Penn Institute for Economic Research
Department of Economics
University of Pennsylvania
3718 Locust Walk
Philadelphia, PA 19104-6297
pier@econ.upenn.edu
http://www.econ.upenn.edu/pier

PIER Working Paper 02-037

"Unstable Relationships"

by

Kenneth Burdett, Ryoichi Imai, and Randall Wright

UNSTABLE RELATIONSHIPS*

Kenneth Burdett Ryoichi Imai Randall Wright

May 2, 2002

Abstract

We analyze models where agents search for partners to form relationships (employment, marriage, etc.), and may continue searching for different partners while matched. Matched agents are less inclined to search if their match yields more utility and if it is more stable. If one partner searches the relationship is less stable, so the other is more inclined to search, potentially making instability a self-fulfilling prophecy. We show this can generate a multiplicity – indeed, a continuum – of steady state equilibria. In any equilibrium there tends to be too much turnover, unemployment, and inequality, compared to the efficient outcome. A calibrated version of the model explains 1/2 to 2/3 of reported job-to-job transitions.

Keywords: search, matching, marriage, unemployment, inequality

*Burdett and Wright: University of Pennsylvania, 3718 Locust Walk, Philadelphia PA 19104; Imai: Nagoya University of Commerce and Business Administration, Nissin, Komenoki, Sagamine 4-4, Aichi 470-0193, Japan. We thank seminar participants at Arizona State, Teaxs-Austin, Essex, McGill, Kyoto, Waseda, Yokohama, the Cleveland Fed, the Japanese Economics Association, the Kansai Labor Economics Conference, the Southeastern Economic Theory meetings, the New York Area Macro Workshop, and the Canadian Macro Study Group. We also thank David Andolfatto, Gadi Barlevy, Gwen Eudey, John Kennan, Ian King, Dale Mortensen, Giuseppe Moscarini, Rob Shimer and Tracy Webb for their input. The NSF provided financial support.

1 Introduction

We analyze situations where agents search for partners to form bilateral relationships, such as marriage or employment relationships, with the following feature: while matched, an individual may at some cost continue to search for a different partner. Matched agents are less inclined to search – that is, more inclined to be "faithful" to their current partner – if the match is better in the sense of instantaneous utility and also if it is more stable. What lends stability to a relationship? If your partner is searching, your relationship is less secure because you are more likely to be abandoned, and so you will be more inclined to search. Hence, instability can be a self-fulfilling prophecy. We show these considerations can lead to multiple equilibria, and indeed to a continuum of steady state equilibrium. We also study the implications for efficiency. According to our results, there tends to be excessive turnover, unemployment and inequality in equilibrium.

The source of multiplicity and inefficiency is new to the search literature, and has nothing to do with the thick-market or congestion effects that have been understood at least since Diamond (1982), Hosios (1990), and related work. For example, the usual thick-market effect works as follows: assuming increasing returns in the matching technology, if there is more search activity then it is easier to meet people, and this makes you more inclined to search. To emphasize the distinction here we assume constant returns to scale in the matching technology, so that your probability of meeting someone is independent of aggregate search behavior, and make some other assumptions to reduce strategic interactions between individuals and the market. This allows us to focus more on strategic interactions within relationships and concentrate more on endogenous instability.

Our basic framework assumes that when any two agents meet they draw a random x describing the instantaneous utility that each will receive if they form a partnership. The simplest version of the model has a two point distribution, $x \in \{x_1, x_2\}$, with $x_2 > x_1$. In this case, it is relatively easy to characterize the equilibrium set, and describe the parameter values that lead to multiplicity. A simple example (but not the only, nor necessarily the most interesting, example) of multiplicity is the following: for some parameters individuals could either be "faithful" or "unfaithful" in x_1 matches, depending on what they think others are doing. Perhaps surprisingly, for some parameters there can even be equilibria where agents are "unfaithful" in x_2 matches but "faithful" in x_1 matches, despite $x_2 > x_1$, simply as a self-fulfilling prophecy.

Whether or not there are multiple equilibria, there tends to be inefficiently high levels of search, and this leads to inefficiently high levels of turnover, unemployment and inequality. Two distinct types of inefficiency are possible. When there are multiple equilibria, excessive search can be interpreted as a coordination failure: you are "unfaithful" in certain matches because you think others will be, even though a better equilibrium exists where agents are "faithful" in these matches. However, we emphasize that this is not the only situation that can arise: sometimes a unique equilibrium exists, and it also tends to have excessive turnover, unemployment, and inequality. The situation here is analogous to a prisoner's dilemma, not a coordination failure. The potential ways to correct the inefficiencies in these two distinct situations is something we will discuss.

In the case where x is drawn from a general distribution, we consider equilibria of the following class: agents choose a reservation value R such that they enter relationships iff $x \geq R$, and a critical value Q such that they search while matched iff $x \leq Q$. We show that any value of Q in some nondegenerate interval satisfies all of the steady state equilibrium conditions: i.e., there is a continuum of steady state equilibria. This is not a typical outcome.[1] The explanation lies in the value of x playing two roles: it conveys utility, but it can also be used as a signal for agents to coordinate on "faithful" or "unfaithful" behavior. The probability of searching while matched is increasing in Q, and even the best equilibrium tends to have Q above the efficient outcome. Again, equilibrium has excessive turnover, unemployment and inequality.

There is reason to think searching while matched is empirically important. First, the number of job-to-job transitions is big. Fallick and Fleischman (2001), e.g., report that in 1999 more than 4 million workers changed employers in an average month, about the same as the number who left em-

[1] Kehoe et al. (1994) show in a search model, using standard tools from general equilibrium theory, that although there may be many dynamic equilibrium paths there exist a finite number of steady states generically.

ployment to exit the labor force and more than double the number who left for unemployment. Modeling on-the-job search is crucial for understanding this behavior. We can also infer from less formal data that searching while matched is important. For example, 13% of Canadian workers surveyed are actively seeking new jobs, and 45% while not actively searching would be willing to "consider other opportunities or offers" (Galt [2001]). To see how well the model can account for on-the-job search, we calibrate it. It turns out that as long as we want to match the flow from employment to non-employment, the model can explain a sizable fraction (1/2 to 2/3) but not all of the job-to-job transitions in the Fallick and Fleischman data.

Generally, we think that on-the-job search and the implied possibility of endogenous instability seem interesting and have been neglected in the literature. A key new finding that arises here due to on-the-job search is the tendency towards excess turnover, unemployment and inequality. The rest of the paper involves making the model precise and proving our claims. Section 2 presents the basic structure. Section 3 analyzes the case where $x \in \{x_1, x_2\}$. Section 4 discusses robustness to alternative assumptions. Section 5 analyzes the case of a general x distribution. For each version of the model we provide results on the existence and number of equilibria, on the steady state distribution of match quality, and on the comparison between equilibrium and efficient outcomes. Section 5 describes the numerical results. Section 6 provides a brief conclusion. Some of the more technical proofs are relegated to the Appendix.[2]

2 The Basic Framework

There is a $[0,1]$ continuum of infinitely-lived agents who are interested in forming bilateral relationships. While unmatched, they search and receive instantaneous utility b from being single (net of any search costs). While searching they meet other agents according to a Poisson process with arrival rate α. Agents are homogeneous ex ante, but matches are heterogeneous ex post: when a pair meet they draw a random variable x giving the instantaneous utility that each would receive if they form a partnership.[3] The distribution of *potential* match quality is $F(x)$, and it is exogenous. The distribution of match quality across *actual* existing relationships is $G(x)$, and is endogenous because agents may accept some values of x and reject others.

If you search while matched, you pay cost $d > 0$ and continue to meet new agents at rate α; if you do not search you pay no cost and meet no

[2]In terms of related literature, on-the-job search is discussed in many places, including Burdett (1977), Mortensen (1978,1988), Pissarides (1994), Webb (1998,1999), Burdett and Mortensen (1998), Burdett and Coles (1999), Byeonju (1999), and Moscarini (2002). See Mortensen and Pissarides (1998) for additional references. Those papers do not address the main issues studied here, however, including endogenous instability and the implications for inequality and welfare. There is a also body of work that studys partnership formation and cooperation in a different context, including Kranton (1996), Ghosh and Ray (1996), Ramey and Watson (1997), and Eeckhout (2000). The models and the questions addressed in this literature are very different from what we do, however.

[3]One can assume that each meeting generates a random surplus X, and agents bargain in such a way that each gets $x = X/2$ (as would follow from standard theory). Alternatively, one can assume nontrannsferrable utility. Most of what we do here does not depend on this distinction (but see Burdett and Wright [1998] for an example of a realted model where things depend a lot on whether utility is transferrable).

one new. If you are matched and meet someone new, we make the following base assumption: you must first leave your current partner before you draw the value of x associated with the new person, and you may then form a new relationship, or reject the new person and become unmatched, but you cannot go back to your old partner. This may or may not be realistic, depending on the application, but there are several reasons why it is a good assumption for our purposes. First, the alternative model, where you are allowed to go back, is much more complicated for a general distribution F. Even for a simple specification of F, the analysis is less straightforward in the alternative model, and it turns out that the results are qualitatively the same as under the base assumption.

Moreover, our base assumption is appropriate if one wants to focus on the strategic interactions *within* relationships. To explain this, consider the opportunities available when you search. First, your arrival rate of meetings α could depend on the number of agents searching, if the matching technology has non-constant returns. As this has been studied extensively in the past, we assume constant returns. However, even given a constant α, if we allow a matched agent who meets someone new to choose between the new person and his current partner, your *effective* arrival rate will still be endogenous. To see this, note that if matched agents are allowed to choose, they will only enter into a relationship with you if you beat their current value of x, and to know your chances of this you need to know the endogenous distribution G. Under our assumptions you do not need to know G: everyone you meet is effectively unmatched and generates a random draw from F. This allows us to solve the model recursively.

So in our base model agents can take effective arrival rates as given, and therefore concentrate on what is happening within relationships. A critical aspect of what is happening in your relationship is the search behavior of your partner, who could meet someone else and leave.[4] In addition to these endogenous separations, we also assume that there are exogenous terminations at rate σ, simply to guarantee that there is always a positive fraction of the population unemployed – i.e., single – in steady state. All of these terminations could be considered involuntary. You can also voluntarily move from one match directly to another, and you can move from a match to unemployment when meet someone new but find them worse than being single. Hence, the model has well-defined notions of layoffs, quits and job-to-job transitions.

3 A Simple Model

In this section we assume $x = x_2$ with probability π and $x = x_1 < x_2$ with probability $1 - \pi$. Agents can then be in one of three states: unmatched, in an x_1 relationship, or in an x_2 relationship. The fraction in each state

[4] Given someone is searching while matched, when they meet someone new there is no additional decision to leave or stay with their current partner (i.e., to leave or stay *without* observing the new x; once they observe the new x they are not allowed to stay). This is because all new people look identical ex ante, so if an agent would not leave with one new person they would not leave with any, and therefore would not be engaged in costly search in the first place.

is denoted N_0, N_1, and N_2, where $N_0 + N_1 + N_2 = 1$. Let the payoff, or value, function of an agent in each state be V_0, V_1, and V_2. Agents need to choose strategies for deciding when to accept a match and when to search while matched. Let A_j be the probability that a representative agent agrees to enter into an x_j match, and let S_j be the probability that he searches while in an x_j match, $j = 1, 2$. Sometimes we will be more explicit by saying that you choose a_j and s_j taking as given that others choose A_j and S_j; in equilibrium, of course, $a_j = A_j$ and $s_j = S_j$. We focus for now on pure strategy equilibria (mixed strategies are considered later).

The value functions satisfy the standard continuous time dynamic programming equations,

$$\begin{aligned} rV_0 &= b + \alpha\pi A_2(V_2 - V_0) + \alpha(1-\pi)A_1(V_1 - V_0) \\ rV_1 &= x_1 + (\sigma + S_1\alpha)(V_0 - V_1) + s_1\Sigma_1 \\ rV_2 &= x_2 + (\sigma + S_2\alpha)(V_0 - V_2) + s_2\Sigma_2, \end{aligned} \tag{1}$$

where Σ_i is the net gain from searching while in an x_i match,

$$\begin{aligned} \Sigma_1 &= \alpha\pi[A_2V_2 + (1-A_2)V_0 - V_1] + \alpha(1-\pi)(1-A_1)(V_0 - V_1) - d \\ \Sigma_2 &= \alpha\pi(1-A_2)(V_0 - V_2) + \alpha(1-\pi)[A_1V_1 + (1-A_1)V_0 - V_2] - d. \end{aligned}$$

For example, the third equation in (1) equates the flow value rV_2 to the sum of three terms. The first is the instantaneous utility x_2. The second is the probability you become involuntarily unmatched, either because of an exogenous separation or because your partner meets someone new, $\sigma + S_2\alpha$, times the capital loss $V_0 - V_2$. The final term is your search decision s_2 times the net gain from searching while matched, Σ_2.[5]

A *steady state equilibrium* is a list including value functions (V_0, V_1, V_2) satisfying the dynamic programming equations (1), a steady state (N_0, N_1, N_2) satisfying conditions to be given below, and strategies (A_1, A_2, S_1, S_2) satisfying the following best response conditions:

$$A_j = \begin{cases} 1 & \text{if } \Delta_j > 0 \\ [0,1] & \text{if } \Delta_j = 0 \\ 0 & \text{if } \Delta_j < 0 \end{cases}, \qquad S_j = \begin{cases} 1 & \text{if } \Sigma_j > 0 \\ [0,1] & \text{if } \Sigma_j = 0 \\ 0 & \text{if } \Sigma_j < 0 \end{cases} \tag{2}$$

where $\Delta_j = V_1 - V_0$ is the net gain from accepting an x_j match and Σ_j is the net gain from searching in an x_i match defined above. Notice that we do not actually need to know the steady state to analyze search behavior, since the N_j's do not enter (1) or (2) due to our simplifying assumptions. Hence we can solve things recursively: first determine (A_1, A_2, S_1, S_2) and (V_0, V_1, V_2), and later find (N_0, N_1, N_2).

As we will show, of all possible pure strategy profiles, only five potentially constitute equilibria. First, there is a "degenerate" or *type D* equilibrium where agents reject all matches: $A_1 = A_2 = 0$ (and S_i is irrelevant). Second, there is what we call a "choosy" or *type C* equilibrium where agents accept x_2 but reject x_1 and, since they would not accept x_1, do not search in x_2 matches: $A_1 = 0$, $A_2 = 1$ and $S_2 = 0$. Third, there is a "faithful" or *type F*

[5] One could be more explicit in terms of a_j and A_j, but we have written the equations in the text using the fact that, given x, the two parties always agree on whether to form a relationship. We ignore the possibility of a degenerate outcome where, for any x both agents reject the match because they believe the other will, since this would not be subgame perfect in a game where agents move sequentially.

equilibrium where agents accept x_1 as well as x_2 and do not search in either case: $A_1 = A_2 = 1$ and $S_1 = S_2 = 0$. Fourth, there is an "unfaithful" or *type U* equilibrium, in which agents accept both matches and continue to search in x_1 matches: $A_1 = A_2 = 1$, $S_1 = 1$ and $S_2 = 0$. Finally, there is a "perverse" or *type P* equilibrium where agents accept both and, perhaps counter to intuition, search in x_2 but not x_1 matches: $A_1 = A_2 = 1$, $S_1 = 0$ and $S_2 = 1$.

The following proposition describes exactly when each of the different types of equilibria exist. The proof is given in the Appendix, but the method is straightforward: for any possible strategy profile, simply solve for the value functions and check the best response conditions..

Proposition 1 *There are five potential types of equilibria:*

$$\begin{array}{lll}
\textit{type } D: & A_1 = A_2 = 0 & \text{exists iff } x_2 \leq b \\
\textit{type } C: & A_1 = 0, A_2 = 1, S_2 = 0 & \text{exists iff } x_1 \leq b + d,\ x_2 \geq y_1 \\
\textit{type } F: & A_1 = A_2 = 1, S_1 = S_2 = 0 & \text{exists iff } x_2 \leq y_1, y_2 \\
\textit{type } U: & A_1 = A_2 = 1, S_1 = 1, S_2 = 0 & \text{exists iff } x_1 \leq b + d,\ x_2 \geq y_3 \\
\textit{type } P: & A_1 = A_2 = 1, S_1 = 0, S_2 = 1 & \text{exists iff } b + d \leq x_2 \leq y_4
\end{array}$$

where the critical values y_j are given by

$$\begin{aligned}
y_1 &= \frac{r+\sigma+\alpha\pi}{\alpha\pi}x_1 - \frac{r+\sigma}{\alpha\pi}b \\
y_2 &= x_1 + \frac{r+\sigma}{\alpha\pi}d \\
y_3 &= \frac{r+\sigma+\alpha}{r+\sigma+2\alpha}x_1 + \frac{\alpha}{r+\sigma+2\alpha}b + \frac{(r+\sigma)(r+\sigma+2\alpha)+\alpha^2\pi}{\alpha\pi\,(r+\sigma+2\alpha)}d \\
y_4 &= \frac{r+\sigma+2\alpha}{r+\sigma+\alpha}x_1 - \frac{\alpha}{r+\sigma+\alpha}b - \frac{(r+\sigma)(r+\sigma+2\alpha)+\alpha^2(1-\pi)}{\alpha(1-\pi)(r+\sigma+\alpha)}d.
\end{aligned}$$

There are no other pure strategy equilibria.

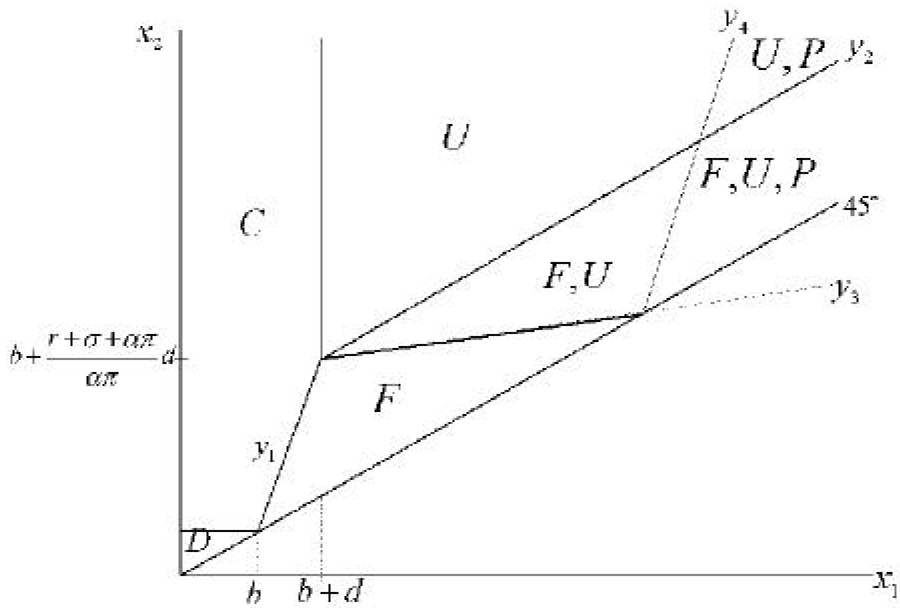

Figure 1: Regions of different equilibria

The regions where the different equilibria exist are depicted in (x_1, x_2) space in Figure 1 (*type D* equilibrium exists in the region labeled D, and so on). Notice, for example, that for the *type F* equilibrium to exist we need x_1 big enough so that agents accept it, and x_2 small enough so that agents stop searching once they do accept x_1. For the *type U* equilibrium, x_2 must be large enough to make agents search in x_1 matches, but we also need x_1 large enough that agents prefer to accept x_1 and search, rather that reject x_1 and continue to search while unmatched. Observe that when $x_2 \in (y_3, y_2)$ the *type F* and *type U* equilibria coexist; hence agents in x_1 matches may either be "faithful" or "unfaithful" depending on what other agents are doing. This is endogenous instability.[6]

Endogenous instability is sufficiently powerful that for some parameters there exists a "perverse" equilibrium where $V_1 > V_2$, even though $x_1 < x_2$, simply because agents believe that x_2 matches will be unstable and these matches are in fact unstable because people search while in x_2 matches. This is only possible if x_1 is not too much less than x_2, however, since agents will only sacrifice so much utility for security, and if x_1 and x_2 are large relative to $b + d$, since it is a high cost of separation that makes security important. Also notice that whenever the *type P* equilibrium exists there coexists another equilibrium (either a *type U* equilibrium, or both a *type U* and *type F* equilibria). In any case, we do not intend to dwell on "perverse" outcomes, but mention the *type P* equilibrium as an extreme example of endogenous instability.[7]

We proceed to consider efficiency, defined in terms of a standard social planner's welfare criterion for this class of models, average lifetime income:

$$W = N_0V_0 + N_1V_1 + N_2V_2. \tag{3}$$

[6] When *type F* and *type U* equilibria coexist, one can also construct a mixed-strategy equilibrium where agents in x_1 matches search with probability $S_1 \in (0, 1)$. We omit the routine calculations in the interests of space, but details are available on request.

[7] A special case of a *type P* equilibrium occurs when $x_1 = x_2$, which means that there is no fundamental difference between matches but people simply believe that certain relationships will be unstable. This has an interpretation in terms of *discrimination*. For example, suppose that agents are distinguished by one of two identifiable but otherwise irrelevant characteristics, say black and white. It is possible for individuals to believe that black-white relationships will be less stable and hence less desirable than black-black or white-white relationships, and for this belief to be true in equilibrium, even if color has no intrinsic impact on payoffs. When $x_2 > x_1$ these can be equilibrium beliefs even though black-white realtionships are intrinsically better.

As $x_2 < b$ implies the efficient outcome is obviously $A_1 = A_2 = 0$, we assume here $x_2 > b$. Then any efficient outcome clearly entails $A_2 = 1$ and $S_2 = 0$. Hence, the *type P* equilibrium cannot possibly be efficient, so we ignore it for now and ask when the planner prefers the *type C*, *type F* or *type U* strategies. The answer is given below, where again the proof is in the Appendix.

Proposition 2 *Given $x_2 > b$, the strategies that maximize W are as follows:*

$$\begin{array}{lll} \textit{type C:} & A_1 = 0 & \text{if } x_1 \leq b + d,\ x_2 \geq y_A \\ \textit{type F:} & A_x = 1, S_1 = 0 & \text{if } x_2 \leq y_A,\ x_2 \leq y_S \\ \textit{type U:} & A_x = 1, S_1 = 1 & \text{if } x_1 \geq b + d,\ x_2 \geq y_S, \end{array}$$

where the critical values y_j are given by

$$\begin{aligned} y_A &= \frac{\alpha\pi + \sigma}{\alpha\pi} x_1 - \frac{1}{\alpha\pi} b \\ y_S &= \frac{\pi(\sigma + 2\alpha) + \sigma}{\pi(\sigma + 2\alpha)} x_1 - \frac{\sigma}{\pi(\sigma + 2\alpha)} b + \frac{\sigma(\sigma + \alpha)}{\alpha\pi(\sigma + 2\alpha)} d. \end{aligned}$$

To compare equilibria and efficient outcomes, it is best to begin with the limiting case $r \to 0$, since then the y_1 defined in Proposition 1 coincides with the y_A defined in Proposition 2. This means that the region where $A_1 = 1$ is an equilibrium coincides with the region where it is efficient, and given that A_1 is efficient, we can concentrate on the efficiency of S_1. Figure 2 shows a version of Figure 1 drawn with $r \approx 0$, and highlights two regions in which the equilibrium differs from the planner's solution (in all other regions, equilibrium is efficient). In the region labeled 1, the planner chooses the *type F* strategy but the unique equilibrium is *type U*. Here, there is unambiguously too much search, in the sense that $S_1 = 1$ is the unique equilibrium but the efficient solution is $S_1 = 0$. In the region labeled 2, the planner again chooses the *type F* strategy, but now there are multiple equilibria, including *type F* but also including *type U*.

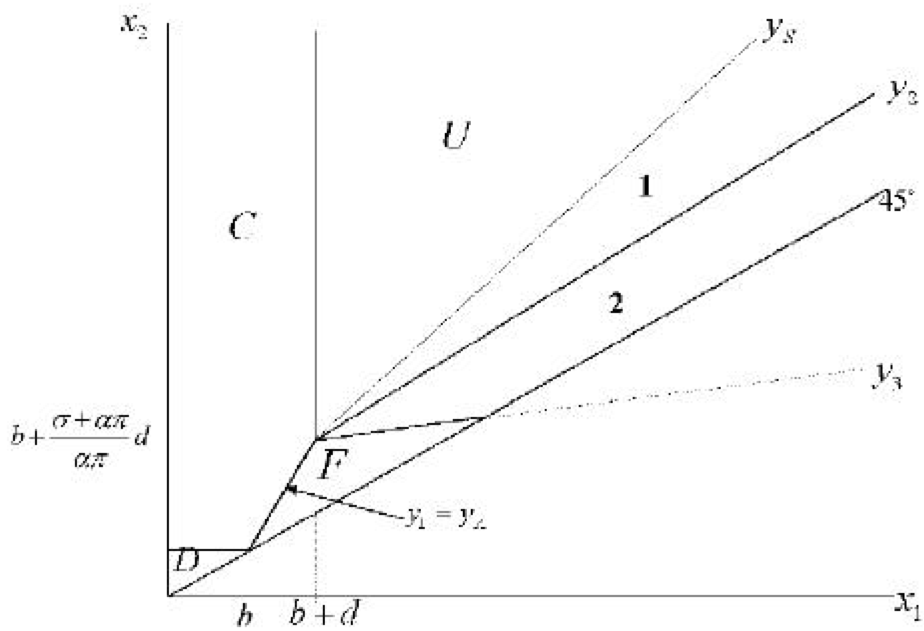

Figure 2: Regions of inefficiency, $r \approx 0$

The conclusion is that, at least when r is not too big, there is a tendency towards too much search. Heuristically, the reason is that if your partner decides to search while matched, they take into account their own costs and benefits but neglect the cost on you.[8] Notice that the inefficiency takes the

[8]One may think there is an additional effect, which is that when other agents search they might meet your partner, who will then abandon you. While this could be true without constant returns in matching, it is not true here. Thus, when other agents search, they increase your arrival rate since they might meet you, but also decrease it since they

form of a coordination failure in region 2, where there are both good and bad equilibria: you will search in an x_1 match, counter to efficiency, if and only if your partner does. In contrast, in region 1 the inefficiency takes the form of a prisoner's dilemma: you search in an x_1 match, counter to efficiency, regardless of what everyone else does.

Before discussing the welfare results for large r, we first want to compare the distributions across the equilibrium and efficient outcomes. Begin by writing the net flow of agents into x_1 matches as

$$\dot{N}_1 = N_0\alpha(1-\pi)A_1 + N_2 S_2\alpha(1-\pi)A_1 - N_1\psi_1, \tag{4}$$

where $\psi_1 = \sigma + S_1\alpha + S_1\alpha(1-\pi)(1-A_1) + S_1\alpha\pi$. The first term represents single agents who find an x_1 match and accept. The second term represents agents in x_2 matches who search, find an x_1 match and accept. The final term is flow out of x_1 matches, including agents whose relationships break up exogenously, those who are abandoned by their partners, those who are searching and get an x_1 draw and reject it, and those who are searching and get an x_2 draw whether they accept or not.

There is a symmetric expression for $\dot{N}_2$, and steady state solves $\dot{N}_1 = \dot{N}_2 = 0$. Letting superscripts indicate the steady state in a particular equilibrium (e.g., N_0^C is the number of unmatched agents in *type C* equilibrium), the steady state in each type of equilibrium is described in the following

might meet others who could have met you, and these effects exactly cancel under constant returns. This is standard in search theory, and is exactly why we assume constant returns – so that we can ignore these effects acrss macthes and focus on intra-match effects.

Proposition. As these results follow from routine algebra, we omit a proof.

Proposition 3 *There is a unique steady corresponding to each type of equilibrium, given by*

$$
\begin{array}{lll}
N_0^D = 1 & N_1^D = 0 & N_2^D = 0 \\
N_0^C = \dfrac{\sigma}{\alpha\pi + \sigma} & N_1^C = 0 & N_2^C = \dfrac{\alpha\pi}{\alpha\pi + \sigma} \\
N_0^F = \dfrac{\sigma}{\alpha + \sigma} & N_1^F = \dfrac{\alpha(1-\pi)}{\alpha + \sigma} & N_2^F = \dfrac{\alpha\pi}{\alpha + \sigma} \\
N_0^U = \dfrac{\sigma(\sigma + \alpha + \alpha\pi)}{\kappa_U} & N_1^U = \dfrac{\sigma\alpha(1-\pi)}{\kappa_U} & N_2^U = \dfrac{\alpha\pi(\sigma + 2\alpha)}{\kappa_U} \\
N_0^P = \dfrac{\sigma(\sigma + 2\alpha - \alpha\pi)}{(\sigma + 2\alpha)\kappa_P} & N_1^P = \dfrac{\alpha(1-\pi)}{\kappa_P} & N_2^P = \dfrac{\sigma\alpha\pi}{(\sigma + 2\alpha)\kappa_P}
\end{array}
$$

where $\kappa_U = (\sigma + 2\alpha)(\alpha\pi + \sigma)$ *and* $\kappa_P = \alpha(1-\pi) + \sigma$.

The key observation is that $N_0^U > N_0^F$, $N_2^U > N_2^F$ and $N_1^U < N_1^F$. The is relevant because when the equilibrium and efficient outcomes differ, it is because we are in a *type U* equilibrium but the planner's preferred strategies are *type F*. Hence, the planner prefers fewer unmatched agents, fewer x_2 matches, and more x_1 matches than obtain in equilibrium. In other words, the efficient outcome involves less unemployment and less inequality. Intuitively, too much search comes with too much inequality, simply because what agents are searching for is to move up in the distribution, and also too much unemployment, because one way they move up in the distribution is by abandoning their current partners.

The above welfare (but not steady state) results are for the case $r \approx 0$. The effect that generates too much search is there for any r, but for big r

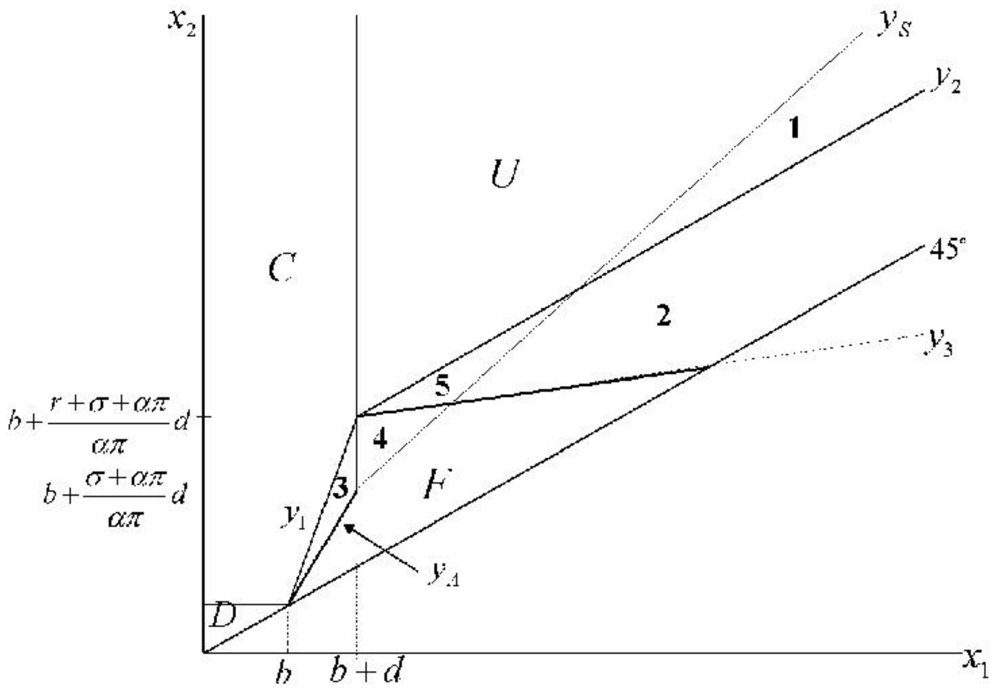

Figure 3: Regions of inefficiency, $r > 0$

there is another effect going the other way. This other effect implies too little search, according to W, because impatient agents are less inclined than the planner to reject an x_1 match and to search in an x_1 match simply because their private gains from doing so accrue only in the future but the cost is paid today. Figure 3 provides a version of Figure 2 with $r > 0$, and shows that, in addition to the regions 1 and 2 with too much search, there are three new regions with too little search.[9] This effect is due to differences in

[9] In the region labeled 3, the planner chooses the *type C* strategy but the unique equilibrium is *type F*; in the region labeled 4, the planner chooses the *type U* strategy but the unique equilibrium is *type F*; and in the region labeled 5, the planner chooses the *type U* strategy but *type U* and *type F* equilibria exist.

discount rates between the planner and private agents, is well understood, and is not especially interesting. What is novel here is the effect that tends to generate too much search, which in any event always dominates for small r. We summarize what has been shown in the following statement.

Proposition 4 *At least for r not too big, either the equilibrium is efficient, or the equilibrium has $S_1 = 1$ while the efficient outcome has $S_1 = 0$, which means the equilibrium has too much search, too much unemployment, and too much inequality.*

4 Robustness

In this section we discuss some robustness issues. To begin, we sketch the effect of changing the base assumption that you cannot stay with your old partner when you meet someone new. If we change this assumption, then in principle we need to generalize acceptance strategies by letting A_j^i be the probability that an agent enters into an x_j match *conditional* on currently being in state $i = 0, 1, 2$. In the base model we did not need to condition acceptance decisions on the agent's state, since everyone is effectively single when they decide whether to accept x_j. This complicates things, in general, although for the two-point distribution $\{x_1, x_2\}$ one can show that $A_j^i = A_j$ in equilibrium.[10]

[10] First note that we have the same five types of equilibria in the alternative model as we had under the base assumption. Now, clearly, in a *type D* equilibrium we have $A_j^i = 0$ for all i, j. In a *type C* equilibrium, $A_1^0 = 0$ and $A_2^0 = 1$, since $V_2 > V_0 > V_1$. This implies $A_1^2 = 0$ and $A_2^1 = 1$. So we have $A_1^i = 0$ and $A_2^i = 1$ in this case. In a *type F*, *type U* or

Given this, the analogue of (1) is

$$\begin{aligned} rV_0 &= b + \alpha(q_0+q_1)\pi A_2(V_2-V_0) + \alpha(q_0+q_2)(1-\pi)A_1(V_1-V_0) \\ rV_1 &= x_1 + [\sigma + S_1\alpha(q_0+q_1)\pi A_2](V_0-V_1) + S_1\Sigma_1 \\ rV_2 &= x_2 + [\sigma + S_2\alpha(q_0+q_2)(1-\pi)A_1](V_0-V_2) + S_2\Sigma_2, \end{aligned} \tag{5}$$

where

$$\begin{aligned} \Sigma_1 &= \alpha(q_0+q_1)\pi A_2(V_2-V_1) - d \\ \Sigma_2 &= \alpha(q_0+q_2)(1-\pi)A_1(V_1-V_2) - d. \end{aligned}$$

In these expressions q_0 and q_i are the probabilities that you meet an unmatched agent and an agent in a x_i match, respectively:

$$q_0 = \frac{N_0}{N_0+S_1N_1+S_2N_2} \text{ and } q_i = \frac{S_iN_i}{N_0+S_1N_1+S_2N_2},\ i=1,2.$$

This model is not recursive: the N_i's appear in (5), and so we need to solve for steady state before characterizing equilibria.[11]

Although this version is much messier, it turns out the results are qualitatively the same as those shown in Figure 1. Indeed, the *type D*, *type C*

---

type P equilibrium, $A_1^0 = A_2^0 = 1$. If $A_2^1 = 1$ then $A_2^0 = 1$ and if $A_2^1 = 0$, then $S_1 = 0$ and A_2^1 is irrelevant. For A_1^2, we have a similar result. So again we can set $A_j^i = A_j$ in all the relevant cases. The establishes the claim.

[11]For the record, in *type D*, *type C* or *type F* equilibria the steady state is the same as in the base model, and in the *type U* and *type P* equilibria we have

$$\begin{array}{lll} N_0^U = \frac{\sigma}{\alpha\pi+\sigma}\left[1-\frac{2\alpha+\sigma-\xi_U}{2\alpha(1-\pi)}\right] & N_1^U = \frac{\sigma(2\alpha+\sigma-\xi_U)}{(\alpha\pi+\sigma)[2\alpha(1-\pi)]} & N_2^U = \frac{\alpha\pi}{\alpha\pi+\sigma} \\ N_0^P = \frac{\sigma}{\alpha(1-\pi)+\sigma}\left[1-\frac{2\alpha+\sigma-\xi_P}{2\alpha\pi}\right] & N_1^P = \frac{\alpha(1-\pi)}{\alpha(1-\pi)+\sigma} & N_2^P = \frac{\sigma(2\alpha+\sigma-\xi_P)}{2\alpha\pi[\alpha(1-\pi)+\sigma]} \end{array}$$

where $\xi_U = \sqrt{(\sigma+2\alpha\pi)(\sigma+4\alpha-2\alpha\pi)}$ and $\xi_P = \sqrt{(\sigma+2\alpha+2\alpha\pi)(\sigma+2\alpha-2\alpha\pi)}$. Substituting the N_j's into the Bellman equations yields a complicated but not intractable system (details are available upon request).

and *type F* equilibria exist in exactly the same regions of parameter space, while the *type U* and *type P* equilibria exist for smaller regions than in the base model. It may be surprising that the *type U* and *type P* equilibria exist for smaller regions, since it would seem that our base assumption discourages on-the-job search. However, with a two point x distribution it really does not. Suppose, e.g., you are in an x_1 match and searching. Since next person you meet cannot generate any less than x_1, the base assumption that you cannot stay with your old partner is not binding, and simply serves as a tie-breaking rule. However, there is a general equilibrium effect: if the tie-breaking rule is to go with the new person, relationships are less stable and agents are therefore more inclined to search.

We conclude several things. First, the alternative model is more complicated. Second, at least for the case where x has a two-point distribution the two models generate the same qualitative implications – the exact regions where some of the equilibria exist differ slightly, but the basic theorems on existence, multiplicity, and efficiency continue to hold; in particular, it is still the case that equilibrium is either efficient or involves too much search. Finally, for the case of a general x distribution the alternative model is really very complicated. These considerations, as well as the fact that our base model focuses attention on within-relationship effects, suggest that when we consider a general distribution $F(x)$ in the next section it makes sense to stick with the base assumption.[12]

[12] Another robustness issue concerns the effects of allowing agents to sign *contracts* spec-

5 The General Model

The general match quality distribution is $F(x)$. If it is differentiable we denote the density by $f(x)$, but we do not especially need differentiability. We will focus on equilibria with the following property: unmatched agents accept partners iff $x \geq R$ where R is called the reservation match quality; and matched agents search iff $x \leq Q$ where Q is called the critical match quality.[13] Moreover, we concentrate on equilibria with $Q > R$, since we want some on-the-job search (sufficient conditions for this are given below). An equilibrium now is defined as a list including the value functions for unmatched and matched agents, $[V_0, V(x)]$, a steady state described by the number of unmatched agents and the distribution of match values across existing relationships, $[N_0, G(x)]$, and strategies as summarized by (R, Q), satisfying conditions to be given below.

To begin the analysis, observe that V_0 satisfies the standard equation from search theory

$$rV_0 = b + \alpha \int_R^\infty [V(z) - V_0] dF(z), \tag{6}$$

ifying what happens when a relationship breaks up (prenuptial agreements or severance packages). As we discuss this in the Conclusion, sometimes this will change the results and sometime it will not. In any event, there are certainly situations where such contracts are not enforceable – e.g., a worker or a spouse can simply run off – and the model applies without qualification.

[13] Although this class of equilibria seems very natural, that there can be equilibria not in this class can be seen from the *type P* equilibrium the previous section. In any case, we will show that there are plenty of equilibria even with this class.

where R satisfies $V(R) = V_0$. Similarly, $V(x)$ satisfies

$$rV(x) = x + (\sigma + S\alpha)[V_0 - V(x)] + s\Sigma(x) \tag{7}$$

where

$$\Sigma(x) = \alpha[V_0 - V(x)] + \alpha \int_R^\infty [V(z) - V_0]dF(z) - d.$$

In any equilibrium with $Q > R$, agents search in relationships at the reservation match value (i.e., $x = R$ implies $s = 1$). Therefore,

$$rV(R) = R + \alpha \int_R^\infty [V(z) - V_0]dF(z) - d. \tag{8}$$

Comparing (8) and (6), one sees immediately that $R = b + d$, and so unmatched agents form relationships iff the instantaneous return net of search cost, $x - d$, exceeds b.[14]

We next derive a quasi-reduced-form version of $V(x)$. First rewrite (7) as

$$rV(x) = x - sd + (\sigma + S\alpha + s\alpha)[V_0 - V(x)] + s\alpha I,$$

where $I = \int_{b+d}^\infty [V(z) - V_0]dF(z)$ does not depend on x. Then insert $V_0 = (b + \alpha I)/r$ and rearrange to yield

$$V(x) = \frac{r(x - sd) + (\sigma + S\alpha + s\alpha)b + (\sigma + S\alpha + s\alpha + sr)\alpha I}{r(r + \sigma + S\alpha + s\alpha)}. \tag{9}$$

For future reference, note that

$$I = \int_{b+d}^\infty [1 - F(z)]V'(z)dz = \int_{b+d}^Q \frac{[1 - F(z)]dz}{r + \sigma + 2\alpha} + \int_Q^\infty \frac{[1 - F(z)]dz}{r + \sigma}dz \tag{10}$$

[14]The result $R = b + d$ is only true in general in an equilibrium with $Q > R$. Recall from the previous section that there could exist a *type F* equilibrium where agents accept matches with $x < b + d$, which means $R < b + d$; however, in this equilibrium agents do not search in these matches, so $Q < R$, contrary to the maintained assumption in this section.

after integrating by parts and inserting $V'(x)$ from (9). This gives $I = I(Q)$ as a function of Q; otherwise, I depends only on exogenous variables.

Equation (9) expresses a matched agent's payoff as a function of match quality x, his search behavior s, the search behavior of his partner S, and the equilibrium value of Q, which determines the behavior of other agents since it is taken as given that they search while matched iff $x \leq Q$. Denoting this by $v_{sS}(x, Q)$, we have:

$$\begin{aligned}
v_{11}(x,Q) &= \frac{r(x-d) + (\sigma + 2\alpha)b + (\sigma + 2\alpha + r)\alpha I(Q)}{r(r + \sigma + 2\alpha)} \\
v_{01}(x,Q) &= \frac{rx + (\sigma + \alpha)b + (\sigma + \alpha)\alpha I(Q)}{r(r + \sigma + \alpha)} \\
v_{10}(x,Q) &= \frac{r(x-d) + (\sigma + \alpha)b + (\sigma + \alpha + r)\alpha I(Q)}{r(r + \sigma + \alpha)} \\
v_{00}(x,Q) &= \frac{rx + \sigma b + \sigma\alpha I(Q)}{r(r + \sigma)}.
\end{aligned}$$

Figure 4 shows the four v_{ij} functions. Notice that they are linear in x, with $\partial v_{00}/\partial x > \partial v_{01}/\partial x = \partial v_{10}/\partial x > \partial v_{11}/\partial x$. Also, as shown in the figure, we have $v_{10} = v_{11} > v_{01}, v_{00}$ at $x = b + d$, which follows from $Q > R = b + d$. Given all agents search iff $x \leq Q$, your value function $V(x)$ is given by $\max\{v_{01}, v_{11}\}$ when your partner is searching and $\max\{v_{00}, v_{10}\}$ when your partner is not searching, as shown by the thick line in the diagram.

For any $Q > b + d$, it should be clear from Figure 4 that there is a unique $q_0(Q) > b + d$ such that $v_{00} = v_{10}$; that is, given your partner is not searching, your best response switches from searching to not searching when x crosses $q_0(Q)$. Similarly, there is a unique $q_1(Q) > b + d$ such that $v_{01} = v_{11}$; that

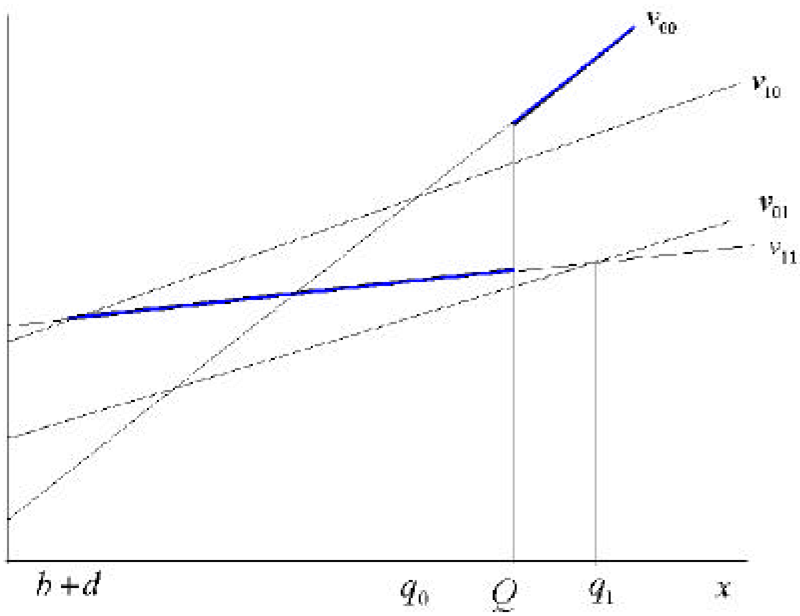

Figure 4: Value function

is, given that your partner is searching, your best response switches from searching to not searching as x crosses $q_1(Q)$. Figure 4 depicts a situation where Q is arbitrary except that $b+d < q_0(Q) < Q < q_1(Q)$ (we show below this will be the case). We claim that this implies it is an equilibrium for all agents to search while matched iff $x \leq Q$. To see why, suppose that everyone else searches iff $x \leq Q$: then if you are in a match with $x \leq Q$, your partner searches, and so you want to search because $x < q_1(Q)$; and if you are in a match with $x > Q$, your partner does not search, and so you do not because $x > q_0(Q)$. This establishes the claim.

It is not obvious that the situation depicted in Figure 4, $b+d < q_0(Q) <$

$Q < q_1(Q)$, will actually arise. To show that it does we first equate $v_{0j} = v_{1j}$ and rearrange to yield

$$\begin{aligned} q_0(Q) &= b - (\sigma + r)d/\alpha + (\sigma + \alpha + r)I(Q) \\ q_1(Q) &= b - (\sigma + \alpha + r)d/\alpha + (\sigma + 2\alpha + r)I(Q). \end{aligned}$$

As shown in Figure 5, the functions $q_0(Q)$ and $q_1(Q)$ are strictly decreasing. Therefore each has at most one fixed point. We now make the following mild parameter restriction, which always holds if d is not too big:

$$\frac{\alpha}{r+\sigma}\int_{b+d}^{\infty}[1 - F(z)]dz > d. \tag{11}$$

This basically says F is such there is some net gain to searching at $x = b + d$. It is not hard to verify that (11) guarantees $q_0(b + d) > b + d$, and so the fixed point of $q_0(Q)$, call it $\underline{q}$, satisfies $\underline{q} > b + d$. Also, $\underline{q} > b + d$ guarantees $q_1(\underline{q}) > \underline{q}$, and so the fixed point of $q_1(Q)$, call it $\overline{q}$, satisfies $\overline{q} > \underline{q}$.

Summarizing the above analysis, we have the following.

Proposition 5 *Assuming (11) holds, we have $b + d < \underline{q} < \overline{q}$, as in Figure 5. Then we can choose any $Q \in [\underline{q}, \overline{q}]$ and have $b + d < q_0(Q) \leq Q \leq q_1(Q)$, as in Figure 4. This means that any $Q \in [\underline{q}, \overline{q}]$ is consistent with equilibrium.*

What is behind this continuum of equilibria? One answer goes along the following lines. Consider a simple, static, two-player game where agents can either cooperate or defect, parameterized by ρ, representing, say, the payoff to joint cooperation. Suppose that the game has multiple equilibria, one

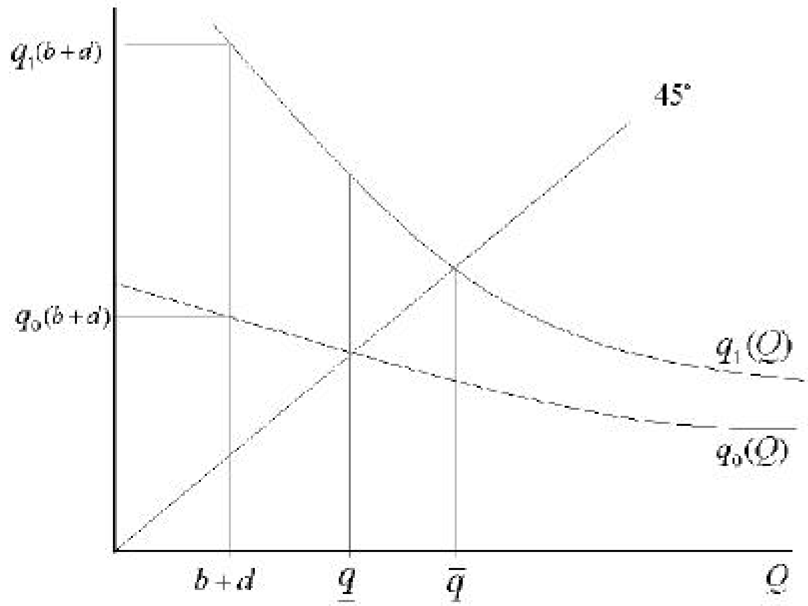

Figure 5: Determining $\underline{q}$ and $\overline{q}$

where both agents cooperate and another where they both defect, for all ρ in some interval (ρ_1, ρ_2). Then imagine agents choosing strategies before ρ is observed of the following form: cooperate if $\rho \geq \hat{\rho}$ and defect if $\rho < \hat{\rho}$. A symmetric equilibrium obtains if they choose the same $\hat{\rho}$. It should be clear that there can be a continuum of such equilibria: for any $\hat{\rho} \in (\rho_1, \rho_2)$, this strategy of cooperating iff $\rho \geq \hat{\rho}$ constitutes a best response to itself (see Frankel, Morris and Pauzer [1999] and the references therein).

Intuitively, whatever fundamental impact ρ may have in this game, it can also be used as a signal or coordinating device determining which equilibrium to play once ρ is realized. The same thing applies in our model. The realization of x has fundamental value as the utility you get from the match, but it can also be a signal indicating whether to be "faithful" in the match. Since we know from the previous section that there can be multiple equilibria in the game where partners choose whether to search, using the value of x relative to Q as a coordination device is consistent with equilibrium for any Q in some range. Of course, if x is too small (big), then agents will (will not) want to search regardless of what their partner is doing; this is why the range of equilibrium Q is bounded.[15]

We also want to reconcile the results for the special model where $x \in \{x_1, x_2\}$ with the above Proposition. In the general case, we define an equilibrium in terms of the value of Q, and we have established that there is a continuum of equilibrium values of Q. However, if F puts weight on only a discrete number of values of x, then some of these equilibrium values of Q are observationally equivalent. For example, whether you stop searching at $Q = 140$ or $Q = 160$ does not matter for the outcome if $x \in \{100, 200\}$. Hence, with a two point distribution we could define an equilibrium in terms whether or not you search in an x_j match, S_j, and there are a finite number of these outcomes.

So far we know that $R = b + d$ and that any Q in $[\underline{q}, \overline{q}]$ is consistent with equilibrium. To complete the characterization we need to describe the distri-

[15]This interpretation also suggests the result is *not* due to the fact that we assume search is a discrete choice. If s were a continuous variable, as long as their are multiple equilibria in the game of deciding how much to search while matched, it should be possible to generate a continuum of equilibria along very similar lines.

bution of agents across states. The following is one of the main results in the paper, as it yields the mapping between the exogenous $F(x)$ and the endogenous N_0 and $G(x)$, and hence gives the key to discussing how unemployment and inequality vary across equilibria. The proof, which involves manipulating differential equations for the numbers of agents in different states, then imposing stationarity and solving, is in the Appendix.

Proposition 6 *Given Q and R, the steady state unemployment rate is*

$$N_0 = \frac{\sigma[\sigma + 2\alpha - \alpha F(Q) + \alpha F(R)]}{(\sigma + 2\alpha)[\sigma + \alpha - \alpha F(Q)]}, \tag{12}$$

and the distribution of x across existing matches is

$$G(x) = \begin{cases} \dfrac{\sigma[F(x) - F(R)]}{\sigma[1 - F(R)] + 2\alpha - 2\alpha F(Q)} & \textit{if } x \in [R, Q] \\[2ex] \dfrac{\sigma[F(x) - F(R)] + 2\alpha[F(x) - F(Q)]}{\sigma[1 - F(R)] + 2\alpha - 2\alpha F(Q)} & \textit{if } x > Q. \end{cases} \tag{13}$$

Notice $G(x) \leq F(x)$ for all x with strict inequality on the interior of the support (stochastic dominance). Moreover, G is continuous at $x \neq Q$ as long as F is, but in any case G has a kink at Q. Thus, if F has density f then G has density g, where:

$$g(x) = \begin{cases} \dfrac{\sigma f(x)}{\sigma[1 - F(R)] + 2\alpha - 2\alpha F(Q)} & \text{if } x \in [R, Q] \\[2ex] \dfrac{(\sigma + 2\alpha) f(x)}{\sigma[1 - F(R)] + 2\alpha - 2\alpha F(Q)} & \text{if } x > Q \end{cases} \tag{14}$$

Increasing Q from Q_1 to Q_2 shifts g up for $x < Q_1$ and $x > Q_2$, and shifts g down for $x \in (Q_1, Q_2)$ (see the next section, where we provide an explicit example). Hence, equilibria with higher Q entail more inequality. Also, from (12), increasing Q raises N_0. Hence, equilibria with higher Q entail more unemployment.

It remains to discuss welfare. In the class of outcomes under consideration, given R and Q average lifetime income can be expressed as

$$W = N_0 V_0 + (1 - N_0)\left[\int_R^Q v_{11}(x,Q)dG(x) + \int_Q^\infty v_{00}(x,Q)dG(x)\right]. \quad (15)$$

With some work, this can be simplified to

$$W = \frac{\chi(Q,R)b + \alpha\sigma\int_R^Q (x-d)dF(x) + \alpha(\sigma+2\alpha)\int_Q^\infty x dF(x)}{r(\sigma+2\alpha)[\sigma+\alpha-\alpha F(Q)]}, \quad (16)$$

where $\chi(Q,R) = \sigma[\sigma + 2\alpha - \alpha F(Q) + \alpha F(R)]$. We now solve the planner's problem of maximizing W by choosing R and Q. To ease the presentation assume F has density f. Also, let us concentrate on the case $Q > R$, since otherwise there is no on-the-job search.[16] In the Appendix we prove:

Proposition 7 *Given $Q > R$, the solution to the planner's problem is $R = b + d$ and the value of Q that solves $T^o(Q) = 0$, where*

$$T^o(Q) \;=\; 2\alpha\left[\sigma+\alpha-\alpha F(Q)\right]Q - \alpha\sigma\left[1+F(R)\right]b + \sigma\left[\sigma+\alpha-\alpha F(R)\right]d$$

[16] A sufficient condition for $Q > R$ is

$$\int_{b+d}^\infty [1 - F(x)]dx > \frac{\sigma d}{\alpha}.$$

To see this, note that in the proof of the proposition we show the solution to the planner's problem entails $R = b + d$ and $Q = Q^o$ where Q^o satisfies $T^o(Q^o) = 0$. Since T^o is an increasing function, and the above condition is merely a simplification of $T^o(b+d) < 0$, it guarantees that $Q^o > b + d = R$.

$$-\alpha\sigma\int_R^Q (x-d)dF(x) - \alpha(\sigma+2\alpha)\int_Q^\infty xdF(x). \tag{17}$$

One thing this implies is that the equilibrium value of $R = b + d$ is efficient. We now show that the on-the-job search decision is not efficient, and indeed that Q is always too high, at least in the limiting case where $r \to 0$. Hence, exactly as in the previous section, there tends to be too much search in equilibrium when r is not too big. Again, the proof is in the Appendix.

Proposition 8 *Given $Q > R$, the planner's choice of Q is below $\underline{q}$ and therefore below any equilibrium Q, at least for r not too big.*

Summarizing the results in this section, here is what has been established. There exist values $\underline{q}$ and $\overline{q} > \underline{q}$ such that any $Q \in [\underline{q}, \overline{q}]$ is consistent with equilibrium. Given the mild parameter condition in (11), we have $\underline{q} > b + d$ and therefore $Q > b + d$ in any equilibrium. We know that $R = b + d$ is efficient, but at least for r not too big Q is inefficient – all equilibria entail too much on-the-job search. Therefore, given the solution for N_0 and $G(x)$, we know that all equilibria also have too much unemployment and too much inequality.

6 A Calibrated Example

To see quantitatively how much on-the-job search and endogenous instability might matter, in this section we provide numerical results to compare the model's predictions to some labor market data (it would also be interesting to confront marriage market data, but we start here with the labor market). As mentioned in the Introduction, the numbers concerning employment flows, especially job-to-job transitions, are important, and it seems interesting to see what this model has to say about them. While some of the model parameters are calibrated more easily than others, it turns out that when we were unsure of their values we tried a variety of choices and the results reported below are fairly robust.

We assume the exogenous distribution $F(x)$ is log-normal, where $Ex = 100$ and we set the log of the standard deviation somewhat arbitrarily to 1. The functional form of F is not particularly important for the key statistics reported below.[17] We calibrate the model to a period 1 month in order to facilitate comparison with the labor market data discussed below. This would suggest setting r so that $(1+r)^{12} = 1 + \hat{r}$ where $\hat{r}$ is the observed annual interest rate. We do report results for $\hat{r} = .04$ below, but for our base model we set $\hat{r} \approx 0$ simply so that our theoretical welfare results in Proposition 8 apply unambiguously. We will see below that this does not matter very much for the main point, although we did find that for big r there can be too little as opposed to too much on-the-job search in equilibrium, as we know from

[17] What is true is that G inherits properties of F, as is obvious from Proposition 6, and if one wants to take seriously the notion that G is the income or wage distribution this could help pin down a good functional form for F. But for the empliyment flows, the parametric form of F is not so important. Of course, for any F, dispersion matters for the well-know reason that a mean-preserving spread encourages more search. Again, since we concentrate on flows this is not so important: a change in dispersion means recalibrating some other parameters, but then the net results are similar.

the theoretical analysis.

Two key parameters are b and d. We set $d \approx 0$ since: (i) this allows us to avoid worrying about whether the out-of-pocket cost of search is higher on or off the job; and (ii) we already have a high implicit cost of on-the-job search built into the model, due to the assumption that when you meet someone new you cannot go back to your current partner. We then set $b = 37$ (compare to $Ex = 100$) according to the following logic. In equilibrium a certain fraction of new entrants to the unemployment pool come from unstable matches ($x < Q$) and the rest from stable matches ($x > Q$). The former typically have been matched for a very short time (a few months), and so we would like to give them a UI payment of $b = 0$; the latter have typically been matched for a much longer time, and we would like to give them b equal to half their previous earnings. Since we do not want to complicate things by having b depend on employment experience, we give every unemployed agent the same b, calibrated so that on average it yields the right replacement ratio.

The remaining parameters are calibrated from labor market numbers. Fallick and Fleischman (2001) use CPS data from January 1994 to December 2000 to generate measures of the flows between unemployment, employment, and out of the labor force, and also job-to-job transitions. Consolidating unemployment and out of the labor force into one state N_0, the monthly probability of transiting from N_0 to employment is $P_{0e} = .068$, and the probability of transiting the other direction is $P_{e0} = .041$ in their sample.

They also find a job-to-job transition rate of $P_{ee} = .027$. Given the transition rates, one can solve the steady state condition $N_0 P_{0e} = (1 - N_0) P_{e0}$ for the stock of people not employed, $N_0 = .376$.[18] We do not calibrate to P_{ee} or N_0; rather, given F, b and d we simply set α and σ to match P_{ue} and P_{eu}, and check later to see what the model predicts about the other numbers. Of course, since there are multiple equilibria we need to choose one; for calibration purposes we choose the best (the one with the lowest Q), which implies $\alpha = .099$ and $\sigma = .0101$.

Although we calibrate to the best equilibrium, we will also report results for the worst, to see just how much endogenous instability can matter. It is straightforward to solve the model numerically for the best and worst equilibrium values of Q, which we called $\underline{q}$ and $\overline{q} > \underline{q}$ in the previous section, by solving for the fixed points of $q_0(Q)$ and $q_1(Q)$ (Figure 5). We then use Proposition 6 to compute N_0 and the distribution of x across existing matches in steady state, $G(x)$. We use Proposition 7 to compute the efficient Q (which is unambiguously below $\underline{q}$, given $r \approx 0$). As we know from the previous section, the planner picks the same value of R as the equilibrium, $R = b + d = 37$. In terms of on-the-job search, the planner picks $Q = 228$, the best equilibrium is $\underline{q} = 233$, and the worst is $\overline{q} = 300$. Figure 6 shows

[18] The data suggest that the labort market was close to if not quite in steady state during the period (there was a slight decrease in N_0 over the sample). In any case, one can argue the Fallick and Fleischman number $P_{ee} = .027$ may too high for several reasons. For example, some reported job-to-job transitions actually involve a spell of unemployment of less than the month in between interviews. This is actually good news for the model: we will see it is hard to get job-to-job transitions high enough to match these data.

$g(x)$ in the best and worst equilibrium (the efficient g looks very similar to the best equilibrium since the planner's Q is close to $\underline{q}$, so it is not shown).

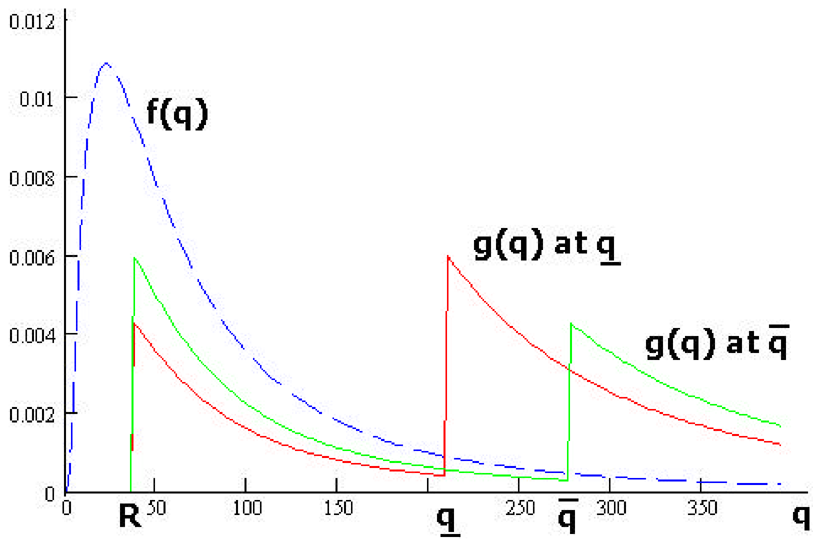

Figure 6: Distributions of Match Quality

Some things are common across all equilibria, so we describe these first. Given any Q, the model predicts the expected duration in the N_0 state is about 15 months – which matches the data, of course, since this is effectively what we calibrate to. What is more interesting is the expected durations in unstable or unstable relationships, since we only calibrate to the average; these are about 5 and 99 months, respectively. Hence, stable (unstable) relationships are very stable (unstable) here. In any equilibrium, an unmatched agent encounters an offer each month with probability about .10 and accepts it with probability .69. When an unstable match breaks up, 52% of the time it is because you are abandoned (a layoff), 33% of the time you move directly to a new partner (a job-to-job transition), and 15% of the time you leave your partner to check out someone new but find them lacking and end up unmatched (one might call this a quit).

Comparing across equilibria, in the best (worst) equilibrium, 87% (92%) or *new* relationships are unstable. Hence, almost everyone starts in an unstable relationship, although since they tend to be short the stock of stable relationships is much bigger in steady state. We denote the stock of agents in unstable ($x < Q$) and stable ($x > Q$) relationships by N_1, and N_2 – which fits with the notation in the simple version of the model, at least in the class of equilibria under consideration. These numbers are given below, along with some statistics on earnings, in Table 1. Notice for example that Ex is over 4 times higher in stable relationships, implying a strong earnings-tenure correlation. We do not report measures of inequality across equilibria but it is obvious from Figure 6 which way it goes. The table also compares W (average income over all agents, employed and unemployed); notice bad equilibria have lower W, despite a higher Ex, since N_0 is much bigger.[19]

[19]We also report that 18% of the newly unemployed come from stable matches, and calibrating to a replacement ratio of 1/2 for them is how we set b.

	N_0	N_1	N_2	$\frac{Ex}{x<Q}$	$\frac{Ex}{x>Q}$	$\frac{Ex}{\text{all}}$	W	P_{ee}
Base model								
$Q = \underline{q}$	.382	.153	.465	95	409	331	219	.017
$Q = \overline{q}$	.454	.196	.349	104	500	357	212	.024
Discounting								
$Q = \underline{q}$	.374	.131	.496	95	380	320	157	.014
$Q = \overline{q}$	.455	.177	.369	105	474	354	154	.022

Table1: Calibration Results

Returning to stocks, the best equilibrium comes close to the actual values of $N_0 = .376$ and $N_1 + N_2 = .624$. What the Fallick and Fleischman data do not give us is the breakdown of N_1+N_2 into stable and unstable relationships. The survey data reported by Galt (2001), although perhaps to be taken with a grain of salt, shed light on this. As mentioned earlier, these data indicate that 13% of workers are actively seeking new jobs and an additional 45% although not actively searching "would consider other opportunities or offers." We find 25% (36%) of relationships are unstable in the best (worse) equilibrium. In any case, it may be a better test is to see what the model predicts about job-to-job transitions, which Fallick and Fleischman report as $P_{ee} = .027$. In the best (worst) equilibrium we get .017 (.024).

Hence, in the equilibrium to which we calibrated we can account for only 63% of the reported job-to-job transitions. We do better (nearly 90%) in the worst equilibrium, but then we have too high a transition rate between employment and unemployment, .057 instead of .041. This feature of the model is quite robust. For example, the table also reports results with discounting at rate $\hat{r} = .04$ per year (which means we need to re-calibrate $\alpha = .106$,

$\sigma = .012$ and $b = 42$ in order to match P_{ue}, P_{eu} and the desired replacement ratio). The results are similar, although naturally impatient agents search less and hence end up with lower average earnings. Also, we explain a little less of P_{ee}, only about 1/2, again because impatient agents search less. Still, the results are roughly similar, and again the model cannot match both P_{eu} and P_{ee}.

Summarizing, although the model does quite well, in general, a key finding is that if we calibrate to P_{eu} we can account for around 1/2 to 2/3 of P_{ee}. This is perhaps not too surprising, as our theory is highly stylized and abstracts from many things. For instance, advance notice of an impending layoff could cause agents to change jobs, even if they would not search while matched if the job was not known to be terminating shortly. Moreover, as we mentioned above, there are reasons to suspect that $P_{ee} = .027$ may be an overestimate. Finally, notice that our hands are somewhat tied here, since every job-to-job transition in the model generates a transition of at least one person and possibly two people from employment to unemployment. Doing better at matching both P_{eu} and P_{ee} may well require rethinking this aspect of the model, say by explicitly distinguishing between workers and firms, or perhaps trying to calibrate a marriage market version where the assumption is closer to reality.[20]

[20] If an agent switches partners here, he abandons a previous partner and his new parter also abandons someone if they were matched. In a model with two types – say workers and firms – whenever a worker changes jobs he creates a vacancy (assuming the employer he leaves keeps the position open), and either takes another vacancy off the books if the position was previously vacant or creates a new unemployed person if it was previously

7 Conclusion

Our analytic results suggest that endogenous instability is a interesting force that can lead to multiple (indeed to a continuum of) steady state equilibria, and can lead to inefficiencies including excessive turnover, unemployment and inequality. Our numerical results suggest that modeling on-the-job search in this way may help to explain certain aspects of the data. Of course, the model is special, and many extensions and generalizations are possible, but we think the basic message is sound and that some of the analytic methods we have developed – say, for characterizing the equilibrium distribution G – should be applicable in a variety of alternative specifications.

In terms of extensions, one could allow for different arrival rates for unemployed and employed agents, or more generally for a continuous choice of search intensity, in order to endogenize arrival rates. Our discrete choice model, where you either search or you don't, made some things easier but the main results should be robust. It may be interesting to allow the match value x to vary over time within a relationship (Mortensen and Pissarides [1994]), or to have learning about x (Jovanovic [1979]; Wright [1986]), since these provide additional sources of endogenous job destruction. One would like to know how much job destruction is due to changes in x, to learning, and to on-the-job search; one would presumably also like to know how much is efficient, as some versions of the first two models imply, and how much is

filled. Similarly, whenever a firm changes workers it creates a new unemployed person and either takes an unemployed worker off the books or causes a new vacancy to open.

inefficient, as our model implies.

It might be interesting to introduce ex ante – as opposed to match specific – heterogeneity, or for some other reason have partners potentially disagree about the value of the match. This would force one to think about counteroffers when one's partner meets someone new, and perhaps also about offering partners enough to keep them off the market in the first place. For these as well as other considerations one would like to introduce transferrable utility. For labor market analysis it may also be important to extend things to explicitly include both firms and workers, and for the marriage market to explicitly include men and women. We thought it was useful to first work out the case with one type, although clearly the generalization would be interesting theoretically and quantitatively.

A big issue to consider is the robustness of our welfare conclusions. We believe the effects the model demonstrates concerning excessive search and turnover are real and important, but naturally there are forces going the other way. In particular, societies may develop institutions to mitigate this type of inefficiency. The church, the extended family, and related institutions all typically strive to encourage "faithfulness," for instance. One can imagine that investment in match-specific capital (perhaps children) would make the cost of separation higher, which could reduce search while matched. Of course, instability also makes investments in such capital more risky. Reputational effects could also come into play; e.g., a scarring effect of separations as a social norm could discourage on-the-job search, even if there is nothing fundamental signaled by past separations. Whether or not the relevant inefficiencies can be generally eliminated by such institutions, or by contracts, is worth studying in detail.

Our preliminary work on contracts indicates the following: Suppose that agents can sign binding agreements for severance (alimony) upon separation – if i leaves j for whatever reason, i pays j a lump sum Z. In the case of a coordination failure, such a contract can eliminate the inefficiency even if we set $Z = 0$; all that is required is for the two agents to negotiate to not to search on-the-job. In the coordination failure case, such an agreement is self-enforcing. In the case where the inefficiency is along the lines of a prisoner's dilemma, however, this is not true: the inefficiency can still be eliminated, but this requires $Z > 0$. Still, there are many situations where such agreements are simply not possible or not enforceable. Moreover, it is only in a very special situation where bilateral contracts can eliminate all the inefficiencies. With non-constant returns in the matching technology, e.g., we would generally require multilateral contracts, including agreements between agents who have not yet met. Could this rationalize government-imposed severance pay or firing restrictions?

As we said, this is worth studying in much more detail, but we leave it and the other extensions for future research. The goal here was to illustrate how endogenous instability works in a relatively simple context, to develop some tools for analyzing its implications, and to see what a numerical version of the model implied.

8 Appendix

Proof of Proposition 1: To begin, we can easily rule out anything other than the five cases that are listed. For example, suppose $A_1 = 0$, $A_2 = 1$, and $S_2 = 1$. These strategies imply $\Sigma_2 = \alpha(1-\pi)(V_0 - V_2) - dS_1$ and $\Delta_2 = V_2 - V_0$; but the best response conditions require $\Sigma_2 > 0$ and $\Delta_2 > 0$, which is a contradiction. The other cases are similar. The next step is to derive parameter values for which each of the remaining five candidate equilibria exist.

Consider first a *type D* equilibrium, where $A_1 = A_2 = 0$ and $rV_0 = b$. We use the unimprovability principle: to check whether strategies constitute an equilibrium, it suffices to show the payoffs from using these strategies cannot be improved by deviating, in any possible contingency, once and then reverting to the candidate strategies. Consider the payoff to deviating from $A_2 = 0$ by accepting an x_2 match. It cannot be optimal to accept the match and then search, given $A_1 = A_2 = 0$, so we know $S_2 = 0$. This means $rV_2 = x_2 + \sigma(V_0 - V_2)$, which implies Δ_2 is proportional to $x_2 - b$. Hence, $\Delta_2 \leq 0$, and $A_2 = 1$ does not improve your payoff, iff $x_2 \leq b$. Similarly, $A_1 = 1$ does not improve your payoff iff $x_1 \leq b$, which is not binding given $x_2 \geq x_1$. Hence, the *type D* equilibrium exists iff $x_2 \leq b$, as claimed.

Now consider a *type F* equilibrium, where $A_1 = A_2 = 1$ and $S_1 = S_2 = 0$. This implies

$$rV_0 \;\; = \;\; b + \alpha\pi(V_2 - V_0) + \alpha(1-\pi)(V_1 - V_0)$$

$$\begin{aligned} rV_1 &= x_1 + \sigma(V_0 - V_1) \\ rV_2 &= x_2 + \sigma(V_0 - V_2). \end{aligned}$$

For this to be an equilibrium, we require $\Delta_1 \geq 0$, which can be seen from straightforward algebra to hold iff $x_2 \leq y_1$ where y_1 is defined above, and $\Delta_2 \geq 0$, which is not binding. We also require $\Sigma_1 \leq 0$, which holds iff $x_2 \leq y_2$ where y_2 is defined above, and $\Sigma_2 \leq 0$, which is not binding. Hence, this equilibrium exists iff $x_2 \leq y_1$ and $x_2 \leq y_2$, as claimed.

The *type U* and *type P* equilibria are symmetric, in terms of the algebra, since in each case $A_1 = A_2 = 1$ and $S_j = 0$ for one of the two types of matches. The *type U* equilibrium implies

$$\begin{aligned} rV_0 &= b + \alpha\pi(V_2 - V_0) + \alpha(1-\pi)(V_1 - V_0) \\ rV_1 &= x_1 + (\sigma + \alpha)(V_0 - V_1) + \alpha\pi(V_2 - V_1) - d \\ rV_2 &= x_2 + \sigma(V_0 - V_2). \end{aligned}$$

The relevant inequalities can be shown to hold iff $x_1 \geq b + d$ and $x_2 \geq y_3$, as claimed. Similarly, the relevant inequalities for the *type P* equilibrium can be shown to hold iff $x_2 \geq b + d$ and $x_2 \leq y_4$.

Finally, consider a *type C* equilibria. This is a little more complicated because, although we know $A_1 = 0$, $A_2 = 1$ and $S_2 = 0$ in a *type C* equilibria, we need to specify S_1 differently depending on parameters – i.e., even though no one accepts an x_1 match in equilibrium, it matters what agents believe about S_1 off the equilibrium path. First, the value functions satisfy

$$
\begin{aligned}
rV_0 &= b + \alpha\pi(V_2 - V_0) \\
rV_1 &= x_1 + (\sigma + S_1\alpha)(V_0 - V_1) \\
rV_2 &= x_2 + \sigma(V_0 - V_2).
\end{aligned}
$$

Now suppose we set $S_1 = 1$ if $x_2 \geq y_5$, $S_1 = 0$ if $x_2 \leq y_6$, and $S_1 \in (0,1)$ if $x_2 \in (y_6, y_5)$, where

$$
\begin{aligned}
y_5 &= \frac{(r+\sigma+\alpha\pi)}{\pi(2\alpha+\sigma+r)}x_1 + \frac{\alpha\pi-(1-\pi)(r+\sigma)}{\alpha\pi(2\alpha+\sigma+r)}b + \frac{(r+\sigma+\alpha)(r+\sigma+\alpha\pi)}{\alpha\pi(2\alpha+\sigma+r)}d \\
y_6 &= \frac{(r+\sigma+\alpha\pi)}{\pi(\alpha+\sigma+r)}x_1 - \frac{(1-\pi)(r+\sigma)}{\pi(\alpha+\sigma+r)}b + \frac{(r+\sigma)(r+\sigma+\alpha\pi)}{\alpha\pi(\alpha+\sigma+r)}d.
\end{aligned}
$$

Given S_1 is set in this way, it is a matter of algebra to verify that all the conditions for the *type* C equilibrium hold iff $x_1 \leq b + d$ and $x_2 \geq y_1$. This completes the proof. ■

Proof of Proposition 2: To begin, insert V_i and N_i, as well as $A_2 = 1$ and $S_2 = 0$, into (3), and rearrange to express the objective function in terms of A_1 and S_1:

$$
rW = \frac{\left\{\sigma\alpha S_1[2-(1-\pi)A_1]+\sigma^2\right\}b+\sigma\alpha(1-\pi)A_1(x_1-S_1 d)+\alpha\pi(\sigma+2\alpha S_1)x_2}{\sigma\alpha(1-\pi)(1-S_1)A_1+(\sigma+\alpha\pi)(\sigma+2\alpha S_1)}. \tag{18}
$$

If the planner chooses the *type* C strategy, then

$$
rW = \frac{\alpha\pi x_2 + \sigma b}{\alpha\pi + \sigma}.
$$

If he chooses the *type* F strategy, then

$$
rW = \frac{\alpha(1-\pi)x_1 + \alpha\pi x_2 + \sigma b}{\alpha + \sigma}.
$$

And if he chooses the *type U* strategy, then

$$rW = \frac{\alpha\sigma(1-\pi)[x_1-(b+d)] + (\sigma+2\alpha)(\alpha\pi x_2+\sigma b)}{(\sigma+2\alpha)(\alpha\pi+\sigma)}.$$

It is now a simple matter of solving for the parameter values such that each choice yields the greatest value of W. ■

Proof of Proposition 6: As a preliminary step, we take $G(x)$ as given, and observe that the number of unmatched agents evolves according to:

$$\dot{N}_0 = (1-N_0)\sigma + (1-N_0)G(Q)[\alpha+\alpha F(R)] - N_0\alpha[1-F(R)].$$

This says that the flow into the set of unmatched agents is the number of matched agents who suffer an exogenous separation plus the number of matched agents who are abandoned by their partner or meet someone with $x < b+d$, while the flow out is the number of unmatched agents who meet someone with match value above $b+d$. Setting $\dot{N}_0 = 0$, we have

$$N_0 = \frac{\sigma + G(Q)\alpha[1+F(R)]}{\sigma + G(Q)\alpha[1+F(R)] + \alpha[1-F(R)]}. \tag{19}$$

We now derive $G(x)$. First note that $G(R) = 0$. Next, denote the measure of agents who are matched with match quality $x \le \bar{x}$ by $\mu(\bar{x}) = (1-N_0)G(\bar{x})$. For $x \in [R, Q]$, this evolves according to

$$\begin{aligned}
\dot{\mu}(x) &= N_0\alpha[F(x)-F(R)] + (1-N_0)[G(Q)-G(x)]\alpha[F(x)-F(R)] \\
&\quad -G(x)(1-N_0)[\sigma+2\alpha+\alpha F(R)-\alpha F(x)].
\end{aligned}$$

For $x > Q$, it evolves according to

$$\begin{aligned}
\dot{\mu}(x) &= N_0\alpha[F(x)-F(R)] - (1-N_0)[G(x)-G(Q)]\sigma \\
&\quad -G(Q)(1-N_0)[\sigma+2\alpha+\alpha F(R)-\alpha F(x)].
\end{aligned}$$

Now insert (19) into $\dot{\mu}(x) = 0$ and solve for the steady state $G(x)$, for any $x \geq R$, as a function of $G(Q)$:

$$G(x) = \begin{cases} \dfrac{[F(x) - F(R)][\sigma + 2\alpha G(Q)]}{[1 - F(R)](\sigma + 2\alpha)} & \text{if } x \in [R, Q] \\ \\ \dfrac{\sigma[F(x) - F(R)] + 2\alpha G(Q)[1 - F(x)]}{\sigma[1 - F(R)]} & \text{if } x > Q \end{cases} \tag{20}$$

We can solve for $G(Q)$ by setting $x = Q$ in (20) and rearranging:

$$G(Q) = \frac{\sigma F(Q) - \sigma F(R)}{\sigma[1 - F(R)] + 2\alpha[1 - F(Q)]}. \tag{21}$$

Then we can substitute (21) into (20) to arrive at (13). Similarly, we can substitute (21) into (19) to arrive at (12). This completes the proof. ■

Proof of Proposition 7: To reduce notation slightly, we define $\bar{W} = (\sigma + 2\alpha) rW$ and maximize $\bar{W}$. The first order condition with respect to R is

$$\frac{\partial \bar{W}}{\partial R} = \frac{(b + d - R)\, \sigma\alpha F'(R)}{\sigma + \alpha - \alpha F(Q)} = 0,$$

which immediately implies $R = b + d$. The first order condition for Q can be written, after simplification,

$$\frac{\partial \bar{W}}{\partial Q} = \frac{\bar{W} - \sigma(b + d) - 2\alpha Q}{\sigma + \alpha - \alpha F(Q)} \alpha F'(Q) = 0,$$

which leads to (17) by straightforward algebra. Notice that

$$\frac{\partial T^o}{\partial Q} = 2\alpha\, [\sigma + \alpha - \alpha F(Q)] + \alpha\sigma d F'(Q) > 0.$$

Hence, there is a unique Q satisfying this condition. To check that it constitutes a maximum, we check the second order conditions. After simplification using the first order conditions, the Hessian matrix conveniently reduces to

$$\begin{bmatrix} \frac{-\sigma\alpha F'(R)}{\sigma+\alpha-\alpha F(Q)} & 0 \\ 0 & \frac{-2\alpha^2 F'(R)}{\sigma+\alpha-\alpha F(Q)} \end{bmatrix}.$$

Hence, the second order conditions hold, and the proof is complete. ■

Proof of Proposition 8: We know the efficient Q is the solution to (17). We also know that Q is an equilibrium if and only if it is in the interval $[\underline{q}, \overline{q}]$, where $\underline{q}$ is the solution to $q_0(Q) = Q$. In the limiting case $r = 0$, $q_0(Q) = Q$ can be rewritten $T^e(Q) = 0$, where

$$\begin{aligned} T^e(Q) &= \alpha[\sigma + 2\alpha - 2\alpha F(Q)]Q - \alpha\sigma F(R)b + \sigma[\sigma + 2\alpha - \alpha F(R)]d \\ &\quad -\alpha\sigma \int_R^Q x dF(x) - \alpha(\sigma + 2\alpha) \int_R^\infty x dF(x) \end{aligned}$$

after integrating by parts to replace $\int (1 - F)dx$ terms with $\int x dF$ terms. Observe that T^o and T^e are both increasing in Q, and that given $R = b + d$,

$$T^o(Q) - T^e(Q) = \sigma\alpha(Q - R).$$

Hence, T^o lies above T^e for all $Q > R$, and therefore $\underline{q}$ exceeds the planner's solution. Hence, any equilibrium Q exceeds the planner's solution. ■

References

Burdett, Kenneth [1977] "On-the-Job Search," *American Economic Review.*

Burdett, Kenneth, and Melvyn G. Coles [1999] "The Simple Analytics of Match Formation," mimeo.

Burdett, Kenneth, and Randall Wright [1998] "Two-Sided Search with Non-Transferrable Utility," *Review of Economic Dynamics*.

Diamond, Peter A. [1982] "Aggregate Demand Management in Search Equilibrium," *Journal of Political Economy*.

Eeckhout, Jan [2000] "Competing Norms of Cooperation," mimeo.

Fallick, Bruce and Charles Fleischman [2001] "The Importance of Employer-to-Employer Flows in the U.S. Labor Market," mimeo.

Frankel, David, Stephen Morris and Ady Pauzer [1999] "Equilibrium Selection in Global Games with Strategic Complementarities," mimeo.

Galt, Virginia [2001] "Are Workers Wedded to Their Jobs? Not Likely," *Toronto Globe and Mail*, August 30.

Ghosh, Parikshit and Debraj Ray [1996] "Cooperation in Community Interaction Without Information Flows," *Review of Economic Studies.*

Hosios, Arthur J. [1990] "On the Efficiency of Matching and Related Models of Search and Unemployment." *Review of Economic Studies* 57, 279-298.

Jeong, Byeonju [1999] "Why are there Mobility Restrictions?" mimeo.

Jovanovic, Boyan [1979] "Job Matching and the Theory of Turnover," *Journal of Political Economy* 87, 972-90.

Kehoe, Timothy J., Nobuhiro Kiyotaki and Randall Wright [1994] "More on

Money as a Medium of Exchange," *Economic Theory*.

Kranton, Rachel E. [1996] "The Formation of Cooperative Relationships," *Journal of Law, Economics and Organization*.

Mortensen, Dale T. [1978] "Specific Capital and Labor Turnover," *Bell Journal of Economics.*

Mortensen, Dale T. [1988] "Matching: Finding a Partner for Life or Otherwise," *American Journal of Sociology* 94, S215-S240.

Mortensen, Dale T. and Christopher A. Pissarides [1994] "Job Creation and Job Destruction in the Theory of Unemployment," *Review of Economic Studies* 61, 397-415.

Mortensen, Dale T., and Christopher A. Pissarides [1998] "New Developments in Models of Search in the Labor Market," mimeo.

Moscarini, Giuseppe [2002]. "Skill and Luck in the Theory of Turnover," mimeo.

Pissarides, Christopher A. [1994] "Search Unemployment with On-the-Job Search," *Review of Economic Studies.*

Ramey, Gary and Joel Watson [1997] "Contractual Fragility, Job Destruction, and Business Cycles," *Quarterly Journal of Economics.*

Webb, Tracy [1998] "Unemployment and Turnover in a model with On-the-Job Search," mimeo.

Webb, Tracy [1999] "Marriage and Divorce," mimeo.

Wright, Randall [1986] "Job Search and Cyclical Unemployment," *Journal of Political Economy* 94, 38-55.