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PIER Working Paper 02-038

"Crime, Inequality, and Unemployment"

by

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http://ssrn.com/abstract_id=339160

Crime, Inequality, and Unemployment*

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May 3, 2001

Abstract

There has been much discussion of the relationships between crime, inequality and unemployment. We construct a model where all three are endogenous. Introducing crime into otherwise standard models affects the labor market in several interesting ways. For example, we show how the crime rate affects the unemployment rate and vice-versa; how the possibility of criminal activity can lead to wage inequality among homogeneous workers; and how the possibility of crime can generate multiple equilibria in natural but previously unexplored ways. In particular, two fundamentally identical neighborhoods may easily end up with different levels of unemployment, inequality, and crime. The model can be used to study the equilibrium effects of anti-crime policies, such as changes in apprehension rates or jail sentences, as well as more traditional labor market policies such unemployment insurance.

^{*}We thank participants in several seminars and conferences, especially James Albrecht and Hugo Hopenhayn, for their comments and suggestions. Lagos thanks STICERD and Wright the NSF for financial support. The usual disclaimer applies.

1. Introduction

In homage to Joe Friday, we begin with "just the facts." At the turn of the millennium, 6.3 million people in the US were in jail or prison, on probation and on parole. Of these, 2.1 million were in prison or jail. As one might expect, the numbers differ across sex (only 91,000 females are in prison), and race (3.4% of black males, 1.3% of Hispanic males, and 0.4% of white males are in prison). These numbers are high, and rising: during the 1990s the average annual increase in state and federal prisons was 6.5%, with a 13.4% increase in federal prisons in 1999 alone. At the same time, crime rates have been falling (see Merlo [2001] and the references contained therein). Given the large numbers of people incarcerated, and by implication the large number committing crimes, it seems interesting to analyze the interactions between this and more conventional economic activities. In this paper, we focus on theoretical models of crime in the context of the labor market, and in particular on the relationship between crime, unemployment, and inequality.

There is of course a previous theoretical and empirical literature on the relationships between these variables.¹ A novel element of this project is that crime, inequality, and unemployment are all endogenous. The framework illuminates the relationships between these variables (e.g., we can show how the crime rate affects the unemployment rate and vice-versa). It can also be used to study the equilibrium effects of alternative policies to combat crime, such as changes in apprehension rates or jail sentences, as well as more traditional labor market policies such unemployment insurance. Moreover, we find that introducing the possibility of criminal activity into otherwise standard models can alter the normal functioning of the labor market in a big way. For example, once crime is incorporated, models that would otherwise

¹In terms of models, in addition to the seminal work of Becker (1968), examples include Sah (1991), Benoît and Osborne (1995), Tabarrok (1997), Fender (1999), and İmrohoroğlu et al. (1999, 2000). In terms of the data, the positive correlation between labor-income inequality and property crime in the US is discussed in İmrohoroğlu et al. (1999), while both time-series and cross-sectional studies generally find that unemployment is positively correlated with crime, although the link is not always strong (see Freeman [1996] for a survey).

predict a single wage can generate wage dispersion. Even in models that otherwise generate wage inequality, the nature of inequality can change qualitatively once crime is introduced.

A key finding is that in models that otherwise yield a unique equilibrium, once crime is allowed we can generate multiple equilibria in natural but previously unexplored ways. Hence, two intrinsically identical neighborhoods may end up with very different amounts of crime. This is especially interesting in light of recent empirical work (e.g., Glaeser et al. [1996]) that finds it is very difficult to account for the high variance of crime rates across locations based on observable characteristics. One channel of multiplicity that we emphasize, and one which could only arise in a model with endogenous wage setting, is this: Suppose there are lots of good jobs available – i.e., lots of firms paying enough to dissuade their workers from crime. With market conditions favorable in this sense, all agents are less inclined to criminal activity. This makes it easier for any given firm to pay wages that keep its workers honest.

Hence, there is a strategic complementarity in wage setting that can give rise to multiple equilibria, although only for some parameters. In particular we find that jail sentences have to relatively long on average for multiple equilibria to arise through the above channel. Intuitively, favorable market conditions are more likely to dissuade workers from crime when conviction takes one out of the market for a long time. We also explore other channels of multiplicity. For example, suppose the probability of being victimized depends on the number of criminals. Then the less crime there is, the higher is the relative attractiveness of legitimate versus criminal activity, and hence the less crime there will be. This effect seems quite general and should be relevant for any analysis in the economics of crime, independent of the actual details of the model.²

²Other sources of multiplicity, such as congestion in law enforcement, have been discussed in the literature (see e.g., Fender 1999, Tabarrok 1997, Sah 1991, and Murphy et al. 1993). While these may be empirically relevant, we focus here on channels that have not been analyzed before.

Our base model workers randomly encountering firms that post wages. Workers also randomly encounter opportunities to commit crimes and become victims of crime. Criminals face a probability of being caught, which may lead to a fine, a prison sentence, and the loss of one's job if currently employed. The decision to accept a job depends not only on the usual labor market considerations (the wage, the distribution of wages across other firms, unemployment insurance, etc.), but also on parameters describing the nature of crime and punishment (the opportunities for engaging in criminal activity, the probability of getting caught, the probability and costs of being victimized, etc.). The decision to commit a crime similarly depends on all of these things, as well as one's current employment status, current income, and general economic conditions. Moreover, firms' wage setting decisions are also influenced by criminal behavior, since higher wages discourage workers from crime and thus reduce turnover.

While the emphasis in this paper is mainly on endogenous wages, the first thing we do is analyze the individual worker's decision problem for a given distribution of wages. This can be interpreted as the natural extension of the textbook job search model to incorporate criminal activity. Generally, behavior in the standard model is fully characterized by the reservation wage, above which workers accept any offer; here behavior is characterized by the reservation wage plus the crime wage, above which employed workers prefer to remain legitimate rather than commit crime. For any given distribution of wage offers, we derive the steady state distribution of wages paid across employed workers, as well as the steady state unemployment and crime rates. This would fully describe the equilibrium, and can be used to analyze the effects of changes in policy on crime, unemployment, etc., if wage offers are exogenous.

However, we are actually more interested in the case where wages are determined within the model by profit-maximizing firms. In equilibrium, of course, if two distinct wages are posted they must imply the same profit. Assuming no on-the-job search, we show that there will be at most two distinct wages in any equilibrium: firms either post the reservation wage or the crime wage. This can be understood as the natural generalization of results in Diamond (1971), which imply that in fairly general search models without crime (and, again, no on-the-job search) there is a unique equilibrium and it implies all firms post the reservation wage. Here some firms may pay above the reservation wage to keep workers honest. This is in some respects similar to what happens in efficiency wage models, such as Shapiro and Stiglitz (1984), where higher wages discourage shirking; however, as we discuss in more detail below, the crime model generates some results that are quite different from efficiency wage models, including wage dispersion and multiple equilibria.

The above results hold even if the probability of being victimized is independent of the decision to commit crime. However, we also show how to close the model by setting the probability of being a victim equal to the endogenous crime rate. This allows us to analyze the second source of multiple equilibria discussed above. We also generalize the results to the case where there is on-the-job search. This is interesting because, among other reasons, the on-the-job search model generates wage dispersion even without crime, and we show that the nature of wage inequality changes qualitatively once crime is introduced. We also consider some other extensions, including the case where workers have the bargaining power, and the case where the rewards to criminal activity depend on the average wage. The essential findings are robust.

The rest of the paper is organized as follows. In Section 2 we present the worker's problem taking wages as given. In Section 3 we determine the equilibrium wage distribution assuming no on-the-job search. In Section 4 we endogenize the crime rate. In Section 5 we extend the model to include on-the-job search. In Section 6 we discuss the other extensions. Section 6 concludes.

2. Workers

There is a [0, 1] continuum of infinite-lived and risk-neutral workers. There is also a [0, N] continuum of infinite-lived and risk-neutral firms, so that N is the firm-worker ratio. For the present, all that we need to say about firms is that to each firm there is associated a wage w, it pays w to all of its employees, and it is willing to hire at that wage any worker who is willing to accept. As wages may not be constant across firms, let F(w) denote the distribution of wage offers from which workers will be sampling. Later, F will be endogenized, but for now it is taken as given so that we can concentrate on worker behavior. In any case, the distribution of wages paid to employed workers, call it G(w), will not generally be the same as the distribution of wages offered and needs to be determined endogenously.

Workers are ex ante identical, but at any point in time they can be in one of three distinct states, employed (at some wage w), unemployed, or in jail, and we let the numbers of such workers be e, u, and n. Let the payoffs, or value functions, in the different states be $V_1(w)$, V_0 , and J. While unemployed, workers get a flow payment b and receive job offers at rate λ_0 , each of which is a random draw from F(w). While employed, workers get their wage w, receive new offers (independent draws from F) at rate λ_1 , and, in addition to leaving jobs for endogenous reasons analyzed below, have their jobs destroyed for exogenous reasons at rate δ . Agents in jail get a flow payment z, which could well be negative, are released into the unemployment pool at rate ρ , and receive no job offers until they are released. We assume for simplicity that the release rate ρ does not depend on time served, and that ex convicts face the same market opportunities as other unemployed workers.

We introduce criminal activity into the model as follows. First, unemployed workers encounter opportunities to commit crimes at rate μ_0 while employed workers encounter such opportunities at rate μ_1 . A crime opportunity is a chance to steal some amount, which is fixed for now but later will be endogenized. Let ϕ_0 and $\phi_1(w)$ be the probabilities with which unemployed workers and workers employed at w commit crimes, respectively. Given you have just committed a crime, let π be the probability of being sent to jail and κ the probability of receiving a fine ψ . For analytic convenience, we assume that you are either caught instantly or not at all – there are no long investigations resulting in eventual prosecution and conviction. We also assume the probability is 0 that two or more events, such as a job offer and a crime opportunity, occur simultaneously (as would necessarily follow if these events occur according to independent Poisson arrivals).

Let g be the expected gain from crime net of the expected fine (i.e., the amount you expect to get away with minus $\kappa \psi$). Then the net expected payoffs from crime for unemployed and for employed workers are given by

$$K_0 = g + \pi J + (1 - \pi)V_0 \tag{2.1}$$

$$K_1(w) = g + \pi J + (1 - \pi)V_1(w).$$
(2.2)

Clearly, an unemployed worker commits a crime if $K_0 > V_0$ and not if $V_0 > K_0$, while a worker employed at w commits a crime if $K_1(w) > V_1(w)$ and not if $V_1(w) > K_1(w)$. Hence, the crime decisions satisfy the following best response conditions:

$$\phi_0 = \begin{cases} 1 & \text{if } V_0 - J < \frac{g}{\pi} \\ 0 & \text{if } V_0 - J > \frac{g}{\pi} \end{cases} \text{ and } \phi_1(w) = \begin{cases} 1 & \text{if } V_1(w) - J < \frac{g}{\pi} \\ 0 & \text{if } V_1(w) - J > \frac{g}{\pi} \end{cases}$$
(2.3)

Whether employed or not, all workers fall victim to crime at rate γ . The crime rate will be endogenized below by setting the total number of victims $(e + u)\gamma$ equal to the number of crimes $u\mu_0\phi_0 + e\mu_1 \int \phi_1(w) dG(w)$, but it facilitates the discussion to first analyze things taking γ as given. We could rationalize this by saying the group of agents under consideration is small, so that whether or not they engage in criminal activity has a negligible effect the overall crime rate; in any event, we will soon make γ endogenous. In terms of losses, unemployed victims of crime suffer g_0 while employed victims suffer $g_1(w)$. They suffer these losses whether or not the perpetrator is caught, and the losses are exogenous for now although they will also be endogenized below. Even when the losses are endogenous, the expected gain from stealing g is independent of a victim's employment status or wage because we assume criminals cannot observe this; however, this expected gain will depend below on average income of the noninstitutionalized population, $\omega = \frac{u}{e+u}b + \frac{e}{e+u}\int w dG(w).^3$

If r is the rate of time preference, the flow Bellman equation for an unemployed worker is

$$rV_0 = b - \gamma g_0 + \mu_0 \phi_0 (K_0 - V_0) + \lambda_0 E_x \max\left\{V_1(x) - V_0, 0\right\}.$$
 (2.4)

In words, the flow return to being unemployed equals instantaneous income net of losses due to crime, plus the value of receiving a random crime opportunity, plus the value of receiving a random job offer. Similarly, for an agent employed at wage w,

$$rV_{1}(w) = w - \gamma g_{1}(w) + \delta [V_{0} - V_{1}(w)] + \mu_{1}\phi_{1}(w)[K_{1}(w) - V_{1}(w)] \quad (2.5)$$
$$+\lambda_{1}E_{x} \max \{V_{1}(x) - V_{1}(w), 0\},$$

where the final term represents the expected value of receiving a new offer x while employed at w. Finally, for an agent in jail,

$$rJ = z + \rho (V_0 - J).$$
 (2.6)

There are two aspects to a worker's strategy: his decision to accept a job, and his decision to commit a crime. We now describe a general method for analyzing these strategies. In terms of the offer acceptance strategy, first note that $V_1(w)$ is increasing in w. Hence, an employed worker should accept any offer above his current wage, and

³Agents here do not need to make career choices to either become full-time criminals or legitimate citizens, since we generally allow them to encounter criminal opportunites whether working in a legitimate job or not. This makes some aspects of the analysis easier, and also seems consistent with the evidence that criminals come from both the unemployed and employed population (see e.g., Merlo 2001). However, to the extent that one likes the career choice model, we could always set $\mu_1 = 0$. In this case, only the unemployed have crime opportunities, which means that when a job offer comes along the agent needs to decide whether to stay unemployed and potentially a criminal, or to go legit. Also note that our way of modeling things may seem to better capture robbery, mugging, etc., rather than crimes of pure violence such as murder, rape or assault; in principle, however, the analysis applies to any criminal activity that is goal directed and subject to cost-benefit calculations. In any case, only 8.2% of crimes are crimes of pure violence (Federal Bureau of Investigation [1992]).

an unemployed worker should accept any offer above his reservation wage R, defined by $V_1(R) = V_0$. In terms of the crime decision, note that $K_1(w) - V_1(w)$ is decreasing in w, and also that $K_0 - V_0 = K_1(R) - V_1(R)$. The former fact implies that workers are less likely to engage in crime when their wages are higher, and the latter implies that the unemployed engage in crime if workers employed at the reservation wage engage in crime. Therefore, we proceed by first checking ϕ_0 . On the one hand, if $\phi_0 = 0$ is a best response, then $\phi_1(w) = 0$ for all w. On the other hand, if $\phi_0 = 1$ is a best response, then $\phi_1(w) = 1$ if w < C, where C > R is called the crime wage, and satisfies $K_1(C) = V_1(C)$, or equivalently

$$g = \pi \left[V_1 \left(C \right) - J \right].$$
 (2.7)

To derive the reservation wage equation, we proceed in the standard way. First, equate (2.4) to (2.5) evaluated at w = R, and rearrange to yield

$$R = b + \gamma [g_1(R) - g_0] + (\lambda_0 - \lambda_1) \Delta(R) + (\mu_0 - \mu_1) \phi_0 [g - \pi (V_0 - J)], \quad (2.8)$$

where $\Delta(R) = \int_{R}^{\infty} [V_1(x) - V_0] dF(x) = \int_{R}^{\infty} V'_1(x) [1 - F(x)] dx$, after integrating by parts. Suppose that $\phi_0 = 0$, which as we said above implies $\phi_1(w) = 0$ for all $w \ge R$, and also makes the final term in (2.8) vanish. Then to simplify the reservation wage equation, differentiate (2.5) and insert $V'_1(x)$ to rewrite

$$\Delta(R) = \int_{R}^{\infty} \frac{\left[1 - F\left(x\right)\right]}{r + \delta + \lambda_1 \left[1 - F\left(x\right)\right]} dx.$$
(2.9)

Inserting (2.9) into (2.8) yields one equation in R, taking F as given, the solution to which fully characterizes worker strategies in the case $\phi_0 = 0$. Indeed, if we also set $\gamma = 0$, this case reduces to the standard job search model since no one commits or is the victim of crime.

Now suppose that $\phi_0 = 1$, which means $\phi_1(w) = 1$ iff w < C for some C > R. Then the procedure leading (2.9) now yields

$$\Delta(R) = \int_{R}^{C} \frac{[1 - F(x)] \, dx}{r + \delta + \mu_1 \pi + \lambda_1 \, [1 - F(x)]} + \int_{C}^{\infty} \frac{[1 - F(x)] \, dx}{r + \delta + \lambda_1 \, [1 - F(x)]}.$$
 (2.10)

Also, since the final term in (2.8) now does not vanish (except in the special case $\mu_0 = \mu_1$), we need to subtract (2.6) and (2.4) to compute

$$V_0 - J = \frac{b - z - \gamma g_0 + \mu_0 g + \lambda_0 \Delta(R)}{r + \rho + \mu_0 \pi}.$$
 (2.11)

Inserting (2.11) and (2.10) into (2.8) again yields the reservation wage equation. In this case we also need the crime wage equation. Manipulation of (2.5) and (2.7)implies

$$C = z + \gamma g_1(C) + (r+\delta)\frac{g}{\pi} + (\rho-\delta)(V_0 - J) - \lambda_1 \Delta(C).$$
(2.12)

Inserting (2.11) and (2.10) into (2.12) yields the desired result.

We can summarize the above analysis as follows. First, one checks if $K_0 - V_0 < 0$, which implies $\phi_0 = 0$ and $\phi_1(w) = 0$ for all employed workers. This basically reduces to the standard model with no crime. However, if $K_0 - V_0 > 0$, then $\phi_0 = 1$ and at least the unemployed commit crime. In this case we need to compute the crime wage C in addition to the reservation wage, and an employed worker engages in crime iff his wage is below C. Then the criminal behavior of employed workers depends on the wage distribution; e.g., if F(C) = 0 then no employed worker commits a crime. Once we endogenize F, it will naturally depend on worker behavior as described by R and C, and so we have an fixed point problem that is analyzed in the next section.

Even taking F as given, we think this is an interesting extension of the usual search model. For instance, it can be used to derive predictions about the effects of many variables, including policy variables such as π , ρ , z and b, on R and C, and hence on things like unemployment and crime rates. This is especially simple in the case $\mu_0 = \mu_1$ and $\lambda_0 = \lambda_1$, since then the reservation wage equation reduces nicely to $R = b + \gamma [g_1(R) - g_0]$. Indeed, if we also assume that the loss due to crime is a linear function of current income, $g_1(w) = \alpha w + d$ and $g_0 = \alpha b + d$, say, then R = b, and we can focus on the effects of variables like π , ρ , and z on the crime wage. Even in the general case, the comparative static results are not difficult to derive, but we leave this as an exercise in order to concentrate on models with endogenous wage offers.

Before analyzing wage determination, however, we need to discuss the distribution of workers across states for a given F, as this will be an input into the firms' problem. Let e_L the number of workers employed at w < C, $e_H = e - e_L$ the number employed at $w \ge C$, and $\sigma = 1 - F(C)$ the fraction of firms offering at least C. For the case $\phi_0 = 1$, the labor market flows are shown in Figure 2.1, under the assumption that $w \ge R$ with probability 1 (as we will soon see, this is always true in equilibrium with endogenous wages). One can easily solve for the steady state

$$e_{H} = (\delta + \lambda_{1} + \mu_{1}\pi) \sigma \rho \lambda_{0} / \Omega$$

$$e_{L} = (1 - \sigma) \rho \delta \lambda_{0} / \Omega$$

$$u = (\delta + \lambda_{1}\sigma + \mu_{1}\pi) \rho \delta / \Omega$$

$$n = \mu_{0} (\delta + \mu_{1}\pi + \lambda_{1}\sigma) \pi \delta / \Omega$$
(2.13)

where $\Omega = (\delta + \sigma \lambda_1) (\rho \delta + \rho \lambda_0 + \mu_0 \pi \delta) + \mu_1 \pi (\sigma \rho \delta + \rho \lambda_0 + \mu_0 \pi \delta)$. This gives the steady state as a function of σ , which will be determined below from the firm side of the model. In the other case, $\phi_0 = 0$, there is no crime, and we have $u = \delta/(\delta + \lambda_0)$ and $e = e_H = \lambda_0/(\delta + \lambda_0)$.

We also need the distribution of wages paid across employed workers. In the case $\phi_0 = 1$, it is convenient to define the conditional distributions above and below C:

$$F_H(w) = F(w|w \ge C) \text{ and } F_L(w) = F(w|w < C),$$

 $G_H(w) = G(w|w \ge C) \text{ and } G_L(w) = G(w|w < C).$

Then one can derive:⁴

⁴To derive these results, let us work under the assumption that firms offer $w \ge R$ with probability 1 (as we said, this will always be true when wages are endogenous). Given any w < C, the number of workers employed at a wage no greater than w is $G_L(w) e_L$. Then the distribution G_L evolves



Figure 2.1: Flows Across States

$$G_L(w) = \frac{\lambda_0 (1-\sigma) F_L(w) u}{\{\delta + \mu_1 \pi + \lambda_1 \sigma + \lambda_1 (1-\sigma) [1-F_L(w)]\} e_L}$$
(2.14)

$$G_H(w) = \frac{(\lambda_0 u + \lambda_1 e_L) \sigma F_H(w)}{\{\delta + \lambda_1 \sigma [1 - F_H(w)]\} e_H}.$$
(2.15)

Eliminating u, e_L and e_H using (2.13), these conditions fully describe the distribution of wages paid given the offer distribution, as parameterized by σ , F_L and F_H . In the other case, $\phi_0 = 0$, all employed workers earn above C and so we have

$$G(w) = \frac{\lambda_0 u F(w)}{\left\{\delta + \lambda_1 \left[1 - F(w)\right]\right\} e}.$$
(2.16)

Again, eliminating u and e yields the distribution of wages paid as a function of F. through time according to

 $\frac{d}{dt}G_L(w)e_L = \lambda_0(1-\sigma)F_L(w)u - \{\delta + \mu_1\pi + \lambda_1\sigma + \lambda_1(1-\sigma)[1-F_L(w)]\}e_LG_L(w).$

Similarly, ${\cal G}_{\cal H}$ evolves according to

$$\frac{d}{dt}G_{H}(w)e_{H} = (\lambda_{0}u + \lambda_{1}e_{L})\sigma F_{H}(w) - \{\delta + \lambda_{1}\sigma [1 - F_{H}(w)]\}e_{H}G_{H}(w)$$

Setting the time derivatives equal to 0 and simplifying yields the expressions in the text.

To the extent that one is willing to take F exogenously, these conditions describe the induced inequality in wages paid and how it depends on parameters, including the crime parameters π , μ_1 , etc.

3. Equilibrium without On-the-Job Search

In this section we make wages endogenous in a relatively simple version of the model with $\lambda_1 = 0$ and $\lambda_0 = \lambda > 0$. Also, to reduce notation here we set $\mu_0 = \mu_1 = \mu$, and set $g_1(w) = g_0 = g$ (lump sum theft). Our model of wage determination is as follows: each firm has linear technology with the common marginal product p > b, and posts a wage at which it commits to hire all workers that it contacts. Each firm takes as given the wages of other firms as described by F, as well as worker behavior as described by (R, C). For simplicity, we assume firms maximize steady state profit, which can be understood as the limiting case of maximizing the present value of the profit flow when $r \approx 0$ (this is standard in many models of wage posting; see Burdett and Mortensen [1998] and Coles [1999]). Also, for the time being we keep γ and gexogenous.

It is easy to show that, in this model, with no on-the-job search, there will be at most two wages paid in equilibrium: each firm either pays the reservation wage or the crime wage. The argument is as follows. First, firms paying w < R attract no workers. Hence, all firms pay at least R, and therefore every worker they contact is willing to accept the job. Suppose $\phi_0 = 0$. Then $\phi_1(w) = 0$ for all $w \ge R$, and so no firm has an incentive to pay above R. Hence, if $\phi_0 = 0$ then all firms post w = R; this is the Diamond (1971) result. Now suppose $\phi_0 = 1$. Then $\phi_1(w) = 0$ iff $w \ge C$ for some C > R. All firms can still hire every worker they contact, but firms paying below C lose workers faster since their workers with positive probability end up going to jail. That is, $w \ge C$ implies you lose workers at the (exogenous) rate δ , while and w < C implies you lose workers at rate $\delta + \mu\pi$. Hence, any firm paying w > C could make more profit if it reduced its wage to C, while any firm paying $w \in (R, C)$ could make more profit by lowering its wage to R. Hence, $\phi_0 = 1$ implies all firms post either R or C.⁵

Summarizing, when $\phi_0 = 0$ all firms post R, and when $\phi_0 = 1$ the equilibrium distribution F can be summarized by three numbers, R, C, and the fraction σ posting C. In the case of $\phi_0 = 1$, a firm posting R hires workers at rate $\lambda u/N$ (since the number of workers contacting a firm is λu and there are N firms per worker) and loses workers at rate $\delta + \mu \pi$. Hence, in steady state it will have $l_R = \lambda u/(\delta + \mu \pi)N$ workers. Similarly, a firm posting C will have $l_C = \lambda u/\delta N$ workers. Steady-state profits are therefore $\Pi_R = (p-R) l_R$ and $\Pi_C = (p-C) l_C$. Since firms are ex ante identical, profit maximization implies

$$\sigma = \begin{cases} 1 & \text{if } \Pi_C > \Pi_R \\ 0 & \text{if } \Pi_C < \Pi_R. \end{cases}$$
(3.1)

Clearly, wage dispersion $(0 < \sigma < 1)$ requires equal profits, which means $(p - C) (\delta + \mu \pi) = (p - R)\delta^{6}$.

Given there are at most two wages posted, write V_R and V_C for the workers' value functions, and ϕ_R and ϕ_C for the crime decisions, at w = R and w = C. Of course, we already know that in general $\phi_0 = \phi_R$ and $V_0 = V_R$. The Bellman equations can now be simplified a lot. Consider first the case where $\phi_0 = 0$, which we call a Type N equilibrium (N for "no crime"). In this case, all firms post R and we have

$$rV_0 = b - \gamma g \tag{3.2}$$

$$rV_R = R - \gamma g. \tag{3.3}$$

Then $V_0 = V_R$ implies R = b, exactly as in the Diamond model. This constitutes an

⁵Notice that tie-breaking rules always go the right way for workers deciding to accept a job or to commit a crime – namely, with probability 1 they accept a job when w = R and forego crime when w = C. This must be so in equilibrium, since, e.g., if workers are indifferent but decide to reject w = R with positive probability, a firm could do better by posting $R + \varepsilon$ for any $\varepsilon > 0$ because this makes them accept for sure.

⁶One can obtain an analogous condition by assuming that each firm can hire only one worker, and setting the value of posting a vacancy at wage R equal to the value of posting a vacancy at wage C (see Mortensen [2000] for a comparison of the two approaches in a model without crime).

equilibrium iff the best response condition (2.3) holds at $\phi_0 = 0$; i.e., iff $V_0 - J \ge \frac{g}{\pi}$. Subtracting (3.2) and (2.6), we see that (when $r \approx 0$)

$$V_0 - J = \frac{b - z - \gamma g + \mu \phi_0 g + \lambda \sigma g/\pi}{\rho + \mu \pi \phi_0 + \lambda \sigma}.$$

Hence, $\phi_0 = 0$ is a best response iff $b \ge \overline{b}$, where

$$\overline{b} = z + \gamma g + \rho g / \pi. \tag{3.4}$$

This means that the Type N equilibrium exists iff $b \ge \overline{b}$. It will also follow from the analysis of other possible cases below that no other type of equilibrium can exist when $b > \overline{b}$. Hence, for some parameters we find that there cannot be equilibrium with crime, while for others it must be the case that at least the unemployed commit crime. We will say more about the economics underlying the relevant parameters below, after we discuss other possible types of equilibria. For now we summarize what we know about the equilibrium with no crime as follows.⁷

Proposition 1. If $b > \overline{b}$ there is a unique equilibrium and it is Type N. If $b < \overline{b}$ there is no Type N equilibrium.

Let us now proceed to cases where $\phi_0 = 1$, which means the unemployed commit crime and the employed do at w = R but not at w = C > R. The Bellman equations become

$$rV_0 = b - \gamma g + \lambda \sigma \left(V_C - V_0 \right) + \mu \left[g + \pi \left(J - V_0 \right) \right]$$
(3.5)

$$rV_{R} = R - \gamma g + \mu \left[g + \pi \left(J - V_{0}\right)\right]$$
(3.6)

$$rV_C = C - \gamma g + \delta \left(V_0 - V_C \right). \tag{3.7}$$

It is easy to solve for the value functions given (R, C, σ) , and then solve the reservation and crime wage equations, $V_0 = V_R$ and $V_C - J = g/\pi$, for R and C as functions of σ . Then result is

⁷Notice that in this result, as in most of the Propositions stated in the paper, to ease the presentation we ignore nongeneric cases, like $b = \overline{b}$.

$$R(\sigma) = b + \frac{\lambda\sigma}{\rho + \mu\pi + \lambda\sigma} \left(\overline{b} - b\right)$$
(3.8)

$$C(\sigma) = \overline{b} + \frac{\delta - \rho}{\rho + \mu\pi + \lambda\sigma} (\overline{b} - b).$$
(3.9)

Note that $\phi_0 = 1$ is equivalent to C > R, which is equivalent to $b < \overline{b}$ where \overline{b} was defined in Proposition 1.

Moreover, $\Pi_C - \Pi_R$ is proportional to

$$T(\sigma) = p - \overline{b} - \frac{(\rho^* - \rho)(\overline{b} - b)}{\rho + \mu\pi + \lambda\sigma},$$
(3.10)

where $\rho^* = \delta (\delta + 2\mu\pi) / \mu\pi$. The first term is (proportional to) the difference between the total revenues of firms paying C and R, which is positive because the former have more workers in steady state, while the second term is (proportional to) the difference between the wage bills. Thus, given $b < \overline{b}$, the possible equilibria can be described as follows. A Type L equilibrium (L for "low crime") has $\sigma = 1$, and requires T(1) > 0. In this case no employed workers, but only unemployed workers, commit crime. A Type H equilibrium (H for "high crime") has $\sigma = 0$, and requires T(0) < 0. In this case everyone engages in crime. A Type M equilibrium (M for "medium crime") has $\sigma \in (0, 1)$, and requires $T(\sigma) = 0$. This case implies a nondegenerate wage distribution, and some low wage workers commit crime while high wage workers do not.

For describing when each type of equilibrium with $\phi_0 = 1$ exists, it is convenient to define

$$p_0(b) = \overline{b} + \frac{(\rho^* - \rho)(\overline{b} - b)}{\rho + \mu\pi}$$
(3.11)

$$p_1(b) = \overline{b} + \frac{(\rho^* - \rho)(b - b)}{\rho + \mu\pi + \lambda}.$$
(3.12)

We will break things into two cases, $\rho > \rho^*$ and $\rho < \rho^*$, which determines the sign of $p_0(b)-p_1(b)$ in the relevant range.⁸ At this stage, existence of each type of equilibrium

⁸We also maintain the assumption that $V_0 \ge J$ (i.e., the value of being in jail is no greater than

simply involves checking the sign of T(1) and T(0). Moreover, it is clear that there can never be more than one equilibrium follows of a given type, since $T(\sigma)$ is monotone, although in the case T(0) < 0 < T(1) we will have one equilibria each of Type L, Type H and Type M. Hence, we have the following results, the proof of which involve straightforward algebra.

Proposition 2. Suppose $b < \overline{b}$ and $\rho > \rho^*$. Then, $p_0(b) < p_1(b)$ and

- (a) if $p > p_1(b)$ there is a unique equilibrium and it is Type L;
- (b) if $p < p_0(b)$ there is a unique equilibrium and it is Type H;
- (c) if $p_0(b) there is a unique equilibrium and it is Type M.$

Proposition 3. Suppose $b < \overline{b}$ and $\rho < \rho^*$. Then, $p_0(b) > p_1(b)$ and

- (a) if $p > p_1(b)$ there is a unique equilibrium and it is Type L;
- (b) if $p < p_0(b)$ there is a unique equilibrium and it is Type H;
- (c) if p₁ (b) 0</sub> (b) there are three equilibrium, one of each of Type L, Type H and Type M.

The sets of equilibria in the two cases are shown in Figures 3.1 and 3.2. In either case, $b > \overline{b}$ implies no crime. Since $\overline{b} = z + \gamma g + \rho g/\pi$, no crime is an equilibrium if, for example, z is low relative to $b - \gamma g$ (jail is lot worse than unemployment), ρ is low (sentences are long), or π is high (apprehension rates are high). When $b < \overline{b}$ at least the unemployed will commit crime, and the employed do iff p is relatively low. Intuitively, low p means firms are not willing to pay enough to keep them honest, while high p means they are willing since then turnover is quite costly. Another way to say it is this: an increase in p raises revenue faster at high wage firms than low wage firms, since the former have more workers, and therefore for large p all firms

the value of unemployment) which holds iff $b \ge z + \gamma g - (\mu \pi + \lambda \sigma) \frac{g}{\pi}$. If this condition were not met unemplyed agents would volunteer for jail.

will find it profitable to post C. How high does p have to be to eliminate crime? This depends on parameters, but also on beliefs: at least when $\rho < \rho^*$ there are multiple equilibria for intermediate values of p. We will discuss the intuition for this more below.



Figure 3.1: Regions of Different Equilibria: $\rho > \rho^*$

The value of σ in Type M equilibrium is given by

$$\sigma^* = \frac{\left(\rho^* - \rho\right)\left(\overline{b} - b\right) - \left(\rho + \mu\pi\right)\left(p - \overline{b}\right)}{\lambda\left(p - \overline{b}\right)}.$$

This can be inserted into (3.8) and (3.9) to derive the reduced form for R and C in the Type M equilibrium; in the Type L equilibrium or Type H equilibrium, insert $\sigma = 1$ or $\sigma = 0$. We can now also solve explicitly for the number of agents in each state in each type of equilibrium. As remarked in Section 2, if $\phi_0 = 0$ then $u = \delta/(\delta + \lambda)$ and $e = \lambda/(\delta + \lambda)$. In the cases with $\phi_0 = 1$ we can substitute σ^* into (2.13) do derive the results in the following table,

where $\Sigma = \delta \lambda \rho (1 - \sigma) + \lambda \rho (\delta + \mu \pi) \sigma + \delta \rho (\delta + \mu \pi) + [\delta + \mu \pi + (1 - \sigma) \lambda] \delta \mu \pi$.



Figure 3.2: Regions of Different Equilibria: $\rho < \rho^*$

	e_L	e_H	u	n
Type L	0	$\frac{\lambda ho}{\lambda ho + \delta(ho + \mu \pi)}$	$rac{\delta ho}{\lambda ho + \delta(ho + \mu \pi)}$	$rac{\delta\mu\pi}{\lambda ho+\delta(ho+\mu\pi)}$
Туре Н	$\frac{\rho\lambda}{(\rho+\mu\pi)(\delta+\lambda+\mu\pi)}$	0	$\frac{\rho(\delta + \mu\pi)}{(\rho + \mu\pi)(\delta + \lambda + \mu\pi)}$	$\frac{\mu\pi}{ ho+\mu\pi}$
Type M	$\frac{\delta\lambda\rho(1-\sigma)}{\Sigma}$	$\frac{\lambda\rho(\delta+\mu\pi)\sigma}{\Sigma}$	$\frac{\delta \rho(\delta + \mu \pi)}{\Sigma}$	$\frac{[\delta + \mu \pi + (1 - \sigma)\lambda]\delta\mu\pi}{\Sigma}$

Table 3.1: Steady States in Each Type of Equilibrium

The unemployment rate in a Type J equilibrium is $U_J = u_J/(1 - n_J)$ where u_J and n_J are the numbers unemployed and in jail (the institutionalized population are not in the labor force). We have $U_N = U_L < U_M < U_H$. Also, letting c_J be the number of crimes per non-institutionalized worker in a Type J equilibrium, we have $c_N = 0, c_L = \mu \delta / (\delta + \lambda), c_H = \mu$, and

$$c_M = \frac{\left[\delta + \mu\pi + (1 - \sigma)\lambda\right]\delta\mu}{\left(1 - \sigma\right)\delta\lambda + \left(\delta + \mu\pi\right)\left(\delta + \lambda\sigma\right)}.$$
(3.13)

Notice that $c_N < c_L < c_M < c_H$ (and so the labels make sense). Also notice that c_J is ranked the same way across equilibria as U_J . Thus, crime and unemployment go together, although we cannot say one causes the other since both are endogenous.

In terms of inequality, we have the following: in Type N and Type H equilibria there is no inequality, as all employed workers earn w = b, the same as unemployment income; in Type L equilibrium all employed workers earn C(1) > b; and in Type M the fraction $\sigma^* \in (0, 1)$ of employed workers earn $C(\sigma^*)$ while the rest earn $R(\sigma^*)$, where $b < R(\sigma^*) < C(\sigma^*)$.

As we have seen, for intermediate values of p, we either have a unique equilibrium with wage dispersion or multiple equilibria, depending on the release rate from jail, ρ . To see why the possibility of multiple equilibria depends on ρ , note that it determines when $T'(\sigma) > 0$ is possible. To explain this, first note that when σ increases, V_0 increases. This implies that R increases, and C increases iff $\rho > \delta$ (since $\rho > \delta$ implies that agents transit to unemployment from jail faster than from employment). For the relative profit of a high-wage firm to be decreasing in σ we need $C'(\sigma)$ positive and large relative to $R'(\sigma)$, which holds if ρ large. Hence, large ρ implies the relative profits of high-wage firms fall with σ and equilibrium must be unique. However, if ρ is small, then $C'(\sigma)$ is either negative or at least small relative to $R'(\sigma)$. This means that it becomes cheaper to pay the crime wage relative to the reservation wage as the fraction of high-wage firms rises. In this case the relative profit of high-wage firms is increasing in σ , and multiple equilibria can arise.

This possibility of multiple equilibria seems interesting since it implies that two intrinsically identical neighborhoods can end up in very different situations. In particular, one can end up in a Type L equilibrium where all employed workers earn C > b, and there is little crime or unemployment, while the other can end up in a Type H equilibrium where all workers earn b and there is lots of crime and unemployment. It is the very fact that there are lots of good wage jobs (i.e., σ is high) in the first neighborhood that deters workers from crime, and it is the fact that workers are less inclined to criminal activity that makes it profitable to offer C rather than R. Clearly, this effect can only arise in a model with endogenous wage setting. It also may be interesting in light of the empirical work cited in the Introduction comparing crime rates across locations.

To close this section we comment on the connection between our model and efficiency wage models like Shapiro and Stiglitz (1984). In those models, by paying an efficiency wage above the reservation wage firms keep workers from shirking, which affects profits since shirkers produce less (or no) output. In our model, by paying a crime wage above the reservation wage firms keep workers from engaging in criminal activity, which also affects profit but in this case through turnover since criminals may be sent to jail. If we had only a fine ψ and convicted criminals did not lose their jobs, then crime would not affect profit. Note that in the standard efficiency wage model there is always a unique equilibrium, and all firms always pay the same wage, while in the crime model there can be multiple equilibria if ρ is small, and the can be may wage dispersion for any ρ at least for intermediate values of p.⁹

4. Endogenous Crime Rate

So far we have assumed that the rate at which workers fall victim to crime, γ , is independent of the rate at which they commit crimes – as would be the case, say, if the labor market under consideration is small relative to the whole economy. In this section we endogenize γ by relating the total number of crimes committed to the total number of victims. Now in any equilibrium with $\phi_0 = 0$, there is no crime, so $\gamma = 0$. In any equilibrium with $\phi_0 = 1$, recall that the rate which workers commit crime in

⁹The standard efficiency wage model does not generate multiple equilibria because the efficiency wage is always increasing in the value of unemployment, and hence in the fraction of firms paying the efficiency wage; i.e., there is no strategic complementarity in wage setting. Also, note that in efficiency wage models someone caught shirking only loses their job – they are not sent to jail – and we have seen that not only going to jail but going to jail for a relatively long time is key for multiplicity. Finally, note this aspect of our model helps to avoid a well-known problem with the standard efficiency wage literature, which is that if a worker is caught shirking the firm really has no incentive to follow through with the threat of firing and the parties should renegotiate. In our model, the relevant threat is enfoced by the criminal justice system, and so renegotiation is not an option.

Type M equilibrium as a function of $\sigma \in (0, 1)$ is given by (3.13), which implies

$$\gamma(\sigma) = \frac{\left[\delta + \mu\pi + (1 - \sigma)\lambda\right]\delta\mu}{\left(1 - \sigma\right)\delta\lambda + \left(\delta + \mu\pi\right)\left(\delta + \lambda\sigma\right)}.$$
(4.1)

This yields the crime rates in Type H and L equilibria as special cases, $\gamma(0) = \mu$ and $\gamma(1) = \mu \delta / (\delta + \lambda)$.

The method for analyzing the model is the same as before, except we replace γ with the endogenous crime rate. The same qualitative types of equilibria can exist: if $\phi_0 = 0$ there is no crime (Type N); and if $\phi_0 = 1$ there are three subcases, $\sigma = 1$ (Type L), $\sigma = 0$ (Type H) and $\sigma \in (0,1)$ (Type M). In the previous section, whether $\phi_0 = 1$ or not depends on b relative to the exogenous threshold $\overline{b} = z + \gamma g + \rho g/\pi$, but here \overline{b} is endogenous:

$$\overline{b} = \hat{b}(\gamma(\sigma)) = z + \gamma(\sigma)g + \rho g/\pi.$$
(4.2)

For example, if agents believe there is no crime $(\gamma = 0)$ then $\phi_0 = 1$ iff $b < \hat{b}(0) = z + \rho g/\pi$; but if they believe that everyone engages in crime, then $\phi_0 = 1$ iff $b < \hat{b}(\gamma(0)) = z + \rho g/\pi + \mu g$. Generally, if they believe all the unemployed plus a fraction σ of those employed engage in crime, then $\phi_0 = 1$ iff $b < \hat{b}(\gamma(\sigma))$. Note that $\hat{b}(0) < \hat{b}(\gamma(1)) < \hat{b}(\gamma(0))$.

The following result describing when the unique equilibrium involves no crime provides a version of Proposition 1 for endogenous crime rates. The proof simply involves checking that the restriction $b > \hat{b}(\gamma(0)) = z + \rho g/\pi + \mu g$ makes $\phi_0 = 0$ a dominant strategy. However, unlike the previous section, if this restriction does not hold we cannot yet say that a Type N equilibrium does not exist (see below).

Proposition 4. If $b > \hat{b}(\gamma(0))$ there is a unique equilibrium and it is of Type N.

Now consider equilibria with $\phi_0 = 1$. Let \hat{T} be as in (3.10) except that \overline{b} is replaced by $\hat{b}(\gamma(\sigma))$:

$$\hat{T}(\sigma) = p - \hat{b}(\gamma(\sigma)) - \frac{(\rho^* - \rho) \left[\hat{b}(\gamma(\sigma)) - b \right]}{\rho + \mu\pi + \lambda\sigma}.$$

The relevant equilibrium condition is now $\sigma = 1$ if $\hat{T}(1) > 0$ and $\sigma = 0$ if $\hat{T}(0) < 0$. Notice that

$$\hat{T}'(\sigma) = T'(\sigma) - \left(\frac{\rho^* + \mu\pi + \lambda\sigma}{\rho + \mu\pi + \lambda\sigma}\right)\frac{d\hat{b}}{d\sigma} > T'(\sigma).$$

It is convenient to define

$$\hat{p}(b,\gamma(\sigma)) = \hat{b}(\gamma(\sigma)) + \frac{(\rho^* - \rho)\left[\hat{b}(\gamma(\sigma)) - b\right]}{\rho + \mu\pi + \lambda\sigma},$$

so that in general, $\hat{T}(\sigma) = p - \hat{p}(b, \gamma(\sigma))$. We know that we can have multiple equilibria even with γ exogenous, for some values of ρ , for reasons discussed in the previous section. To focus on the impact of an endogenous crime rate, we now set $\rho = \rho^*$ to eliminate any possibility of multiple equilibria from the channel analyzed above.¹⁰ This implies $\hat{T}(\sigma) = p - \hat{b}(\gamma(\sigma))$.

The following results show that for low values of b the outcome is very much as in the case shown in Figure 3.2 in the previous section: for high p no employed workers are criminals, for low p they all are, and for intermediate values of p both these equilibria exist, along with one where there is wage dispersion and some workers are criminals while others are not. For intermediate values of b, however, it now turns out that these outcomes can coexist with the Type N equilibrium. We present the results in three Propositions, which apply for progressively higher values of b.

Proposition 5. Let $\rho = \rho^*$ and suppose $b < \hat{b}(0)$. Then,

- (a) if $p > \hat{b}(\gamma(0))$ there is a unique equilibrium and it is Type L;
- (b) if $p < \hat{b}(\gamma(1))$ there is a unique equilibrium and it is Type H;
- (c) if $\hat{b}(\gamma(1)) there are three equilibria, one of each of Type L, Type H and Type M.$

¹⁰Notice that with $\rho = \rho^*$, the function T of the previous section is constant. This means that as σ rises, both C and R rise, but in such a way that the profit differential between high and low-wage firms is constant.

Proof. First note that $b < \hat{b}(0)$ implies the unemployed choose $\phi_0 = 1$ for any γ . Also, $\rho = \rho^*$ implies $\hat{T}(\sigma) = p - \hat{b}(\gamma(\sigma))$, so conditions (a), (b), and (c) are equivalent to $\hat{T}(0) \ge 0$, $\hat{T}(1) \le 0$, and $\hat{T}(0) < 0 < \hat{T}(1)$ respectively. But $\rho = \rho^*$ also implies $\hat{T}' > 0$ which guarantees uniqueness and concludes the proof.

Proposition 6. Let $\rho = \rho^*$ and suppose $\hat{b}(0) < b < \hat{b}(\gamma(1))$. Then all the equilibria in Proposition 5 exist under the same conditions, plus there always exists a Type N equilibrium.

Proof. Since $\hat{b}(0) < b$, the unemployed choose $\phi_0 = 0$ if $\gamma = 0$. Hence for any p > b there is an equilibrium of Type N. Since $b < \hat{b}(\gamma(1))$, the unemployed will choose $\phi_0 = 1$ for any $\gamma(\sigma) \ge \gamma(1)$. So in addition to the equilibrium of Type N, we can construct an equilibrium of Type L if $\hat{T}(1) > 0$, an equilibrium of Type H if $\hat{T}(0) < 0$ and an equilibrium of Type M if $\hat{T}(0) < 0 < \hat{T}(1)$. These three conditions are equivalent to (a), (b) and (c), respectively.

Proposition 7. Let $\rho = \rho^*$ and suppose $\hat{b}(\gamma(1)) < b < \hat{b}(\gamma(0))$. Then,

(a) if $p > \hat{b}(\gamma(0))$ there is a unique equilibrium and it is Type N;

(b) if $p < \hat{b}(\gamma(0))$ there are three equilibria, one each of Type N, Type H and Type M.

Proof. Since $\hat{b}(0) < \hat{b}(\gamma(1)) < b$, an equilibrium of Type N can be constructed for any p > b, as shown in the proof of Proposition 6. The restriction in (a) is equivalent to $\hat{T}(0) > 0$, and $\hat{b}(\gamma(1)) < b$ implies that the unemployed choose $\phi_0 = 0$ if $\gamma = \gamma(1)$, so an equilibrium of Type L cannot exist. Hence the Type N equilibrium is unique under the conditions in (a). Next, note that $b < \hat{b}(\gamma(0))$ implies the unemployed choose $\phi_0 = 1$ if $\gamma \ge \gamma(0)$. This together with the condition in (b), which is equivalent to $\hat{T}(0) < 0 < \hat{T}(1)$, implies an equilibrium of Type H exists if $p < z + \rho g/\pi + \mu g$. To finish the proof of (b), we construct a Type M equilibrium. First note that $\hat{T}(0) < 0 < \hat{T}(1)$ implies there exists a unique σ satisfying $\hat{T}(\sigma) = 0$, or equivalently, satisfying $p = z + \rho g/\pi + \gamma(\sigma) g$. Finally, the unemployed set $\phi_0 = 1$ because $b < \hat{b}(\gamma(\sigma)) = z + \rho g/\pi + \gamma(\sigma) g = p$, and therefore an equilibrium of Type M exists.

The results in are illustrated in Figure 4.1. The effects of parameter changes are all quite reasonable; e.g., making jail worse makes crime less likely. The basic difference between this model and the one with γ fixed is that with γ fixed there is a unique level of $b = \overline{b}$ such that $\phi_0 = 1$ iff $b < \overline{b}$. With γ endogenous, whether $\phi_0 = 1$ is a best response also depends on beliefs about γ . Economically, the idea is simple: if you live in a neighborhood with lots of crime then life on the street, while still better than jail, is not so much better than jail. Hence, you are more likely to commit crime yourself. This effect seems quite general and will be relevant to any model (not just search models) where the crime rate is endogenously set to equate the number of criminals to the number of victims.¹¹

5. On-The-Job Search

In this section study wage determination with on-the-job search, as in Burdett and Mortensen (1998). Thus, at rate $\lambda_1 > 0$, employed workers contact other firms, and move iff the other firms pay higher wages. One reason to study this version is that on-the-job search models have been used extensively in recent empirical work. To the extent that one is interested in quantitative implications, methods for estimating on-the-job search models have been developed, and estimates for many of the key parameters are readily available. Moreover, this model generates wage dispersion even without crime (because higher w increases the rate at which you hire workers

¹¹So far we have been modelling a victim's loss as a lump sum g. If we assume victims employed at $w \log \alpha w + d$ and the unemployed lose $\alpha b + d$, then there would be yet another source of multiple equilibria, but operating through the distribution of wages. The reason is that when the loss is increasing in your income, a reduction in γ induced by the increase in σ allows all firms to offer lower wages, and in particular, it allows high-wage firms to reduce C relative to R. To verify this, one can re-write (3.8) and (3.9) with proportional theft and verify that $\partial C/\partial \gamma > \partial R/\partial \gamma > 0$. Intuitively, in this case crime acts like a proportional tax on income, and increases in σ cause a reduction in the marginal tax rate γ . All agents benefit from a reduction in γ but workers employed at high wages profit relatively more than low-wage workers.



Figure 4.1: Equilibria with Endogenous γ

away from competitors and lower the rate at which you lose workers to competitors). It seems interesting to see what the possibility of criminal activity does in a model that already generates wage inequality for other reasons. Finally, it is always good to know the extent to which one's results are robust to relaxing restrictions, like no on-the-job search.

We take γ as given here in order to concentrate on complimetarities in wage setting as highlighted in Section 3 (although it could be endogenized as in Section 4; details available on request). Wage setting here is more complicated with $\lambda_1 > 0$ because of the considerations alluded to above: higher w affects the rate at which you steal workers from and lose worker to competitors. This leads to wage dispersion even without crime. Still, our general method outlined above still applies, and the types of possible equilibria mirror those of the model with $\lambda_1 = 0$. First we check when $\phi_0 = 0$ is a best response, which yields a Type N equilibrium. Then we check when $\phi_0 = 1$ is a best response, which yields the three subcases: $\sigma = 0$ (Type L equilibrium); $\sigma = 1$ (Type H equilibrium); and $\sigma \in (0, 1)$ (Type M equilibrium). In each case, the reservation and crime wages are given by (2.8) and (2.12), and the distribution F will be derived from equal profit conditions.

In the case $\phi_0 = 0$, there is no crime and things look much like the standard on-the-job search model (except that we do have to check that $\phi_0 = 0$ satisfies the best response condition). Again, letting l(w) be the steady state number of workers employed at a firm paying w, we assume $r \approx 0$ so that firms maximize steady state profit, $\Pi(w) = (p - w) l(w)$. They choose w taking as given worker behavior and the wages of other firms, as summarized by F. In equilibrium, all wages on the support of F yield equal profit Π^* , and any wage off the support yields $\Pi(w) \leq \Pi^*$. First we provide some properties of the equilibrium F and the lower and upper bounds of its support, denoted by \underline{w} and \overline{w} . We omit the proof of these results, which can be found in Burdett and Mortensen (1998), and in any case are very similar to the results for the case $\phi_0 = 1$ presented in detail below (see Lemma 2).

Lemma 1. Suppose $\phi_0 = 0$. Then we know the following: (a) F has no mass points; (b) $\underline{w} = R$; (c) $\overline{w} < p$; (d) there are no gaps between \underline{w} and \overline{w} .

Since all wages on the support of F earn equal profits, including $\underline{w} = R$, we have

$$(p-w) l(w) = (p-R) l(R) \text{ for all } w \in [R,\overline{w}].$$

$$(5.1)$$

Also, since the number of workers at a firm paying w must equal the number of workers earning w divided by the number of firms paying w, we have $l(w) = eG'(w)/F^{0}(w)$ where e is the number of employed workers.¹² Using the expression (2.16) for G in terms of F derived in Section 2, we have

$$l(w) = \frac{(\delta + \lambda_1) \lambda_0 u}{\left\{\delta + \lambda_1 \left[1 - F(w)\right]\right\}^2}.$$
(5.2)

 $^{^{12}}$ This assumes F and G are differentiable, which as we will see turns out to be true. Burdett and Mortensen (1998) provide a more detailed discussion.

Substituting l(w) as well as the steady state u into (5.1) and using F(R) = 0, we get

$$\frac{(p-w)}{\left\{\delta + \lambda_1 \left[1 - F\left(w\right)\right]\right\}^2} = \frac{(p-R)}{\delta^2} \text{ for all } w \in [R, \overline{w}].$$

The previous expression can be rearranged as

$$F(w) = \frac{\delta + \lambda_1}{\lambda_1} \left(1 - \sqrt{\frac{p - w}{p - R}} \right) \text{ for all } w \in [R, \overline{w}], \tag{5.3}$$

which is the unique equilibrium wage distribution given $\phi_0 = 0$. The upper bound is found by solving $F(\overline{w}) = 1$. The lower bound R is found by solving the reservation wage equation, which can be integrated explicitly once we know F(w) is given by $(5.3)^{.13}$ This fully describes the equilibrium when $\phi_0 = 0$. It is a matter of algebra now to check that $\phi_0 = 0$ satisfies the best response condition iff $p \ge \hat{p}_0(b)$, where $\hat{p}_0(b)$ is the linear function defined as the solution

$$\left(\frac{\lambda}{\delta+\lambda}\right)^2 p + \left[1 - \left(\frac{\lambda}{\delta+\lambda}\right)^2\right] b = \left(\frac{\rho}{\pi} + \gamma\right)g + z.$$

Hence, very similar to what we found before, a Type N equilibrium exists as long as p is big.

We now move to the case with $\phi_0 = 1$. Let the number of workers employed by firms paying wages above and below C be denoted $l_H(w)$ and $l_L(w)$. Let \underline{w}_H and \overline{w}_H be the lower and upper bounds of the support of F_H , and \underline{w}_L and \overline{w}_L the lower and upper bounds of the support of F_L . Then we have the following analog to Lemma 1. Note that we state the results for a general σ , with the understanding that some cases are vacuous; e.g., if $\sigma = 0$ then any statements about \underline{w}_H and \overline{w}_H do not apply.

Lemma 2. Suppose $\phi_0 = 1$. Then we know the following: (a) F has no mass points; (b) $\underline{w}_L = R$ and $\underline{w}_H = C$; (c) $\overline{w}_L < C$ and $\overline{w}_H < p$; (d) there are no gaps between \underline{w}_L and \overline{w}_L or between \underline{w}_H and \overline{w}_H , although there must be a gap between \overline{w}_L and \underline{w}_H .

¹³Although this can be done in general, it is particularly easy when $\lambda_0 = \lambda_1 = \lambda$, which guarantees R = b (recall from Section 2 that one also needs $\mu_0 = \mu_1 = \mu$ and assumptions on g_1 and g_0 to guarantee R = b, in general, but given $\phi_0 = 0$ these are not relevant).

Proof. To show (a), suppose there is a mass point at w' < p. Then any firm paying w' could earn strictly greater profit by paying $w' + \varepsilon$ for some $\varepsilon > 0$, since this would imply a discrete increase in the number of workers it employs. It implies a discrete increase because now the firm can hire workers currently earning w', and it meets workers earning w' with positive probability given the mass point. Hence, there can be no mass point at w' < p. There cannot be a mass point at $w \ge p$ since no firm offers $w \ge p$ (see below). To show (b), first suppose w' is the lowest wage in F_L . Clearly $w' \ge R$ since a firm offering less than R earns 0 profit. But if w' > R then the firm earns more profit by offering R since it hires and loses workers at the same rate (agents still accept iff they are unemployed and leave for any other firm). This means $\underline{w}_L = R$. Now suppose w' > C is the lowest wage in F_H . Given w' cannot be a mass point, the firm paying w' can strictly increase its profit by paying C, because in doing so it does not lose workers any faster. Hence, $\underline{w}_H = C$.

To show (c), first suppose that $\overline{w} = C - i.e.$ there are firms offering less than Cbut arbitrarily close to C. But then they can earn greater profit by offering C since they discretely reduce the rate at which they lose workers to jail. Hence, $\overline{w}_L < C$. Now suppose $\overline{w}_H \ge p$; as this implies non-positive profit, we have $\overline{w}_H < p$. Finally, to show (d), suppose there is an non-empty interval $[w^0, w^{00}]$, with $C \notin [w^0, w^{00}]$, with some firm paying w'' and no firm paying $w \in [w^0, w^{00}]$. Then the firm paying w'' can make strictly greater profit by paying $w^{00} - \varepsilon$ for some $\varepsilon > 0$. This is because such a firm loses no more workers than it did before and still hires at the same rate.

We can summarize the cases with $\phi_0 = 1$ as follows. First, assume $\sigma \in (0, 1)$, so that some firms pay above and some below C. Then F necessarily entails a connected support between $\underline{w}_L = R$ and \overline{w}_L , a gap between \overline{w}_L and $\underline{w}_H = C$, and a connected support between C and \overline{w}_H , as shown by the density in the final panel of Figure 5.1. The cases where $\sigma = 0$ or $\sigma = 1$ shown in panels 2 and 3 of the Figure can be considered special cases of the same thing, where one branch of F becomes degenerate. Also, show in the first panel is the case where $\phi_0 = 0$, analyzed above, which is observationally equivalent to the standard model. The key difference that arises due to crime is that firms paying in w < C not only lose workers to competitors paying higher wages, they also lose workers to jail. This explains the gap: firms paying less than C must pay strictly less than any firm paying at least C since the rate at which they lose workers is strictly higher.¹⁴



Figure 5.1: Different Equilibria With On-the-Job Search

The same logic that implies (5.2) now implies

$$l_L(w) = \frac{\lambda_0 \left(\delta + \lambda_1 + \mu_1 \pi\right) u}{\left\{\delta + \mu_1 \pi + \lambda_1 \sigma + \lambda_1 \left(1 - \sigma\right) \left[1 - F_L(w)\right]\right\}^2}$$
(5.4)

$$T_H(w) = \frac{\left(\delta + \lambda_1 \sigma\right) \left(\lambda_0 u + \lambda_1 e_L\right)}{\left\{\delta + \lambda_1 \sigma \left[1 - F_H(w)\right]\right\}^2}.$$
(5.5)

¹⁴Another way to understand the result is this. In the model with $\lambda_1 = 0$ in Section 3, we get a single wage without crime and possibly two wages with crime. In the model with $\lambda_1 > 0$ we get a distribution of wages without gaps without crime and possibly two distributions without gaps once crime is introduced.

Similarly, we now have an equal profit condition as in (5.1) for all firms paying below C and above C:

$$(p - \underline{w}_L) l_L(\underline{w}_L) = (p - w) l_L(w) \text{ for all } w < C$$
(5.6)

$$(p-C) l_H(C) = (p-w) l_H(w) \text{ for all } w \ge C.$$
 (5.7)

Substituting the l_L and l_H into the equal profit conditions and rearranging, we have the following versions of (5.3)

$$F_L(w) = \frac{\delta + \lambda_1 + \mu_1 \pi}{\lambda_1 (1 - \sigma)} \left(1 - \sqrt{\frac{p - w}{p - R}} \right)$$
(5.8)

$$F_H(w) = \frac{\delta + \lambda_1 \sigma}{\lambda_1 \sigma} \left(1 - \sqrt{\frac{p - w}{p - C}} \right).$$
(5.9)

The upper bounds are found by solving $F_L(\overline{w}_L) = F_H(\overline{w}_H) = 1$:

$$\overline{w}_L = p - (p - R) \left(\frac{\delta + \mu_1 \pi + \lambda_1 \sigma}{\delta + \mu_1 \pi + \lambda_1}\right)^2$$
(5.10)

$$\overline{w}_H = p - (p - C) \left(\frac{\delta}{\delta + \lambda_1 \sigma}\right)^2.$$
(5.11)

We now have F_L and F_H in terms of R, C, and σ . To determine σ , we compare profit across firms paying above and below C. Thus, define $T(\sigma) = \Pi(C) - \Pi(R)$, as in the model with no on-the-job search. Using (5.4) and (5.5), $T(\sigma) = 0$ reduces to

$$\frac{p-R}{\delta+\mu_1\pi+\lambda_1} = \frac{\left(\delta+\mu_1\pi+\lambda_1\right)\left(p-C\right)}{\left(\delta+\sigma\lambda_1\right)\left(\delta+\mu_1\pi+\sigma\lambda_1\right)},\tag{5.12}$$

a quadratic equation in σ with at most one positive root. This fully describes F, given (R, C), when $0 < \sigma < 1$. The cases $\sigma = 0$ or $\sigma = 1$ can be considered special cases. Finally, we can get R and C explicitly in terms of σ by integrating the reservation and crime wage equations for this F. Inserting these into (5.12) reduces the model to one equation in σ .

At this point the analysis mimics the model with no on-the-job search in terms of describing when the different equilibria exist. While one could proceed more generally, to reduce notation we focus here on the case where $\lambda_0 = \lambda_1 = \lambda$, $\mu_0 = \mu_1 = \mu$, and

 $g_1(w) = g_0 = g$, which guarantees R = b. Then it is relatively easy to solve for the crime wage C and insert it into $T(\sigma)$ to reduce things to one equation in σ . Consider first the Type L equilibrium. We need to check $\phi_0 = 1$ and $\sigma = 1$. Given $\sigma = 1$, $\phi_0 = 1$ satisfies the best response condition iff $p \leq \hat{p}_0(b)$, where $\hat{p}_0(b)$ is defined above. It remains to check that $T(1) \geq 0$ holds iff $p \geq \hat{p}_1(b)$, where $\hat{p}_1(b)$ solves

$$\frac{(\delta+\lambda)(\lambda+\mu\pi)^2 + [\rho\delta+\lambda(\rho-\delta)]}{(\delta+\lambda)(\delta+\lambda+\mu\pi)^2} p + \left\{1 - \frac{(\delta+\lambda)(\lambda+\mu\pi)^2 + [\rho\delta+\lambda(\rho-\delta)]}{(\delta+\lambda)(\delta+\lambda+\mu\pi)^2}\right\} b = \left(\frac{\rho}{\pi} + \gamma\right)g + z.$$

Similarly, for a Type H equilibrium we need to check $\phi_0 = 1$ and $\sigma = 0$. The best response condition for $\phi_0 = 1$ now holds iff $p \leq \hat{p}_2(b)$, where $\hat{p}_2(b)$ solves

$$\left(\frac{\lambda}{\delta+\lambda+\mu\pi}\right)^2 p + \left[1 - \left(\frac{\lambda}{\delta+\lambda+\mu\pi}\right)^2\right] b = \left(\frac{\rho}{\pi} + \gamma\right)g + z$$

The condition for $\sigma = 0$, $T(0) \le 0$, holds iff $p \le \hat{p}_3(b)$ where $\hat{p}_3(b)$ solves

$$\frac{(\lambda+\mu\pi)^2+\rho(\mu\pi+2\lambda)}{(\delta+\lambda+\mu\pi)^2}p + \left[1 - \frac{(\lambda+\mu\pi)^2+\rho(\mu\pi+2\lambda)}{(\delta+\lambda+\mu\pi)^2}\right]b = \left(\frac{\rho}{\pi} + \gamma\right)g + z.$$

One can work out similar conditions for the Type M equilibrium.

The key results are summarized by saying a Type N equilibrium exists if $p \ge \hat{p}_0(b)$, a Type L equilibrium exists if $\hat{p}_1(b) \le p \le \hat{p}_0(b)$, and a Type H equilibrium exists if $p \le \min \{\hat{p}_2(b), \hat{p}_3(b)\}$. Moreover, it is again possible to show explicitly that there are multiple equilibria for low enough ρ .¹⁵ The bottom line is that the model with on-the-job search delivers very similar results. Of course, the on-the-job search model is more complicated, perhaps because it is more realistic along certain dimensions concerning labor market activity, but in terms of the economics of crime it behaves much like the simpler models studied above.

6. Other Extensions

While there are clearly many ways in which one could extend and generalize the model, in this section we explore two natural ones: we consider an alternative mechanism

¹⁵ To see this, first notice that $\hat{p}_2(b) > \hat{p}_0(b)$. Then it is a matter of algebra to show that $\hat{p}_0(b) < \hat{p}_3(b)$, and hence that an equilibrium of Type N coexists with one of Type H, as long as $(\mu \pi + 2\lambda)\rho < \left[\frac{\lambda(\delta+\lambda+\mu\pi)}{\delta+\lambda}\right]^2 - (\delta+\mu\pi)^2$.

for determining wages, and we endogenize the expected amount one gets from theft. Thus, instead of assuming firms post wages, we now suppose the wage is set by bargaining. In principle, one could use the generalized Nash solution, where workers have any degree of bargaining power $\theta \in [0, 1]$; for simplicity, however, we concentrate here on the case $\theta = 1$, which immediately implies w = p. In some sense this is the polar opposite of wage posting. Also since we have seen that on-the-job search does not affect the basic results, here we set $\lambda_1 = 0$ and $\lambda_0 = \lambda$. Finally, also for simplicity we set $\mu_0 = 0$ and $\mu_1 = \mu$. This means the unemployed do not get crime opportunities, which means that we only have to determine $\phi = \phi_1(p)$ and not ϕ_0 .

We endogenize the returns to crime by setting $g_0 = \alpha b$, $g_1(w) = \alpha w$, and $g = \alpha \omega$ where $\omega = \frac{ep+ub}{e+u}$ is the expected income of a random victim. Hence, the victim loses and the perpetrator gets a fraction α of the former's current income, but the agents cannot observe income when deciding to commit the crime. We also endogenize the crime rate, as in Section 4, by setting $\gamma = \frac{e\mu\phi}{e+u}$. With these assumptions, the Bellman equations are

$$rV_0 = (1 - \alpha\gamma) b + \lambda (V_1 - V_0)$$

$$rV_1 = (1 - \alpha\gamma) p - \delta (V_1 - V_0) + \mu\phi [\alpha\omega - \pi (V_1 - J)].$$

The steady states are $e = \lambda \rho / \Psi$, $u = (\delta + \mu \pi \phi) \rho / \Psi$, and $n = \mu \pi \lambda \phi / \Psi$, where $\Psi = \mu \pi \lambda \phi + \lambda \rho + (\delta + \mu \pi \phi) \rho$. The crime rate is

$$\gamma = \frac{\mu\lambda\phi}{\delta + \lambda + \mu\pi\phi}$$

and expected income is

$$\omega = \frac{\lambda p + (\delta + \mu \pi \phi) b}{\delta + \lambda + \mu \pi \phi}.$$

The only thing left to determine is ϕ , which comes from knowing the sign of $V_1 - J - \frac{\alpha \omega}{\pi}$. Since this term is proportional to

$$S(\phi) = [(\delta + \lambda + \mu \pi \phi - \alpha \mu \lambda \phi) (r + \rho + \lambda) \pi - (r + \rho) (\delta + \rho + \lambda) \alpha \lambda] p +$$

$$[(\delta + \lambda + \mu \pi \phi - \alpha \mu \lambda \phi) (\delta - \rho) \pi - (r + \rho) (\delta + \rho + \lambda) \alpha (\delta + \mu \pi \phi)] t$$
$$-\pi (r + \delta + \lambda) (\delta + \lambda + \mu \pi \phi) z,$$

the equilibrium is fully described by the best response condition

$$\phi = \begin{cases} 0 & \text{if } S(0) > 0\\ \phi^* \in [0,1] & \text{if } S(\phi) = 0\\ 1 & \text{if } S(1) < 0 \end{cases}$$

The results are similar to those shown in either Figure ?? or ??, depending on parameters. In particular, for a given b, for low p we have a unique Type H equilibrium, for high p we have a unique Type L equilibrium, and for intermediate values of p we either have a unique Type M equilibrium or all three equilibria. It is not hard to construct explicit examples with multiplicity.

Hence, the basic results survive under alternative mechanisms for determining the wage and under endogenizing the expected returns to theft. The multiplicity in this section of course comes from the channel of endogenous γ , since with our extreme bargaining rule the wage distribution is basically fixed (w = p with probability 1), but this would not necessarily be true with general bargaining. In any event, the overall conclusion is that the basic economics of the model, including multiple equilibria through a variety of channels, wage dispersion for some parameterizations, and predictions about how crime, unemployment, wages etc. depend on policy and other parameters, seems quite robust.

7. Conclusion

This paper analyzed several versions of economic model of crime. In addition to criminal behavior, unemployment and wages dispersion are endogenous. We think that the model usefully illuminates the interrelations between these variables. It also sheds new light on some standard models of the labor market. For example, once crime is introduced, in models that otherwise predict a single we can generate wage inequality, and in models that otherwise imply wage dispersion the nature of this dispersion changes qualitatively. A key result is that the crime model generates multiple equilibria in interesting ways and through different channels. We think that versions of this model can be used in the future to address the empirical and policy issues in more detail.

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