller posting q and like his good only a little, accept iff $q \geq q_H$ where $\delta q_H = V_m - V_0$.

We can now show that for $K = 2$, in any equilibrium $F(q)$ puts positive probability on at most two points. Thus, given any $F(q)$ all buyers choose the same reservation values q_H and q_L. Any seller posting $q_j < q_L$ never makes a sale, which cannot be a best response. Any seller posting $q_j > q_H$ sells to all buyers, but he could still sell to all buyers if he lowered q_j towards q_H, so $q_j > q_H$ cannot be a best response. Finally, any agent posting $q_j \in (q_L, q_H)$ sells only to the fraction λ of buyers who like his output a lot, but he could still sell to the same set of buyers if he lowered q_j towards q_L, so $q_j \in (q_L, q_H)$

cannot be a best response. Hence, no seller will post anything other than q_L or q_H. In what follows we let θ denote the fraction of firms posting q_H. Summarizing, we have.

Proposition 1 *Given $U = q$ with probability λ and $U = \delta q$ with probability $1-\lambda$, where $\delta < 1$, any equilibrium distribution $F(q)$ must have the following property: $q = q_H$ with probability θ and $q = q_L$ with probability $1-\theta$, where $q_L = \delta q_H = V_m - V_0$.*

These results allow us to reduce (1) to

$$rV_m = (1 - M)\lambda\theta(q_H + V_0 - V_m). \tag{2}$$

There is only one term in (2) because the only time a buyer realizes positive gains from trade is when he locates a good he likes a lot posted at q_H. Also, the value of being a seller can be written $V_0 = \max\{V_L, V_H\}$, where V_k is the value of posting q_k. These satisfy

$$rV_L = M\lambda[V_m - V_L - c(q_L)] \tag{3}$$

$$rV_H = M[V_m - V_H - c(q_H)], \tag{4}$$

which makes clear the relevant trade-off: a higher probability of trading in each single coincidence meeting comes with a higher cost of production and hence lower profit.

One can now see why in the standard model, where the preference parameter δ is nonrandom, a price-setting equilibrium cannot have valued money.

Simply let $\lambda = 1$, which means that all buyers trade iff $q \geq V_m - V_0 = q_L$. This means that all sellers post q_L, so there is a single price and buyers never realize any gains from trade. If buyers get no surplus then $V_m = 0$ – but then no seller would offer $q > 0$ to get a dollar as long as production is costly. This is why the standard model needs to assume buyers have bargaining power $\beta > 0$. In our model, the random δ_k potentially allows money to be valued because meetings are heterogeneous, even though buyers are homogeneous in the search process, in the sense that they all draw δ_k from the same distribution and therefore all have the same value of V_m.

One might think that any heterogeneity in buyers would do the trick. For example, Albrecht and Axell (1984) assume workers are intrinsically different in terms of their value of leisure in order to generate wage-posting equilibria with dispersion. Following their lead, what if we assume the fraction λ_1 of agents are type 1 and *always* get utility $U = \delta_1 q$ in a single-coincidence trade while the fraction λ_2 *always* get $U = \delta_2 q$? Clearly there are still two reservation values, say q_1 and q_2, and all sellers will post either q_1 or q_2. Suppose $q_2 < q_1$ (the other case is symmetric). Then type 1 buyers never get any gains from trade when they have money, and so they would never post a positive q in order to get money. But then in steady state all buyers in the market are type 2, and hence all sellers post q_2, which means type 2 buyers also get no gains from trade and the monetary equilibrium has broken down.[5]

This unraveling of monetary equilibria obviously works for any number of heterogeneous types. Given this, let us see what we can get with the purely match-specific differences in δ_k. Proposition 1 says that with $K = 2$ there are only three possible types of equilibria: $\theta = 0$, $\theta = 1$, or $\theta \in (0, 1)$. The first case is trivial: if $\theta = 0$ then all sellers set $q = q_L$, and so $V_m = 0$. Hence, there is no monetary equilibrium in this case, and we have only two interesting cases to consider: equilibria with a single price, $q = q_H$ with probability $\theta = 1$, and equilibria with price dispersion, $\theta \in (0, 1)$.

Consider first an equilibrium with $\theta = 1$. Given all sellers post q_H, (2) can be written

$$rV_m = (1 - M)\lambda(q_H + V_H - V_m). \tag{5}$$

Equilibrium requires q_H solve the reservation equation, $V_m - V_H = \delta q_H$. Using the Bellman equations to eliminate the value functions, this can be reduced to $e(q_H) = 0$ where

$$e(q) \equiv [(r + M)\delta - (1 - M)\lambda(1 - \delta)]q - Mc(q). \tag{6}$$

Notice $e(0) = 0$ and $e(q) < 0$ for large q. In Appendix A we show $e'(q) < 0$ for any $q > 0$ such that $e(q) = 0$, and so there can exist no more than one positive solution to $e(q) = 0$. There exists a positive solution, call it q^e, iff

[5] Note that in Albrecht and Axell workers with the highest reservation wage also get no surplus, but are assumed to keep searching rather than drop out. Of course, if there were strictly positive search costs, no matter how small, they would drop out.

$e'(0) > 0$, which holds iff

$$(r+M)\delta - (1-M)\lambda(1-\delta) > 0. \tag{7}$$

If (7) does not hold there cannot exist a single-price monetary equilibrium. If (7) does hold, there is a unique $q_H = q^e$ that is a candidate equilibrium. To check that it is an actual equilibrium, we need to check that no seller wants to deviate from $q_H = q^e$ to $q_L = \delta q^e$. From the Bellman equations we see that $V_H \geq V_L$, and therefore no one wants to deviate, iff $f(q^e) \geq 0$, where

$$f(q) \equiv \delta(1-\lambda)q - c(q) + \lambda c(\delta q). \tag{8}$$

In Appendix A we show that there always exists a unique positive solution to $f(q) = 0$, call it q^f, and that $f'(q^f) < 0$. Thus, $f(q^e) \geq 0$ and no one wants to deviate iff $q^e \leq q^f$. The left panel of Figure 1 shows the functions $e(q)$ and $f(q)$, drawn so that q^e exists and satisfies $q^e \leq q^f$, which means that it constitutes an equilibrium for all sellers post $q_H = q^e$. The right panel shows the regions of parameter space where this obtains, but we postpone discussion of this until we describe the other equilibrium.

In an equilibrium with $\theta \in (0,1)$, we must have $V_L = V_H$. It is easy to see that, for any θ, $V_L = V_H$ iff $f(q_H) = 0$ where f was defined in (8); hence, $V_L = V_H$ iff $q_H = q^f$. We also require $\delta q_H = V_m - V_H$. Rearranging the Bellman equations, $\delta q_H = V_m - V_H$ iff $\theta = \theta(q_H)$ where

$$\theta(q) \equiv \frac{(r+M)\delta q - Mc(q)}{(1-M)\lambda(1-\delta)q}. \tag{9}$$

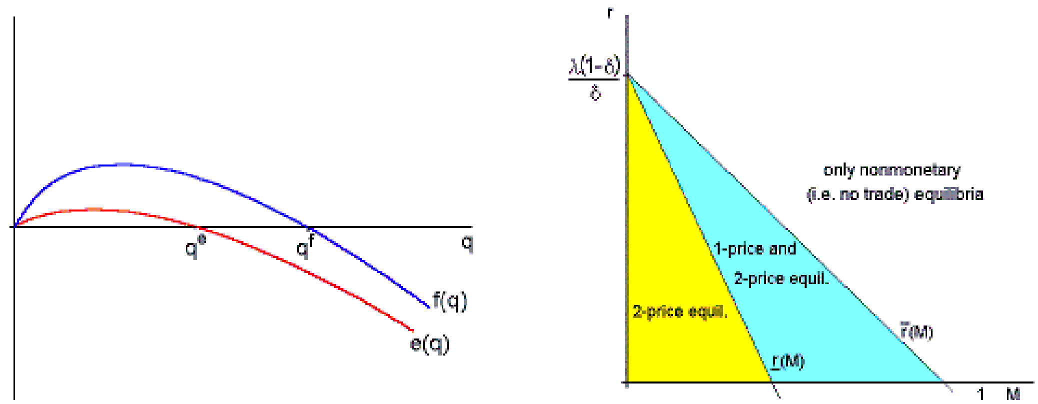

Figure 1: Functions $e(q)$ and $f(q)$ and Existence Regions

Therefore, $q_H = q^f$ implies that sellers are indifferent between posting q_H and $q_L = \delta q_H$, and given q^f we know that $\theta = \theta\left(q^f\right)$ means $\delta q_H = V_m - V_H$. The only thing left to check is $0 < \theta\left(q^f\right) < 1$. In Appendix A we show this holds iff $e\left(q^f\right) < 0$. We can now easily describe the parameter regions where this and the other equilibrium exist.

Proposition 2 *There are two linear functions of M, $\underline{r}$ and $\bar{r}$, with $\underline{r} < \bar{r}$ for all $M > 0$, as show in the right panel of Figure 1, with the following properties: (a) a single-price equilibrium, where $q = q^e$ with probability 1, exists iff $\underline{r} < r \leq \bar{r}$; (b) a two-price equilibrium, where $q = q^f$ with probability $\theta\left(q^f\right) > 0$ and $q = \delta q^f$ with probability $1 - \theta\left(q^f\right) > 0$, with $\theta(q)$ defined in (9), exists iff $r < \bar{r}$; and (c) these are the only (steady state) monetary equilibria.*

Proof: We have established so far the following. If on the one hand (7) fails, then $e(q) < 0$ for all q and in particular $e\left(q^f\right) < 0$; this means the single-price monetary equilibrium does not exist and the two-price equilibrium does. If on the other hand (7) holds, then both equilibria exist iff $e\left(q^f\right) < 0$. We can rewrite (7) as

$$r > \underline{r} \equiv \frac{\lambda(1-\delta)}{\delta} - \left[\frac{\delta + (1-\delta)\lambda}{\delta}\right] M. \tag{10}$$

Hence, if $r \leq \underline{r}$ the two-price equilibrium exists and the one-price equilibrium does not. If $r > \underline{r}$, then q^e exists as in Figure 1, and $e\left(q^f\right) < 0$ iff $q^e < q^f$. Using (8) to write $q^f = \frac{c\left(q^f\right) - \lambda c\left(\delta q^f\right)}{\delta(1-\lambda)}$, inserting this into (6) and rearranging, we see that this holds iff

$$r < \bar{r} \equiv \frac{\lambda(1-\delta)}{\delta} - \left[\frac{\lambda c\left(q^f\right) - \{\delta + \lambda(1-\delta)\}\lambda c\left(\delta q^f\right)}{\delta\{c\left(q^f\right) - \lambda c\left(\delta q^f\right)\}}\right] M. \tag{11}$$

By virtue of (8), q^f is independent of M and r, and so $\bar{r}$ is also a decreasing linear function of M. One can see that $\bar{r}$ has the same intercept as $\underline{r}$, that $\bar{r}$ has a flatter slope, and that $\bar{r} < 0$ when $M = 1$. Therefore, the situation is as depicted in Figure 1. ■

Several economic results emerge from the analysis. First, monetary equilibria only exist if r and M are not too big, as is standard. Second, in the two-price equilibrium, as M increases $q_H = q^f$ and $q_L = \delta q^f$ do not change while θ rises (Appendix A), so the average quantity $\bar{q} = \theta q_H + (1-\theta) q_L$ is increasing and the price $\bar{p} = 1/\bar{q}$ decreasing in M. Also, in the single-price equilibrium, as M increases $q_H = q^e$ rises (Appendix A), so prices are again decreasing in M. However one interprets an increase in M, these results are curious and nonstandard. For example, Trejos and Wright (1995) provide one model where $\partial p/\partial M > 0$ for all parameters, and another where $\partial p/\partial M > 0$ for all but extreme parameters, while here $\partial p/\partial M < 0$ for all parameters such that monetary equilibria exist. Finally, perhaps the most interesting thing to observe is that the two-price equilibrium is more robust than the single-price equilibrium: the latter exists for a strict subset of the parameters for which the former exists.[6]

We also want to study welfare, given by $W = MV_m + (1-M)V_0$. For the single-price equilibrium, one can derive

$$W^S = \frac{(1-M)q^e}{r}[\lambda(1-\delta) - \delta r].$$

Hence, W^S is proportional to $(1-M)q^e$, and since q^e is increasing in M the net result is that welfare is non-monotonic in M (see below). For the two-price equilibrium, one can derive

$$W^D = \frac{M}{(1-\lambda)r}\left[(r+\lambda)c\left(q^f\right) - (r+1)\lambda c(\delta q^f)\right].$$

Since q^f is independent of M, W^D is linearly increasing in M up to the point

[6] Consider the following (algebraic) intuition. To construct a single-price equilibrium, we first solve for the q_H that satisfies the reservation equation $\delta q_H = V_m - V_H$, and then *hope* the implied value functions satisfy the pricing condition $V_H \geq V_L$. To construct a two-price equilibrium, we first solve for the q_H that makes the price setting condition $V_H = V_L$ hold, and then find the θ that satisfies the reservation equation. Algebraically, it is easier to solve for an endogenous variable that makes the pricing condition hold than to hope it holds at $\theta = 1$.

where the equilibrium breaks down, which is at

$$\overline{M} = \frac{[\lambda(1-\delta) - r\delta]\left[c\left(q^f\right) - \lambda c\left(\delta q^f\right)\right]}{c\left(q^f\right) - [\delta + \lambda(1-\delta)]\lambda c\left(\delta q^f\right)}.$$

In general, it is not possible to rank W^S and W^D, as we now show by example.

The only functional form we need is $c(q) = q^\varepsilon$, $\varepsilon > 1$. Given this, we can compute

$$\begin{aligned}
q^e &= \left[\frac{(r+M)\delta - (1-M)\lambda(1-\delta)}{M}\right]^{\frac{1}{\varepsilon-1}} \\
q^f &= \left[\frac{\delta(1-\lambda)}{1-\lambda\delta^\varepsilon}\right]^{\frac{1}{\varepsilon-1}} \\
\theta &= \frac{(r+M)\delta - M\frac{\delta(1-\lambda)}{1-\lambda\delta^\varepsilon}}{(1-M)\lambda(1-\delta)}.
\end{aligned}$$

The left panel of Figure 2 shows the value functions and welfare in the single-price equilibrium and in the equilibrium with price dispersion as functions of M (given $\delta = 0.15$, $\lambda = 0.25$, $\varepsilon = 2$ and $r = 0.1$). The curves are only drawn for values of M such that the relevant equilibria exist – for the two-price equilibrium this means $M < \overline{M}$, while for the single-price equilibrium this means $M \in (\underline{M}, \overline{M})$. As one can see, when the equilibria co-exist we can have $W^S > W^D$ or vice-versa, depending on M. The right panel shows q as a function of M. As one can see, it is not generally possible to rank prices across equilibria either.[7]

This completes the analysis of the model with $K = 2$. To reiterate, the main results are as follows. With no variability in δ, or with differences in

[7] One might conjecture that welfare is higher iff the average price if lower, but the example shows this is *not* true.

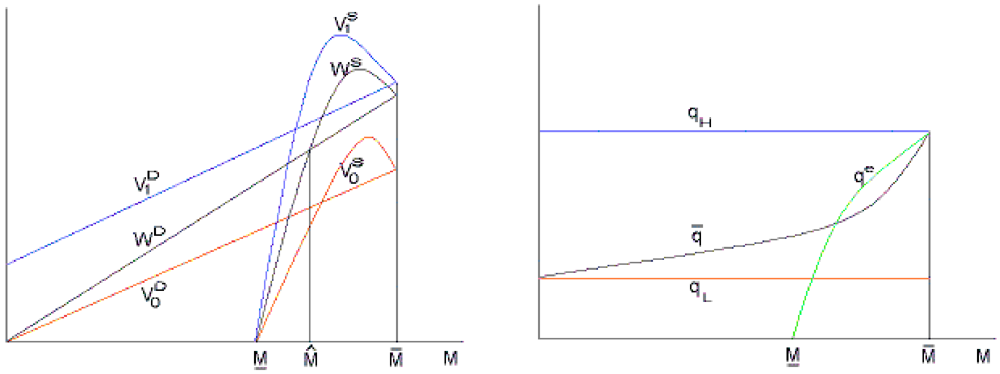

Figure 2: Welfare and prices in the example

δ that are permanent across agents, a price-posting equilibrium cannot have valued fiat currency. When δ varies across meetings, however, it is possible and even simple to construct monetary equilibria. Such equilibrium may or may not entail price dispersion, although the obvious generalization of Diamond's result implies that there can never be more than 2 prices when $K = 2$. In fact we found that the two-price equilibrium exists on a strictly larger subset of parameter space than the single-price equilibrium. We also showed the different equilibria cannot generally be ranked in terms of welfare or prices.

4 The General Model

We now move to the general case where in a random meeting the buyer has utility function $\delta_k q$ with probability λ_k, $k = 1, 2, \ldots, K$, with $\delta_1 < \delta_2 < \ldots <$

δ_K. Given any distribution $F(q)$ there will now be K reservation values, one for each δ_k, and therefore in equilibrium there can be at most K different q's posted with positive probability, say $q_1, q_2, \ldots, q_K$, where we order these so that $q_1 < q_2 < \ldots < q_K$.[8] The highest reservation value q corresponds to the lowest δ, $\delta_1 q_K = V_m - V_0$; the second highest q corresponds to the second lowest δ, $\delta_2 q_{K-1} = V_m - V_0$; and so on, until we reach the lowest q which corresponds to the highest δ, $\delta_K q_1 = V_m - V_0$. In general, we see that $\delta_{K+1-k} q_k = V_m - V_0$ for all k, and so

$$q_k = \frac{\delta_K}{\delta_{K+1-k}} q_1 \text{ for } k = 1, 2, \ldots, K. \tag{12}$$

A seller posting q_1 will sell to only buyers realizing the highest δ, which occurs in a given meeting with probability λ_K; a seller posting q_2 will sell to buyers realizing the highest or second highest δ, which occurs with probability $\lambda_K + \lambda_{K-1}$; and so on. In general, therefore, Bellman's equation for a seller posting q_k is

$$rV_k = M \sum_{j=K+1-k}^{K} \lambda_j [V_m - V_k - c(q_k)]. \tag{13}$$

When a buyer meets a seller posting q_k, he only derives gains from trade if he realizes δ_j with $j > K + 1 - k$. Hence, if θ_j denotes the fraction of sellers posting q_j, Bellman's equation for a buyer is

$$rV_m = (1 - M) \sum_{k=1}^{K} \theta_k \sum_{j=K+2-k}^{K} \lambda_j [\delta_j q_k + V_0 - V_m]. \tag{14}$$

[8] In case there is any doubt, the argument is this: any seller posting $q < q_1$ makes no sales, and any seller posting $q > q_1$ such that q is not one of the reservation values can earn more profit per sale without losing any sales by lowering q slightly.

These equations are all we need to prove the law of two prices.

Proposition 3 *Suppose $U = \delta_k q$ with probability λ_k, $k = 1, 2, \ldots, K$, for any K. Then for generic parameter values an equilibrium distribution $F(q)$ must have the following property: $\theta_k > 0$ for at most two values of k.*

Proof: Suppose $\theta_i > 0$, $\theta_j > 0$, and $\theta_k > 0$ for distinct i, j, and k. Since these must yield equal profit,

$$V_i = V_j = V_k = V_0. \tag{15}$$

Now use $V_m - V_0 = \delta_K q_1$ and (12) to rewrite (13) as

$$rV_k = M \sum_{j=K+1-k}^{K} \lambda_j \left[\delta_K q_1 - c\left(\frac{\delta_K}{\delta_{K+1-k}} q_1 \right) \right] \equiv g_k(q_1), \tag{16}$$

where $g_k(q_1)$ depends only on q_1, k and exogenous variables. By (15), $g_i(q_1) = g_j(q_1) = g_k(q_1)$. For generic parameter values, one cannot find a value of q_1 satisfying both of these equalities. ■

To develop some intuition for the result, consider Figure 3, which shows the value of being a seller as a function of the posted q, say $v_0(q)$, taking as given all other sellers' behavior as summarized by $F(q)$. Every time q crosses a reservation value q_k, $v_0(q)$ jumps discretely because the seller now gets customers with probability $\sum_{j=K+1-k}^{K} \lambda_j$ instead of $\sum_{j=K-k}^{K} \lambda_j$ – i.e., the probability of a sale in each single coincidence meeting increases by $\lambda_{K+1-k} > 0$. Now, *if* the reservation values were exogenous, $v_0(q)$ would generically be maximized at a single point in the set $\{q_1, ...q_K\}$. However, we know from the previous section that we can construct equilibria where two points in $\{q_1, ...q_K\}$ both maximize $v_0(q)$ by adjusting the endogenous reservation values.

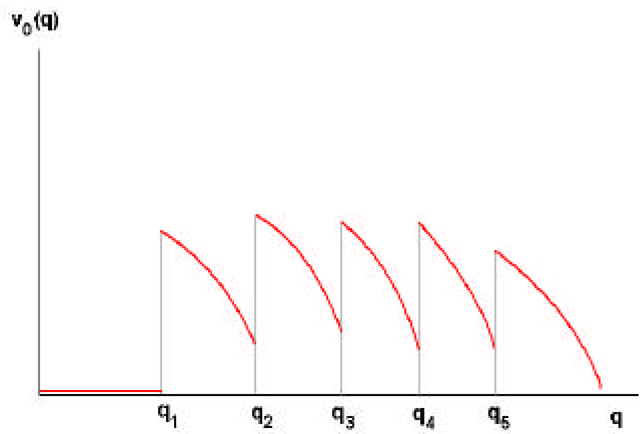

Figure 3: The function $v_0(q)$

Recall from the $K = 2$ case how this works: first pick q_H (and implicitly $q_L = \delta q_H$) so that $V_L = V_H$ and then chose θ so that q_H satisfies the reservation condition (i.e. $\delta q_H = V_m - V_0$). Given $K \geq 3$, let us try to pick three distinct points in $\{q_1, ...q_K\}$, say q_i, q_j and q_k, such that $V_i = V_j = V_k$. But note that we cannot pick q_i, q_j and q_k independently: all reservation values are proportional by virtue of (12). Hence, for generic parameter values, we can potentially pick q_i so that $V_i = V_j$, even though q_i and q_j are proportional, as we did in the case of $K = 2$. But then q_k is also pinned down since it is also proportional to q_i, and it would be a pure fluke if $V_k = V_i$.[9]

At this point there are several issues to consider. For one thing, it seems important to know which of the assumptions are critical for the result, and to what extent the result carries over to other models. We take this up in the next section. To close this section we want to address some technical points. First, although we know that θ_k can be positive for at most two values of k, we do not know which two – for example, must they be the two highest reservation values, the two lowest, or two consecutive values? Second, we would like to know if two-price equilibria are common – and in particular are they more common that single-price equilibria, as we found with $K = 2$? Although it may be hard to sort these issues out for the general case, we can learn a lot by looking at $K = 3$.

The method when $K = 3$ is the same as $K = 2$ except messier, so we will sketch the analysis briefly. First, (14) can be written

$$\begin{aligned} rV_m &= (1-M)\theta_2\lambda_3\left(\frac{\delta_3\delta_1 q_3}{\delta_2} + V_0 - V_m\right) \\ &\quad +(1-M)\theta_3[\lambda_2(\delta_2 q_3 + V_0 - V_m) + \lambda_3(\delta_3 q_3 + V_0 - V_m)], \end{aligned} \tag{17}$$

where using (12) we have substituted for q_1 and q_2 in terms of q_3. Similarly,

[9]Note that this logic does not depend on not any properties of the cost function $c(q)$, and in particular it does not require convexity. In Figure 3 the curvature of $c(q)$ is only relevant for determining the curvature of $v_0(q)$ between the points of $\{q_1, ...q_K\}$, which is not important since $\arg\max v_0(q)$ always lies in $\{q_1, ...q_K\}$. This is not to say that the curvature of $c(q)$ is irrelevant for all properties of the equilibrium set, such as the number of equilibria of a given type. For example, we do use the convexity of c in Appendix A to show there is a unique solution to $e(q) = 0$ and to $f(q) = 0$. The point is that the law of two prices does not depend on the convexity of c.

using (12) we can write the Bellman equations for sellers in terms of only q_3:

$$
\begin{aligned}
rV_1 &= M\lambda_3 \left[V_m - V_1 - c\left(\frac{\delta_1 q_3}{\delta_3}\right)\right] && (18)\\
rV_2 &= M(\lambda_2 + \lambda_3) \left[V_m - V_2 - c\left(\frac{\delta_1 q_3}{\delta_2}\right)\right] && (19)\\
rV_3 &= M[V_m - V_3 - c(q_3)]. && (20)
\end{aligned}
$$

These expressions lead to the following results:

$$
\begin{aligned}
V_3 - V_1 &\propto f_{31}(q_3) \equiv (1-\lambda_3)\delta_1 q_3 + \lambda_3 c\left(\frac{\delta_1 q_3}{\delta_3}\right) - c(q_3) && (21)\\
V_3 - V_2 &\propto f_{32}(q_3) \equiv \lambda_1 \delta_1 q_3 + (\lambda_2 + \lambda_3) c\left(\frac{\delta_1 q_3}{\delta_2}\right) - c(q_3). && (22)
\end{aligned}
$$

The virtue of (21)-(22) is that they can be used to tell whether $V_j - V_k$ is positive or negative for any j, k (e.g., the sign of $V_2 - V_1$ equals the sign of $f_{21} \equiv f_{31} - f_{32}$), which is what we need to check to see if any seller wants to deviate in a candidate equilibrium. Analogous to the function $f(q)$ in Section 3, there exists a unique positive solution to $f_{ij}(q_3) = 0$, say $q_3 = q^{f_{ij}}$, as shown in Figure 4 (note the figure shows $q^{f_{31}} > q^{f_{32}}$, but the reverse is also possible). We will use these relations to characterize the set of equilibria. With $K = 3$, there are exactly 6 candidate equilibria: 3 single-price equilibria and 3 two-price equilibria. However, there is no monetary equilibrium with $\theta_1 = 1$, since as in Section 3 if all sellers set the lowest reservation q then $V_m = 0$. Hence, there are 5 cases to analyze.

Consider first an equilibrium with $\theta_3 = 1$. We require two things: $V_3 = \max(V_k)$, and $\delta_1 q_3 = V_m - V_3$. Using (17) and (20), we can reduce the latter

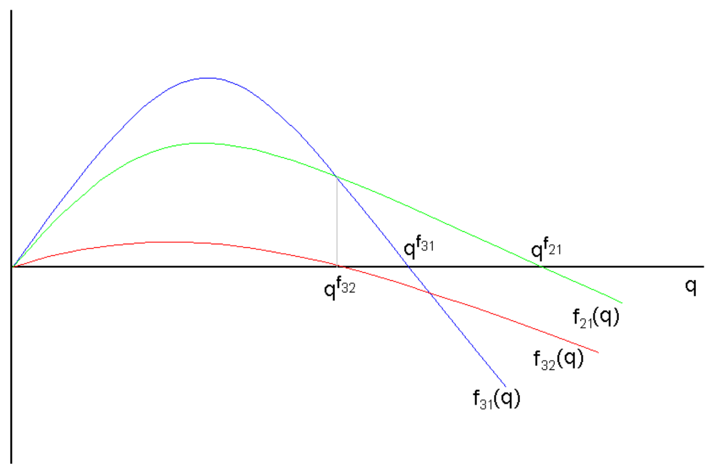

Figure 4: Functions f_{ij} (the case with $q^{f_{31}} > q^{f_{32}}$)

condition to $e_3(q_3) = 0$, where

$$e_3(q_3) \equiv \{(r+M)\delta_1 - (1-M)[\lambda_2(\delta_2 - \delta_1) + \lambda_3(\delta_3 - \delta_1)]\}q_3 - Mc(q_3).$$

One shows there can exist no more than one positive solution to $e_3(q_3) = 0$, and there exists a positive solution, say $q_3 = q^{e_3}$, iff $e_3'(0) > 0$ which holds iff

$$(r+M)\delta_1 - (1-M)[\lambda_2(\delta_2 - \delta_1) + \lambda_3(\delta_3 - \delta_1)] > 0. \tag{23}$$

If (23) does not hold there cannot exist an equilibrium with $\theta_3 = 1$. If (23) does hold, there exists a unique solution $q_3 = q^{e_3}$ to $e_3(q_3) = 0$, which is a candidate equilibrium. To verify that it is an equilibrium we check that no seller wants to deviate from q_3 to either $q_1 = \frac{\delta_1}{\delta_3}q_3$ or $q_2 = \frac{\delta_1}{\delta_2}q_3$. Using f_{31} and f_{32} from (21) and (22), this is true iff $f_{31}\left(q^{e_3}\right) \geq 0$ and $f_{32}\left(q^{e_3}\right) \geq 0$, or equivalently $e_3\left(q^{f_{31}}\right) \leq 0$ and $e_3\left(q^{f_{32}}\right) \leq 0$.

The region of the (M, r) plane in which the equilibrium conditions (23), $e_3\left(q^{f_{31}}\right) \leq 0$, and $e_3\left(q^{f_{32}}\right) \leq 0$ are all satisfied is given by

$$r > \underline{r}_3 \equiv A_3 - B_3 M \tag{24}$$

$$r \leq \bar{r}_{31} \equiv A_3 - B_{31} M \tag{25}$$

$$r \leq \bar{r}_{32} \equiv A_3 - B_{32} M, \tag{26}$$

where the constants A_3, B_3, etc. are given in Appendix B; all that concerns us here is that $\underline{r}_3$, $\bar{r}_{31}$ and $\bar{r}_{32}$ are decreasing linear functions of M with the same intercept, $\bar{r}_{31}$ and $\bar{r}_{33}$ have flatter slopes than $\underline{r}_3$, and $\bar{r}_{31}, \bar{r}_{32} < 0$ at $M = 1$ (see below). A similar argument implies that $\theta_2 = 1$ is an equilibrium when

$$r > \underline{r}_2 \equiv A_2 - B_2 M \tag{27}$$

$$r \leq \bar{r}_{21} \equiv A_2 - B_{21} M \tag{28}$$

$$r \leq \bar{r}_{23} \equiv A_2 - B_{23} M \tag{29}$$

where the constants are also given in Appendix B, and $\underline{r}_2$, $\bar{r}_{21}$ and $\bar{r}_{23}$ have similar properties to the previous case.

This exhausts the possible single-price equilibria. We now turn to two-price equilibria. Consider first $\theta_1, \theta_3 > 0$. This requires $V_3 - V_1 = 0$, which means $q_3 = q^{f_{31}}$, and $\delta_1 q_3 = V_m - V_3$, which holds iff $\theta_3 = \theta_3\left(q^{f_{31}}\right)$. The closed form solution for θ_3 is given in Appendix B, from which one can check $\theta_3 \in (0, 1)$ iff $r < \bar{r}_{31}$. The last thing we require is that no seller has an incentive to deviate to q_2, which is true iff $f_{32}\left(q^{f_{31}}\right) \geq 0$, which is true iff $r \leq \bar{r}_{32}$. Similarly, an equilibrium with $\theta_1, \theta_2 > 0$ exists iff $r < \bar{r}_{21}$ and $r \leq \bar{r}_{23}$, and an equilibrium with $\theta_2, \theta_3 > 0$ exists iff $\bar{r}_{23} < r < \bar{r}_{32}$ and $r < \bar{r}_{31}$. This exhausts the possible equilibria.

Figure 5 shows the regions of parameter space where the different equilibria exist. As things depend a lot on f_{31} and f_{32}, there are two cases to consider: $q^{f_{31}} > q^{f_{32}}$, which implies $\underline{r}_{32} < \underline{r}_{31}$ and $\underline{r}_{23} < \underline{r}_{21}$ as in the left panel; and $q^{f_{31}} < q^{f_{32}}$, which implies $\underline{r}_{32} > \underline{r}_{31}$ and $\underline{r}_{23} < \underline{r}_{21}$ as in the right panel (see Appendix B for the proof of the relevant inequalities). In the first case we have the following equilibria: $\theta_1, \theta_2 > 0$ exists in regions 1, 2 and 3; $\theta_2, \theta_3 > 0$ exists in regions 3, 4 and 5; $\theta_2 = 1$ exists in region 2; and $\theta_3 = 1$ exists in region 5. In the second case we have the following equilibria: $\theta_1, \theta_3 > 0$ exists in regions A, B and C; $\theta_2 = 1$ exists in region B; and $\theta_3 = 1$ exists in region C. In either case, these are all the (steady state, monetary) equilibria.

Notice in one case that the equilibrium with $\theta_1, \theta_3 > 0$ exists if any monetary equilibria exist, while in the other this equilibrium does not exist at all but the other two two-price equilibria both exist for some parameters. Both of the possible single-price monetary equilibria exist for some parameters in each case. So all of the possible equilibria can exist, and in particular a two-price equilibrium can involve the two highest reservation values, the two lowest, or the highest and the lowest. Further notice that there can co-exist

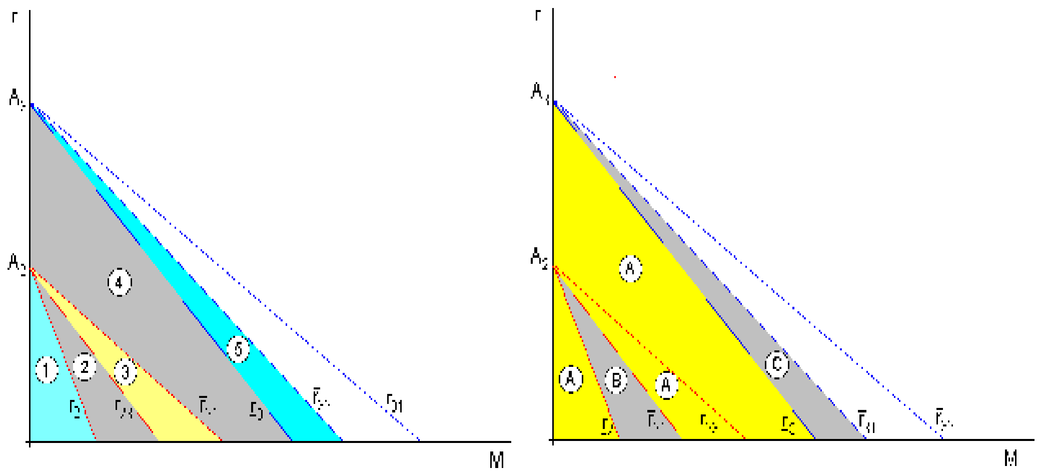

Figure 5: Existence regions with $K = 3$

multiple two-price equilibria. Finally, notice that, as in we found in Section 3, whenever a single-price equilibrium exists so does a two-price equilibrium but not vice-versa. Hence, although we cannot say for sure that it is true for any K, at least we know the finding that two-price equilibria are more common that single-price equilibria is not only true when $K = 2$.

5 Discussion

Here we discuss the role of the key assumptions and study some alternative specifications. The first thing to mention is that we have only looked at discrete random variables, $\delta = \delta_k$ with probability λ_k, $k = 1, 2, ...K$. In a related model, Jafarey and Masters (2000) assume δ is uniformly distributed on some interval. It turns out that this implies their model has only single-price equilibria. Of course, this is not inconsistent with our law of two prices, which only says there will be two or fewer prices, but what it does indicate is that we cannot say that two prices are *always* more common than one. The key to their result is that there is now a continuum of reservation values, one corresponding to each realization of δ, and therefore the function $v_0(q)$ in Figure 3 will not have any discrete jumps. Indeed, one can easily check that $v_0(q)$ is strictly concave when δ is uniform, and so there must be a single price in equilibrium.

Masters (2000) argues that $v_0(q)$ is strictly concave, and hence equilibrium must involve a single price, more generally whenever the density of δ is non-decreasing. He also shows that when δ is discrete, in the special case where $\lambda_k = 1/K$ for all k, $v_0(q)$ is strictly concave across the points of $\{q_1, ...q_K\}$. However, this is not true in our model, due to a technical difference in assumptions.[10] This is important, since the strict concavity of $v_0(q)$ across the points of $\{q_1, ...q_K\}$ would imply that any possible two-price equilibrium must involve two consecutive reservation values, and in the previous section we constructed an equilibrium with $K = 3$ where we had $\theta_1 > 0$ and $\theta_3 > 0$. In any case, we leave for future work the derivation of more results

[10] He assumes sellers pay $c(q)$ ex ante, before entering the search process, while in our model they only pay $c(q)$ upon making an actual trade. His assumption makes concavity more likely since $v_0(q)$ is additively separable between the cost of production and the probability of a sale, while in our specification these terms interact.

with other special discrete distributions and with continuous distributions.

The next thing we do is to consider a nonmonetary version of the model, so see which results are particular to economies where trade uses fiat currency. While there are many potentially interesting ways to set up a nonmonetary alternative, including labor market applications, we stick to a model of consumer search to keep things otherwise close to the above specification. As before, there are M buyers and $1-M$ sellers, but now sellers offer q units of the good in exchange for p units of a general good for which utility is linear, rather than money (i.e., it is a transferable utility model). Moreover, the sellers stay in the market forever, while the buyers stay only until they make a trade, at which point they exit and get replaced by new buyers.[11] We assume for now that sellers post an endogenous q in exchange for a fixed p, which provides the most natural comparison with our earlier model, but we also consider below models where they post p for a fixed q.

As above, in each meeting with a seller the buyer realizes utility function $U=\delta_k q$ with probability λ_k, $k=1,2,...K$. Given any distribution $F(q)$, a buyer's value function satisfies

$$rV_b=(1-M)\sum_{i=1}^{K}\lambda_i\int\max\{\delta_i q-p-V_b,0\}dF(q). \tag{30}$$

Clearly there is a reservation value corresponding to each realization of δ,

[11] The results were similar in other formulations we tried in terms of whether different agents trade once or stay in the market forever. We had all agents stay in the market forever in previous sections because this is standard in monetary search models; in nonmonetary models it is common to have one or both sides exit after trade, and so we adopted assumptions that make the algebra easier.

satisfying $\delta_k q_k = p + V_b$, and no seller would ever post anything other than one of these reservation values. At this point it is easy to use the same strategy we used for Proposition 2 to show that generically there cannot be more than two values of q_k actually posted: we cannot generically find three values for q_k that yield equal profit, because they are not independent since they all satisfy $\delta_k q_k = p + V_b$.

We conclude that our law of two prices has nothing to do with monetary exchange, per se. However, some things do depend on money. For instance, suppose $K = 2$ and let $\delta_2 = 1$, $\delta_1 = \delta < 1$, and $\lambda_2 = \lambda \in (0, 1)$. Letting θ be the fraction of sellers setting $q_H = p + V_b$, Bellman's equations are now

$$\begin{aligned} rV_b &= (1 - M)\lambda\theta(q_H - p - V_b) && (31) \\ rV_H &= M[p - c(q_H)] && (32) \\ rV_L &= M\lambda[p - c(q_L)]. && (33) \end{aligned}$$

We can solve (31) for V_b in terms of q_H, which can then be combined with the reservation condition $q_H = p + V_b$ to yield

$$q_H = Q(\theta) \equiv \frac{rp}{r\delta - (1 - M)\lambda\theta(1 - \delta)}. \tag{34}$$

Substituting q_H and $q_L = \delta q_H$ into the sellers' Bellman equations, we find $V_H - V_L$ is proportional to

$$E(\theta) = p(1 - \lambda) - c[Q(\theta)] + \lambda c[\delta Q(\theta)]. \tag{35}$$

An equilibrium requires either: $\theta = 1$ and $E(1) \geq 0$; $\theta = 0$ and $E(0) \leq 0$; or $\theta \in (0,1)$ and $E(\theta) = 0$.

It is immediate that equilibrium always exists, and is unique because $E'(\theta) < 0$ for all $\theta \in (0,1)$. We can get any of the three types of equilibria, depending on parameters; e.g., it is easy to work out an example with $c(q) = q^{\varepsilon}$ and verify that $\theta = 1$ for small p, $\theta = 0$ for large p, and $\theta \in (0,1)$ for intermediate p. Hence, price dispersion is possible here as it was with fiat currency. What is different is that now we always have a unique equilibrium. It is not surprising that the monetary economy is more likely to display multiplicity, but it is interesting in this context because that multiplicity allowed us to conclude (at least for $K = 2$ or 3) that two-price equilibria are more robust than single-price equilibria, in the sense that they exist on a strictly larger subset of parameter space.[12]

For completeness, to facilitate comparison with the literature, and because we use it below, we also sketch the model where we fix $q = 1$ and let sellers post p. Buyers' Bellman equation is now

$$rV_b = (1 - M) \sum_{i=1}^{K} \lambda_i \int \max\{\delta_i - p - V_b, 0\} dF(p). \tag{36}$$

For each δ_k there is a reservation price $p_k = \delta_k - V_b$, but again the law of two prices holds for any K. In the $K = 2$ case, which makes this very similar

[12] We also report the following results: $\theta = 0$ implies q_H and q_L are independent of M; $\theta \in (0,1)$ again implies they are independent of M, but since θ is increasing in M so is the average $\bar{q}$; and $\theta = 1$ implies q_H and q_L are decreasing in M. So the unusual comparative static results from the monetary model carry over here in the $\theta \in (0,1)$ equilibrium but not the $\theta = 1$ equilibrium.

to the model in Diamond (1987), it is easy to show there exists a unique equilibrium and it may entail $\theta = 0$, $\theta = 1$, or $\theta \in (0, 1)$, depending on parameters. Hence, this model behaves much like the one where sellers set q – but it is worth presenting it because it provides the easiest vehicle within which to address the next issue.

The next issue is in some sense the most critical assumption in all of the above models: the assumption that δ is purely match specific, and not an intrinsic characteristic of an agent. We already argued that when different individuals have permanently different values of δ monetary equilibria must unravel, but this is not necessarily a problem in a nonmonetary economy. Thus, we now consider a model like the one in the previous paragraph, except that there are now K distinct types of buyers each with a permanently different utility parameter δ_k (with $K = 2$ this is exactly Diamond [1987]). Let V_b^k be the value function for a buyer of type k. His reservation price solves $p_k = \delta_k - V_b^k$, which differs from the reservation price equation in the previous model, $p_k = \delta_k - V_b$, since now the value functions differ across types. Still, there will be at most K prices posted in equilibrium, by the usual argument.

To make the point it suffices to consider $K = 3$ and $c(q) \equiv 0$. Letting θ_i now be the fraction of sellers setting p_i, we will construct an equilibrium with $\theta_i > 0$ for all i. The Bellman equations for the three different types of buyers are

$$
\begin{aligned}
rV_b^1 &= 0 && (37)\\
rV_b^2 &= (1-M)\theta_1(\delta_2 - p_1 - V_b^2) && (38)\\
rV_b^3 &= (1-M)\theta_1(\delta_3 - p_1 - V_b^3) + (1-M)\theta_2(\delta_3 - p_2 - V_b^3). && (39)
\end{aligned}
$$

These equations have a recursive structure: (37) implies immediately $p_1 = \delta_1 - V_b^1 = \delta_1$. Substituting this into (38), we can solve for V_b^2 as a function of θ_1 and use $p_2 = \delta_2 - V_b^2$ to determine $p_2 = p_2(\theta_1)$, given in Appendix C. Then we can substitute p_1 and p_2 into (39) and use $p_3 = \delta_3 - V_b^3$ to determine $p_3 = p_3(\theta_1, \theta_2)$, also given in Appendix C. It is now a matter of algebra to write down the value functions for sellers who set the three different prices and then use $V_1 = V_2 = V_3$ to determine the θ's, again reported in Appendix C.

Since the results are somewhat messy we numerically calculate the θ's for various values of M and display the outcome in Figure 6. Whenever $\theta_i > 0$ for all three i we have an equilibrium with more than two prices. As shown, this is indeed possible for a range of M. Therefore we conclude that our law of two prices *does* depend on δ_k being idiosyncratic to meetings and not a permanent characteristics of an individual. It should not have been too surprising that our result would not hold in all possible models, of course, since for one thing there are examples in the literature of endogenous price

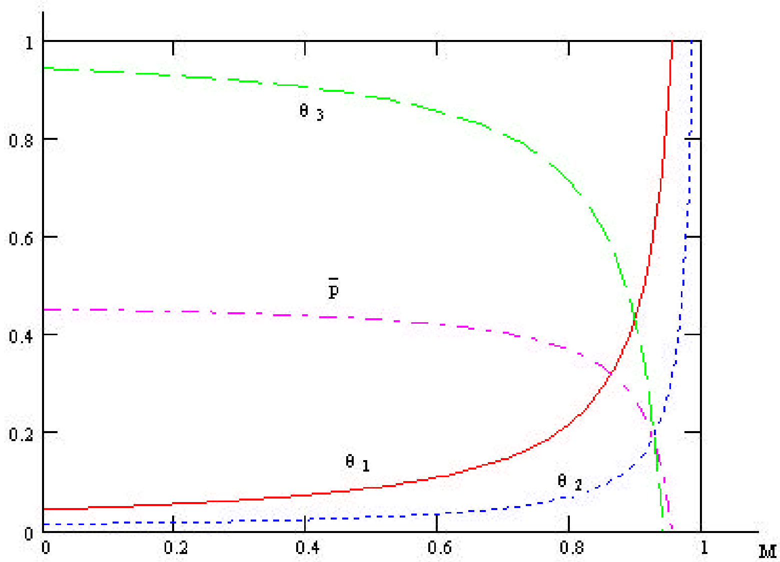

Figure 6: Equilibrium with $\theta_i > 0$, $i = 1, 2, 3$

or wage distributions with more than two prices.[13] We think the law of two prices is interesting even though there are alternative models in which it does not hold; in any case it is good to know what assumptions are behind it.

6 Conclusion

To sum up, we have introduced a framework that combines search, money, price setting, and preference parameters that differ randomly across buyer-

[13] A leading example is Burdett and Mortensen (1998), where they show in an on-the-job search model that the unique equilibrium has a continuous wage distribtuion even though workers are ex ante homogeneous. Intuitively, although workers are ex ante homogenous, if two workers have different wages they are effectively heterogeneous in their search for better jobs. Given a continuous wage distribution there is effectively a continuous distribution of worker types, which supports a continuous wage distribution as an equilibrium.

seller meetings, and we have used the model to analyze the value of money and the distribution of prices. As a contribution to monetary economics, we showed that price-setting equilibria exist with valued fiat currency. This is not true when preferences are constant, nor when preferences differ permanently across individuals – we really do need them to vary across meetings. One may or may not be surprised by these results, although it does seem worthwhile to try to understand monetary models with price posting, instead of the standard ex post bargaining, especially given our preference structure. Still, perhaps the contribution is not so much to integrate price posting into search models of money, but to extend the literature on price dispersion.

Along this dimension, we showed that equilibria may not only violate the law of one price, as others have shown in different contexts, but that two prices are more common than one in our base model. Perhaps the most interesting result is that there can be at most two prices, given that buyers are homogenous ex ante but preferences differ across meetings. This result holds also holds in nonmonetary models, although some things do differ once fiat money is introduced (e.g., multiple equilibria with different price distributions). The result does not necessarily hold if preferences differ permanently across individuals, but it still seems interesting to understand the nature of price dispersion when agents are homogeneous but preferences are random. While one might have guessed that price dispersion was possible in such models, it was surprising to us that such economies imply the law of two prices.

Appendix A

Here we provide the technical results used in Section 3.

Lemma 1 $e'(q^e) < 0$.

Proof: $e'(q) = (r+M)\delta - (1-M)\lambda(1-\delta) - Mc'(q_H)$. If q^e solves $e(q^e) = 0$ then $q^e = Mc(q^e)/[(r+M)\delta - (1-M)\lambda(1-\delta)]$, and therefore $e'(q^e) = \frac{M}{q^e}[c(q^e) - qc'(q^e)] < 0$, since for any convex function $c(q)$, $c(q) - qc'(q)$ is negative. ■

Lemma 2 $f'\left(q^f\right) < 0$.

Proof: First observe that $f(0) = 0$ and $f'(0) = \delta(1-\lambda) > 0$. Hence, $f(q) > 0$ for some small $q > 0$. Recalling that $\hat{q} = c(\hat{q})$, notice $f\left(\frac{\hat{q}}{\delta}\right) = (1-\lambda)\hat{q} - c\left(\frac{\hat{q}}{\delta}\right) + \lambda c(\hat{q}) = \hat{q} - c\left(\frac{\hat{q}}{\delta}\right) < 0$. By continuity, there exists a $q^f \in \left(0, \frac{\hat{q}}{\delta}\right)$ such that $f\left(q^f\right) = 0$. Rearranging $f\left(q^f\right) = 0$ yields $q^f = \frac{c\left(q^f\right) - \lambda c\left(\delta q^f\right)}{\delta(1-\lambda)}$, which implies

$$\begin{aligned} f'\left(q^f\right) &= \delta(1-\lambda) - c'\left(q^f\right) + \lambda\delta c'\left(\delta q^f\right) \\ &= \left[c\left(q^f\right) - q^f c'\left(q^f\right)\right] - \lambda\left[c\left(\delta q^f\right) - \delta q^f c'\left(\delta q^f\right)\right]. \end{aligned}$$

For any convex function $c(q)$, $c(q) - qc'(q)$ is not only negative it is also decreasing; hence, the first term in the previous expression is more negative than the second, and we conclude $f'\left(q^f\right) < 0$. ■

Lemma 3 $0 < \theta(q^f) < 1$ *iff* $e\left(q^f\right) < 0$.

Proof: Clearly, $\theta(q) > 0$ iff $(r+M)\delta q > Mc(q)$. Since $f\left(q^f\right) = 0$ implies $q^f = \frac{c\left(q^f\right)-\lambda c\left(\delta q^f\right)}{\delta(1-\lambda)}$, we have $\theta\left(q^f\right) > 0$ iff $(r+M)\left[c\left(q^f\right) - \lambda c\left(\delta q^f\right)\right] > Mc\left(q^f\right)(1-\lambda)$. This last condition is equivalent to $(r+\lambda M)c\left(q^f\right) > (r+M)\lambda c\left(\delta q^f\right)$, which is always true. Hence $\theta(q^f) > 0$ for all parameters. It is a matter of algebra to show $\theta(q^f) < 1$ iff $e\left(q^f\right) < 0$. ■

Lemma 4 *$\partial\theta/\partial M > 0$ in the two-price equilibrium and $\partial q/\partial M > 0$ in the one-price equilibrium.*

Proof: In the two-price equilibrium, we have $\partial\theta/\partial M = A[(1+r)\delta q^f - c\left(q^f\right)]$, where $A > 0$. Using $q^f = \frac{c\left(q^f\right)-\lambda c\left(\delta q^f\right)}{\delta(1-\lambda)}$, we see that $\partial\theta/\partial M$ takes the same sign as $(r+\lambda)c\left(q^f\right) - \lambda(r+1)c\left(\delta q^f\right) > 0$. In the single-price equilibrium, $\partial q^e/\partial M = B[\delta + \lambda(1-\delta)]q^e - Bc\left(q^e\right)$, where $B > 0$. Inserting $c\left(q^e\right) = q^e[(r+M)\delta - (1-M)\lambda(1-\delta)]/M$, we see that $\partial q^e/\partial M$ takes the same sign as $\lambda(1-\delta) - r\delta$, which is positive as long as $r < \bar{r}$, which must be the case for the single-price equilibrium to exist. ■

Appendix B

Here we provide a few results related to the analysis of the case $K = 3$. First, the constants in (24)-(29) are given by

$$A_3 = \frac{\lambda_2(\delta_2-\delta_1)+\lambda_3(\delta_3-\delta_1)}{\delta_1} \qquad B_3 = \frac{\lambda_2(\delta_2-\delta_1)+\lambda_3(\delta_3-\delta_1)+\delta_1}{\delta_1}$$

$$B_{31} = \frac{[\lambda_2(\delta_2-\delta_1)+\lambda_3\delta_3]c\left(q^{f31}\right) - \lambda_3[\delta_1+\lambda_2(\delta_2-\delta_1)+\lambda_3(\delta_3-\delta_1)]c\left(\frac{\delta_1 q^{f31}}{\delta_3}\right)}{\delta_1 c\left(q^{f31}\right) - \delta_1\lambda_3 c\left(\frac{\delta_1 q^{f31}}{\delta_3}\right)}$$

$$B_{32} = \frac{(\lambda_2\delta_2+\lambda_3\delta_3)c\left(q^{f32}\right)-(\lambda_2+\lambda_3)[\delta_1+\lambda_2(\delta_2-\delta_1)+\lambda_3(\delta_3-\delta_1)]c\left(\frac{\delta_1 q^{f32}}{\delta_2}\right)}{\delta_1 c\left(q^{f32}\right)-\delta_1(\lambda_2+\lambda_3)c\left(\frac{\delta_1 q^{f32}}{\delta_2}\right)},$$

and

$$A_2 = \frac{\lambda_3(\delta_3-\delta_2)}{\delta_2} \qquad B_2 = \frac{\lambda_2\delta_2+\lambda_3\delta_3}{\delta_2}$$

$$B_{21} = \lambda_3\frac{\delta_3(\lambda_2+\lambda_3)c\left(\frac{\delta_1 q^{f21}}{\delta_2}\right)-(\lambda_2\delta_2+\lambda_3\delta_3)c\left(\frac{\delta_1 q^{f21}}{\delta_3}\right)}{\delta_2(\lambda_2+\lambda_3)c\left(\frac{\delta_1 q^{f21}}{\delta_2}\right)-\delta_2\lambda_3 c\left(\frac{\delta_1 q^{f21}}{\delta_3}\right)}$$

$$B_{23} = \frac{(\lambda_2\delta_2+\lambda_3\delta_3)c\left(q^{f23}\right)-(\lambda_2+\lambda_3)[\lambda_3\delta_3+\delta_2(1-\lambda_3)]c\left(\frac{\delta_1 q^{f23}}{\delta_2}\right)}{\delta_2 c\left(q^{f23}\right)-\delta_2(\lambda_2+\lambda_3)c\left(\frac{\delta_1 q^{f23}}{\delta_2}\right)}.$$

We also report the probabilities in the two-price equilibria. In the equilibrium with $\theta_2 = 0$, we have

$$\theta_3 = \frac{(r+M\lambda_3)\delta_1 c\left(q^{f31}\right)-(r+M)\delta_1\lambda_3 c\left(\frac{\delta_1 q^{f31}}{\delta_3}\right)}{(1-M)[\lambda_3(\delta_3-\delta_1)+\lambda_2(\delta_2-\delta_1)]\left[c(q^{f31})-\lambda_3 c(\frac{\delta_1 q^{f31}}{\delta_3})\right]};$$

in the equilibrium $\theta_3 = 0$ we have

$$\theta_2 = \frac{(r+M\lambda_3)\delta_2(\lambda_2+\lambda_3)c\left(\frac{\delta_1 q^{f21}}{\delta_2}\right)-[r+M(\lambda_2+\lambda_3)]\delta_2\lambda_3 c\left(\frac{\delta_1 q^{f21}}{\delta_3}\right)}{(1-M)\lambda_3(\delta_3-\delta_2)\left[(\lambda_2+\lambda_3)c\left(\frac{\delta_1 q^{f21}}{\delta_2}\right)-\lambda_3 c\left(\frac{\delta_1 q^{f21}}{\delta_3}\right)\right]};$$

and in the equilibrium with $\theta_1 = 0$ we have

$$\theta_3 = \frac{[r\delta_2-\lambda_3(\delta_3-\delta_2)+M(\lambda_2\delta_2+\lambda_3\delta_3)]\delta_1 c\left(q^{f32}\right)}{(1-M)(\lambda_2\delta_2+\lambda_3\delta_3)(\delta_2-\delta_1)\left[c\left(q^{f32}\right)-(\lambda_2+\lambda_3)c\left(\frac{\delta_1 q^{f32}}{\delta_2}\right)\right]}$$
$$-\frac{\{r\delta_2-\lambda_3(\delta_3-\delta_2)+M[\lambda_3(\delta_3-\delta_1)+\delta_2]\}\delta_1(\lambda_2+\lambda_3)c\left(\frac{\delta_1 q^{f32}}{\delta_2}\right)}{(1-M)(\lambda_2\delta_2+\lambda_3\delta_3)(\delta_2+\delta_3)\left[c\left(q^{f32}\right)-(\lambda_2+\lambda_3)c\left(\frac{\delta_1 q^{f32}}{\delta_2}\right)\right]}.$$

Given these results it is matter of algebra to verify most of the claims in the text, such as finding the parameter restrictions that imply $\theta_j \in (0,1)$. The only thing left to establish is the claim used in drawing Figure 5.

Lemma 5 *(a)* $q^{f_{31}} > q^{f_{32}}$ *implies* $\overline{r}_{31} > \overline{r}_{32}$ *and* $\overline{r}_{23} < \overline{r}_{21}$*; (b)* $q^{f_{31}} < q^{f_{32}}$ *implies* $\overline{r}_{31} < \overline{r}_{32}$ *and* $\overline{r}_{23} > \overline{r}_{21}$*.*

Proof: For part (a), first observe that $q^{f_{31}} > q^{f_{32}}$ implies $f_{31}(q) > 0$ whenever $f_{32}(q) > 0$; see Figure 4. Now, recall that for a $\theta_3 = 1$ equilibrium, we have two no-deviation constraints: $r \leq \overline{r}_{32}$ (which guarantees $f_{32} \geq 0$) and $r \leq \overline{r}_{31}$ (which guarantees $f_{31} \geq 0$). By the first observation, only the former is binding, which means that $\overline{r}_{31} > \overline{r}_{32}$. A similar argument verifies $\overline{r}_{23} < \overline{r}_{21}$. The proof of part (b) is symmetric. ■

Appendix C

Here we give some details related to the model in Section 5 with permanent differences in utility. First we have the reservation prices for types 2 and 3:

$$\begin{aligned} p_2 &= p_2(\theta_1) \equiv \tfrac{r\delta_2+(1-M)\theta_1\delta_1}{r+(1-M)\theta_1} \\ p_3 &= p_3(\theta_1,\theta_2) \equiv \tfrac{[r+(1-M)\theta_1]r\delta_3+(1-M)\theta_2 r\delta_2+(1-M)\theta_1[r+(1-M)(\theta_1+\theta_2)]\delta_1}{[r+(1-M)\theta_1][r+(1-M)(\theta_1+\theta_2)]}. \end{aligned}$$

Then $V_1 = V_2$ yields

$$\theta_1 = \tfrac{r}{\lambda_1(1-M)}\left[\tfrac{(\lambda_2+\lambda_3)\delta_2-\delta_1}{\delta_1}\right],$$

$V_1 = V_3$ yields

$$\theta_2 = \tfrac{r}{\lambda_1(1-M)}\left(\tfrac{\lambda_1^2(\delta_1-\lambda_3\delta_2)\delta_1+\{(\lambda_2+\lambda_3)\delta_2-\delta_1\}\{\lambda_1(2+\lambda_3)\delta_1+(1-\lambda_3)[(\lambda_2+\lambda_3)\delta_2-\delta_1]-\lambda_1\lambda_3\delta_3\}}{(\lambda_3-1)[(\lambda_2+\lambda_3)\delta_2-\delta_1]-\lambda_1\delta_1+\lambda_1\lambda_3\delta_2}\right),$$

and $\theta_3 = 1 - \theta_1 - \theta_2$. It is now a matter of checking when all three θ's are positive.

References

Albrecht, James and Bo Axell (1984) "An Equilibrium Model of Search Unemployment," *Journal of Political Economy* 92, 824-40.

Albrecht, James and Susan Vroman (2000) "Wage Dispersion in a Job Search Model with Time-Varying Unemployment Benefits," mimeo.

Boyarchenko, Svetlana (2000) "A Monetary Search Model with Heterogeneous Agents," mimeo.

Camera, Gabriel and Dean Corbae (1999) "Money and Price Dispersion," *International Economic Review* 40, 985-1008.

Peter Diamond (1970) "A Model of Price Adjustment," *Journal of Economic Theory* 3, 156-168.

Peter Diamond (1987) "Consumer Differences and Prices in a Search Model," *Quarterly Journal of Economics* 102, 429-436.

Green, Edward and Ruilin Zhou (1998) "A Rudimentary Model of Search with Divisible Money and Prices," *Journal of Economic Theory* 81, 252-71.

Jafarey, Saquib and Adrian Masters (2000) "Money and Prices," mimeo.

Kiyotaki, Nobuhiro and Randall Wright (1989) "On Money as a Medium of Exchange," *Journal of Political Economy* 97, 927-54.

Kiyotaki, Nobuhiro and Randall Wright (1991) "A Contribution to the Pure Theory of Money," *Journal of Economic Theory* 53, 215-235.

Kudoh, Noritaka (2000) "Matching, Bargaining, and Dispersed Values of Fiat Currency," mimeo.

Molico, Miguel (1998) "The Distribution of Money and Prices in Search Equilibrium," mimeo.

Masters, Adrian (2000) "Nonexistence of Dispersed Wage Equilibria," mimeo.

Shi, Shouyong (1995) "Money and Prices: A Model of Search and Bargaining," *Journal of Economic Theory* 67, 467-496.

Trejos, Alberto and Randall Wright (1995) "Search, Bargaining, Money and Prices," *Journal of Political Economy* 103, 118-141.

Zhou, Ruilin (1999) "Individual and Aggregate Real Balances in a Random Matching Model," *International Economic Review* 401009-1038

Wallace, Neil and Ruilin Zhou (1997) "A Model of a Currency Shortage," *Journal of Monetary Economics* 40, 555-572.