



Penn Institute for Economic Research
Department of Economics
University of Pennsylvania
3718 Locust Walk
Philadelphia, PA 19104-6297
pier@econ.upenn.edu
<http://www.econ.upenn.edu/pier>

PIER Working Paper 02-015

“From Individual to Aggregate Labor Supply: A Quantitative Analysis based on a Heterogeneous-Agent Macroeconomy”

by

Yongsung Chang and Sun-Bin Kim

http://ssrn.com/abstract_id=318922

From Individual to Aggregate Labor Supply: A Quantitative Analysis based on a Heterogeneous-Agent Macroeconomy*

Yongsung Chang
Department of Economics
University of Pennsylvania
3718 Locust Walk
Philadelphia, PA 19104-6297

Sun-Bin Kim
Department of Economics
Concordia University
1455 de Maisonneuve Blvd, W.
Montreal, QC, H3G 1M8

July 2, 2002

Abstract

We investigate the mapping from individual to aggregate labor supply using a general equilibrium heterogeneous-agent model with incomplete market. Heterogeneity of the workforce is designed such that the evolution of wages, worker flows between employment and nonemployment, and cross-sectional earnings distribution are consistent with micro data. We find that the aggregate labor-supply elasticity of such an economy is around 1, bigger than micro estimates but smaller than those often assumed in aggregate models.

Keywords: Aggregate Labor-Supply Elasticity, Heterogeneity, Indivisible Labor

JEL Classifications: E24, E32, J21, J22

*We thank Robert Shimer for sharing his monthly worker transition-rate data with us. We thank Cesairé Meh and José-Víctor Ríos-Rull for helpful comments. Chang gratefully acknowledges the financial support of the University of Pennsylvania Research Foundation.

1 Introduction

Despite enormous heterogeneity in the workforce, economists often postulate and analyze an economy populated by identical agents for its simplicity and tractability. A fully specified representative-agent model has become a workhorse in macroeconomics, and it is a common practice to rely on micro evidence to pin down the key parameters of highly aggregated models (Kydland and Prescott, 1982; Prescott, 1986; King, Plosser, and Rebelo, 1988).

Unfortunately, however, this practice often creates a tension between micro and macro observations.¹ A stylized fact in business-cycle fluctuations is that total hours vary greatly over the business cycle without much variation in wages. The explanations range from equilibrium to disequilibrium approaches. According to the intertemporal substitution hypothesis, pioneered by Lucas and Rapping (1969), workers are willing to substitute leisure over the business cycle.² To be consistent with the observed movement in hours and wages, this hypothesis requires an elastic labor supply beyond admissible estimates from micro data (Ghez and Becker, 1975; MaCurdy, 1981; Altonji, 1986; Abowd and Card, 1989).³ Even in a disequilibrium approach where the role of labor supply is dismissed in the short run, its slope is still important for the welfare cost departing

¹See Browning, Hansen, and Heckman (1999) that raises warning flags about the current use of micro evidence in calibrating macro models.

²An alternative equilibrium approach is to introduce shifts in labor supply as well as in demand through shifts in preference (Bencivenga, 1992), home technology (Benhabib, Rogerson, and Wright, 1991; Greenwood and Hercowitz, 1991) or government spending (Christiano and Eichenbaum; 1992).

³For example, Pencavel (1986) reports most estimates between 0 and 0.45 for men. In their parallel survey of research on labor supply of women, Killingsworth and Heckman (1986) present a wide range of estimates, from -0.3 to 14; they do not venture a guess as to which is correct but conclude that the elasticity is probably somewhat higher for women than men. See Blundell and MaCurdy (1999) for a more recent review of the literature. See also Mulligan (1998) on how the current micro estimates may underestimate the workers' willingness to substitute leisure over time.

from an equilibrium.⁴

The participation margin, the so-called extensive margin, has been recognized as a potential resolution. With heterogeneity among workers, hours fluctuations are accounted for mainly by movement in and out of employment by workers (Coleman, 1984; Heckman, 1984) with different reservation wages. Then, the slope of aggregate labor supply curve has little to do with intertemporal substitution but rather with the distribution of reservation wages across workers. The well-known lottery economy by Rogerson (1988) and Hansen (1985) is a special case where the reservation-wage distribution is degenerate, yielding a very high elasticity—in fact, infinity.

In this paper, we investigate the mapping from individual to aggregate labor supply using a general equilibrium model economy. Workers face idiosyncratic productivity shocks and the capital market is incomplete. Heterogeneity of the workforce in the model is disciplined by the three sets of micro data. The stochastic process of idiosyncratic productivity is estimated by the panel data from the Panel Study of Income Dynamics (PSID) for 1979-1992. The gross worker flows in and out of employment are consistent with those observed in the Current Population Survey (CPS) for 1967:II-2000:IV. The cross-sectional distributions of earnings and wealth are comparable to those from the PSID and Survey of Consumer Finance (SCF).

We find the aggregate labor-supply elasticities of such an economy ranging from 0.8 to 1.7, depending on the degree of heterogeneity and employment rates. These values are bigger than typical micro estimates, but smaller than those often assumed in aggregate models. We also compare the cyclical fluctuation of our economy to those of representative-agent economies. When the stochastic productivity shocks are introduced, we find that the hours response of our economy is

⁴The importance of labor-supply elasticity is not limited to the business-cycle analysis; for example, it plays an important role for the timing and effect of taxes and government spending (Aucherbach and Kotlikoff, 1987; Judd, 1987).

comparable to those of the representative-agent economy with intertemporal substitution elasticity of leisure between 1 and 2.⁵

Kydland (1984) and Cho (1995), among others, are the closest to our work as they focus on labor-supply elasticity in a stochastic growth model. Kydland constructs an economy with two types of workers, skilled and unskilled, and reproduces some labor-market regularity in relative wages and hours. However, this approach does not reflect the participation margin, a dominant source of variation in total hours in the data. Cho first incorporates the extensive margin into the real-business-cycle model in which workers are *ex ante* identical and *ex post* heterogeneous. This considerably simplifies the computation as consumption is shared among workers. It is, however, clear in the data (e.g., Consumer Expenditure Survey) that persons with greater hours or greater earnings per hour consume more.

Other important works on the labor-market heterogeneity in the context of stochastic general equilibrium include Andolfatto and Gomme (1996), Castañeda, Díaz-Gimenez, and Ríos-Rull (1998), Merz (1999), den Haan, Ramey, and Watson (2000), and Gomes, Greenwood, and Rebelo (2001). Andolfatto and Gomme study the Canadian unemployment insurance policy; Castañeda et al. the income distribution and unemployment spells; Merz the cyclical behavior of labor turnover; den Haan et al. the propagation mechanism under labor-market matching and job destruction; Gomes et al. the cyclical behavior of unemployment rates.

The paper is organized as follows. Section 2 presents the model. Section 3 calibrates the model consistent with various micro data. In Section 4, we investigate the aggregate labor supply of the

⁵An economy with indivisibility at the micro level may be approximated by a representative-agent economy with divisible labor, as the indivisibility is smoothed by an aggregation over heterogeneous agents. While this point is well illustrated in Mulligan (2001), we have yet to investigate its quantitative implications because the mapping from the micro to the macro function depends crucially on the nature of heterogeneity in the workforce.

model in both steady state and fluctuations. Comparison with the representative-agent model is also provided. Section 5 is the conclusion.

2 The Model

2.1 Environment

The model economy is a version of the stochastic-growth model with a large (measure one) population of infinitely lived workers. Individual workers differ from each other in productivity. Each worker maximizes the expected discounted lifetime utility:

$$U = \max_{\{c_t, h_t\}_{t=0}^{\infty}} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u(c_t, h_t) \right\},$$

with

$$u(c_t, h_t) = \ln c_t + B \frac{(1 - h_t)^{1-1/\gamma}}{1 - 1/\gamma},$$

where $E_0[\cdot]$ denotes the expectation operator conditional on information available at time 0, β is the discount factor, c_t consumption, and h_t hours worked at time t . The utility is separable between consumption and leisure and across times. The assumption about the form of utility is popular in both business-cycle analysis (Hansen, 1985; King, Plosser, and Rebelo, 1988) and the empirical labor-supply literature (MaCurdy, 1981; Altonji, 1986). The parameter γ denotes the intertemporal substitution elasticity of leisure. Log utility in consumption supports a balanced growth path.

According to our production technology, which will be specified below, labor input enters simply as an effective unit. Thus, a worker who supplies h_t units of time earns $w_t x_t h_t$, where w_t is the market wage rate for the efficiency unit of labor, and x_t represents the worker's productivity. We assume that individual productivity x_t exogenously varies over time according to a stochastic process with a transition probability distribution function $\pi_x(x'|x) = \Pr(x_{t+1} \leq x' | x_t = x)$. Since

participation is the dominant source of variation in total hours worked, we abstract from an hours choice and assume that labor supply is indivisible; i.e., h_t takes either zero or $\bar{h} (< 1)$. A worker can save and borrow by trading a claim for physical capital, which yields the rate of return r_t and depreciates at rate δ . Workers face a borrowing constraint; the level of asset holding, a_t , cannot be negative at any time. The capital market is incomplete; the physical capital is the only asset available to insure against idiosyncratic risks in x . A worker's budget constraint is:

$$c_t = w_t x_t h_t + (1 + r_t) a_t - a_{t+1},$$

and

$$a_{t+1} \geq 0.$$

Firms produce output according to constant-returns Cobb-Douglas technology in capital, K_t , and effective labor, L_t :

$$Y_t = F(L_t, K_t, \lambda_t) = \lambda_t L_t^\alpha K_t^{1-\alpha},$$

where λ_t is aggregate productivity, following a stochastic process with a transition probability distribution function, $\pi_\lambda(\lambda'|\lambda) = \Pr(\lambda_{t+1} \leq \lambda' | \lambda_t = \lambda)$.

It is useful to consider a recursive equilibrium. Suppose $\mu(a, x)$ denotes the distribution (measure) of workers.⁶ Let V^E and V^N denote the values of employed and nonemployed. If a worker decides to work, she solves the following Bellman equation by choosing the next period asset holding a' :

$$V^E(a, x; \lambda, \mu) = \max_{a' \in \mathcal{A}} \left\{ u(c, 1 - \bar{h}) + \beta E \left[\max \{ V^E(a', x'; \lambda', \mu'), V^N(a', x'; \lambda', \mu') \} | x, \lambda \right] \right\} \quad (1)$$

⁶Let \mathcal{A} and \mathcal{X} denote sets of all possible realizations of a and x , respectively. The measure $\mu(a, x)$ is defined over a σ -algebra of $\mathcal{A} \times \mathcal{X}$.

subject to

$$c = wx\bar{h} + (1 + r)a - a',$$

$$a' \geq 0,$$

and

$$\mu' = \mathbf{T}(\lambda, \mu).$$

where \mathbf{T} denotes a transition operator for μ .

If the worker decides not to work, her Bellman equation is:

$$V^N(a, x; \lambda, \mu) = \max_{a' \in \mathcal{A}} \left\{ u(c, 1) + \beta E \left[\max \{ V^E(a', x'; \lambda', \mu'), V^N(a', x'; \lambda', \mu') \} | x, \lambda \right] \right\} \quad (2)$$

subject to

$$c = (1 + r)a - a',$$

$$a' \geq 0,$$

and

$$\mu' = \mathbf{T}(\lambda, \mu).$$

Having solved (1) and (2), it is straightforward to deal with worker's labor-supply decision:

$$V(a, x; \lambda, \mu) = \max_{h \in \{\bar{h}, 0\}} \left\{ V^E(a, x; \lambda, \mu), V^N(a, x; \lambda, \mu) \right\}. \quad (3)$$

2.2 Equilibrium

Equilibrium consists of a set of decision rules for consumption, asset holding, and labor supply,

$\{c(a, x; \lambda, \mu), a'(a, x; \lambda, \mu), h(a, x; \lambda, \mu)\}$, aggregate inputs, $\{K(\lambda, \mu), L(\lambda, \mu)\}$, factor prices, $\{w(\lambda, \mu), r(\lambda, \mu)\}$,

and a law of motion for the distribution $\mu' = \mathbf{T}(\lambda, \mu)$ such that:

1. Individual optimization:

Given $w(\lambda, \mu)$ and $r(\lambda, \mu)$, the individual decision rules $c(a, x; \lambda, \mu)$, $a'(a, x; \lambda, \mu)$, and $h(a, x; \lambda, \mu)$ solve (1), (2), and (3).

2. The firm's profit maximization:

$$w(\lambda, \mu) = F_1(L(\lambda, \mu), K(\lambda, \mu), \lambda) \quad (4)$$

$$r(\lambda, \mu) = F_2(L(\lambda, \mu), K(\lambda, \mu), \lambda) - \delta \quad (5)$$

for all (λ, μ) .

3. The goods market clears:

$$\int \{a'(a, x; \lambda, \mu) + c(a, x; \lambda, \mu)\} d\mu = F(L(\lambda, \mu), K(\lambda, \mu), \lambda) + (1 - \delta)K \quad (6)$$

for all (λ, μ) .

4. Factor markets clear:

$$L(\lambda, \mu) = \int xh(a, x; \lambda, \mu) d\mu \quad (7)$$

$$K(\lambda, \mu) = \int ad\mu \quad (8)$$

for all (λ, μ) .

5. Individual and aggregate behaviors are consistent:

$$\mu'(A^0, X^0) = \int_{A^0, X^0} \left\{ \int_{\mathcal{A}, \mathcal{X}} \mathbf{1}_{a'=a'(a,x;\lambda,\mu)} d\pi_x(x'|x) d\mu \right\} da' dx' \quad (9)$$

for all $A^0 \subset \mathcal{A}$ and $X^0 \subset \mathcal{X}$.

3 Calibration

A key element in the mapping from individual to aggregate labor supply is the nature of heterogeneity in the workforce. We assume that individual productivity x follows an AR(1) process in logs:

$$\ln x' = \rho_x \ln x + \varepsilon_x, \quad \varepsilon_x \sim N(0, \sigma_x^2). \quad (10)$$

We view x reflecting a broad measure of earnings ability and opportunity in the market. Wage data from the PSID for the period of 1979-1992 are used to estimate this process. Appendix A.1 describes the PSID data we use in detail. According to the model, the log wage for individual i at time t , denoted by $\ln w_t^i$, can be written as $\ln w_t^i = \ln w_t + \ln x_t^i$. When quasi-differenced, individual wage evolves as:

$$\ln w_t^i = \rho_x \ln w_{t-1}^i + (\ln w_t - \rho_x \ln w_{t-1}) + \varepsilon_{x,t}^i. \quad (11)$$

Equation (11) is estimated with year dummies capturing aggregate effects including $\ln w_t - \rho_x \ln w_{t-1}$. The OLS estimate for ρ_x is .817 with a standard error of .0025 ($R^2 = .69$).⁷ The corresponding quarterly persistence is .95(= .817^{.25}). Given the estimate of ρ_x , we compute the standard deviation of innovation consistent with the cross-sectional wage distribution in the PSID as described in Appendix A.2: $\sigma_x = .194$.

In general, the dispersion of productivity in the population is larger than that of the wage distribution because workers at the very low end of the distribution are less likely to participate.

⁷As x reflects a broad measure of earnings ability, we do not control for individual characteristics. When we include age, sex, and years of schooling in the regression, the estimate for ρ_x decreases from .817 to .758. These estimates are slightly lower than—but comparable to—the persistence of idiosyncratic earnings risks in Storesletten, Telmer, and Yaron (1999). The difference is due to the fact that they decompose idiosyncratic shocks into a persistent AR(1) and a purely temporary *i.i.d.* components whereas we assume a single AR(1) process.

To understand the magnitude of this selection bias, consider the following two extreme cases. Suppose employment is randomly determined regardless of productivity. In this case, the standard deviation of wages is a good proxy for σ_x . Consider the other extreme case where employment is completely ordered by the current productivity—that is, a worker with the highest productivity is hired first and so forth. In this case, the (observed) wage distribution is a truncated distribution of x . Under log-normality, when the bottom 40 percent, the average nonemployment rate in the CPS for 1967-200, is truncated, the standard deviation of the underlying distribution is larger than that of the truncated distribution by a factor of 1.5 (See Maddala, 1983). For a benchmark case, we use $\sigma_x = .2425$ by multiplying the standard deviation of wages by 1.25, the mid point of the two extreme cases. In the analysis below, we also consider the lower bound, $\sigma_x = .194$ (the standard deviation the wage distribution in the PSID), and upper bound, $\sigma_x = .291(.194 \times 1.5)$.⁸

Other parameters of the model are in accord with business-cycle analysis and empirical labor-supply literature. According to the Michigan Time-Use Survey, a typical household allocates about 33 percent of its discretionary time for paid compensation (Hill, 1984; Juster and Stafford, 1991): $\bar{h} = 1/3$. Most micro estimates of intertemporal substitution elasticity of leisure fall between 0 and .5: we use $\gamma = .2$.⁹ The labor share, α , is .64, and the quarterly depreciation rate, δ , is 2.5 percent. We search for the weight parameter on leisure, B , such that the steady-state employment rate is

⁸Although we explore a range of σ_x , we argue that a fairly high persistence is desirable for two reasons. First, individual earnings ability, reflected in wages, are very persistence. Second, when we use transient idiosyncratic shocks, the model generates unreasonably high worker flows between employment and nonemployment. For example, with $\rho_x = .5$, about 16% of population move between employment and non-employment each quarter, whereas the average flow is between 6.82% and 7.01%. With $\rho_x = .95$, the worker flows from the model is comparable to those in the CPS. This is discussed in detail in the next section.

⁹With discrete choice of hours of work, the value of γ is not so important for the aggregate labor-supply elasticity since it mostly depends on the shape of the reservation-wage distribution.

60 percent, the average from the CPS. The discount factor β is chosen so that the quarterly rate of return to capital is 1 percent.¹⁰ Table 1 summarizes the parameter values.

Finally, when we investigate the response of hours with aggregate fluctuations, we introduce exogenous shifts in labor demand through aggregate technology shocks whose stochastic process is consistent with the post-war total factor productivity: λ can take two values, $\ln \lambda \in \{-.0224, .0224\}$, and its transition matrix between the two states is

$$\pi_\lambda = \begin{bmatrix} .95 & .05 \\ .05 & .95 \end{bmatrix}.$$

4 Results

4.1 Steady State

We first characterize the steady state of the model economy where there is no aggregate uncertainty and the distribution of worker $\mu(x, a)$ is invariant. A detailed description of computing the equilibrium is provided in Appendix A.4. Even in the absence of aggregate fluctuations there are constant flows of workers in and out of employment due to individual productivity shocks. Table 2 presents employment rate, gross-worker flows, and hazard rates from the model and the CPS. As described in the previous section, employment rate is 60 percent, matching the average in the CPS for 1967:II - 2000:IV. The quarterly gross-worker flows are computed using Robert Shimer's monthly hazard rates as described in Appendix A.3. On average, 6.82 percent of the population moved from employment to nonemployment each quarter; 7.01 percent of the population moved in

¹⁰The discount factor is lower than that in the representative-agent model, because market incompleteness increases savings as noted in Aiyagari (1994).

the opposite direction, from nonemployment to employment.¹¹ In our benchmark case, this flow is 6.35 percent, slightly lower than those in the data. With different degrees of idiosyncratic shocks they are 6.87 ($\sigma_x=.194$) and 6.33 ($\sigma_x=.291$).¹² Although we did not calibrate the model to match these values, the worker flows and hazard rates are fairly close to those in the CPS.

The wealth and earnings, apart from preference and non-market opportunity which are hard to measure, are probably the most important factors for participation decision. Figure 1 exhibits the Lorenz curves of wealth from the 1984 PSID and three models.¹³ Family wealth in the PSID reflects the net worth of house, other real estate, vehicles, farms and businesses owned, stocks, bonds, cash accounts, and other assets. According to Table 3, the Gini coefficient of wealth is .76 in the PSID, whereas those from the models are .57 ($\sigma_x=.194$), .60 ($\sigma_x=.2425$) .62. ($\sigma_x=.291$), respectively. Overall, wealth is more concentrated in the data.

Figure 2 shows the Lorenz curves of earnings. The data is based on family earnings (earnings of head and wife) also from the 1984 PSID. The model and the PSID exhibit similar inequality except that there are more zero earners at the bottom of the distribution in the model; 40 percent of population in the model and 20 percent in the PSID. In the PSID, a family with at least one member worked at some point during the survey year recorded a positive earnings, whereas the model is calibrated to match the average employment rate of 60 percent in the CPS. This makes

¹¹Nonemployment includes both unemployment and non-labor-force. According to Blanchard and Diamond (1990), the flows between employment and non-labor force are as big as those between employment and unemployment.

¹²For each model, we adjust the utility parameter B and the discount factor β so that the employment rate is 60 percent and quarterly rate of return to capital is 1 percent in steady state.

¹³In the PSID, information on family wealth is available for 1984, 1989, and 1996 survey years. We use 1984 survey because it is around the mid point of our sample. The degree of inequality does not vary significantly across three surveys.

Gini index of the model, between .60 and .69, somewhat higher than .54 in the PSID (See Table 3). However, when we use positive earnings only, the Gini indices are .34 ($\sigma_x = .194$), .42 ($\sigma_x=.2425$), and .48 ($\sigma_x=.291$), comparable to .42 in the PSID.

Table 4 summarizes detailed information on wealth and earnings from the SCF, PSID, and benchmark model.¹⁴ Since the wealth-earnings distributions between the PSID and SCF are similar, we discuss the comparison between the model and PSID only. For each quintile group of the wealth distribution, we calculate the wealth share, the ratio of group average to economy-wide average, and the earnings share of the group. Both in the data and model, the poorest 20 percent own almost nothing. In fact, households in the first quintile of the wealth distribution in the 1984 PSID are in debt. Those in the model own less than one percent, .37 percent, of total wealth. On the contrary, households in the 4th and 5th quintile of the PSID own 18.74 and 76.22 percent of total wealth, respectively. According to the model, they own 23.14 and 62.16 percent, respectively. The average wealth of the 4th and 5th quintile are, respectively, .93 and 3.81 times larger than that of a typical household, while these ratios are 1.15 and 3.11 according to our model. The 4th and 5th quintile group of the wealth distribution earn, respectively, 24.21 and 38.23 percent of total earnings in the PSID. The corresponding groups earn 23.70 and 33.44 percent, respectively, in the model. Overall, the wealth is more concentrated in the data. In particular, the model fails to match the highly concentrated wealth in the right tail of the distribution. About half of total wealth—43 and 53 percent in the PSID and SCF, respectively—is held by the top 5 percent of the population (not shown in the Table). In our model, only 18.4 percent of total wealth is held by them. However,

¹⁴In terms of Gini indices, the wealth and earnings distributions from the PSID are slightly less concentrated than those in the SCF. According to Díaz-Giménez, Quadrini, and Ríos-Rull, Gini indices are .78 and .63 for wealth and earnings, respectively, in the 1992 SCF.

our primary objective is not to explain the behavior of the top 1 or 5 percent of population.¹⁵ We argue that the model economy possess a reasonable heterogeneity to study the average response of hours, as the stochastic process of wages is estimated from the panel data and the cross-sectional earnings distribution is, by and large, consistent with the data.

The shape of reservation-wage distribution is crucial for the mapping from individual to aggregate labor supply. In Figure 3, we draw the reservation wage schedule for the benchmark model. At a given asset level, workers with wage (productivity) above the line choose to work. The reservation wage increases as the asset increases. To be informative, we adjust the unit of the model such that the mean asset matches the average net asset of the families in the 1984 PSID survey, \$60,524.¹⁶ Thus, the values are in 1983 dollars. Consider a worker whose asset is \$30,887, the median of the wealth distribution from the model. According to the model, he is indifferent in working at quarterly earning of \$3,634. A worker whose asset is equivalent to the average asset holding of the economy, \$60,524—which belongs to the 67th percentile of the wealth distribution in our model and to the 72th percentile in the PSID—is indifferent in working at \$4,830 per quarter.

The reservation-wage schedule and invariant distribution $\mu(x, a)$, allows us to uncover the aggregate labor-supply curve of the economy. Figure 4 exhibits the aggregate labor-supply curves for three model economies: the benchmark, the lower-bound and upper-bound heterogeneity. Labor supply is less elastic for an economy with a large heterogeneity as the reservation wage distribution is more dispersed. We calculate the elasticity at employment rates of 55, 60, and 65 percent in Table 5. For the benchmark economy, the elasticity is 1.02 at the steady-state employment rate

¹⁵For studies on the wealth distribution in a dynamic general equilibrium environment, see Huggett (1996), Krusell and Smith (1998), or Quadrini (2000) among others.

¹⁶The mean asset in our model is 13.32 unit. The reservation wages in the x-axis reflect the quarterly earnings, the reservation wage rate multiplied by \bar{h} .

of 60 percent. They are 1.32 and .80 at the employment rates of 55 percent and 65 percent, respectively. With a smaller degree of heterogeneity ($\sigma_x = .194$), the elasticities are 1.68, 1.19, and .98 at employment rates of 55, 60, and 65 percent, respectively. With a larger heterogeneity ($\sigma_x = .291$), the elasticities are smaller as the reservation wage distribution is more dispersed; they are 1.01, .80, and .75 at employment rates of 55, 60, and 65 percent, respectively. While these values are higher than most micro estimates, usually less than .5, they remain at moderate range. In particular, a very high aggregate elasticity—in fact, infinity—generated by the employment lottery, such as Hansen and Rogerson, does not survive a serious heterogeneity.

4.2 Fluctuations

While the labor supply elasticity plays an important role in many areas of macroeconomics including issues like timing of taxes and government spending, it has been debated most extensively in the business-cycle literature (Hall, 1980, Hansen, 1985; Mankiw, Rotemberg, and Summers, 1985; Christiano and Eichenbaum, 1992; Hansen and Wright, 1992). In this section, we examine the fluctuations of total hours by introducing exogenous shifts in aggregate productivity. We do not take a stand on the sources of the business cycle here, but we intentionally exclude other types of aggregate disturbances, especially those that shift the labor-supply curve. Aggregate productivity shocks serve for an instrument, shifting the labor-demand curve, to identify the slope of the aggregate labor-supply curve.

Computing the equilibrium fluctuations of an economy like this requires a considerable degree of approximation. We use the so-called “bounded rationality method” developed by Krusell and Smith, in which agents are assumed to make use of a finite set of moments of the distribution μ . The justification of this method is that by using partial information about μ , households do almost as well as by using all the information in μ when predicting future prices. In fact, Krusell

and Smith show that use of the first moment only provides a good approximation in a stochastic-growth model. The same method is used in Gomes et al. and Castañeda et al. However, our method differs from that of Gomes et al. As suggested in Ríos-Rull (1999), we explicitly solve for an labor-market equilibrium, whereas Gomes et al. assume a backyard technology to reduce the computational burden. The details of computation including the accuracy of prediction functions of aggregate prices are provided in Appendix A.4.

Since the representative-agent economies serves a reference in literature, we compare our benchmark economy to the representative-agent (with divisible labor) models with various degrees of intertemporal substitution of leisure. The representative-agent economies have the same parameter values as our benchmark economy in Table 1 except for γ .¹⁷ We consider four representative economies with γ equaling to .5, 1, 1.5, and 2. Reference to the real-business-cycle analysis, Prescott (1986) corresponds to $\gamma = 1$ and Hansen (1985) to $\gamma = \infty$.

Table 6 displays the statistics of five model economies (our benchmark economy and four representative-agent economies with divisible labor) and the U.S economy. The upshot is that the response is similar to those from the representative-agent economies with γ between 1 and 2. The volatility of output of our benchmark economy is 1.81, slightly smaller than that of the economy with $\gamma=1$ (1.93). The volatility of consumption (.49) is the same as that with $\gamma=1.5$. The volatility of investment (5.92) is somewhat smaller than that with $\gamma = 1$ (6.51) and greater than that with $\gamma = .5$ (5.69). The volatility of hours in our model is 1.24 which is close to that with $\gamma = 1.5$ (1.28). The volatility of hours relative to labor productivity is 1.51, almost similar to that with $\gamma = 1.5$ (1.52). The volatility of hours relative to output is .69, slightly higher than that with $\gamma = 2$ (.66).

A common way to characterize the intertemporal substitution of labor is using a Frisch function

¹⁷For each representative-agent economy, the parameter B is also adjusted to yield $\bar{h} = 1/3$ in the steady state.

(Frisch, 1959).¹⁸ For a representative agent economy with the same type of utility function as ours, the Frisch labor-supply function linearized around the steady state is

$$\widehat{h}_t = \psi(\widehat{w}_t - \widehat{c}_t), \quad \psi = \frac{1 - \bar{h}}{\bar{h}}\gamma, \quad (12)$$

where the circumflexes denote the variable's percentage deviation from its steady-state value. The Frisch labor-supply elasticity, ψ , also known as compensated labor-supply elasticity, represents the elasticity of hours with respect to wages holding consumption (or wealth) constant. Due to incomplete market, the aggregation theorem does not hold in our model economy. Yet it is still of interest to estimate Equation (12) pretending that the data were generated by a representative agent. Since there is only one aggregate disturbance which exogenously shifts the labor demand curve, a simple OLS (without any instrumental variable) is sufficient to reveal the slope of labor supply.¹⁹ When we estimate (12) using the aggregate time series from our benchmark economy, the estimate for ψ is 1.08 consistent with the value obtained from the steady-state reservation-wage distribution.

The response of aggregate hours to shifts in demand is moderate as the reservation-wage distribution is scattered. For example, the dispersion of individual productivity, measured by the cross-sectional standard deviation of log wages in the PSID (.606), is larger than that of aggregate productivity, measured by the time-series standard deviation of aggregate TFP (.0224) by a factor of nearly 27.²⁰

¹⁸See Heckman (1974), MaCurdy (1982) or McLaughlin (1995) for relationships among various measures of labor supply elasticities.

¹⁹Hall (1980) estimates aggregate labor-supply elasticity using instruments such as military spending, political party of the president, and oil prices. However, as Barro (1980) points out in his discussion of Hall, it is hard to find a good instrument for labor supply at aggregate level.

²⁰We abstract from the variation of hours per worker to isolate the effect of participation margin only. Allowing

5 Conclusion

The labor supply is at the heart of macroeconomic research. It is a cornerstone of the equilibrium approach that relies on intertemporal substitution of leisure. In a disequilibrium approach, in which the role of labor supply is dismissed in the short run, the labor supply is still crucial for the welfare loss of the economy departing from the equilibrium. Aggregate models often assume elastic labor supply, despite the low estimates from empirical studies based on individual data. The fact that fluctuations of hours are mainly accounted for by participation suggests that the labor supply in aggregate has little to do with intertemporal substitution rather with the distribution of reservation wages.

We construct a model economy where heterogeneous agents decide on labor-market participation and the capital market is incomplete. Heterogeneity of the workforce is designed such that evolution of wages, gross worker flows, and cross-sectional income distribution are consistent with the micro data. We find the aggregate labor supply elasticity of such an economy around 1, higher than typical micro estimates but smaller than those often assumed in aggregate models.

for an intensive margin may generate a bigger response of labor supply. However, under the small intertemporal substitution elasticity of leisure, the effect on aggregate labor supply would be small.

References

- [1] Abowd, John. M., and David Card, 1989, "On the Covariance Structure of Earnings and Hours Changes," *Econometrica*, 57, 411-445.
- [2] Aiyagari, Rao S., 1994, "Uninsured Idiosyncratic Risk and Aggregate Savings," *Quarterly Journal of Economics*, CIX, 659-683.
- [3] Altonji, Joseph, 1986, "Intertemporal Substitution in Labor Supply: Evidence from Micro Data," *Journal of Political Economy* 94, s176-s215.
- [4] Andolfatto, David and Paul Gomme, 1996, "Unemployment Insurance and Labor Market Activity in Canada," *Carnegie-Rochester Conference Series on Public Policy*, 44, 47-82.
- [5] Auerbach, A. J. and L. J. Kotlikoff, 1987, *Dynamic Fiscal Policy* Cambridge, U.K.: Cambridge University Press.
- [6] Barro, Robert, 1980, "Comments," *Carnegie-Rochester Conference Series on Public Policy*, 12, 34-40.
- [7] Bencivenga, Valerie, 1992, "An Econometric Study of Hours and Output Variation with Preference Shocks," *International Economic Review* 33, 449-71.
- [8] Benhabib, Jess, Richard Rogerson, and Randall Wright (1991) "Homework in Macroeconomics: Household Production and Aggregate Fluctuations," *Journal of Political Economy* 99, 1166-1187.
- [9] Blanchard, Olivier J., and Peter Diamond, 1990, "The Cyclical Behavior of the Gross Flows of U.S. Workers," *Brookings Papers on Economic Activity*, 85-155.

- [10] Blundell, Richard and Thomas MaCurdy, 1999, "Labor Supply: a Review of Alternative Approaches," *Handbook of Labor Economics* Vol. 3A, edited by O. Ashenfelter and D. Card. Amsterdam: North Holland.
- [11] Browning, Martin, Lars Hansen, and James Heckman, 1999, "Micro Data and General Equilibrium Models," in J. Taylor and M. Woodford eds., *Handbook of Macroeconomics*, Vol 1A, 543-633.
- [12] Castañeda, Ana, Javier Díaz-Giménez, and José-Victor Ríos-Rull, 1998, "Exploring the Income Distribution Business Cycle Dynamics," *Journal Of Monetary Economics* (42)1, 93-130.
- [13] Cho, Jang-Ok, 1995, "Ex-post Heterogeneity and the Business Cycle," *Journal of Economic Dynamics and Control* 19, 533-551.
- [14] Christiano, L. and M. Eichenbaum, 1992, "Current Real-Business-Cycle Theories and Aggregate Labor-Market Fluctuations," *American Economic Review* 82, 430-450.
- [15] Coleman, T., 1984, *Essays on Aggregate Labor Market Business Cycle Fluctuations*, Ph. D. thesis, University of Chicago.
- [16] den Haan, Wouter J., Garey Ramey, and Joel Watson, 2000, "Job Destruction and Propagation of Shocks." *American Economic Review*, 90 (3), 482-98.
- [17] Díaz-Giménez, Javier, Vincenzo Quadrini, and José-Víctor Ríos-Rull, 1997, "Dimensions of Inequality: Facts on the U.S. Distributions of Earnings, Income, and Wealth," *Quarterly Review* 21 (2), 3-21.
- [18] Frisch, Ragnar, 1959, "A Complete Scheme for Computing All Direct and Cross Demand Elasticities in a Model with Many Sectors," *Econometrica* 27, 177-196.

- [19] Ghez, G. Ricardo. and Gary Becker, 1975, *The Allocation of Time and Goods over the Life Cycle*, New York, National Bureau of Economic Research, Columbia University Press.
- [20] Gomes, Joao, Jeremy Greenwood, and Sergio Rebelo, 2001, "Equilibrium Unemployment," *Journal of Monetary Economics*, 48, 109-152.
- [21] Greenwood, Jeremy, and Zvi Hercowitz, 1991, "The Allocation of Capital and Time over the Business Cycle," *Journal of Political Economy* 99, 1188-1215.
- [22] Hall, Robert E., 1980, "Labor Supply and Aggregate Fluctuations," *Carnegie-Rochester Conference Series on Public Policy*, 12, 7-33.
- [23] Hansen, Gary D., 1985, "Indivisible Labor and the Business Cycle," *Journal of Monetary Economics*, 16, 309-327.
- [24] Hansen, Gary D. and Randall Wright, 1992, "The Labor Market in Real Business Cycle Theory," *Federal Reserve Bank of Minneapolis Quarterly Review*, Spring, 2-12.
- [25] Heckman, James, 1974, "Life Cycle Consumption and Labor Supply: An Explanation on the Relationship between Income and Consumption over the Life Cycle," *American Economic Review* 64, 188-194.
- [26] Heckman, James, 1984, "Comments on Ashenfelter and Kydland Papers," *Carnegie-Rochester Conference Series on Public Policy* 21, 209-224.
- [27] Hill, M. S. 1984, "Pattern of Time Use," in *Time, Goods and Well-Being*, F. Thomas Juster and Frank P. Stafford, eds., Ann Arbor, University of Michigan Press.
- [28] Huggett, Mark, 1996, "Wealth Distribution in Life-Cycle Economies," *Journal of Monetary Economics*, 38, 469-94.

- [29] Judd, K. L., 1987, "The Welfare Cost of Factor Taxation in a Perfect Foresight Model," *Journal of Political Economy* 95(4), 675-709.
- [30] Juster, F. T., and F. P. Stafford, 1991, "The Allocation of Time: Empirical Findings, Behavior Models, and Problems of Measurement," *Journal of Economic Literature* 29, 471-522.
- [31] Killingsworth, M. and J. Heckman, 1986, "Female Labor Supply," *Handbook of Labor Economics*, Vol. 1, edited by O. Ashenfelter and R. Layards. Amsterdam: North Holland, 103-204.
- [32] King, Robert, Charles Plosser, and Sergio Rebelo, 1988, "Production, Growth, and Business Cycles I: Basic Neo-classical Model" *Journal of Monetary Economics* 21, 195-232.
- [33] Krusell, Per, and Anthony Smith, 1998, "Income and Wealth Heterogeneity in the Macroeconomy," *Journal of Political Economy*, 106, 867-896.
- [34] Kydland, Finn, 1984, "Heterogeneity in Labor Market and Business Cycles," *Carnegie-Rochester Conference Series on Public Policy* 21, 173-208.
- [35] Kydland, Finn, and Edward Prescott, 1982, "Time to Build and Aggregate Fluctuations," *Econometrica* 50, 1345-1370.
- [36] Lucas, Robert E. Jr., and Leonard Rapping, 1969, "Real Wages, Employment, and Inflation," *Journal of Political Economy* 77, 721-754.
- [37] MaCurdy, Thomas, 1981, "An Empirical Model of Labor Supply in a Life Cycle Setting," *Journal of Political Economy*, 88, 1059-1085.
- [38] Maddala, G. S., 1983, *Limited-dependent and qualitative variables in economics*, Cambridge University Press.

- [39] McLaughlin, K. J. 1995, "Intertemporal Substitution and lamda-constant Comparative Statistics," *Journal of Monetary Economics* (35) 1, 193-213.
- [40] Mankiw, N. Gregory, Julio Rotemberg and Lawrence J. Summers, 1985, "Intertemporal Substitution in Macroeconomics," *Quarterly Journal of Economics* 100, 225-251.
- [41] Merz, Monika, 1999, "Heterogeneous Job-Matches and the Cyclical Behavior of Labor Turnover," *Journal of Monetary Economics*, 43, 91-124.
- [42] Mulligan, Casey B., 1998, "Substitution Over Time: Another Look at Life-Cycle Labor Supply," *NBER Macroeconomics Annual*, 13, 75-133.
- [43] Mulligan, Casey B., 2001, "Aggregate Implications of Indivisible Labor," *Advances in Macroeconomics* Vol. 1: No. 1, Article 4. <http://www.bepress.com/bejm/advances/vol1/iss1/art4>.
- [44] Pencavel, J., 1986, "Labor Supply of Men," *Handbook of Labor Economics*, Vol. 1, edited by O. Ashenfelter and R. Layards. Amsterdam: North Holland, 3-102.
- [45] Prescott, Edward, 1986, "Theory Ahead of Business-Cycle Measurement," *Carnegie-Rochester Conference Series of Public Policy*, 25, 11-44.
- [46] Quadrini, Vincenzo, 2000, "Enterprenuership, Savings and Social Mobility," *Review of Economic Dynamics*, 3(1), 1-40.
- [47] Ríos-Rull, José-Víctor, 1999, "Computation of Equilibria in Heterogeneous-Agents Models," in *Computational Methods for the Study of Dynamic Economies*, Ramon Marimon and Andrew Scott eds., Oxford University Press.
- [48] Rogerson, Richard, 1988, "Indivisible Labor, Lotteries and Equilibrium," *Journal of Monetary Economics*, 21, 3-16.

- [49] Shimer, Robert, 2002, "Equilibrium Unemployment Fluctuations," *mimeo*, Princeton University.
- [50] Storesletten, Kjetil, Chris Telmer, and Amir Yaron, 1999, "The Risk-Sharing Implications of Alternative Social Security Arrangement," *Carnegie-Rochester Conference Series on Public Policy*, 50, 213-260.
- [51] Tauchen, George, 1986, "Finite State Markov-Chain Approximations to Univariate and Vector Autoregressions," *Economics Letters*, 20, 177-181.

A Appendix

A.1 The PSID Data

The PSID sample period used is 1979-1992. The sample consists of heads of households and wives. We start in 1979 because the wage data for wives are available only since 1979. Wages are annual hourly earnings (annual labor incomes divided by annual hours). Nominal wages are deflated by the Consumer Price Index. The base year is 1983. Workers who worked less than 100 hours per year or whose hourly wage rate was below \$1 (in 1983 dollars) are viewed as nonemployed even though their employment status is reported as employed in the survey. We use workers who were employed in non-agricultural sectors and not self-employed. We also restrict the sample to hourly earnings less than or equal to \$500. In the PSID, the wealth data are available for 1984, 1989 and 1996 only. We use 1984 data as it is around the mid point of our sample. The distributions are similar across the three surveys. The wealth is defined as the sum of net worth of all family members resulting from the aggregation of the following components: house (main home), other real estate, vehicles, farms and businesses, stocks, bonds, cash accounts, and other assets. Family earnings is the sum of earnings of head and wife. The descriptive statistics for our PSID data are summarized in Table A.1.

A.2 Conversion between Annual and Quarterly Variances

After controlling for aggregate price of an effective unit of labor, the wage evolves according to x .

Since the wages in the PSID are annual averages:²¹

$$\ln \tilde{x}_\tau = \frac{1}{4} \left\{ \ln x_{(\tau,1)} + \ln x_{(\tau,2)} + \ln x_{(\tau,3)} + \ln x_{(\tau,4)} \right\},$$

²¹Note that $\frac{1}{4} \sum_{q=1}^4 \ln x_{(\tau,q)}$ can be interpreted as a log-linear approximation of the arithmetic average $\ln \tilde{x}_\tau = \ln[\frac{1}{4} \sum_{q=1}^4 x_{(\tau,q)}]$.

Table A.1: Summary Statistics for the PSID

Variable	Mean	S.D.	Obs.
Real Wage Rate (in 1983 \$)	10.95	9.66	45207
Log Real Wage (net of aggregate effect)	0	.606	45207
Annual Hours of work	2079.1	9.65	45207
Age	38.3	10.7	45207
Years of schooling	13.32	2.35	45207
Sex (male =1)	.584	.49	45207
Wealth	60,524.4	231103.1	5515
Earnings	17,933.1	19681.9	6918

Note: Log real wages (net of aggregate effect) are the residuals from the regression of log wages on year dummies. Family wealth and earnings are based on 1984 survey.

where \tilde{x}_τ is annual average and $x_{(\tau,q)}$ denotes the wage of the q th quarter in year τ . According to a AR(1) process for x , the quarterly persistence ρ_x is simply $.95(= .817^{.25})$ where 0.817 is the annual estimate from the PSID. Furthermore,

$$\ln \tilde{x}_\tau = \frac{1}{4} \left\{ (1 + \rho_x + \rho_x^2 + \rho_x^3) \ln \tilde{x}_{\tau-1} + (1 + \rho_x + \rho_x^2) \varepsilon_{x,(\tau,2)} + (1 + \rho_x) \varepsilon_{x,(\tau,3)} + \varepsilon_{x,(\tau,4)} \right\}.$$

The standard deviation of annual average is:

$$\sigma(\ln \tilde{x}) = \frac{1}{4} \sigma_x \left\{ \frac{(1 + \rho_x + \rho_x^2 + \rho_x^3)^2}{1 - \rho_x^2} + (1 + \rho_x + \rho_x^2)^2 + (1 + \rho_x)^2 + 1 \right\}^{1/2}.$$

When we replace $\sigma(\ln \tilde{x})$ with .606, the standard deviation of log annual hourly earnings (net of aggregate effects) from the PSID, we obtain $\hat{\sigma}_x = .194$.

A.3 The Worker-Flow Data

We compute quarterly worker flows from the seasonally adjusted monthly hazard rates in the CPS for 1967-2000, obtained from Robert Shimer as follows. There are three possible labor-market status: employment, unemployed, and non-labor-force, denoted by e , u , and n , respectively. The

flow of workers from labor-market status i to j during the quarter, denoted by f_{ij} :

$$f_{ij} = \bar{i} \times \left\{ \sum_{k,l \in \{e,u,n\}} h_{ik}^1 h_{kl}^2 h_{lj}^3 \right\} \quad i, j \in \{e, u, n\},$$

where \bar{i} denotes the number of workers in status i in the beginning of the quarter, and h_{kl}^m the monthly hazard rate from status k to l in the m -th month of the quarter. This takes into account all possible paths, direct and indirect, from i to j during a quarter, and avoids a potential double counting in a simple sum of monthly flows. Because of survey redesigns and privacy restrictions, hazard rates are not available in January 1976, January 1978, July 1985, October 1985, January 1994, and June to October 1995. For these months we interpolate with the values from nearby periods.

A.4 Computational Procedures

A.4.1 Steady-State Equilibrium

The distribution of workers $\mu(x, a)$ is invariant in the steady state; so are the factor prices. In finding the invariant μ , we use the algorithm suggested by Ríos-Rull (1999). We search for the discount factor β that clears the capital market given the quarterly rate of return of 1 percent. Computing the steady-state equilibrium amounts to finding the value functions, the associated decision rules, and time-invariant measure of workers. Details are as follows:

1. First, we choose the grid points for a and x . The number of grids are denoted by N_a and N_x , respectively: $N_a = 936$, $N_x = 17$. The asset holding a_i is in the range of $[0, 260]$, where the average asset holding is 13.3. The grid points of assets are not equally spaced. We assign more points on the lower asset range to better approximate savings decisions of workers with lower assets. For productivity, x_j , we construct grid vectors of N_x equally spaced points in which $\ln x_j$'s lie on the range of $\pm 3\sigma_x / \sqrt{1 - \rho_x^2}$.

2. Given β , we solve the individual optimization problem in (1), (2), and (3) at each grid point of the individual states. In this step, we also obtain the optimal decision rules for asset holding $a'(a_i, x_j)$ and labor supply $h(a_i, x_j)$. This step involves the following procedure:

(a) Initialize value functions $V_0^E(a_i, x_j)$, $V_0^N(a_i, x_j)$, and $V_0(a_i, x_j)$.

(b) Update value functions by evaluating the discretized versions of (1), (2), and (3):

$$V_1^E(a_i, x_j) = \max \left\{ u(w\bar{h}x_j + (1+r)a_i - a', 1 - \bar{h}) \right. \\ \left. + \beta \sum_{j'=1}^{N_x} \max [V_0^E(a', x_{j'}), V_0^N(a', x_{j'})] \pi_x(x_{j'}|x_j) \right\}, \quad (\text{A.4.1})$$

$$V_1^N(a_i, x_j) = \max \left\{ u((1+r)a_i - a', 1) \right. \\ \left. + \beta \sum_{j'=1}^{N_x} \max [V_0^E(a', x_{j'}), V_0^N(a', x_{j'})] \pi_x(x_{j'}|x_j) \right\}, \quad (\text{A.4.2})$$

and

$$V_1(a_i, x_j) = \max \left\{ V_1^E(a_i, x_j), V_1^N(a_i, x_j) \right\}, \quad (\text{A.4.3})$$

where $\pi_x(x_{j'}|x_j)$ is the transition probabilities of x , which is approximated using Tauchen's (1986) algorithm.

(c) If V_1 and V_0 are close enough for all grid points, then we found the value functions.

Otherwise, set $V_0^E = V_1^E$ and $V_0^N = V_1^N$, and go back to step 2-(b).

3. Using $a'(a_i, x_j)$, $\pi_x(x_{j'}|x_j)$ obtained from step 2, we obtain time-invariant measures $\mu^*(a_i, x_j)$ as follows:

(a) Initialize the measure $\mu_0(a_i, x_j)$.

(b) Update the measure by evaluating the discretized version of (9):

$$\mu_1(a_{i'}, x_{j'}) = \sum_{i=1}^{N_a} \sum_{j=1}^{N_x} \mathbf{1}_{a_{i'}=a'(a_i, x_j)} \mu_0(a_i, x_j) \pi_x(x_{j'}|x_j) \quad (\text{A.4.4})$$

- (c) If μ_1 and μ_0 are close enough for all grid points, then we found the time-invariant measure. Otherwise, replace μ_0 with μ_1 , and go back to step 3(b).
4. We calculate the real interest rate as a function of β , i.e., $r(\beta) = \alpha(K(\beta)/L(\beta))^{1-\alpha} - \delta$, where $K(\beta) = \sum_{i=1}^{N_a} \sum_{j=1}^{N_x} a_i \mu^*(a_i, x_j)$ and $L(\beta) = \sum_{i=1}^{N_a} \sum_{j=1}^{N_x} h(a_i, x_j) \mu^*(a_i, x_j)$. Other aggregate variables of interest are calculated using μ^* and decision rules. If $r(\beta)$ is close enough to the assumed value of the real interest rate, we found the steady state. Otherwise, choose a new β and go back to step 2.

A.4.2 Equilibrium with Aggregate Fluctuations

Approximating the equilibrium in the presence of aggregate fluctuations requires us (i) to include the measure of workers and the aggregate productivity shock in the list of state variables, and (ii) to keep track of the evolution of the measure μ over time. Since μ is an infinite dimensional object, it is almost impossible to implement these tasks as they are. We follow the procedure suggested by Krusell and Smith (1998); agents are assumed to make use of its first moment only in predicting the law of motion for μ . Therefore, computing the equilibrium with aggregate fluctuations amounts to finding the value functions, decision rules, and law of motion for the aggregate capital within the class of a parametric form of the log linear in K and λ . While the same method is used in Gomes et al. in their analysis on equilibrium unemployment rates, our method differ from theirs as we explicitly solves for the labor market equilibrium as suggested in Ríos-Rull (1999). Technically speaking, this requires an additional do-loop step which finds a fixed point for the labor market equilibrium. Details are as follows:

1. In addition to the grids for individual state variables specified above, we choose 7 grid points for the aggregate capital K in the range of $[\.93K^*, 1.07K^*]$, where K^* denotes the steady-state aggregate capital. In our numerous simulations, the capital stock has never reached the

upper or lower bound. The stochastic process for the aggregate productivity shock $\pi_\lambda(\lambda'|\lambda)$ is described in the text.

2. Let the parametric law of motion for the aggregate capital take a log linear in K and λ :

$$\ln K_{t+1} = \kappa_0^0 + \kappa_1^0 \ln K_t + \kappa_2^0 \ln \lambda_t. \quad (\text{A.4.5})$$

In order for individuals to make their decisions on savings and labor supply they have to know (or predict) the interest rate and wage rate for an effective unit of labor. While the factor prices depend on aggregate capital and labor aggregate labor input is not known to individuals at the moment when they make decisions. Thus, individuals need to predict the factor prices. These predictions on factor prices, in turn, must be consistent with the outcome of individual actions—the factor market clearing in (7) and (8). We also assume that individuals predict the market wage and the interest rate using a log-linear function of K and λ :

$$\ln w_t = b_0^0 + b_1^0 \ln K_t + b_2^0 \ln \lambda_t. \quad (\text{A.4.6})$$

and

$$\ln r_t = d_0^0 + d_1^0 \ln K_t + d_2^0 \ln \lambda_t \quad (\text{A.4.7})$$

3. We choose the initial values for the coefficients κ^0 's, b^0 's and d^0 's. Good initial values may come from a representative-agent model.
4. Given the law of motion for the aggregate capital and the prediction functions for factor prices, we solve the individual optimization problem in (1), (2), and (3). This step is analogous to step 2 in the steady-state computation:

- (a) We have to solve for the value functions and the decision rules over a bigger state space.

Now the state variables are (a, x, K, λ) .

- (b) Computation of the conditional expectation involves the evaluation of the value functions not on the grid points along the K dimension since K' predicted by (A.4.5) need not be a grid point. We polynomial-interpolate the value functions along K dimension when necessary.
5. Using $a'(a_i, x_j, K_l, \lambda_m)$, $h(a_i, x_j, K_l, \lambda_m)$, $\pi_x(x_{j'}|x_j)$, $\pi_\lambda(\lambda_{m'}|\lambda_m)$, and the assumed law of motion for the aggregate capital, we generate a set of artificial time series data $\{K_t, w_t, r_t\}$ of the length of 5,000 periods. Each period, $\{K_t, w_t, r_t\}$ is calculated by aggregating labor supply and assets of 50,000 individuals.
6. We obtain new values for coefficients κ^1 's, b^1 's and d^1 's by the OLS from the simulated data. If κ^1 's, b^1 's and d^1 's are close enough to κ^0 's, b^0 's, and d^0 's, respectively, we found the law of motion. Otherwise, update coefficients by setting $\kappa^0 = \kappa^1$, $b^0 = b^1$'s and $d^0 = d^1$'s, and go back to step 4.

The estimated law of motion for capital and prediction functions and their accuracy, measured by R^2 for the prediction equations are as follows.

- the law of motion for aggregate capital in equation (A.4.5):

$$\ln K_{t+1} = .1297 + .9500 \ln K_t + .1099 \ln \lambda_t, \quad R^2 = .9998$$

- the market wage rate in equation (A.4.6):

$$\ln w_t = -.3110 + .4539 \ln K_t + .7988 \ln \lambda_t, \quad R^2 = .9894$$

- the interest rate in equation (A.4.7):

$$\ln r_t = .182 - .0282 \ln K_t + .0476 \ln \lambda_t, \quad R^2 = .9687$$

The law of motion for aggregate capital provides the highest accuracy. The wage function is more accurate than the interest rate function. Overall, predictions functions are fairly precise as R^2 's are close to 1.

Table 1: Parameters of the Benchmark Economy

Parameter	Description
$\alpha = .64$	Labor share in production function
$\beta = .97724$	Discount factor
$\gamma = .2$	Intertemporal substitution elasticity of leisure
$B = 1.019$	Utility parameter
$\bar{h} = 1/3$	Steady-state hour
$\rho_x = .95$	Persistence of idiosyncratic productivity shock
$\sigma_x = .2425$	Standard deviation of innovation to idiosyncratic productivity

Table 2: Labor Market Steady States

Variable	CPS	Model		
		$\sigma_x = .194$	$\sigma_x = .2425$	$\sigma_x = .291$
Employment rate	60.02	60.05	60.02	60.33
Flow out of employment	6.82	6.87	6.35	6.33
Flow into employment	7.01	6.87	6.35	6.33
Hazard rate out of nonemployment	17.52	17.18	15.87	15.83
Hazard rate out of employment	11.36	11.45	10.58	10.55

Note: All variables are in percentage. Data are quarterly averages for 1967:II-2000:IV as described in Appendix A.3.

Table 3: Gini Indices for Wealth and Earnings

Variable	PSID	<u>Model</u>		
		$\sigma_x = .194$	$\sigma_x = .2425$	$\sigma_x = .291$
Wealth	.76	.57	.60	.62
Earnings	.53	.60	.65	.69
Earnings (non-zeros)	.42	.34	.42	.48

Note: Data statistics are family wealth and labor income from the PSID 1984.

Table 4: Characteristics of Wealth Distribution

	<u>Quintile</u>					Total
	1st	2nd	3rd	4th	5th	
<u>SCF</u>						
Share of wealth	-.39	1.74	5.72	13.43	79.49	100
Group average/population average	-.02	.09	.29	.67	3.97	1
Share of earnings	7.05	14.50	16.48	20.76	41.21	100
<u>PSID</u>						
Share of wealth	-.52	.50	5.06	18.74	76.22	100
Group average/population average	-.02	.03	.25	.93	3.81	1
Share of earnings	7.51	11.31	18.72	24.21	38.23	100
<u>Benchmark Model</u>						
Share of wealth	.37	3.80	10.54	23.14	62.16	100
Group average/population average	.02	.19	.53	1.15	3.11	1
Share of earnings	9.43	14.72	18.70	23.70	33.44	100

Note: The statistics for the SCF are from Diaz-Gimenez, Quadrini, and Rios-Rull (1997).

Table 5: Aggregate Labor Supply Elasticities

Model	Elasticity at Employment Rate		
	$E = 55\%$	$E = 60\%$	$E = 65\%$
$\sigma_x = .194$	1.68	1.19	.98
$\sigma_x = .2425$	1.32	1.02	.80
$\sigma_x = .291$	1.01	.80	.75

Note: Elasticities are evaluated at employment rates of 55%, 60% and 65%, respectively.

Table 6: Comparison with Divisible Labor Economies

	Benchmark	Divisible Labor				U.S. Data 1955:I-1998:4
		$\gamma = .5$	$\gamma = 1$	$\gamma = 1.5$	$\gamma = 2$	
$\sigma(Y)$	1.81	1.70	1.93	2.08	2.17	2.10
$\sigma(C)$.49	.42	.47	.49	.51	.84
$\sigma(I)$	5.92	5.69	6.51	7.04	7.40	4.77
$\sigma(N)$	1.24	.69	1.05	1.28	1.44	1.58
$\sigma(N)/\sigma(Y)$.69	.41	.54	.62	.66	.75
$\sigma(N)/\sigma(Y/N)$	1.51	.67	1.15	1.52	1.78	1.35

Note: Data are from the DRI data base. Y = GDP-government spending; C = consumption of non durables and services; I = non-residential fixed investment; N = total employed hours in non-agricultural private sector from the establishment survey. All variables are divided by civilian noninstitutional population over 16. All variables are detrended by the H-P filter.

Figure 1: Lorenz Curves for Wealth

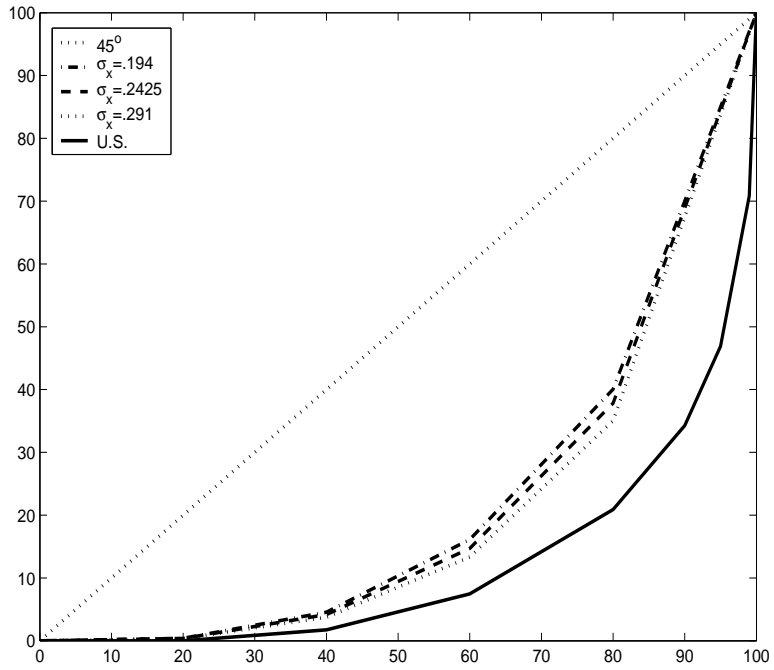


Figure 2: Lorenz Curves for Earnings

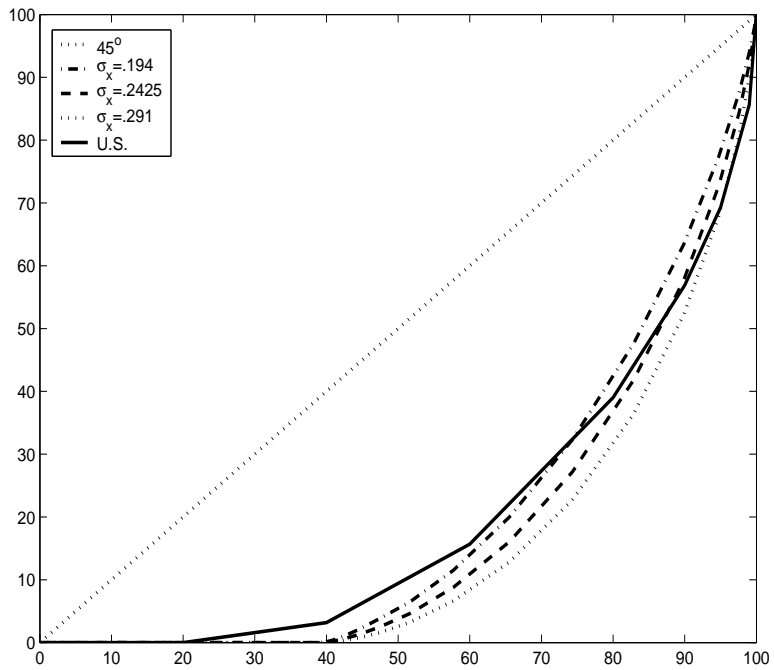


Figure 3: Working Decisions

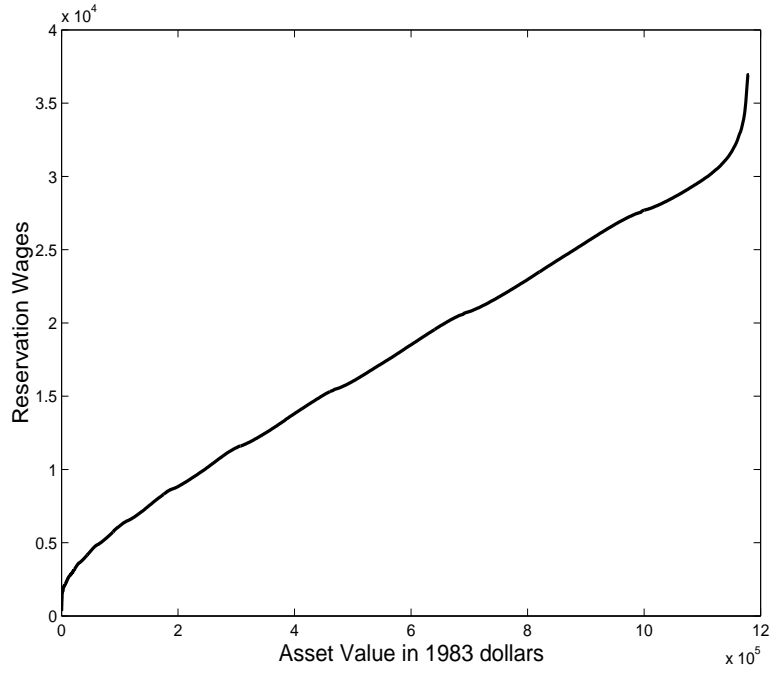


Figure 4: Aggregate Labor Supply Curves

