



Penn Institute for Economic Research  
Department of Economics  
University of Pennsylvania  
3718 Locust Walk  
Philadelphia, PA 19104-6297  
[pier@econ.upenn.edu](mailto:pier@econ.upenn.edu)  
<http://www.econ.upenn.edu/pier>

## ***PIER Working Paper 02-032***

“Product Innovations and Quality-Adjusted Prices”

by

Arthur Fishman and Rafael Rob

[http://ssrn.com/abstract\\_id=335181](http://ssrn.com/abstract_id=335181)

# Product Innovations and Quality-Adjusted Prices

Arthur Fishman

Bar Ilan University

52900 Ramat Gan

ISRAEL

e-mail: afishman@biu.ac.il

fax: (+) 972-3- 535-3180

Rafael Rob\*

University of Pennsylvania

3718 Locust Walk

Philadelphia, PA. 19104-6297

USA

e-mail: rrob@econ.sas.upenn.edu

fax: (+) 1-215-573-2057

May 7, 2002

## Abstract

A striking characteristic of high-tech products is the rapid decrease of their quality-adjusted prices. Empirical studies show that the rate of decrease of QAPs is typically not constant over time; QAPs decrease rapidly at early stages of the product and then the rate of decrease tapers off. Studies also suggest that the QAP is positively correlated with the rate of product introductions: The faster new products are introduced, the faster is the rate of decrease in their QAPs. This paper presents a dynamic model of product innovations consistent with these empirical regularities.

**Keywords:** Quality Adjusted Prices, Frequency of Innovations, Durable Goods, Obsolescence.

**JEL Classification numbers:** C73, L12, L63, O31.

---

\*Acknowledges the support of the NSF under grant numbers 98-12681 and 01-36922.

## 1. Introduction

When a durable product is introduced, several features characterize its evolution. First, new and higher quality versions of the product appear every so often, and are referred to as new “models” or new “generations.” Second, the “quality-adjusted price,” which is ordinarily defined as the price paid by consumers divided by the product quality, decreases over time. Thus, if we consider consumers as buying quality units, rather than products which embody these quality units, consumers are getting a much better value when they buy a new-generation product as compared to buying an old-generation product. These phenomena have been documented for several classes of products: Automobiles (Griliches (1971), Roff and Trajtenberg (1995)), industrial equipment (Mansfield (1968)), mainframe computers (Chow (1967), Greenstein (1994)), minicomputers (Brendt and Griliches (1993), Gordon (1995)), and software packages (Gandal (1994)).

While these studies focus on different products, several common features of their evolution are revealed. First, the frequency with which new products are introduced varies a great deal and depends on the cost of product introductions: The more costly it is to introduce a product, the less frequently new models of it are introduced. Second, the rate of decrease of the QAP is typically not constant over time; QAPs decrease rapidly at early stages of the product and then the rate of decrease tapers off. Third, the rate of decrease of the QAP is positively correlated with the rate of product introductions: The faster new products are introduced, the faster is the rate of decrease of their QAPs. Thus Greenstein (1994) reports that new generations of mainframe computers are introduced every 6 years (on average) and their QAPs decrease at an average rate of 20%, while Berndt and Griliches (1993) report that minicomputers are introduced every 2 or 3 years and the average rate of decrease of their QAP is 30% per year.

This paper presents a dynamic analysis of product innovations, based on the model of Fishman and Rob (2000), which is consistent with these empirical facts. The setting is a market for a durable product, where new models of the product are introduced periodically and replace old models. The extent to which a new model improves upon its predecessor depends on the length of time that is allowed to elapse between successive introductions.

While the introduction of a new model renders previous models of the same product technolog-

ically obsolete, consumers have the option of continuing to use their old models. Their decision to adopt the state-of-the-art model, therefore, depends on the incremental quality that the latter delivers, on its price, and on consumers' expectations about the new model's longevity - the duration for which the new model will be on the technological frontier before being rendered obsolete by a still better model. Consequently, new models of the product have to be appropriately priced, to make it worthwhile for consumers to adopt them. It follows that successful innovation must balance all these factors: Technological factors and development costs, on the one hand, and consumers' demands on the other hand.

Fishman and Rob (2000) analyzed this scenario in the monopoly context, and derived a Markov-perfect equilibrium path of product introductions and prices. Here we build on their analysis to identify the way technological and other determinants govern the path of technical progress, including the frequency of product introductions, the determination of QAPs, and how QAPs vary over time and in a cross-section of products.

The rest of the paper is organized as follows. The next section reviews a simplified version of the Fishman and Rob (2000) model of repeated product introductions and presents the equilibrium. Section 3 analyzes the evolution of QAPs implied by this equilibrium and relates them to the empirical literature.

## 2. The Model

Consider a perfectly durable product which, for concreteness, will be referred to as a computer. All computers are introduced and sold<sup>1</sup> by a single, infinitely-lived monopoly firm. There is a continuum of infinitely-lived consumers of measure 1. The utility a consumer derives from a computer depends on its quality, denoted by  $q$ . A computer of quality  $q$  delivers  $q$  worth of utility per period. Consumers' demand for computers is discrete: they buy either zero or one unit. A consumer cannot use more than one computer at a time. Time is continuous and indexed by  $t \in [0, \infty)$ . The

---

<sup>1</sup>The assumption that computers are sold rather than rented accords well with the legal environment of several companies. For example, the IBM corporation used to exclusively (or almost exclusively) rent its mainframe computers; however, after lengthy anti-trust litigation it was determined that it must make computers available for sale.

instantaneous interest rate,  $r > 0$ , is constant and the same for consumers and producers.

New and improved computers can be introduced (repeatedly) over time. The extent to which a new computer improves upon its predecessor depends on the length of time which has elapsed since the last product introduction - the “gestation period,”  $t$ . The bigger is  $t$ , the higher is the quality of the new product and the higher is the price the monopolist can charge for it. However, the bigger is  $t$ , the longer the monopolist has to wait before realizing his profit. The monopolist balances these two effects, choosing when to optimally introduce a new product. Each time a new product is introduced, the state-of-the-art technology is re-defined, and the process of knowledge growth is re-started from that state.

The way previous quality interacts with incremental knowledge is additive. If the quality of the last computer is  $q$  and if  $t$  units of time have elapsed since its introduction, then it is possible to introduce a new computer of quality  $q + g(t)$ . We refer to  $g$  as the (knowledge) growth function;  $t$  is considered as an input into it. It is assumed that  $g(0) = 0$  and that  $g(\bullet)$  is bounded, monotonically increasing, twice continuously differentiable and strictly concave.

The monopoly incurs a fixed cost,  $F$ , termed the implementation cost, each time a new product is introduced.<sup>2</sup> This puts a limit on the frequency with which it is profitable to introduce new products.  $F$  might be the cost of translating theoretical knowledge to commercial applications, or, it might be the cost of building a new plant or acquiring new equipment needed for starting work on the next product. Variable costs of production are assumed to be constant and equal across different-generation products. For convenience, the constant variable cost is set at zero.

A strategy for the monopolist consists of an infinite sequence of gestation periods,  $t_1, t_2, \dots$ , so that new products are introduced at  $T_1 = t_1$ ,  $T_2 = t_1 + t_2$ , etc. Thus,  $T_i$ s are the calendar date at which new products are introduced while  $t_i$ s are the lengths of time between introductions. The computer introduced at  $T_i$  is of quality  $\sum_{j=i}^i g(t_j)$ , and consumers will use it over  $[T_i, T_{i+1})$ . A strategy for a consumer specifies, for each introduction, at what prices to buy.

Fishman and Rob (2000) derive the Markov-Perfect Equilibria (MPE) of this game in which

---

<sup>2</sup>In Fishman and Rob (2000) the monopolist incurs an additional R&D expenditure flow which is paid continuously. Here we simplify by assuming that the fixed cost  $F$  subsumes all costs.

the time between successive introductions is a constant. The net result is as follows:

**Proposition 2.1.** (i) *There exists a unique MPE in which the monopoly chooses constant gestation periods,  $t_1 = t_2 = \dots = t^m$ . The equilibrium  $t^m$  is the unique solution to*

$$\left[1 - e^{-rt}\right] \left[g(t) - g'(t) \frac{1 - e^{-rt}}{r}\right] = rF. \quad (2.1)$$

(ii) *Each generation of the product sells for the price:*

$$g(t^m) \frac{1 - e^{-rt^m}}{r},$$

*which is constant and independent of product quality.*

As explained in Fishman and Rob (2000) the reason for the constancy of price is that consumers are only willing to pay for the quality increment,  $g(t^m)$ , discounted over the period,  $t^m$ , they are going to use the new product. Since  $t^m$  is constant so is the price.

Given equation (2.1) one comparative static property of the equilibrium is as follows:

**Proposition 2.2.** *The equilibrium  $t^m$  is decreasing in  $F$ .*

**Proof.** *Consider equation (2.1). It can be verified that LHS is increasing in  $t$  while the RHS is constant. Now, if we increase  $F$  the RHS increases and the LHS remains intact. Hence, the solution to (2.1) becomes bigger. ■*

### 3. Quality Adjusted Prices

The price behavior of durable products has been the subject of extensive empirical research. In the literature, the QAP is defined as the ratio of the nominal price to an index of product quality.<sup>3</sup> The body of empirical research estimates the QAPs for several products and, for each product, it estimates the QAP for several time periods. Examination of this body of data reveals some common empirical regularities.

---

<sup>3</sup>One task for empirical research is to quantify “product quality,” given that there are many attributes which determine quality (in our model quality is *unidimensional*, so this is not an issue). Examples of attributes in the case of PCs are MB of RAM or MB of HD capacity or the computing speed as measured by Mhz. The usual procedure is to estimate the hedonic prices of these attributes and then combine them into an overall measure of quality.

First, QAPs decrease over time as products with the same characteristics sell for a lower nominal price and/or as the nominal price remains constant but products contain enhanced characteristics. Second, the rate of decrease of the QAP is typically not constant over time; QAPs decrease rapidly at early stages of the product and then the rate of decrease tapers off. Third, as reported by Chow (1967), Berndt and Griliches (1993) and Greenstein (1994), the QAP of mainframe computers had decreased at an average rate of 20% per year, while the QAP of PCs had decreased at an average rate of 30% per year. At the same time, new models of mainframes had been introduced less frequently than PCs. Thus, the rate of decrease of the QAP is positively correlated with the rate of product introductions: the faster new products are introduced, the faster is the rate of decrease in their QAPs.

We show that the equilibrium described in proposition 2.1 is consistent with these empirical regularities. Define the quality-adjusted price as the ratio of the nominal price to the product quality. Proposition 1 states that the nominal price of each generation of technology remains constant over time,  $p(n) = g(t^m) \frac{1-e^{-rt^m}}{r}$ , for the  $n^{\text{th}}$  generation technology. At the same time each generation is of higher quality than its predecessor,  $q(n) = ng(t^m)$ . Hence, the QAP of computers decreases over time. The following proposition establishes that our model also parallels the other empirical findings.

**Proposition 3.1.** (i) *The faster is the equilibrium rate of product introductions,  $1/t^m$ , the faster will be the rate of decrease in the QAP.* (ii) *The rate of decrease of the QAP is monotonically decreasing as the product ages.*

**Proof.** Consider the date,  $T$ , at which the  $n^{\text{th}}$  generation of the product is introduced,  $T = nt^m$ . Then the QAP of this product is:

$$\frac{p(n)}{q(n)} = \frac{g(t^m)(1 - e^{-rt^m})/r}{ng(t^m)} = \frac{1 - e^{-rt^m}}{nr}.$$

Consider now two distinct dates,  $T_1 = n_1 t^m$  and  $T_2 = n_2 t^m$ , with  $n_2 > n_1$  and with QAPs of  $\frac{1-e^{-rt^m}}{n_i r}$ ,  $i = 1, 2$ . Then the rate of decrease in the QAP between  $T_1$  and  $T_2$  is the solution,  $\lambda$ , to the equation  $\frac{1-e^{-rt^m}}{n_1 r} e^{-\lambda(T_2-T_1)} = \frac{1-e^{-rt^m}}{n_2 r} e^{-\lambda(n_2-n_1)t^m} = \frac{1-e^{-rt^m}}{n_2 r}$ . Solving this equation we obtain:

$$\lambda = \frac{1}{(n_2 - n_1)t^m} \log \frac{n_2}{n_1}.$$

Therefore: (i)  $\lambda$  is proportional to  $1/t^m$ , i.e., the more frequently products are introduced (the smaller is  $t^m$ ), the larger is the rate of decrease,  $\lambda$ , in their QAP. And (ii) fixing the difference  $n_2 - n_1$ , the ratio  $\frac{n_2}{n_1}$  is decreasing as  $n_1$  increases, i.e., as the product ages,  $\lambda$  decreases. ■

It is worth emphasizing that the correlation between the QAP and the rate of product introductions is not a *causality* relationship. Rather, the QAP and the rate of product introduction are endogenous variables and are determined - in equilibrium - by the “fundamentals”,  $r$ ,  $F$  and  $g(t)$ . As the fundamentals change, the endogenous variables change as well, and this co-variation generates the positive correlation.



## References

- [1] Berndt, E., and Griliches, Z., “Price Indices for Microcomputers: An Exploratory Study” in Foss, M., Manser, M., and Young, A., (editors) *Price Measurements and their uses*, Chicago: University of Chicago Press, 1993.
- [2] Chow, G., “Technological Change and the Demand for Computers,” *American Economic Review* 57 (1967), 1117-1130.
- [3] Gandal, N., “Hedonic Price indexes for Spreadsheets and an empirical test for Network Externalities,” *The Rand Journal of Economics* 25 (1994), 160-170.
- [4] Gordon, R., *The measurement of Durable Good Prices*, Chicago: University of Chicago Press, 1990.
- [5] Greenstein, S. H., “Did Computer Technology Diffuse Quickly?: Best and Average Practice in Mainframe Computers, 1968-1983”, NBER working paper # 4647, February 1994.
- [6] Griliches, Z., ”Hedonic price indexes for automobiles: An econometric analysis of quality change,” in *The price statistics of the federal government. General Series n° 73*. New York: Columbia University Press (for the National Bureau of Economic Research), 1961.
- [7] Mansfield, E. “Industrial Research and Technological Innovation” W.W. Norton: New-York, 1968.
- [8] Fishman, A. and Rob, R. “Product Innovation by a Durable Good Monopoly,” *Rand Journal of Economics*, 31 (2) (2000), 237-252.
- [9] Roff, D. and Trajtenberg, M., “Quality-Adjusted prices for the American Automobile Industry, 1906-1940,” NBER working paper, 1995.