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"Informational Size and Efficient Auctions"

by

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Informational Size and Efficient Auctions*

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Abstract

We develop an auction model for the case of interdependent values and multidimensional signals in which agents' signals are correlated. We provide conditions under which a modification of the Vickrey auction which includes payments to the bidders will result in an ex post efficient outcome. Furthermore, we provide a definition of informational size such that the necessary payments to bidders will be arbitrarily small if agents are sufficiently informationally small.

Keywords: Auctions, Incentive Compatibility, Mechanism Design, Interdependent Values.

JEL Classification: C70, D44, D60, D82

1 Introduction

The efficiency of market processes has been a central concern in economics since its inception. Auction mechanisms constitute a very important class of market processes, yet the analysis of auctions has typically focused on their revenue generating properties rather than their efficiency properties. This is partly due to the fact that, for many of the problems typically studied, efficiency is trivial. When bidders have private values, a standard Vickrey auction guarantees that the object will be sold to the buyer with the highest valuation for the object. In the case of pure common values - that is, when all buyers have the same value for the object - any outcome

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that with probability one assigns the object to *some* bidder will be efficient. The intermediate case in which bidders' values are not identical but may depend on other bidders' signals is more problematic. When bidders' values are interdependent in this way, any single bidder's value may depend on the information of other agents and, hence, he may not even know his own value. It is not clear what it would mean for an agent to bid his "true" value, even before we ask if it is optimal for him to do so.

Several papers have studied efficient auctions with interdependent valuations and independent types. In the case of two bidders, Maskin (1992) extended the Vickrey auction to the case of interdependent values in a way that assures an efficient outcome. Dasgupta and Maskin (1998) and Perry and Reny (1998) use the basic idea in Maskin (1992) to construct auction mechanisms that guarantee efficient outcomes for the case in which there are multiple units to be sold. In these papers, an agent's information regarding the value of the object(s) to be sold must be represented by a one dimensional signal. Dasgupta and Maskin provide simple examples showing that, if agents' types are independent, there may not exist mechanisms that are efficient when signals are multidimensional. Jehiel and Moldovanu (1998) prove a general theorem about the generic impossibility of efficient mechanisms when bidders have independent types and multidimensional signals.

The exclusion of multidimensional types is quite restrictive. For many problems, individuals have private information of two very different kinds: information about the qualitative features of the object being sold and information about themselves that affects their personal valuation of an object with particular physical characteristics, but does not affect others' valuations. Potential bidders for an oil tract may have information about the size and nature of the oil field and, in addition, information regarding their own cost of retrieving and processing the oil. Agents bidding in a spectrum auction may have information about the number and characteristics of the individuals covered by the license being sold, as well as information about the value to their company from serving that population. In such problems, the information (type) of an agent is multidimensional and, hence, existing papers on efficient auctions provide no guidance.

The work described above on the possibility/impossibility of efficient auction mechanisms restricts attention to the case in which agents' types are independent. While this is a natural place to begin, the independence assumption is not compelling for many problems. Often, the general structure of the problem corresponds to that described in the previous paragraph where the value to a given prospective buyer of the object(s) being sold depends on two qualitatively different things: objective characteristics of the object itself (the quantity and quality of oil in a tract to be auctioned off or the demographic characteristics of the consumers covered by a license in a spectrum auction), and idiosyncratic characteristics of the buyer (his cost of extracting the oil in the field or his cost of serving the customers covered by a given spectrum license). When bidders' types include information about objective characteristics of the object, it is likely that their types are correlated.

Cremer and McLean (1985,1988) showed that, when agents' types are correlated, mechanisms can be designed to induce truthful revelation of private information, and that information can be used to ensure efficient outcomes.[1] When agents' types are correlated, the multidimensionality of information poses no problems for Bayes-Nash implementation. However, mechanisms that rely on correlation of types to induce truthful revelation are sometimes criticized on the grounds that in such mechanisms, the payments to and from agents can be very large. The use of very large payments makes it clear that such mechanisms will not be of use in the presence of limited liability or nonlinear preferences over money. We use techniques similar to those employed by Cremer and McLean, but we restrict attention to reward schemes in which agents do not make payments. When agents' types are statistically dependent, we show that there exist efficient auction mechanisms for interdependent value auction problems that are essentially Vickrey auctions augmented by payments to (not from) the agents. Most importantly, we link the payment made to an agent to that agent's "informational size", as formulated in McLean and Postlewaite (2002) in a model of exchange economies in which agents had private information about the state of the world.

If all agents are receiving signals correlated with the common but unobservable value of the object, then any single agent's signal may add little to the information contained in the aggregate of the other agents' signals. Informally, we can think of an agent as being informationally small if it is unlikely that the probability distribution of the objective characteristics of the object is very sensitive to that agent's information, given the information of others. When agents are informationally small, the payments necessary for our augmented Vickrey auction will be small. Hence, agents' "informational rents" - as represented by the payments made to them - are linked to their informational size. However, we should emphasize that we are not proposing that agents are *necessarily* informationally small and, consequently, that efficient outcomes can always be assured with small augmented payments.

Our definition of informational size generalizes the concept of nonexclusive information introduced in Postlewaite and Schmeidler (1986). Nonexclusive information was introduced to characterize informational problems in which incentive compatibility would not be an issue. Heuristically, this would be the case when, for any agent and for any information he might have, that agent's information is redundant given the combined information of all other agents'. In the presence of nonexclusive information, it is straightforward to induce truthful revelation. In this case, roughly speaking, the agents' reports will be inconsistent when a single agent misrepresents his information, thus revealing that some agent misreported with probability one.

One can characterize this situation as one in which an agent has no ability to alter the posterior distribution as he contemplates the type he will announce. Our measure

[1] See also McAfee and Reny (1992) for subsequent work.

of informational size extends this concept in the sense that, when an agent has positive informational size, the agent's different types (typically) result in different posterior distributions, given other agents' reported types. When an agent is informationally small, that agent is unlikely to have a large effect on the posterior given other agents' reported types.

Our model is described in Section 2, and in Section 3 we present an example with a simple information structure in which agents receive conditionally independent signals of the state of nature. Section 4 provides an analysis of a more general problem with information structures that include the conditionally independent structure of the example in section 3 as a special case. The analysis in section 4 assumes that agents' types are exogenously specified in a form that separates the part of an agent's information that affects other agents' valuations from the part of the agent's information that affects only his own valuation. In Section 5, we show how the information structure for general incomplete information problems can be represented in a way that decomposes agents' information into these two components. Since for many asymmetric information problems these two aspects of an agent's information are qualitatively different, this decomposition is of some independent interest. Some concluding comments are contained in Section 6 and the proofs are given in Section 7.

2 Auctions

Let $\Theta = \{\theta_1, .., \theta_m\}$ represent the finite set of states of nature. Each $\theta \in \Theta$ represents a complete physical description of the object being sold (e.g., the amount and quality of oil). Let T_i be a finite set of possible types of agent i. As stressed in the introduction, an agent's information may be of two qualitatively different kinds: information about the objective characteristics of the object being sold, and idiosyncratic information about the agent himself. The former is of interest to other agents - and consequently is the cause of the interdependence of agents' valuations - while the latter is irrelevant to other agents in calculating their valuations. The state of nature is unobservable but agent $i's$ information about the physical characteristics of the object to be sold will be captured by the correlation between his type t_i and nature's choice of θ. His type t_i will also capture any idiosyncratic information he may have. Agent $i's$ valuation is represented by a function $v_i : \Theta \times T_i \to R_+$. That is, agent $i's$ value for the object depends on the physical characteristics of the object θ, and his type t_i.

Let $(\widetilde{\theta}, \widetilde{t}_1, \widetilde{t}_2, ..., \widetilde{t}_n)$ be an (n+1)-dimensional random vector taking values in $\Theta \times T (T \equiv T_1 \times \cdots \times T$ and $T_{-i} \equiv \times_{j \neq i} T_j)$ with associated distribution P where

$$P(\theta, t_1, .., t_n) = Prob\{\widetilde{\theta} = \theta, \widetilde{t}_1 = t_1, ..., \widetilde{t}_n = t_n\}.$$

We will make the following full support assumptions regarding the marginal distributions : $P(\theta)$ =Prob$\{\widetilde{\theta} = \theta\} > 0$ for each $\theta \in \Theta$ and $P(t)$ =Prob$\{\widetilde{t}_1 = t_1, ..., \widetilde{t}_n =$

$t_n\} > 0$ for each $t \in T$.

If X is a finite set, let Δ_X denote the set of probability measures on X. The set of probability measures on $\Theta \times T$ satisfying the full support conditions will be denoted $\Delta^*_{\Theta\times T}$

In problems with differential information, it is standard to assume that agents have utility functions $w_i : T \to R_+$ that depend on other agents' types. It is worthwhile noting that, while our formulation takes on a different form, it is equivalent. Given a problem as formulated in this paper, we can define $w_i(t) = \sum_{\theta\in\Theta} [v_i(\theta, t_i)P(\theta|t)]$. Alternatively, given utility functions $w_i : T \to R_+$, we can define $\Theta \equiv T$ and define $v_i(t, t_i) = w_i(t)$. Our formulation will be useful in that it highlights the nature of the interdependence: agents care about other agents' types to the extent that they provide additional information about the physical characteristics of the object being sold.

An *auction problem* is a collection $(v_1, .., v_n, P)$ where $P \in \Delta^*_{\Theta\times T}$. An *auction mechanism* is a collection $\{q_i, x_i\}_{i\in N}$ where $q_i : T \to \mathbb{R}_+$ is the probability that agent i gets the object given a vector of announced types, and $x_i : T \to \mathbb{R}$ are transfer functions.

For any vector of types $t \in T$, let

$$\hat{v}_i(t) = \hat{v}_i(t_{-i}, t_i) = \sum_{\theta\in\Theta} v_i(\theta, t_i)P(\theta|t_{-i}, t_i).$$

Although $\hat{v}$ depends on P, we suppress this dependence for notational simplicity. The number $\hat{v}_i(t)$ represents $i's$ valuation for the object conditional on the informational state $t \in T$.

Definition: An auction mechanism $\{q_i, x_i\}_{i\in N}$ is:

incentive compatible (IC) if for each $i \in N$,

$$\sum_{t_{-i}} [q_i(t_{-i}, t_i)\hat{v}_i(t_{-i}, t_i) - x_i(t_{-i}, t_i)] P(t_{-i}|t_i) \geq \sum_{t_{-i}} [q_i(t_{-i}, t'_i)\hat{v}_i(t_{-i}, t_i) - x_i(t_{-i}, t'_i)] P(t_{-i}|t_i)$$

whenever $t_i, t'_i \in T_i$.

ex post individually rational (XIR) if

$$q_i(t)\hat{v}_i(t) - x_i(t) \geq 0 \text{ for all } i \text{ and all } t \in T.$$

ex post efficient (*XE*) if

$$\hat{v}_i(t) = \max_j \{\hat{v}_j(t)\} \text{ whenever } q_i(t) > 0.$$

For a given auction problem $(v_1, .., v_n, P)$, we will be interested in the second price auction using the conditional values $\hat{v}_i(t)$. For each $t \in T$, let

$$I(t) = \{i \in N | \hat{v}_i(t) = \max_j \hat{v}_j(t)\}$$

and define

$$w_i(t) = \max_{j:j\neq i} \hat{v}_j(t).$$

Formally, we define a *Vickrey auction with conditional values* (Vickrey auction for short) to be the auction mechanism $\{q_i^*, x_i^*\}_{i\in N}$ defined as follows:

$$q_i^*(t) = \begin{cases} \frac{1}{|I(t)|} \text{ if } i \in I(t) \\ 0 \text{ if } i \notin I(t) \end{cases}$$

and

$$x_i^*(t) = q_i^*(t) w_i(t) \ .$$

It is straightforward to show that this Vickrey auction mechanism is ex post efficient and ex post individually rational. It will generally *not* be incentive compatible. However, as we will show below, it is often possible to modify the Vickrey auction payments so as to make truthful revelation an equilibrium when agents are informationally small in a sense to be defined below.

Let $\{z_i\}_{i\in N}$ be an n-tuple of functions $z_i : T \to \mathbb{R}_+$ each of which assigns to each $t \in T$ a nonnegative number, interpreted as a "reward" to agent i. The associated *augmented Vickrey auction with conditional values* (augmented Vickrey auction for short) is the auction mechanism $\{q_i^*, x_i^* - z_i\}_{i\in N}$

We present an example in the next section that illustrates our notion of augmented Vickrey auctions and the relationship between informational size and the payments that agents receive. This example also illustrates the main ideas in the proofs of Theorems 1 and 2 discussed in sections 4 and 5 below.

3 Example

Three wildcatters are competing for the right to drill for oil on a tract of land. It is common knowledge that the amount of oil is either 20 or 30, each equally likely. The state in which the quantity is 20 is denoted θ_L and the state in which the quantity is 30 is denoted θ_H; let $\Theta = \{\theta_L, \theta_H\}$. Each wildcatter i performs a private test that provides information in the form of a noisy signal of the state which we denote s_i. That is, agent i's private test yields a signal H (high) or L (low); for each i, let $S_i = \{H, L\}$. The distribution of the signal for agent i, conditional on the state, is given in the table below ($\rho > 1/2$).

	state	θ_L	θ_H
signal			
L		ρ	$1-\rho$
H		$1-\rho$	ρ

Agents' signals are independent, conditional on the state θ.

In addition to the signal regarding the amount of oil, each of the wildcatters has private information regarding his own cost of extraction. We assume that the extraction cost c_i of wildcatter i is drawn from a finite set. Hence, agent i's type t_i is the pair (s_i, c_i) comprising his privately observed extraction cost c_i and his privately observed signal s_i. We will assume that the vector of extraction costs (c_1, c_2, c_3) is independent of the state-signal vector (θ, s_1, s_2, s_3). The price of oil is 1. Agent i's payoff v_i as a function of the state θ and his type t_i depends only on θ and his private extraction cost c_i. If $t_i = (c_i, s_i)$, then his payoff should he obtain the right to drill is given by:

$$\begin{aligned} v_i(\theta_L, t_i) &= 20 - c_i \\ v_i(\theta_H, t_i) &= 30 - c_i. \end{aligned}$$

Consider the following auction mechanism. Agents announce their types and the posterior distribution on θ given the agents' announcements of their signals is calculated. Let $P_\Theta(\cdot|s_1, s_2, s_3)$ denote this posterior distribution on Θ. Next, compute the agents' expected valuations $\hat{v}_i$ for the object, where

$$\hat{v}_i(t_1, t_2, t_3) = \hat{v}_i(s_1, s_2, s_3, c_i) = v_i(\theta_L, c_i) \cdot P_\Theta(\theta_L|s_1, s_2, s_3) + v_i(\theta_H, c_i) \cdot P_\Theta(\theta_H|s_1, s_2, s_3).$$

Let $\{q_i^*, x_i^*\}_{i \in \{1,2,3\}}$ be the associated Vickrey auction defined in section 2: the drilling rights are awarded to the agent i for whom $\hat{v}_i(s_1, s_2, s_3, c_i)$ is highest and that agent pays a price equal to the higher of the other two agents' valuations. In addition, any agent who has announced a signal equal to that announced by the majority receives a (small) payment $\bar{z} > 0$. Formally, the payments in the augmented Vickrey auction are defined by $z_i(t_1, t_2, t_3) = z_i(s_1, c_1, s_2, c_2, s_3, c_3) = \overline{z}$ if $s_i = s_j$ for at least one $j \neq i$ and zero otherwise. Since z_i does not depend on c, we will simply write i's payment as $z_i(s_1, s_2, s_3)$.

Truth is generally not a dominant strategy for the unaugmented Vickrey mechanism. If agent 3 (for example) announces L when he has in fact received signal H, his announcement of L will lower the expected valuations of all agents. In the event that agent 3 wins the object, he will pay a lower price by announcing L. However, the introduction of the reward $\bar{z}$ in the augmented mechanism will offset this possible gain in expected utility when ρ is close to 1, thus inducing agents to be truthful. In the example, incentive compatibility will be achieved when $\rho \approx 1$ as a result of a subtle interplay of two ideas: *informational size* and the *variability of agent's beliefs*. To explain these ideas in the context of the example, let

$$U_3^*(s_1, c_1, s_2, c_2, s_3', c_3'|s_3, c_3) := q_i^*(s_1, c_1, s_2, c_2, s_3', c_3')\hat{v}_i(s_1, s_2, s_3, c_3) - x_i^*(s_1, c_1, s_2, c_2, s_3', c_3')$$

denote the payoff to agent 3 in the (unaugmented) Vickrey auction $\{q_i^*, x_i^*\}_{i \in N}$ when agent 1 announces $t_1 = (s_1, c_1)$, agent 2 announces $t_2 = (s_2, c_2)$, agent 3 announces

$t'_3 = (s'_3, c'_3)$ and 3's true type is $t_3 = (s_3, c_3)$. Since the conditional valuations of agents 1 and 2 do not depend on agent 3's extraction cost c_3, it follows from the definition of the Vickrey mechanism that

$$U_3^*(s_1, c_1, s_2, c_2, s'_3, c'_3|s_3, c_3) \leq U_3^*(s_1, c_1, s_2, c_2, s'_3, c_3|s_3, c_3).$$

As a result, agent 3 will truthfully reveal his private extraction cost whether or not other agents truthfully reveal their types and whether or not agent 3 himself truthfully reveals his signal s_3.

Hence, to prove incentive compatibility, we must show that the expected gain from a report of (c_3, L) in the augmented Vickrey mechanism $\{q_i^*, x_i^* - z_i\}_{i \in \{1,2,3\}}$ is nonpositive when ρ is sufficiently close to 1. To evaluate this expected gain, we must compute a sum in whose terms are

$$[U_3^*(s_1, c_1, s_2, c_2, L, c_3|H, c_3) - U_3^*(s_1, c_1, s_2, c_2, H, c_3|H, c_3)] + [z_3(s_1, s_2, L) - z_3(s_1, s_2, H)]$$

weighted by the probability $P(s_1, c_1, s_2, c_2|H, c_3)$. Since the independence assumption implies that

$$P(s_1, c_1, s_2, c_2|H, c_3) = P(s_1, s_2|H)P(c_1, c_2|c_3),$$

we will prove incentive compatibility if the sum of the terms

$$[U_3^*(s_1, c_1, s_2, c_2, L, c_3|H, c_3) - U_3^*(s_1, c_1, s_2, c_2, H, c_3|H, c_3)] + [z_3(s_1, s_2, L) - z_3(s_1, s_2, H)]$$

weighted by $P(s_1, s_2|H)$ is nonpositive for each c_1, c_2, c_3 when ρ is sufficiently close to 1.

In the example, agent 3 (and the others as well) is *informationally small* when $\rho \approx 1$ in the sense that, upon observing the signal H, agent 3 concludes that an announcement of L will with high probability have only a small effect on the distribution over states conditioned on the information of all three agents: that is, $P(H, H|H) \approx 1$ and $P_\Theta(\cdot|H, H, H) - P_\Theta(\cdot|H, H, L) \approx 0$ when $\rho \approx 1$. Since $P(H, H|H) \approx 1$, the (total) expected utility gain from a lie will be approximately equal to the gain from a lie when agents 1 and 2 have each reported H[2].

On the other hand, $P_\Theta(\cdot|H, H, L) - P_\Theta(\cdot|H, H, H) \approx 0$. This in turn implies that $\hat{v}_i(H, H, L, c_i) - \hat{v}_i(H, H, H, c_i) \approx 0$ for $i = 1, 2$ and it follows that the utility gain to 3 from a lie of L when the others have reported H will be small. In particular,

$$U_3^*(H, c_1, H, c_2, L, c_3|H, c_3) - U_3^*(H, c_1, H, c_2, H, c_3|H, c_3) \approx 0$$

[2] That is,

$$\begin{aligned} & \sum_{s_1, s_2} [U_3^*(s_1, c_1, s_2, c_2, L, c_3|H, c_3) - U_3^*(s_1, c_1, s_2, c_2, H, c_3|H, c_3)]\, P(s_1, s_2|H) \\ \approx\ & U_3^*(H, c_1, H, c_2, L, c_3|H, c_3) - U_3^*(H, c_1, H, c_2, H, c_3|H, c_3). \end{aligned}$$

when $\rho \approx 1$. In summary, a lie of L when 3 has received signal H may result in a positive gain in expected utility but that gain will be close to zero when ρ is close enough to 1.

The signal of agent 3 can be truthfully elicited if his expected reward from matching the announcement of at least one other agent is at least as large as his expected utility gain. In our mechanism, $P(H, H|H) \approx 1$ when $\rho \approx 1$ so

$$\sum_{s_1 \in S_1} \sum_{s_2 \in S_2} [z_3(s_1, s_2, L) - z_3(s_1, s_2, H)]\, P(s_1, s_2|H) \approx z_3(H, H, L) - z_3(H, H, H) = -\overline{z}.$$

These same arguments can be applied to the situation in which 3 receives the signal L but reports H. Combining these observations, it follows that for any $\overline{z} > 0$, our mechanism will be ex post efficient, ex post individually rational and incentive compatible whenever ρ is sufficiently close to 1. When $\rho \approx 1$, the agents are informationally small and, as a result, the incentive provided by a *small* payment $\bar{z}$ will offset the small expected utility gain from misrepresenting and truthful revelation will be an equilibrium.

In a more general model in which the probabilistic structure is more complex than the conditionally independent noisy signal structure of the example, our ability to find rewards z_i for which the expected gain in reward will dominate the expected gain in utility will depend on the *variability of agents' beliefs*, that is, on the difference between the conditional distributions $P(\cdot, \cdot|H)$ and $P(\cdot, \cdot|L)$ on $S_1 \times S_2$. If, for example, these conditional distributions were equal, then we cannot find a system of rewards satisfying the inequalities

$$\sum_{s_1 \in S_1} \sum_{s_2 \in S_2} [z_3(s_1, s_2, L) - z_3(s_1, s_2, H)]\, P(s_1, s_2|H) < 0$$

and

$$\sum_{s_1 \in S_1} \sum_{s_2 \in S_2} [z_3(s_1, s_2, H) - z_3(s_1, s_2, L)]\, P(s_1, s_2|L) < 0.$$

Even if the expected utility gains are small positive numbers, we will have difficulty constructing an incentive compatible mechanism. Hence, the closeness of 3's beliefs $P(\cdot, \cdot|H)$ and $P(\cdot, \cdot|L)$ on $S_1 \times S_2$ will play a role in our analysis.

In summary, agents must be informationally small and beliefs must be sufficiently variable in order to construct augmented Vickrey auctions that satisfy incentive compatibility. In the next section, we present a model that generalizes several features of this example and formalizes the concepts of informational size and variability.

4 Efficient Auction Mechanisms

4.1 The Model

In this section we will assume that the set of types for agent i has the special product form $T_i = S_i \times C_i$ where $S_1, ..., S_n$ and $C_1, ..., C_n$ are finite sets. An element $s_i \in S_i$ will be referred to as agent i's *signal*. An element $c_i \in C_i$ will be referred to as agent i's *personal* characteristic. Let $S \equiv S_1 \times \cdots \times S_n$ and $S_{-i} \equiv \times_{j \neq i} S_j$. The product sets C and C_{-i} are defined in a similar fashion. We will often write $t = (s, c)$ and $t_i = (s_i, c_i)$ where s and c (s_i and c_i) denote the respective projections of t (t_i) onto S and C (S_i and C_i). Both the signal s_i and the personal characteristic c_i are private information to i with the following interpretations: s_i represents a signal that is correlated with nature's choice of θ and c_i represents a set of other idiosyncratic payoff relevant characteristics of agent i that provide no information about θ or s_{-i} beyond that contained in s_i. In our example, the extraction cost c_i of each wildcatter corresponds to the agent's personal characteristic and, since costs are assumed to be independent of the state and the agents' signals, it is certainly the case that c_i contains no information about θ or s_{-i} beyond that contained in s_i. We assume[3] that the random vectors $(\tilde{\theta}, \tilde{s})$ and $\tilde{c}$ are *stochastically independent*, i.e.,

$$P(\theta, t) \equiv P(\theta, s, c) = P(\theta, s)P(c).$$

We denote by $\Delta^I_{\Theta \times S \times C}$ denote the set of measures in $\Delta^*_{\Theta \times S \times C}$ satisfying this stochastic independence assumption.

4.2 Informational Size and Variability of Beliefs

We now formalize the idea of informational size discussed in section 3 above. Our example indicates that a natural notion of an agent's informational size is the degree to which he can alter this posterior distribution on Θ when other agents are announcing truthfully. Any vector of agents' signals $s = (s_{-i}, s_i) \in S$ induces a conditional distribution on $P_\Theta(\cdot|s_{-i}, s_i)$ on Θ and, if agent i unilaterally changes his announcement from s_i to s'_i, this conditional distribution will (in general) change. If i receives signal s_i but announces $s'_i \neq s_i$, the set

$$\{s_{-i} \in S_{-i} | \; ||P_\Theta(\cdot|s_{-i}, s_i) - P_\Theta(\cdot|s_{-i}, s'_i)|| > \varepsilon\}$$

consists of those s_{-i} for which agent i's misrepresentation will have (at least) an "$\varepsilon-$ effect" on the conditional distribution. (Here and throughout the paper, $|| \cdot ||$ will denote the 1-norm.) Let

$$\nu_i^P(s_i, s'_i) = \min\{\varepsilon \in [0,1] | \text{Prob}\{ \; ||P_\Theta(\cdot|\tilde{s}_{-i}, s_i) - P_\Theta(\cdot|\tilde{s}_{-i}, s'_i)|| > \varepsilon | \tilde{s}_i = s_i\} \leq \varepsilon\}.$$

[3] This assumption can be weakened. See point 11 in the discussion section.

To show that $\nu_i^P(s_i, s_i')$ is well defined, let

$$F(\varepsilon) = \text{Prob}\{||P_\Theta(\cdot|\tilde{s}_{-i}, s_i) - P_\Theta(\cdot|\tilde{s}_{-i}, s_i')|| \leq \varepsilon | \tilde{s}_i = s_i\}.$$

Hence, the set $\{\varepsilon \in [0,1] | 1 - F(\varepsilon) \leq \varepsilon\}$ is nonempty (since $1 - F(1) \leq 1$), bounded and closed (since F is right continuous with left hand limits.)

Finally, define the *informational size* of agent i as

$$\nu_i^P = \max_{s_i, s_i' \in S_i} \nu_i^P(s_i, s_i').$$

Note that $\nu_i^P = 0$ for every i if and only if $P_\Theta(\cdot|s) = P_\Theta(\cdot|s_{-i})$ for every $s \in S$ and $i \in N$. [4]

There are two important features of this definition. First, an agent's informational size depends only on that part of his information that is useful in predicting θ, and second, an informationally small agent may have very accurate information about the state θ.

In our discussion of the example in section 3 above, we indicated that the ability to give agent i an incentive to reveal his information will depend on the magnitude of the difference between $P_{S_{-i}}(\cdot|s_i)$ and $P_{S_{-i}}(\cdot|s_i')$, the conditional distributions on S_{-i} given different signals for agent i. We will refer to this magnitude informally as the *variability of agents' beliefs.*

To define formally the measure of variability, we treat each conditional $P_{S_{-i}}(\cdot|s_i) \in \Delta_{S_{-i}}$ as a point in a Euclidean space of dimension equal to the cardinality of S_{-i}. Our measure of variability is defined as[5]

$$\Lambda_i^{P,S} = \min_{s_i \in S_i} \min_{s_i' \in S_i \setminus s_i} ||P_{S_{-i}}(\cdot|s_i) - P_{S_{-i}}(\cdot|s_i')||^2.$$

4.3 The Result

We now state our first result on the possibility of efficient mechanisms.

Theorem 1: Let $(v_1, .., v_n)$ be a collection of payoff functions.

(i) If $P \in \Delta^I_{\Theta\times S\times C}$ satisfies $\Lambda_i^{P,S} > 0$ for each i, then there exists an incentive compatible Augmented Vickrey auction $\{q_i^*, x_i^* - z_i\}_{i\in N}$ for the auction problem $(v_1, .., v_n, P)$.

(ii) For every $\varepsilon > 0$, there exists a $\delta > 0$ such that, whenever $P \in \Delta^I_{\Theta\times S\times C}$ satisfies

$$\max_i \nu_i^P \leq \delta \min_i \Lambda_i^{P,S},$$

[4] This is essentially the case of nonexclusive information introduced by Postlewaite and Schmeidler (1986) and is discussed further in the last section.

[5] See McLean and Postlewaite (2002) for further discussion of informational size and variability.

there exists an incentive compatible Augmented Vickrey auction $\{q_i^*, x_i^* - z_i\}_{i\in N}$ for the auction problem $(v_1, .., v_n, P)$ satisfying $0 \le z_i(t) \le \varepsilon$ for every i and t.

Part (i) of Theorem 1 states that, if $\Lambda_i^{P,S}$ is positive for each agent i, then there exists an incentive compatible augmented Vickrey mechanism for the auction problem $(v_1, .., v_n, P)$. The hypotheses of part (i) only require that each $\Lambda_i^{P,S}$ be positive and places no lower bound on the magnitude of $\Lambda_i^{P,S}$. Furthermore, the informational size of the agents is not important. On the other hand, the conclusion of part (i) places no upper bound on the size of the reward z_i. These rewards can be quite large.

Part (ii) of the theorem states that there exists an incentive compatible augmented Vickrey mechanism with small payments if, for each i, $\Lambda_i^{P,S}$ is large enough relative to the informational size of agent i. To illustrate part (ii), consider again the example in section 3 where we showed the following: for every $\varepsilon > 0$, there exists a $\tilde{\rho} > 0$ such that, whenever $\tilde{\rho} < \rho < 1$, there exists an incentive compatible augmented Vickrey auction $\{q_i^*, x_i^* - z_i\}_{i\in\{1,2,3\}}$ satisfying $0 \le z_i(t) \le \varepsilon$ for all t. This result can now be deduced as an application of (ii) since, in the example, each $\nu_i^P \to 0$ and each $\Lambda_i^{P,S} \to 1$ as $\rho \to 1$.

While the technical details of the proof are deferred until the last section, we can sketch the ideas here for the special case in which $T_i = S_i$ (i.e., each C_i is a singleton). There are two key steps. First, we show (see Lemmas A.1 and A.2) that for all i , all $s_i, s_i' \in S_i$ and all $s_{-i} \in S_{-i}$,

$$(q_i^*(s)\hat{v}_i(s) - x_i^*(s)) - (q_i^*(s_{-i}, s_i')\hat{v}_i(s) - x_i^*(s_{-i}, s_i')) \ge -M||P_\Theta(\cdot|s_{-i}, s_i) - P_\Theta(\cdot|s_{-i}, s_i')||$$

where

$$M = \max_\theta \max_i \max_{s_i} v_i(\theta, s_i).$$

This result is of some interest in its own right. If $||P_\Theta(\cdot|s_{-i}, s_i) - P_\Theta(\cdot|s_{-i}, s_i')||$ is "small" uniformly in s_i, s_i' and s_{-i}, then truthful reporting is an "approximate" ex post Nash equilibrium in the (unaugmented) Vickrey mechanism $\{q_i^*, x_i^*\}$. For example, if $\tilde{\theta}$ and $\tilde{s}$ are independent, then $\hat{v}_i(s)$ depends only on s_i. In this case, $||P_\Theta(\cdot|s_{-i}, s_i) - P_\Theta(\cdot|s_{-i}, s_i')|| = 0$ for all i , all $s_i, s_i' \in T_i$ and all $s_{-i} \in S_{-i}$ and we deduce the classic result for Vickrey auctions: truthful reporting is a dominant strategy with pure private values.

Of course, $||P_\Theta(\cdot|s_{-i}, s_i) - P_\Theta(\cdot|s_{-i}, s_i')||$ is generally not uniformly small. However, we can use the concept of informational size to show that

$$\sum_{s_{-i}} [(q_i^*(s)\hat{v}_i(s) - x_i^*(s)) - (q_i^*(s_{-i}, s_i')\hat{v}_i(s) - x_i^*(s_{-i}, s_i'))]\, P(s_{-i}|s_i) \ge -3M\hat{\nu}_i^P.$$

If all agents are informationally small, then truthful reporting is "approximately" incentive compatible in the (unaugmented) Vickrey mechanism $\{q_i^*, x_i^*\}$. If $z_i(s)$ is the reward to i when the bidders announce s, then the associated augmented Vickrey auction $\{q_i^*, x_i^* - z_i\}$ will be incentive compatible if and only if

$$\sum_{s_{-i}} [z_i(s_{-i}, s_i) - z_i(s_{-i}, s_i')] \, P(s_{-i}|s_i) - 3M\hat{\nu}_i^P \geq 0$$

for each $s_i, s_i' \in S_i$. This is the generalization of the analysis of the example in section 3.

It can be shown that there exists a collection of numbers $\zeta_i(s)$ satisfying $0 \leq \zeta_i(s) \leq 1$ and

$$\sum_{s_{-i}} [\zeta_i(s_{-i}, s_i) - \zeta_i(s_{-i}, s_i')] \, P(s_{-i}|s_i) > 0$$

for each $s_i, s_i' \in S_i$ if and only if $\Lambda_i^{P,S} > 0$. Part (i) of the theorem now follows: choose $\zeta_i(s)$ to satisfy these inequalities, define $z_i(s) = \alpha\zeta_i(s)$ and choose α large enough so that incentive compatibility is satisfied. Of course, as we mentioned above, the resulting $z_i's$ can be large.

Part (ii) is more delicate. Unfortunately, the optimal value $val_i(P)$ of the linear program

$$\max_{\beta, \zeta_i(s)} \; \beta$$

$$s.t. \; \sum_{s_{-i}} [\zeta_i(s_{-i}, s_i) - \zeta_i(s_{-i}, s_i')] \, P(s_{-i}|s_i) \geq \beta \text{ for all } s_i, s_i'$$

$$0 \leq \zeta_i(s) \leq 1 \text{ for all } s$$

is not bounded from below by a positive number, uniformly in P. If this were the case, then the existence of an incentive compatible augmented Vickrey auction with small payments would depend *only* on informational size. Instead, $val_i(P) \to 0$ as $\Lambda_i^{P,S} \to 0$. In order to prove (ii), we require that each $\Lambda_i^{P,S}$ be large enough relative to the informational size of agent i.

5 Efficient Auction Mechanisms: The General Case

The mechanism in the previous section is successful in achieving an efficient outcome because it deals differently with the component of an agent's information that affects other agents' valuations and with the component that affects only his own valuation. Since second-price auction techniques handle the latter, one need only extract the former to achieve efficient outcomes. The information structure in the previous section assumed that the set of types of an agent could be expressed as the Cartesian product of signals and personal characteristics and that the information structure satisfied stochastic independence. Stated differently, we assumed that assumed that agents' types were exogenously decomposed into "private" and "common" components. General information structures will typically not have this form, and consequently the result in the previous section may not apply. In this section, we show how the information structure for general incomplete information problems, even those without a product structure, can be represented in a way that decomposes agents' information into "signals" and "private characteristics."

In order to extend the ideas of the special model of section 4 to the general problem defined in section 2, we need to define the appropriate generalizations of informational size and variability of beliefs. Let

$$\nu_i^P(t_i, t_i') = \min\{\varepsilon \in [0,1] | \text{Prob}\{ \, ||P_\Theta(\cdot|\tilde{t}_{-i}, t_i) - P_\Theta(\cdot|\tilde{t}_{-i}, t_i')|| > \varepsilon | \tilde{t}_i = t_i\} \leq \varepsilon\}$$

and define the *informational size* of agent i as

$$\nu_i^P = \max_{t_i, t_i' \in T_i} \nu_i^P(t_i, t_i').$$

This is the definition introduced in McLean and Postlewaite (2002). If $T_i = S_i \times C_i$ and if $P \in \Delta^I_{\Theta \times S \times C}$, then the definition of ν_i^P given above coincides with the definition of informational size given in section 4.

To extend the notion of variability of beliefs, we begin with the definition of information decomposition.

Definition: An *information decomposition* (ID) of $P \in \Delta^*_{\Theta \times T}$ is a collection $\mathcal{D}$ consisting of sets $R_1, .., R_n$ and surjections $g_i : T_i \to R_i$ satisfying:

(i) for all i, for all $t_i, t_i' \in T_i$ and for all $t_{-i} \in T_{-i}$,

$$g_i(t_i) = g_i(t_i') \;\Rightarrow P_\Theta(\cdot|t_{-i}, t_i) = P_\Theta(\cdot|t_{-i}, t_i').$$

(ii) for all i, for all $t_i, t_i' \in T_i$ and for all $r_{-i} \in R_{-i}$,

$$g_i(t_i) = g_i(t_i') \Rightarrow \text{Prob}\{g_j(\tilde{t}_j) = r_j \forall j \neq i | \tilde{t}_i = t_i\} = \text{Prob}\{g_j(\tilde{t}_j) = r_j \forall j \neq i | \tilde{t}_i = t_i'\}.$$

We interpret $g_i(t_i)$ as that "part" of an agent's information that is "informationally relevant" for predicting the state of nature θ. Condition (i) has the following interpretation: given a type profile $t_{-i} \in T_{-i}$, a type $t_i \in T_i$ contains no information that is useful in predicting the state θ beyond that contained in the informationally relevant part $g_i(t_i)$. Condition (ii) states that a specific type $t_i \in T_i$ contains no information beyond that contained in $g_i(t_i)$ that is useful in predicting the informationally relevant profile of other agents.

Every measure P has at least one information decomposition: this is the trivial decomposition in which $T_i = R_i$ and $g_i = id$. However, a measure P can have more than one ID. If each $T_i = S_i \times C_i$ as in section 4 and if $P \in \Delta^I_{\Theta \times S \times C}$, then P has a second information decomposition where $R_i = S_i$ and g_i is the projection of T_i onto S_i.

Given an information decomposition $\mathcal{D} = \{R_i, g_i\}_{i \in N}$ for $P \in \Delta^*_{\Theta \times T}$, we let $P^{\mathcal{D}}$ denote the distribution on $R = R_1 \times \cdots \times R_n$ induced by the map $(t_1, .., t_n) \mapsto (g_1(t_1), .., g_n(t_n))$. That is, for each $(r_1, .., r_n) \in R$,

$$P^{\mathcal{D}}(r_1, .., r_n) = \text{Prob}\{\tilde{t}_i \in g_i^{-1}(r_i)\ \forall i \in N\}.$$

Given an information decomposition $\mathcal{D}$, let

$$\Lambda^{P,\mathcal{D}} = \min_{r_i \in R_i} \min_{r'_i \in R_i \backslash r_i} ||P^{\mathcal{D}}_{R_{-i}}(\cdot|r_i) - P^{\mathcal{D}}_{R_{-i}}(\cdot|r'_i)||^2.$$

If each $T_i = S_i \times C_i, R_i = S_i$ and g_i is the projection of T_i onto S_i, then $\Lambda_i^{P,\mathcal{D}}$ coincides with $\Lambda_i^{P,S}$ as defined in section 4.

Using these definitions of informational size and variability of beliefs, we can generalize Theorem 1 as follows.

Theorem 2: Let $(v_1, .., v_n)$ be a collection of payoff functions.

(i) Let $P \in \Delta^*_{\Theta \times T}$. If there exists an information decomposition $\mathcal{D}$ for P with $\Lambda_i^{P,\mathcal{D}} > 0$ for each i, then there exists an incentive compatible Augmented Vickrey auction $\{q_i^*, x_i^* - z_i\}_{i \in N}$ for the auction problem $(v_1, .., v_n, P)$.

(ii) For every $\varepsilon > 0$, there exists a $\delta > 0$ such that, whenever $P \in \Delta^*_{\Theta \times T}$ satisfies

$$\max_i \nu_i^P \leq \delta \min_i \Lambda_i^{P,\mathcal{D}}$$

for some information decomposition $\mathcal{D}$ of P, there exists an incentive compatible Augmented Vickrey auction $\{q_i^*, x_i^* - z_i\}_{i \in N}$ for the auction problem $(v_1, .., v_n, P)$ satisfying $0 \leq z_i(t) \leq \varepsilon$ for every i and t.

Theorem 1 is an immediate corollary of Theorem 2. It is possible that a measure P has only one ID, the trivial decomposition (denoted $\mathcal{D}^0$) where $T_i = R_i$ and $g_i = id$. For this decomposition, it follows from the definitions that

$$\Lambda_i^{P,D^0} = \min_{t_i \in T_i} \min_{t'_i \in T_i \backslash t_i} ||P_{T_{-i}}(\cdot|t_i) - P_{T_{-i}}(\cdot|t'_i)||^2$$

where $P_{T_{-i}}(\cdot|t_i)$ is the conditional on T_{-i} given $\tilde{t}_i = t_i$. For the trivial ID $\mathcal{D}^0$, we have the following corollary to Theorem 2.

Corollary 1: Let $(v_1, .., v_n)$ be a collection of payoff functions.

(i) If $P \in \Delta^*_{\Theta \times T}$ satisfies $P_{T_{-i}}(\cdot|t_i) \neq P_{T_{-i}}(\cdot|t'_i)$ for each $i = 1, ..., n$ and for each $t_i, t_i' \in T_i$ with $t_i \neq t_i'$, then there exists an incentive compatible Augmented Vickrey auction $\{q_i^*, x_i^* - z_i\}_{i \in N}$ for the auction problem $(v_1, .., v_n, P)$.

(ii) For every $\varepsilon > 0$, there exists a $\delta > 0$ such that, whenever $P \in \Delta^*_{\Theta \times T}$ satisfies

$$\max_i \nu_i^P \leq \delta \min_i \Lambda_i^{P,\mathcal{D}^0},$$

there exists an incentive compatible Augmented Vickrey auction $\{q_i^*, x_i^* - z_i\}_{i \in N}$ for the auction problem $(v_1, .., v_n, P)$ satisfying $0 \leq z_i(t) \leq \varepsilon$ for every i and t.

As a final remark on the relationship between our results, we note that Corollary 1 can also be deduced as a special case of Theorem 1 in which each C_i is a singleton and T_i is identified with S_i. If each C_i is a singleton, then stochastic independence is trivially satisfied and Corollary 1 follows from Theorem 1.

6 Discussion

1. As pointed out in the example, truthful revelation is an equilibrium for our augmented Vickrey auction mechanisms, but not the unique equilibrium. One should be able to use the techniques in the literature on exact implementation to construct nonrevelation games that eliminate the multiplicity of equilibria.[6]

2. In this paper, we focus on the augmented Vickrey auction and show that an efficient outcome can be assured with payments to the agents that depend on the agents' informational size. The mechanism that we analyze will not, in general, maximize the net revenue to the seller. In proving our theorem, we demonstrate that for any limit on the total payments to the agents, we can guarantee a structure of payments depending on agents' announcements that will assure incentive compatibility if agents are sufficiently informationally small. Although the payments that we construct will not typically be the minimal payments that induce truthful announcement, it must be the case that any increase in expected net revenue to the seller that can be achieved through optimizing the structure of payments to agents is limited by the total payments identified in our result.

There is a second way the mechanism we analyze may be inefficient that may be more important, however. In our mechanism agents announce their types, and these types are used to calculate agents' conditional values. The agent with the highest conditional value obtains the object at the second highest conditional value, and the difference between the first and second highest valuations constitutes a rent to the winning bidder. Suppose agents' types consist of a signal about θ and a private characteristic. The winning bidder's rent will then depend on his private characteristic and the private characteristic of the agent with second highest conditional value. While the seller may not be able to eliminate this rent when the private characteristics are stochastically independent, we have not made any assumptions regarding such independence. If private characteristics are not independent, there may be scope for extending our techniques to extract this rent. Of course, the possibility of extracting this rent has no bearing on whether the auction mechanism is efficient, which is the focus of this paper.

[6] See, e.g., Postlewaite and Schmeidler (1986), or the surveys of Moore (1992) and Palfrey (1992).

3. We have ignored the Wilson critique in our analysis since the agents' beliefs are used to construct the mechanism. Specifically, the agents face rewards that are functions of the profile of announced types and the construction of these rewards depends on the distribution of agents' types.

In general, the realism of any mechanism which depends on the distribution of agents' types is questionable at the very least, since we expect that such fine details of the environment are likely not available to the seller. While our mechanism is certainly subject to this concern, it is worth discussing the point before passing on.

In our motivating three person example, each agent receives a reward if his announced signal matches at least one other agent's announcement. Suppose that we fix the reward size, say at 1. Then there is a minimum $\bar{\rho}$ (the precision of the signal agents receive) for which truth will be an equilibrium, and truthful announcement continues to be an equilibrium for any higher precision of agents' signals. Hence, one could "propose" the mechanism that is defined in this way: agents announce their types, and the highest conditional value agents receives the object at the second highest conditional valuation, and agents whose announcement matches at least one other agent's announcement receive a reward of 1. This mechanism is not subject to the Wilson critique since its definition is independent of the distribution of agents' types. Of course the mechanism is not generally optimal. For distributions associated with precisions greater than $\bar{\rho}$, the rewards are larger than need be, and for distributions associated with precisions below $\bar{\rho}$, truthful announcement of types will not be incentive compatible. The mechanism may nevertheless be of interest to a seller who cares about efficiency and who has some idea of the information structure, namely that the distribution of agents' types is one of the infinitely many distributions for which agents' signals have precision greater than $\bar{\rho}$. While it may be unrealistic to assume that the seller should know precisely the distribution of agents' types, it is not unreasonable to believe that in some circumstances the seller could have the crude information necessary to choose the mechanism with rewards of 1 for agents who match at least one other announcement.

4. In section 4, we assume that agents' type sets are finite. If the signals and personal characteristics of agents' information are separated, it is only the signal sets that need to be finite. The set of personal characteristics can be finite, a continuum or some combination without affecting the possibility of efficient mechanisms.

5. As mentioned in the introduction, McLean and Postlewaite (2002) introduced a notion of informational size similar to that used in this paper. That paper deals with pure exchange economies with private information in which an agent's utility function depends only on the realized state $\theta \in \Theta$. The preferences in the present paper are more general in the sense that agent i's utility may depend on his type t_i as well as the state θ. The extension of our methods to this case is possible because of the properties of the Vickrey auction for which there are no counterparts in a general equilibrium environment.

6. We treated the case of a single object to be sold. Our techniques can be extended to the problem of auctioning K identical objects when bidders' valuations exhibit "decreasing marginal utility," i.e., when $v_i(k+1, \theta, t_i) - v_i(k, \theta, t_i) \geq v_i(k+2, \theta, t_i) - v_i(k+1, \theta, t_i)$ where $v_i(k, \theta, t_i)$ is the payoff to bidder when the state is θ, his type is t_i and he is awarded k objects.

7. We now expand briefly on the relationship of our paper to those of Cremer and McLean on full surplus extraction (1985,1988). The main point of the Cremer-McLean papers is that correlation of agents' types allows full surplus extraction. In the models in those papers (as in the present paper), players' payoffs include payments that depend on other agents' types. In the Cremer-McLean setup, the type of correlation (for example, the full rank condition in their 1985 paper) permits the construction of announcement dependent lotteries, where truthful revelation generates a lottery with zero conditional expected value while a lie generates negative conditional expected value. If the lotteries are appropriately rescaled, then the incentive for truthful reporting can be made arbitrarily large and an incentive compatible mechanism that extracts the full surplus can be found.

In part (i) of (for example) Corollary 1, we only require that the conditional distribution on T_{-i} be different for different $t_i's$. That is, we only require that $\Lambda_i^{P,\mathcal{D}_0}$ be positive. This is weaker than the full rank condition (and is also weaker than the cone condition in their 1988 paper) and the implication is concomitantly weaker. Our assumption only permits the construction of announcement dependent lotteries where truthful revelation generates a lottery whose conditional expected value exceeds the conditional expected value from a lie. Using the full rank condition and some additional assumptions on the conditional payoff $\hat{v}(t)$, Cremer-McLean construct a mechanism that extracts the full surplus from bidders (see Corollary 2 in Cremer-McLean, 1985). This mechanism is necessarily ex post efficient. Under the weaker conditions of this paper, we construct (in part (i)) a mechanism that is ex post efficient but which may not extract the full surplus. In addition, the payments in a Cremer-McLean mechanism can be positive or negative and they can be large in absolute value. Our paper differs in that we introduce only nonnegative payments. Hence, our techniques do not require unlimited liability on the part of buyers (although the seller may be constrained by the necessary payments that would induce incentive compatibility).

The more interesting part of our results – the ability to induce incentive compatibility with *small* payments when agents are informationally small – has no counterpart in the Cremer-McLean analysis.

8. Many auction papers restrict attention to symmetric problems in which bidders' types are drawn from the same distribution. It should be noted that we make no assumptions on the distribution of bidders' types. However, if agents' beliefs exhibit positive variability, then their types cannot be independent. Several papers analyzing interdependent value auction problems make assumptions regarding the impact of a

bidder's information on his own valuation relative to other bidders' valuations (see, e.g., Maskin (1992), Dasgupta and Maskin (1998) and Perry and Reny (1998). We make no such assumptions.

9. The general mechanism design approach that we use in this paper has been criticized on the grounds that revelation games are unrealistic for many problems. The examples used to illustrate mechanisms typically have simple information structures, as in our example in section 3, in which an agent's type is simply a pair of numbers - the quantity of oil and the cost of extracting it. In general, however, an agent's type encompasses all information he may have, including his beliefs about all relevant characteristics of the object, his beliefs about others' beliefs, etc. When types are realistically described, it seems unlikely that the revelation game could actually be played.

We are sympathetic to this argument, but we want to stress that the underlying logic by which efficient outcomes are obtained in our model does not depend on the particular revelation game we used; similar outcomes might be obtained through a non-revelation game. Consider first the following two-stage game. The second stage is a standard Vickrey auction. In the first stage, agents forecast the highest bid in the second stage, excluding their own bid, and these forecasts are made common knowledge prior to the second stage. An agent is rewarded if the error is his forecast is smaller than some specified level.

Suppose that agents with favorable private information about the value of the object to others forecast high bids. When these forecasts are made public, each agent may be able to infer other agents' information from their forecasts. If they are able to do this, the asymmetry of information will have been eliminated, and the second stage Vickrey auction will assure an efficient outcome. Of course, agents might "manipulate" the system by making strategic rather than naive forecasts that will take into account the effects of their announcements in the second stage auction. However, the effect of strategic forecasting will be small if agents are informationally small. Hence, as in the case our mechanism, the reward for correct forecasting will dominate the potential benefits from strategic forecasting when bidders are informationally small.[7]

10. In this paper we investigated the general problem of the conflict between the extraction of information from agents and the use of that information to ensure efficient allocations. Pesendorfer and Swinkels (2000) analyze a model in which a number of objects are to be auctioned off to a number of bidders. They assume an informational structure that is similar to ours: each agent gets information about a personal taste parameter and a signal about a common value component. Pesendorfer and Swinkels study the problem when the number of agents increases and provide conditions under which the objects are allocated efficiently in the limit. It is easy

[7] **See McLean and Postlewaite (2001) for an investigation of such a mechanism.**

to see that in their framework, each agent's informational size goes to zero as the number of agents goes to infinity.

11. Theorem 1 of Section 4 assumed that the random vectors $(\tilde{\theta}, \tilde{s})$ and $\tilde{c}$ were stochastically independent. However, the conclusions of Theorem 1 will hold under a weaker condition that we call *Informational Independence.* Formally, a probability measure $P \in \Delta^*_{\Theta \times S \times C}$ satisfies *Informational Independence* if for each $(\theta, s, c) \in \Theta \times S \times C$, (i) $P_\Theta(\theta|s, c) = P_\Theta(\theta|s, c_{-i})$ and (ii) $P_{S_{-i}}(s_{-i}|s_i, c_i) = P_{S_{-i}}(s_{-i}|s_i)$. Obviously, informational independence is weaker than the stochastic independence assumption of section 4. Furthermore, it can be shown that, if $P \in \Delta^*_{\Theta \times S \times C}$ satisfies informational independence, then P admits an information decomposition $\mathcal{D} = \{g_i, R_i\}_{i \in N}$ where $R_i = S_i$ and g_i is the projection of T_i onto S_i. As a result, Theorem 1 will still hold under the assumption of Informational Independence.

7 Proofs:

7.1 Preparations for the Proof of Theorem 2:

In this section, we begin with two lemmas that are of some independent interest.

Lemma A.1: Let $(v_1, .., v_n)$ be a collection of payoff functions and let $\{q_i^*, x_i^*\}_{i \in N}$ be the associated Vickrey auction mechanism. For every $i \in N$ and for each $t \in T$ and $t_i' \in T_i$,

$$(q_i^*(t)\hat{v}_i(t) - x_i^*(t)) - (q_i^*(t_{-i}, t_i')\hat{v}_i(t) - x_i^*(t_{-i}, t_i')) \geq -|w_i(t_{-i}, t_i') - w_i(t_{-i}, t_i)|.$$

Proof: Choose $t \in T$ and $t_i' \in T_i$.

Case 1: Suppose that $\hat{v}_i(t_{-i}, t_i') < w_i(t_{-i}, t_i')$. Then

$$q_i^*(t_{-i}, t_i') = x_i^*(t_{-i}, t_i') = 0$$

so

$$\begin{aligned}
&(q_i^*(t)\hat{v}_i(t) - x_i^*(t)) - (q_i^*(t_{-i}, t_i')\hat{v}_i(t) - x_i^*(t_{-i}, t_i')) \\
&\quad = \; q_i^*(t)\hat{v}_i(t) - x_i^*(t) \\
&\quad \geq \; 0 \\
&\quad \geq \; -|w_i(t_{-i}, t_i') - w_i(t_{-i}, t_i)|.
\end{aligned}$$

Case 2: Suppose that $\hat{v}_i(t_{-i}, t_i') > w_i(t_{-i}, t_i')$. Then

$$q_i^*(t_{-i}, t_i')\hat{v}_i(t) - x_i^*(t_{-i}, t_i') = \hat{v}_i(t) - w_i(t_{-i}, t_i').$$

If $\hat{v}_i(t_{-i}, t_i) > w_i(t_{-i}, t_i)$, then

$$q_i^*(t)\hat{v}_i(t) - x_i^*(t) = \hat{v}_i(t) - w_i(t_{-i}, t_i).$$

If $\hat{v}_i(t_{-i}, t_i) \leq w_i(t_{-i}, t_i)$, then

$$q_i^*(t)\hat{v}_i(t) - x_i^*(t) = 0 \geq \hat{v}_i(t) - w_i(t_{-i}, t_i).$$

Therefore,

$$\begin{aligned}
&(q_i^*(t)\hat{v}_i(t) - x_i^*(t)) - (q_i^*(t_{-i}, t_i')\hat{v}_i(t) - x_i^*(t_{-i}, t_i')) \\
&\quad \geq \ (\hat{v}_i(t) - w_i(t_{-i}, t_i)) - (\hat{v}_i(t) - w_i(t_{-i}, t_i')) \\
&\quad = \ w_i(t_{-i}, t_i') - w_i(t_{-i}, t_i) \\
&\quad \geq \ -|w_i(t_{-i}, t_i') - w_i(t_{-i}, t_i)|.
\end{aligned}$$

Case 3: Suppose that $\hat{v}_i(t_{-i}, t_i') = w_i(t_{-i}, t_i')$. Then

$$q_i^*(t_{-i}, t_i')\hat{v}_i(t) - x_i^*(t_{-i}, t_i') = \frac{1}{|I(t_{-i}, t_i')|}\left(\hat{v}_i(t) - w_i(t_{-i}, t_i')\right).$$

If $\hat{v}_i(t_{-i}, t_i) > w_i(t_{-i}, t_i)$, then

$$q_i^*(t)\hat{v}_i(t) - x_i^*(t) = \hat{v}_i(t) - w_i(t_{-i}, t_i) \geq \frac{1}{|I(t_{-i}, t_i')|}\left(\hat{v}_i(t) - w_i(t_{-i}, t_i)\right).$$

If $\hat{v}_i(t_{-i}, t_i) \leq w_i(t_{-i}, t_i)$, then

$$q_i^*(t)\hat{v}_i(t) - x_i^*(t) = 0 \geq \frac{1}{|I(t_{-i}, t_i')|}\left(\hat{v}_i(t) - w_i(t_{-i}, t_i)\right).$$

Therefore,

$$\begin{aligned}
&(q_i^*(t)\hat{v}_i(t) - x_i^*(t)) - (q_i^*(t_{-i}, t_i')\hat{v}_i(t) - x_i^*(t_{-i}, t_i')) \\
&\quad \geq \ \frac{1}{|I(t_{-i}, t_i')|}\left(\hat{v}_i(t) - w_i(t_{-i}, t_i)\right) - \frac{1}{|I(t_{-i}, t_i')|}\left(\hat{v}_i(t) - w_i(t_{-i}, t_i')\right) \\
&\quad = \ \frac{1}{|I(t_{-i}, t_i')|}\left(w_i(t_{-i}, t_i') - w_i(t_{-i}, t_i)\right) \\
&\quad \geq \ -\frac{1}{|I(t_{-i}, t_i')|}|w_i(t_{-i}, t_i') - w_i(t_{-i}, t_i)| \\
&\quad \geq \ -|w_i(t_{-i}, t_i') - w_i(t_{-i}, t_i)|.
\end{aligned}$$

This completes the proof of Lemma 1.

If each $\hat{v}_i(t)$ is a function of t_i only, then $|w_i(t_{-i}, t_i') - w_i(t_{-i}, t_i)| = 0$ and Lemma A.1 yields the familiar result for Vickrey auctions with pure private values: it is a dominant strategy to truthfully report one's type.

Lemma A.2: Let $(v_1, .., v_n)$ be a collection of payoff functions and let $\{q_i^*, x_i^*\}_{i \in N}$ be the associated Vickrey auction mechanism. Let

$$M = \max_{\theta} \max_{i} \max_{t_i} v_i(\theta, t_i)$$

and let $P \in \Delta^*_{\Theta \times T}$. For every $i \in N$ and for each $t_{-i} \in T_{-i}$, $t_i \in T_i$ and $t_i' \in T_i$,

$$|w_i(t_{-i}, t_i') - w_i(t_{-i}, t_i)| \leq M||P_\Theta(\cdot|t_{-i}, t_i) - P_\Theta(\cdot|t_{-i}, t_i')||.$$

Proof: Choose $t_{-i}, t_i, t_i', j \neq i$ and $j' \neq i$ so that

$$w_i(t_{-i}, t_i) = \max_{k \neq i} \sum_{\theta \in \Theta} [v_k(\theta, t_k) P_\Theta(\theta|t_{-i}, t_i)] = \sum_{\theta \in \Theta} [v_j(\theta, t_j) P_\Theta(\theta|t_{-i}, t_i)]$$

and

$$w_i(t_{-i}, t_i') = \max_{k \neq i} \sum_{\theta \in \Theta} [v_k(\theta, t_k) P_\Theta(\theta|t_{-i}, t_i')] = \sum_{\theta \in \Theta} [v_{j'}(\theta, t_{j'}) P_\Theta(\theta|t_{-i}, t_i')] .$$

Note that t_j and $t_{j'}$ are, respectively, the j and j' components of the vector t_{-i}. From the definitions of t_j and $t_{j'}$, it follows that

$$\sum_{\theta \in \Theta} [v_j(\theta, t_j) - v_{j'}(\theta, t_{j'})] P_\Theta(\theta|t_{-i}, t_i) \geq 0$$

and

$$\sum_{\theta \in \Theta} [v_j(\theta, t_j) - v_{j'}(\theta, t_{j'})] P_\Theta(\theta|t_{-i}, t_i') \leq 0.$$

Therefore,

$$\begin{aligned}
&\sum_{\theta \in \Theta} v_{j'}(\theta, t_{j'}) [P_\Theta(\theta|t_{-i}, t_i) - P_\Theta(\theta|t_{-i}, t_i')] \\
&\quad \leq \sum_{\theta \in \Theta} v_{j'}(\theta, t_{j'}) [P_\Theta(\theta|t_{-i}, t_i) - P_\Theta(\theta|t_{-i}, t_i')] + \sum_{\theta \in \Theta} [v_j(\theta, t_j) - v_{j'}(\theta, t_{j'})] P_\Theta(\theta|t_{-i}, t_i) \\
&\quad = w_i(t_{-i}, t_i) - w_i(t_{-i}, t_i') \\
&\quad = \sum_{\theta \in \Theta} v_j(\theta, t_j) [P_\Theta(\theta|t_{-i}, t_i) - P_\Theta(\theta|t_{-i}, t_i')] + \sum_{\theta \in \Theta} [v_j(\theta, t_j) - v_{j'}(\theta, t_{j'})] P_\Theta(\theta|t_{-i}, t_i') \\
&\quad \leq \sum_{\theta \in \Theta} v_j(\theta, t_j) [P_\Theta(\theta|t_{-i}, t_i) - P_\Theta(\theta|t_{-i}, t_i')]
\end{aligned}$$

and we conclude that

$$|w_i(t_{-i}, t_i) - w_i(t_{-i}, t_i')| \leq M||P_\Theta(\cdot|t_{-i}, t_i) - P_\Theta(\cdot|t_{-i}, t_i')||.$$

This completes the proof of Lemma 2.

We prove one final technical result.

Lemma A.3: Let X be a finite set with cardinality k and let $p, q \in \Delta_X$. Then

$$\left[\frac{p}{||p||_2} - \frac{q}{||q||_2}\right] \cdot p \geq \frac{k^{-\frac{5}{2}}}{2} \left[||p-q||\right]^2$$

where $||\cdot||_2$ denotes the 2-norm and $||\cdot||$ denotes the 1-norm.

Proof: Direct computation shows that

$$\left[\frac{p}{||p||_2} - \frac{q}{||q||_2}\right] \cdot p = \frac{||p||_2}{2} \left\|\frac{p}{||p||_2} - \frac{q}{||q||_2}\right\|_2^2$$

The result follows by combining the facts that $||p||_2 \geq 1/\sqrt{k}$, $k(||p-q||_2)^2 \geq ||p-q||^2$ and

$$\left[\left\|\frac{p}{||p||_2} - \frac{q}{||q||_2}\right\|_2\right]^2 \geq \frac{1}{k} \left[||p-q||_2\right]^2.$$

7.2 Proof of Theorem 2:

We prove part (ii) first. Choose $\varepsilon > 0$. Let

$$M = \max_{\theta} \max_{i} \max_{t_i} v_i(\theta, t_i)$$

and let K be the cardinality of T. Choose δ so that

$$0 < \delta < \frac{\varepsilon}{6MK^{\frac{5}{2}}}.$$

Suppose that $P \in \Delta^*_{\Theta \times T}$ has an information decomposition satisfying

$$\max_i \nu_i^P \leq \delta \min_i \Lambda_i^{P,\mathcal{D}}.$$

Define $\hat{\nu}^P = \max_i \nu_i^P$ and $\Lambda^{P,\mathcal{D}} = \min_i \Lambda_i^{P,\mathcal{D}}$. Therefore $\hat{\nu}^P \leq \delta \Lambda^{P,\mathcal{D}}$.

Next, define

$$\zeta_i(r_{-i}, r_i) = \frac{P^{\mathcal{D}}_{R_{-i}}(r_{-i}|r_i)}{||P^{\mathcal{D}}_{R_{-i}}(\cdot|r_i)||_2}$$

for each $(r_1, .., r_n) \in R_1 \times \cdots \times R_n$ and note that

$$0 \leq \zeta_i(r_{-i}, r_i) \leq 1$$

for all i, r_{-i} and r_i. Now we define an augmented Vickrey auction mechanism. For each $t \in T$, let

$$z_i(t) := \varepsilon\zeta_i(g_1(t_1), .., g_n(t_n)).$$

The mechanism $\{q_i^*, x_i^* - z_i\}_{i \in N}$ is clearly ex post efficient. Individual rationality follows from the observations that

$$q_i^*(t)\hat{v}_i(t) - x_i^*(t) \geq 0$$

and

$$z_i(t_{-i}, t_i) \geq 0.$$

To prove incentive compatibility, we consider two cases. First suppose that $g_i(t_i) = g_i(t_i')$. From part (i) of the definition of information decomposition, it follows that $|w_i(t_{-i}, t_i') - w_i(t_{-i}, t_i)| = 0$ for all $t_{-i} \in T_{-i}$ and incentive compatibility is a consequence of Lemma A.1.

Now suppose that $g_i(t_i) = r_i$ and $g_i(t_i') = r_i'$ with $r_i \neq r_i'$. The proof of incentive compatibility will follow from the next two claims.

Claim 1:

$$\sum_{t_{-i}} (z_i(t_{-i}, t_i) - z_i(t_{-i}, t_i'))\, P(t_{-i}|t_i) \geq \frac{K^{-\frac{5}{2}}}{2} \varepsilon \Lambda^{P,\mathcal{D}}.$$

Proof of Claim 1: Part (ii) of the definition of information decomposition implies that

$$\sum_{\substack{t_{-i} \in T_{-i} \\ : g_{-i}(t_{-i}) = r_{-i}}} P(t_{-i}|\hat{t}_i) = P^{\mathcal{D}}_{R_{-i}}(r_{-i}|\hat{r}_i)$$

whenever $g_i(\hat{t}_i) = \hat{r}_i$. Therefore,

$$\begin{aligned}
&\sum_{t_{-i}} (z_i(t_{-i}, t_i) - z_i(t_{-i}, t_i'))\, P(t_{-i}|t_i) \\
&= \sum_{t_{-i}} (\zeta_i(g_{-i}(t_{-i}), g_i(t_i)) - \zeta_i(g_{-i}(t_{-i}), g_i(t_i')))\, P(t_{-i}|t_i) \\
&= \varepsilon \sum_{r_{-i}} [\zeta_i(r_{-i}, r_i) - \zeta_i(r_{-i}, r_i')] \left[\sum_{\substack{t_{-i} \in T_{-i} \\ : g_{-i}(t_{-i}) = r_{-i}}} P(t_{-i}|t_i) \right] \\
&= \varepsilon \sum_{A_{-i}} [\zeta_i(r_{-i}, r_i) - \zeta_i(r_{-i}, r_i')]\, P^{\mathcal{D}}_{R_{-i}}(r_{-i}|r_i) \\
&= \varepsilon \sum_{A_{-i}} \left[\frac{P^{\mathcal{D}}_{R_{-i}}(r_{-i}|r_i)}{||P^{\mathcal{D}}_{R_{-i}}(\cdot|r_i)||_2} - \frac{P^{\mathcal{D}}_{R_{-i}}(r_{-i}|r_i')}{||P^{\mathcal{D}}_{R_{-i}}(\cdot|r_i')||_2} \right] P^{\mathcal{D}}_{R_{-i}}(r_{-i}|r_i)
\end{aligned}$$

$$\begin{aligned} &\geq \frac{\varepsilon K^{-\frac{5}{2}}}{2}\left[||P^{\mathcal{D}}_{R_{-i}}(\cdot|r_i) - P^{\mathcal{D}}_{R_{-i}}(\cdot|r_i')||\right]^2 \\ &\geq \frac{\varepsilon K^{-\frac{5}{2}}}{2}\Lambda_i^{P,\mathcal{D}} \end{aligned}$$

where the last inequality is an application of Lemma A.3.

Claim 2:

$$\sum_{t_{-i}} \left[(q_i^*(t)\hat{v}_i(t) - x_i^*(t)) - (q_i^*(t_{-i}, t_i')\hat{v}_i(t) - x_i^*(t_{-i}, t_i'))\right] P(t_{-i}|t_i) \geq -3M\hat{\nu}^P$$

Proof of Claim 2: Define

$$S_i(t_i', t_i) = \{t_{-i} \in T_{-i}|\ ||P_\Theta(\cdot|t_{-i}, t_i) - P_\Theta(\cdot|t_{-i}t_i')|| > \hat{\nu}^P\}.$$

Since $\nu_i^P \leq \hat{\nu}^P$, we conclude that

$$\text{Prob}\{\tilde{t}_{-i} \in S_i(t_i', t_i)|\tilde{t}_i = t_i\} \leq \nu_i^P \leq \hat{\nu}^P.$$

If $t_{-i} \notin S_i(t_i', t_i)$, then Lemmas A.1 and A.2 imply that

$$\sum_{t_{-i} \notin S_i(t_i', t_i)} \left[(q_i^*(t)\hat{v}_i(t) - x_i^*(t)) - (q_i^*(t_{-i}, t_i')\hat{v}_i(t) - x_i^*(t_{-i}, t_i'))\right] P(t_{-i}|t_i) \geq -M\hat{\nu}^P.$$

Finally, note that

$$|q_i^*(t_{-i}, t_i')\hat{v}_i(t) - x_i^*(t_{-i}, t_i')| \leq M$$

for all i, t_i, t_i' and t_{-i}.

Combining these observations , we conclude that

$$\begin{aligned} &\sum_{t_{-i}} \left[(q_i^*(t)\hat{v}_i(t) - x_i^*(t)) - (q_i^*(t_{-i}, t_i')\hat{v}_i(t) - x_i^*(t_{-i}, t_i'))\right] P(t_{-i}|t_i) \\ &\quad = \sum_{t_{-i} \in S_i(t_i', t_i)} \left[(q_i^*(t)\hat{v}_i(t) - x_i^*(t)) - (q_i^*(t_{-i}, t_i')\hat{v}_i(t) - x_i^*(t_{-i}, t_i'))\right] P(t_{-i}|t_i) \\ &\qquad + \sum_{t_{-i} \notin S_i(t_i', t_i)} \left[(q_i^*(t)\hat{v}_i(t) - x_i^*(t)) - (q_i^*(t_{-i}, t_i')\hat{v}_i(t) - x_i^*(t_{-i}, t_i'))\right] P(t_{-i}|t_i) \\ &\quad \geq -M\hat{\nu}^P - 2M\hat{\nu}^P \\ &\quad = -3M\hat{\nu}^P \end{aligned}$$

and the proof of claim 2 is complete.

Applying Claims 1 and 2, it follows that

$$
\begin{aligned}
&\sum_{t_{-i}} \left[(q_i^*(t)\hat{v}_i(t) - x_i^*(t)) - (q_i^*(t_{-i}, t_i')\hat{v}_i(t) - x_i^*(t_{-i}, t_i')) \right] P(t_{-i}|t_i) \\
&\qquad + \sum_{t_{-i}} (z_i(t_{-i}, t_i) - z_i(t_{-i}, t_i'))\, P(t_{-i}|t_i) \\
\geq\ & \varepsilon \frac{k^{-\frac{5}{2}}}{2} \Lambda^{P,\mathcal{D}} - 3M\hat{\nu}^P \\
\geq\ & 0.
\end{aligned}
$$

and the proof of part (ii) is complete.

Part (i) follows from the computations in part (ii). We have shown that, for any information decomposition $\mathcal{D}$ of P and for any positive number α, there exists an augmented Vickrey auction $\{q_i^*, x_i^* - z_i\}_{i \in N}$ satisfying

$$
\sum_{t_{-i}} \left[(q_i(t)\hat{v}_i(t) - x_i(t)) - (q_i(t_{-i}, t_i')\hat{v}_i(t) - x_i(t_{-i}, t_i')) \right] P(t_{-i}|t_i) \geq \alpha \frac{k^{-\frac{5}{2}}}{2} \Lambda_i^{P,\mathcal{C}} - 3M\hat{\nu}^P
$$

for each i and each t_i, t_i'. If $\Lambda_i^{P,\mathcal{D}} > 0$ for each i, then α can be chosen large enough so that incentive compatibility is satisfied. This completes the proof of part (i).

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