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"Social Assets"

by

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Abstract

We present a model incorporating both social and economic components, and analyze their interaction. We argue that such analysis is necessary for an understanding of the social arrangements that are consistent with underlying economic fundamentals. We introduce the notion of a *social asset*, an attribute that has value only because of the social arrangements governing society. We consider a generational matching model in which an attribute has value in some equilibrium social arrangements (matching patterns), but not in others. We then show that productive attributes (such as education) can have their value increased above their inherent productive value by some social arrangements, leading to the notion of the *social value of an asset*.

Keywords: Social assets, social capital, social arrangements, nonmarket interactions, social norms.

JEL Classification: D20, D31, D5, J41, Z13.

1. Introduction

Economists have long understood that economic decisions are made in a social context, and are affected by that context. Adam Smith wrote in 1759:

> "The rich man glories in his riches, because he feels that they naturally draw upon him the attention of the world, and that mankind are disposed to go along with him in all those agreeable emotions with which the advantages of his situation so readily inspire him. At the thought of this, his heart seems to swell and dilate itself within him, and he is fonder of his wealth, upon this account, than for all the other advantages it procures him."[1]

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[1] Smith (1976 (originally published 1759), Part I, Section III, Chap II).

At the beginning of the last century, Veblen (1899/1934) made concern for the opinion of others the driving force of his analysis of contemporary consumer behavior. Duesenberry (1949), and in the latter half of the century, Easterlin (1974), made a strong case for the proposition that individuals care not just about their own wealth or consumption, but also about how they fare relative to others they know. Further evidence of the importance of the interrelationship of the social and the economic environments was provided by Granovetter (1974), who provided evidence that a majority of people found jobs through information obtained from friends and acquaintances.

The relationship between the social and economic environment is complicated; not only does the social environment affect economic decisions, but also the economic environment affects social decisions. Moreover, the distinction between economic and social decisions is not always a clear one: joining a club whose members include business associates, for example, has aspects of both. In this paper, we present a model that incorporates both the social and economic components of the environment, and analyze the interaction between the two elements.

There is a body of work that begins with the presumption that individuals' economic decisions are made in the presence of other agents, and are affected by them. One strand of this research analyzes how consideration of the social environment in which agents are embedded can shed light on behavior that is anomalous in the absence of such considerations. Examples include Akerlof's work illustrating how social customs can support Pareto inefficient equilibria (Akerlof (1976), (1980)), Corneo and Gruner's argument that class concerns can explain why the poor do not expropriate through taxation the rich (Corneo and Gruner (2000)), the argument of Fershtman and Weiss that social forces can alleviate some externalities (Fershtman and Weiss (1998)), and the possibility that social stigma can influence the choice of welfare or work (Lindbeck, Nyberg, and Weibull (1999)).

These papers focus on the economic decisions of agents and how those decisions are affected by an exogenous social environment. In particular, the nature of the dependence of agent utilities on the social environment is largely taken as exogenous.[2] It is only plausible to take the social environment as given when the relevant aspects of the environment are obvious, and when they are unlikely to change within the time frame of the analysis. For example, the stability of a class system might be undermined by the economic migration of those relegated to the lower classes. Treating the social environment as exogenous constrains the analysis to situations with no significant feedback from the economic behavior that might alter the social environment. Our aim, as outlined above, is to analyze the interaction of the social and economic aspects of society; to do this, one should treat the social environment as endogenous, to be determined—along

[2]For example, Corneo and Gruner (2000) take as given an individual preference for matching with higher class mates.

with economic behavior—in equilibrium.

A second strand of research focuses on the social environment, rather than on economic decisions. This literature endogenizes the social environment, interpreted as the formation of networks.[3] Modelling the social environment as a network does endogenize social arrangements: the pattern of relationships is determined in equilibrium, since people choose connections strategically. However, the network approach typically abstracts from the form of the benefits stemming from being connected to a particular group. Normally, there is no distinction drawn between social and economic benefits. Our focus is on understanding the robustness of social institutions to economic considerations: What economic environments are consistent with particular social arrangements? For a model to provide insight about robustness, it cannot be overly abstract; it should have at least primitive forms of recognizable real institutions.

We introduce a model that combines both the social and the traditional economic aspects of society in a manner that allows for an analysis of the interaction between the social environment and agents' decisions. As argued above, such a model is necessary if we are to understand what social arrangements are consistent with the underlying economic fundamentals of the economy. Toward this end, we introduce the notion of *social assets*. A social asset is an attribute of an agent that has value only because of the social arrangements of society. In other words, the attribute does not have *direct* productive value, yet its possession leads to higher utility. For example, it may be that in a particular society agents with lighter skin or a particular accent enjoy higher consumption than those with darker skin or a less desirable accent, even if those attributes have no productive value.

Just as non-productive attributes may have a value because of social arrangements, productive attributes (such as education) may have their value enhanced because of social arrangements. We refer to this additional value the *social value* of the attribute. Our notions of social assets and social value are not to be confused with currently popular notion of social capital (see, e.g., Putnam (1994)); we return to the distinction in Section 7.

We consider a generational model in which men and women match and have children. Income is random, men and women match, and then jointly consume their income. People get utility from their own consumption and their descendants' consumption. An individual's sole decision is the identity of his or her partner.

Since consumption is joint, a wealthier partner leads to higher consumption. There will then be an equilibrium in which each person's wealth determines completely his or her match. In addition to equilibria of this kind, there may be additional equilibria in which nonproductive attributes affect matching. In particular, in equilibrium, attributes

[3] See, e.g., Dutta and Jackson (forthcominga), and particularly the introductory chapter by Dutta and Jackson (forthcomingb).

that have no fundamental value can have instrumental value. Individuals care about their children's consumption, which depends on the children's (random) income. We assume that it is not possible to insure against this risk; this missing market allows the possibility that social arrangements may arise that ameliorate the resulting inefficiency.

Suppose that there is a heritable attribute that is independent of income, height for example, and suppose further that the attribute does not enter people's utility functions. Suppose that, nevertheless, in this society tall people are considered desirable mates, that is, that people are willing to match with a tall person with slightly less income than a short person. In such a society, people will naturally prefer their children to be tall since, all else equal, they consume more. But if they prefer tall children, and height is a heritable attribute, they will naturally prefer tall mates. In other words, a preference for tall mates may be self-fulfilling. Notice that this has nothing to do with any *intrinsic* desirability for tall people; within this same society it could equally well have been that shortness was a desirable attribute. *Any* heritable attribute might serve as a social asset in this way.[4]

If the social arrangements make height a desirable attribute, we see that the degree of assortativeness of matching on wealth is decreased relative to the case that matching is on wealth alone. When there is no such desirable attribute, wealthy men match only with wealthy women and vice versa. When the social arrangements value an attribute such as height, some wealthy short people match with tall less wealthy people. The consequence of social arrangements that value such assets is that the variance of consumption in society is lower. When people are risk averse, the social arrangements that value attributes that are fundamentally extraneous can be welfare superior to arrangements that ignore such extraneous attributes.

The discussion above focuses on the case in which the attribute is nonproductive, that is, the attribute is completely independent of anything that enters directly into peoples' utility functions. An analogous situation can arise for productive attributes. It may be that height, still the attribute in question, has a productive component; for example, a tall person may be able to reach the top shelves in a storage closet without getting a ladder, thereby being able to do some tasks more quickly than a short person. In such a scenario, height leads to a higher expected income. All people would naturally prefer tall partners in such a world, even if height did not enter directly into utility functions, since people would realize that the children they have with tall partners are more likely to have high income.

Even when the attribute has a productive component, it still may be possible to identify a social component of its value. Since it is productive, people will prefer partners with the attribute to those without, all other things being equal. But if the productive

[4] This story has some similarities with theories of sexual selection that explain, for example, peacock tails (see Ridley (1993, Chapter 5)). We discuss the relationship in Section 7.

advantage is small, there may be two stable matchings corresponding to those described above for the unproductive attribute case. One will have high income people without the attribute matching with like partners, and a second will have high income people without the attribute matching with low income people with the attribute. The situation is as before: it may be that the social arrangements in the society are such that if others in society value the attribute above and beyond its productive value, then it is rational of each individual to do so as well.

We emphasize that we view this model as a parable rather than a serious model of marriage and investment in human capital. The model is designed to demonstrate how the social and economic components of society interact and the role of social assets in such a model. There are many models that accomplish these goals, and we chose the particularly simple one described above for expository ease.[5] The examples of social assets in the description and motivation above were characteristics or traits that were physically embodied in the individual such as accent or height. Some social assets may have such a physical manifestation, but it isn't necessary. For example, our conception of an individual's social assets includes the set of people that one knows personally.

In the next section, we formalize the model described above. In Section 3 we consider the case of genetically transmitted characteristics. We provide conditions under which there are equilibria with nontrivial social assets, and we consider how such social assets can arise. We emphasized in the discussion above that a central concern in this paper was the analysis of the interaction between peoples' decisions and the social environment. In Section 4 we analyze how the social arrangements within a society – that is, what assets have value – can endogenously change over time. We used primarily attributes that were genetically heritable, such as height, as illustrations above. Attributes that are not genetically transmitted, but are rather passed from parents to children socially, such as accents, manners, etc., are arguably even more important than genetically transmitted characteristics. Section 5 treats the case of socially transmitted characteristics. Lastly, we drop the restriction that individuals can only affect their future offsprings' chances of acquiring the attribute through the choice of a mate. We extend our analysis in Section 6 to allow individuals access to a market to influence the chance their children will have attributes that are desirable. We have in mind attributes such as education. We conclude with a discussion section.

2. Model

There is an infinite sequence of two-period lived agents, each of which consists of a continuum of men and women. There is a single non-storable consumption good. In each period, old men and women match and consume their combined wealth (so that the

[5] An alternative, non-matching, model is discussed in Section 3.

good is a public good within couples). In addition, each couple has two offspring. The common consumption utility function for old agents is concave and denoted $U : \Re \to \Re$. Individuals care about their descendants' welfare: the utility to any matched couple is their utility from consumption plus the discounted average utility of their children, with common discount rate β. This means, of course, that their utility depends on the consumption of all future generations.

While agents neither take actions nor receive utility in their first period of life, they may acquire an attribute. We assume (except in Section 6) that agents can only acquire this attribute through their parents: both offspring will have the attribute for sure if both parents possess the attribute, they will surely not have it if neither have it, and they will have it with probability ρ if one parent had the attribute.[6] For simplicity, we assume that either both offspring have the attribute, or neither does. Individuals with the attribute are y agents, while those without the attribute are n agents. This attribute does not enter into agents' utility functions. We distinguish between $\rho = \frac{1}{2}$ and $\rho \neq \frac{1}{2}$. For $\rho = \frac{1}{2}$, the transmission of the attribute may be thought of as genetic and the attribute a characteristic such as height. For $\rho \neq \frac{1}{2}$, the transmission of the attribute may be thought of as having a substantial cultural (or environmental) component and the attribute more like a skill (playing the piano, making other people feel comfortable, etc.). Education is an especially interesting attribute; we consider this attribute in Section 6, and allow parents to expend resources to increase the likelihood of their children having this attribute. At present, however, we assume the transmission is exogenous.

Each agent receives an endowment of the consumption good (income) at the beginning of their second period of life. This income is either high (H) or low (L). The attribute is possibly productive: the probability that a y agent has high income (H) is $\frac{1}{2} + k$, and the probability an n agent has high income is $\frac{1}{2} - k$, $k \geq 0$. The productivity of the attribute is captured by k; if $k = 0$, agents are equally likely to have high or low income, and the attribute is nonproductive.

We assume an agent's income is independent of the parents' incomes. Possible consumption levels for matched pairs are $2H$, $2L$ and $H + L$. We normalize the utility function so that $U(2L) = 0$ and $U(2H) = 1$, and denote the utility of the third possible consumption level, $H + L$, by u; $u \in [1/2, 1)$ since U is concave. An agent's income level and the presence/absence of the attribute together constitute that agent's *characteristic*.

The only decision an agent makes in this economy (except in Section 6) concerns matching. A matching is *stable* if no unmatched pair of agents can increase each of their utilities by matching, taking into account the consequences for their descendants (Roth

[6] Our model of attribute transmission is identical to the vertical transmission model of Cavalli-Sforza and Feldman (1981); they, however, do not consider the incentives agents have to match with different partners.

and Sotomayer (1990)).[7] Any matching induces a matching on agent characteristics in the obvious manner. A matching is *strictly stable* if, for each unmatched pair of agent characteristics, agents with these characteristics would strictly decrease their utilities by matching (taking into account the consequences for their descendants).

We restrict attention to symmetric allocations. An *allocation*, then, in a period is a pair (μ, m), where $\mu \equiv (\mu_y, \mu_n)$ is the distribution of attributes in the economy (μ_y is the fraction of men, and of women, with the attribute, and $\mu_n = 1 - \mu_y$), and m is the matching. Given μ, the distribution of characteristics is determined by the productivity of the attribute, so that, for example, the fraction of the population with high income and the attribute is $(\frac{1}{2} + k)\mu_y$. An *equilibrium* is a specification of $(\mu^t, m^t)_{t=0}^{\infty}$, where μ^t, the distribution of attributes in period t, is induced from the distribution of characteristics and matching in period $t-1$, and where the matching in each period is stable. Note that this notion of equilibrium is anonymous. Parents can only affect the utility of their children through the characteristics they receive; in this sense, the equilibrium is Markov.

When the attribute is genetically transmitted ($\rho = \frac{1}{2}$), in each period half the men and half the women have the attribute, independent of the matching. As a consequence, the distribution over characteristics is independent of the matching. In this case, in order to analyze equilibrium it is enough to describe the stable matchings.

3. Genetic Attributes

3.1. Stable Assortative Matching

In this section and the next, we analyze the case of genetic attributes, i.e., $\rho = \frac{1}{2}$. If the attribute is unproductive, and the distribution over offspring characteristics is independent of parents' characteristics, any matching positively assortative on income will clearly be stable. If the attribute is productive, a matching that is positive assortative on income but not on attribute cannot be stable (since an Hy agent can do better by matching with another Hy agent than with an Hn agent). The *assortative* matching has high income men match with high income women and men with the attribute match with women with the attribute:

Men		Women
Hy	$\longleftrightarrow$	Hy
Hn	$\longleftrightarrow$	Hn
Ly	$\longleftrightarrow$	Ly
Ln	$\longleftrightarrow$	Ln

[7] Since there are no side payments, a matching will only be destabilized if *both* agents in an unmatched pair strictly prefer to match.

It will be convenient to work with average discounted value functions. If an agent has discount factor β, the *average discounted value* of the stream of utilities $\{v_t\}_{t=1}^{\infty}$ is $\sum_{t=1}^{\infty} \beta^{t-1}(1-\beta)v_t$. By rescaling flow utility by the factor $(1-\beta)$, a constant sequence of flow utility v has average discounted value of v.

Denote by V_y^A the average discounted value function for agents who have the attribute when matching is assortative, evaluated before their income has been realized, and by V_n^A the value function of those who do not have the attribute. Note that while matching occurs after income is realized, since matching is assortative on attribute as well as income, if an agent has the attribute, then s/he will match with a partner who also has the attribute, and so their offspring has the attribute with probability 1. The uncertainty over current income translates into uncertainty only over current utility. Then,

$$\begin{aligned} V_y^A &= (\frac{1}{2}+k)[(1-\beta)\,U\,(2H)+\beta V_y^A]+(\frac{1}{2}-k)\left[(1-\beta)\,U\,(2L)+\beta V_y^A\right] \\ &= (\frac{1}{2}+k)\,(1-\beta)+\beta V_y^A, \end{aligned}$$

(recall our normalization that $U\,(2H)=1$ and $U\,(2L)=0$). Solving for V_y^A gives

$$V_y^A = \frac{1}{2}+k.$$

Similarly, we have

$$V_n^A = \frac{1}{2}-k.$$

The value of having the attribute in this equilibrium is $V_y^A - V_n^A = 2k$, which is the flow value of the productivity of the attribute.

Consider now an Hn agent. If he or she matches according to the prescribed assortative matching, the resulting utility will be $1-\beta+\beta V_n^A$, since such a matching yields for sure children without the attribute. If this agent matches instead with an Ly agent, he or she gives up some current consumption utility, but has the chance of producing offspring with the attribute. The resulting utility is $(1-\beta)\,u+\frac{\beta}{2}(V_y^A+V_n^A)$, and consequently he or she would prefer to match with an Ly agent if

$$1-\beta+\beta V_n^A < (1-\beta)\,u+\frac{\beta}{2}(V_y^A+V_n^A),$$

i.e.,

$$(1-u)\,(1-\beta) < \frac{\beta}{2}(V_y^A-V_n^A)=\beta k.$$

The incentive constraint that an Ly prefer to match with Hn rather than another Ly is

$$u\,(1-\beta)+\frac{\beta}{2}(V_y^A+V_n^A) > \beta V_y^A,$$

i.e.,

$$u\,(1-\beta) > \frac{\beta}{2}(V_y^A - V_n^A) = \beta k.$$

Hence, the matching that is perfectly assortative on income and attributes is *not* stable if and only if

$$1 - u < \frac{\beta k}{1-\beta} < u. \tag{1}$$

As we indicated at the beginning of this section, it is clear that assortative matching is stable when the attribute is unproductive ($k = 0$). But the matching is also stable when the attribute is very productive ($k > u\,(1-\beta)\,/\beta$). In this case, there is a sense in which matching is driven primarily by the attribute, and only secondarily by income.

Note also that if $u < \frac{1}{2}$ (i.e., agents are risk-loving), then (1) must always be violated. In order for the assortative matching to fail to be stable, not only must a high income agent without an attribute be willing to give some current utility for the possibility of offspring with the attribute, but a low income agent with the attribute must be willing to sacrifice current utility even though the offspring may, as a result, not have the attribute.

It is clear that agents with the characteristic Hy never have an incentive to deviate, while Ln agents can never induce a matching from agents with other characteristics. Thus, we have the following proposition.

Proposition 1 *Matching assortatively on income and attribute is stable if and only if either*

$$\beta k \leq (1-u)\,(1-\beta)$$

or

$$u\,(1-\beta) \leq \beta k.$$

3.2. Stable Mixed Matching

The second interesting matching is the *mixed matching*:

Men		Women
Hy	$\longleftrightarrow$	Hy
Hn	$\longleftrightarrow$	Ly
Ly	$\longleftrightarrow$	Hn
Ln	$\longleftrightarrow$	Ln

As in the assortative matching, Hy's match with Hy's and Ln's match with Ln's, but unlike that matching, Hn's match with Ly's. The question of stability of this mixed matching immediately arises when the attribute is unproductive. Why would an Hn

give up current consumption by matching with an Ly, who contributes less to current consumption than an Hn? Clearly, if the discount factor β is 0, that is, if parents care only about their personal consumption, mixed matching is not stable: an Hn man and an Hn woman would have higher utility by matching together than they would have if they followed the prescribed matching, whether or not other agents follow the mixed matching prescriptions.

However, if parents care about their children, there is a benefit to an Hn who matches with an Ly when all other agents are following the prescribed mixed matching. An Hn's offspring will have the attribute with probability 1/2 when matched with an Ly, but with probability 0 if he or she matches with another Hn. While the possession of this attribute doesn't affect the child's income, it *does* affect who they will match with. An Ly child will match with a high income agent (Hn), while an Ln child matches with an Ln. Consequently, if other agents are following the prescriptions of mixed matching, the attribute has value in affecting offsprings' matching prospects (and, *a fortiori*, consumption prospects) even when the attribute is nonproductive. The fact that the attribute has value because of its affect on matching doesn't ensure that the mixed matching is stable of course. Stability will be determined by the trade-off that an Hn faces between the lower current consumption that matching with an Ly entails and the expected benefit it will confer on his or her offspring.

The value functions for agents with and without the attribute are denoted V_y^M and V_n^M (the superscript M denotes mixed matching). An agent with the attribute has income H with probability $(\frac{1}{2}+k)$; under mixed matching this agent then matches with an identical agent, jointly consumes $2H$, and has offspring who inherit the attribute with probability 1. An agent with the attribute has income L with probability $(\frac{1}{2}-k)$ and, under mixed matching, matches with an Hn agent. Jointly they consume $H+L$, and their offspring inherit the attribute with probability $\frac{1}{2}$. Thus, V_y^M is given by

$$V_y^M = (\frac{1}{2}+k)[1-\beta+\beta V_y^M] + (\frac{1}{2}-k)[u\,(1-\beta) + \frac{\beta}{2}(V_y^M + V_n^M)]. \tag{2}$$

Similarly, V_n^M is given by

$$V_n^M = (\frac{1}{2}-k)[u\,(1-\beta) + \frac{\beta}{2}(V_y^M + V_n^M)] + (\frac{1}{2}+k)\beta V_n^M, \tag{3}$$

and hence,

$$V_y^M - V_n^M = (\frac{1}{2}+k)[1-\beta+\beta(V_y^M - V_n^M)]$$

and so

$$V_y^M - V_n^M = \frac{(1+2k)\,(1-\beta)}{2-\beta(1+2k)}. \tag{4}$$

For the mixed matching to be stable, an Hn agent must prefer to match with an Ly agent rather than match with another Hn agent, and an Ly agent must prefer to match with an Hn agent rather than with another Ly agent. The incentive constraint for an Hn agent is

$$u\,(1-\beta)+\frac{\beta}{2}(V_y^M+V_n^M)\geq 1-\beta+\beta V_n^M.$$

Similarly, the incentive constraint for an Ly agent is

$$u\,(1-\beta)+\frac{\beta}{2}(V_y^M+V_n^M)\geq \beta V_y^M.$$

Combining these inequalities, a necessary and sufficient condition for the mixed matching to be an equilibrium is

$$(1-u)\,(1-\beta)\leq\frac{\beta}{2}(V_y^M-V_n^M)\leq u\,(1-\beta)\,.$$

Rearranging the inequality and using (4), we have the following proposition.

Proposition 2 *The mixed matching is stable if and only if*

$$1-u\leq\frac{\beta\,(1+2k)}{2\,(2-\beta(1+2k))}\leq u. \tag{5}$$

3.3. Unproductive attributes

The polar case in which the attribute has no productive value, $k=0$, is of particular interest. The corresponding condition for mixed matching to be an equilibrium when the attribute is not productive is

$$1-u\leq\frac{\beta}{2\,(2-\beta)}\leq u.$$

Since $u\geq 1/2$, the second inequality is satisfied for all $\beta\in[0,1]$. Hence, a sufficient condition for mixed matching to be stable when $k=0$ is the first inequality, which is equivalent to

$$u\geq\frac{4-3\beta}{2(2-\beta)}.$$

Figure 1 illustrates the combinations of u and β for which mixed matching is stable.

Recall that the assortative matching is necessarily stable in the case that the attribute is unproductive. Thus, in the unproductive attribute case, there are multiple stable matchings when this inequality is satisfied.

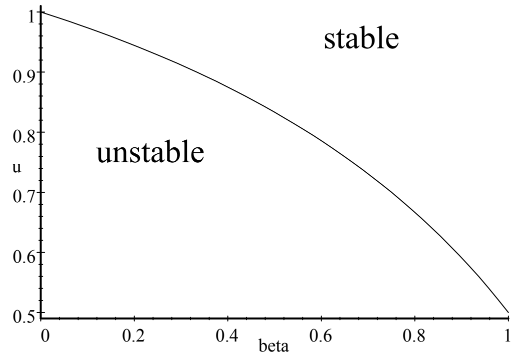

Figure 1: For any β, if u is above the curved line, mixed matching will be stable.

These two matchings have different economic consequences. For the case of genetic attributes, the number of agents with the attribute is independent of the matching, and hence, the distribution of income in the society is independent of the social arrangements: half of the society has income H and half has L. However, since consumption within couples is joint, different matchings may lead to different distributions of *consumption*. That the distribution of consumption differs for the two stable matchings described above is clear. Under assortative matching, high income agents always match with high income, and hence, half of the people consume $2H$, while the other half consume $2L$. On the other hand, in the mixed matching, half of the low income people (those with the attribute) match with high income agents (the high income people without the attribute). Hence, only $\frac{1}{4}$ of the people consume $2H$, $\frac{1}{4}$ consume $2L$, and the remaining half consume $H + L$.

The remainder of this section argues that, as long as agents care sufficiently about their offspring, the mixed matching Pareto dominates the assortative matching. Moreover, it turns out that the condition on β and u is the same as that required to make the mixed matching stable.

The interpretation is straightforward. Higher u corresponds to a more concave utility function over consumption. As in most models similar to ours, the concavity of the utility function is doing double duty, both representing agents' attitude toward risk and their rate of intertemporal substitution. Higher risk aversion translates into higher value for the insurance that is provided by the mixed matching, and simultaneously, lower cost of forgone consumption to obtain the insurance. For a fixed β, if agents' utility functions are sufficiently concave, regardless of income, agents obtain higher utility under mixed matching than under assortative matching.

The roles of β and u are interchangeable, of course. That is, for any $u \in (1/2, 1)$, there is a $\hat{\beta}$ such that for all $\beta \in (\hat{\beta}, 1)$ such that for the problem with u and β, mixed is preferred by all agents to assortative matching. In other words, for any level of risk aversion, if agents care sufficiently about succeeding generations, mixed matching is Pareto superior.

Proposition 3 *Suppose the attribute is unproductive ($k = 0$). The mixed matching (weakly) Pareto dominates the assortative matching if and only if*

$$\frac{4 - 3\beta}{2(2 - \beta)} \leq u, \tag{6}$$

i.e., if and only if the mixed matching is stable.

Proof. Since $k = 0$, the attribute has no value under assortative matching, and so $V_y^A = V_n^A = \frac{1}{2}$. Hence, the utility of a high income agent with the attribute, V_{Hy}^A, equals that of a high income agent without the attribute, V_{Hn}^A:

$$V_{Hy}^A = V_{Hn}^A = (1 - \beta) + \beta \frac{1}{2} = \frac{2 - \beta}{2},$$

and the utility of a low income agent with the attribute, V_{Ly}^A, equals that of a low income agent without the attribute, V_{Ln}^A:

$$V_{Ly}^A = V_{Ln}^A = \frac{\beta}{2}.$$

Under mixed matching, an Hy matches with a similar agent, consumes $2H$ and has two children, each of whom has attribute y. Hence, the utility of such an agent is

$$V_{Hy}^M = 1 - \beta + \beta V_y^M.$$

Similarly, an Ln agent will match with an agent of the same type, consume $2L$ and have two children without the attribute. The utility is

$$V_{Ln}^M = \beta V_n^M.$$

Finally, Hn agents and Ly agents will have the same utility, since they are matched with each other and jointly consume $H + L$ and have children that are equally likely to have attribute y or n. Denoting their utility by V_m^M, we have

$$V_m^M = u\,(1 - \beta) + \frac{\beta}{2}(V_y^M + V_n^M).$$

Solving (2) and (3) when $k = 0$ yields

$$V_y^M = \frac{4 - 3\beta + 2u\,(2-\beta)}{4(2-\beta)} = \frac{(4-3\beta)}{4(2-\beta)} + \frac{u}{2}$$

and

$$V_n^M = \frac{\beta + 2u\,(2-\beta)}{4\,(2-\beta)} = \frac{\beta}{4\,(2-\beta)} + \frac{u}{2}.$$

Thus,

$$\begin{aligned} V_{Hy}^M &= 1 - \beta + \beta \left(\frac{(4-3\beta)}{4(2-\beta)} + \frac{u}{2} \right) \\ &= \frac{8 - 8\beta + \beta^2}{4\,(2-\beta)} + \frac{\beta u}{2}, \end{aligned}$$

$$V_{Ln}^M = \frac{\beta^2}{4\,(2-\beta)} + \frac{\beta u}{2},$$

and

$$\begin{aligned} V_m^M &= u\,(1-\beta) + \frac{\beta}{2}\left(\frac{1}{2} + u\right) \\ &= u\left(1 - \frac{\beta}{2}\right) + \frac{\beta}{4}. \end{aligned}$$

Now,

$$\begin{aligned} V_m^M \geq V_H^A &\Longleftrightarrow \frac{(1-\frac{1}{2}\beta)u + \frac{\beta}{4}}{1-\beta} \geq \frac{2-\beta}{2(1-\beta)} \\ &\Longleftrightarrow u \geq \frac{4-3\beta}{2(2-\beta)}. \end{aligned}$$

So, if $u > \frac{4-3\beta}{2(2-\beta)}$, Hn agents and Ly agents have higher utility under mixed matching than under assortative matching. Since $V_{Hy}^M > V_m^M$, Hy agents also have higher utility under mixed matching.

Finally, Ln agents are also better off if

$$\begin{aligned} V_{Ln}^M \geq V_L^A &\Longleftrightarrow \\ \frac{\beta^2}{4\,(2-\beta)} + \frac{\beta u}{2} \geq \frac{\beta}{2} &\Longleftrightarrow \\ u \geq \frac{4-3\beta}{2(2-\beta)}&. \end{aligned}$$

To summarize, for any $\beta \in (0,1)$, there is $\hat{u} \in (1/2, 1)$ such that for $u \in (\hat{u}, 1)$, mixed matching Pareto dominates assortative matching. ■

3.4. The Emergence of Mixed Matching

While the mixed matching on an unproductive attribute is welfare superior when it is stable, it is natural to ask how or why a society might end up with matching that depends on nonproductive characteristics. Here we briefly outline one possibility: the attribute was at one time productive, and its productivity initially *requires* mixed matching for stability. At a later time, the attribute is no longer productive, and consequently, matching that ignores the attribute may become stable. Nonetheless, the existing mixed matching for which the attribute matters remains stable.

There is a simple intuition why assortative matching that ignores the attribute may not be stable with productive attributes ($k > 0$). When the attribute is sufficiently productive, an Hn agent may find that the increase in expected income for his offspring more than compensates for the decrease in current consumption *independent of any change in matching prospects that might also ensue.*

We now argue that there are configurations of k, β, and u for which (5), (6), and (1) all hold. Recall that (6) is equivalent to $1 - u \leq \beta / [2(2-\beta)]$. Fix $k < \min\{(4-3\beta)/(6\beta), (1-\beta)/\beta\}$. Since $k < (4-3\beta)/(6\beta)$,

$$\frac{\beta}{2}\left[\frac{1+2k}{2-\beta(1+2k)}\right] < 1.$$

Moreover,

$$0 < \frac{\beta}{2(2-\beta)} < \frac{\beta}{2}\left[\frac{1+2k}{2-\beta(1+2k)}\right].$$

Thus, by choosing u large enough all three inequalities will be satisfied.

Consider a world in which the attribute is productive (with parameter k), and that all three inequalities are satisfied. Suppose that in every period there is a small probability p that the attribute becomes unproductive. If p is sufficiently small, the matching must be mixed in every equilibrium before the attribute becomes unproductive. Moreover, after the attribute becomes unproductive, the mixed matching remains stable. One can interpret this as an explanation as to how nonproductive attributes can be valued. They once had productive value, and the environment was such that matching *must* take this attribute into account. The eventual disappearance of the productiveness of the attribute does not upset the stability of the mixed matching.

4. Endogenously Changing Social Arrangements

One of our primary interests is in how social arrangements within a society can change over time. It is often suggested that within some societies, values *do* change through time, evidenced by the common lament that "people just don't care about the things that used to be important." The analysis above showed that in the unproductive attribute case, both assortative and mixed matching can be stable for some values of the parameters u and β. One could simply assert that the change in values is captured by a switch from one equilibrium matching to another, but there are objections to this approach.

The first objection is that we would like the change in norms in a society to be endogenous, that is, we would like the change to arise from the underlying characteristics of the society. Explanations that simply assume that a society switches from one equilibrium to another rely on explanations that are outside the model. Since the explanations don't come from the model itself, they provide no insight into *why* the change took place.

A second objection is less conceptual but more serious. For a matching to be stable, there is an incentive constraint that no unmatched pair of agents would prefer to match rather than follow the suggested matching. The calculations in the determination of the incentive constraints assume the matching is permanent. If agents understand that the matching may change in the future, this should be incorporated into the incentive constraints if we wish to maintain our assumption that agents are fully rational.

More concretely, in the mixed matching, Hn agents prefer to match Ly's. An Hn is trading off the present period utility cost of not matching with another Hn (and getting higher consumption) with the benefit of matching with an Ly (and getting a positive probability of offspring with the desirable attribute, which will assure those offspring higher consumption). The higher expected consumption of offspring that compensates for the immediate lower consumption is less valuable if there is a chance that future generations will not "honor" the claim to higher consumption expected for agents with attribute y.

Our approach is to construct an equilibrium in which the matching specification is stochastic, with the change in matching arising from changes in the environment. The basic idea is that, as we showed above, the possibility that a mixed matching is stable depends on the relationship between u and β. The discount factor β is fixed, but we introduce income growth into the basic model. A high income agent who matches with a low income agent has lower utility from consumption than if he or she had matched with another high income agent. The utility difference, however, will generally depend on the two income levels. If there is rising income, the "risk premium" an agent will pay to ameliorate the riskiness in future generations' consumption may decrease. If this risk premium *does* decrease, it may destabilize mixed matching. We illustrate next how this may occur in equilibrium. For simplicity, the discussion in this section is confined to the case of unproductive attributes.

We maintain the two-point income distribution analyzed above, but allow the possibility of a one-time income increase that occurs at a random time.[8] (Any change in the income process occurs at the end of a period after matching and before the next period's income is realized.) As above, there are initially two income levels, $L < H$. In each period, with probability p, the income levels increase from (L, H) to $(\alpha L, \alpha H)$, $\alpha > 1$. Once the higher income level is reached, it remains at that level permanently.

This particular income growth process preserves relative incomes; only the level changes. If the utility function U exhibits constant relative risk aversion, the incentive constraint for stability of the mixed matching will be satisfied at the initial income level if and only if it is satisfied at the higher level. That is, the introduction of stochastic income growth doesn't affect the stability of the mixed matching.

Suppose, however, that the utility function U exhibits decreasing relative risk aversion. In this case the risk premium associated with the random consumption of future generations will be smaller after the income increase than before, and the incentive constraint requiring a type Hn to prefer matching with a type Ly to matching with another Hn may not be satisfied after the income increase. If this is the case, only assortative matching will be stable after the income increase.

Can it be the case that prior to the income increase the mixed matching can be stable? As mentioned, we maintain rational expectations in the sense that prior to the income increase, the mixed matching must be stable when the agents *know* that there is a chance that the norm will break down in any period, and hence, that it *must* break down eventually. Recall that a matching is *strictly stable* if, for each unmatched pair of agent characteristics, agents with these characteristics would strictly decrease their utilities by matching (taking into account the consequences for their descendants).

Proposition 4 *Suppose the attribute is unproductive, the mixed matching is strictly stable for income levels (L, H), and is not stable for $(\alpha L, \alpha H)$. Suppose income levels begin at (L, H), and in each period there is a probability p of a permanent increase to $(\alpha L, \alpha H)$. There exists $\bar{p} > 0$ such that for $p \in (0, \bar{p})$, it is an equilibrium for matching to be mixed while income is at the level (L, H), and for it to be assortative once income increases to $(\alpha L, \alpha H)$.*

Proof. We denote by $V_i^M(p)$, $i \in \{Hy, Hn, Ly, Ln\}$, the value functions for the agents of each type under mixed matching when incomes start at the level (L, H) and in any period there is a probability p that incomes increase to $(\alpha H, \alpha L)$, and matching changes to assortative matching at that time (with complementary probability, incomes do not increase and matching remains mixed). Denote by $V_i^A(p)$, $i \in \{H, L\}$, the value functions under assortative matching with the higher incomes, $(\alpha H, \alpha L)$. The equations

[8] We discuss the possibility of perpetually increasing incomes below.

for the initial value functions $V_t^M(p)$ are

$$V_{Hy}^M(p) = (1-\beta) + \beta(1-p)\left[\frac{1}{2}V_{Hy}^M(p) + \frac{1}{2}V_{Ly}^M(p)\right] + \beta p\left[\frac{1}{2}V_H^A(p) + \frac{1}{2}V_L^A(p)\right],$$

$$V_{Hn}^M(p) = V_{Ly}^M(p) \equiv V_m^M(p) = u(1-\beta) + \beta(1-p)\left[\frac{1}{4}V_{Hy}^M(p) + \frac{1}{2}V_m^M(p) + \frac{1}{4}V_{Ln}^M(p)\right]$$
$$+\beta p\left[\frac{1}{2}V_H^A(p)) + \frac{1}{2}V_L^A(p)\right],$$

and

$$V_{Ln}^M(p) = \beta(1-p)\left[\frac{1}{2}V_{Hn}^M(p) + \frac{1}{2}V_{Ln}^M(p)\right] + \beta p\left[\frac{1}{2}V_H^A(p) + \frac{1}{2}V_L^A(p)\right].$$

Since the assortative matching value functions are bounded, as $p \to 0$, each value function $V_t^M(p)$ converges to the value function $V_t^M(0)$, that is, the value functions calculated in the previous section. Hence, since the incentive constraint for the case in which income is unchanging is satisfied with strict inequality, for sufficiently low probability p, it will be satisfied for the case in which incomes increase with probability p. ■

To summarize: if at the initial income levels, the mixed matching is stable with a strict inequality in the incentive constraint, and if at the increased income level the incentive constraint is not satisfied, there will be an equilibrium in which matching is based on the mixed matching until incomes increase, at which point the matching must change to the income only ranking.

We can easily generalize this observation to perpetually (stochastically) increasing incomes. In each period there are two income levels. In the first period, the incomes are $H_1 = H$ and $L_1 = L$. In period t, the incomes are $(\alpha_t H, \alpha_t L)$, $\alpha_t \geq 1$. As before, the relative wealth levels stay the same but the incomes grow over time. The income factors α_t are stochastic with $\alpha_t = \alpha_{t-1}$ with probability $1-p \in (0,1)$, and $\alpha_t = \alpha_{t-1} + \gamma$ with probability p.[9] Suppose that the utility function U exhibits decreasing relative risk aversion. Then the value of the insurance to an Hn agent from a match with a high attribute partner is decreasing, and the opportunity cost in terms of forgone current consumption to obtain that insurance is increasing. If at some point, it is not sufficient to offset the immediate utility loss from consumption that results from a match with an Ly agent, mixed matching is not stable, and matching will be assortative. However,

[9] We assume that the increases in income, γ, do not depend on the period or the current income level for expositional ease only. We could allow the size of the increases to depend on these without changing any of the analysis as long as the increases are bounded above. Similarly, the probability that incomes may rise at any time may depend on the period and the current income level; the constraint will be on the maximum probability of an income change in any period.

if the initial income levels are such that the incentive constraint for stability of mixed matching is satisfied with strict inequality, then for sufficiently small p, there will be an equilibrium characterized by mixed matching which will be stable as long as that incentive constraint is satisfied, and assortative matching after that. Furthermore, if $R(x) = -x\frac{U''(x)}{U'(x)} \longrightarrow 0$ as $x \longrightarrow 0$, then the incentive constraint for mixed matching will eventually be violated with probability one.

This result can be interpreted as the sure eventual demise of social arrangements that depend on non-payoff relevant criteria when there is asymptotically vanishing relative risk aversion. It is interesting to note that at the point at which the matching regime changes, there may be only a small change in the income distribution, but a large change in the distribution of consumption. Under assortative matching, all high income agents match with other high income people, while in mixed matching, half the high income agents match with low income agents. The collapse of mixed matching is accompanied by a large increase in the variance of consumption.

5. Culturally Transmitted Attributes

We have thus far focussed on what we called genetic transmission of attributes: when only one parent has the attribute, ρ, the probability that the offspring also have the attribute, is $\frac{1}{2}$. We now extend our analysis to situations where this probability is different from $\frac{1}{2}$, which we refer to as cultural transmission. We maintain the assumption that if neither parent has the attribute, then the offspring will also not have the attribute for sure, and if both parents have the attribute, the offspring also have the attribute for sure.

There are two cases with very different properties, corresponding to whether ρ is smaller or larger than $\frac{1}{2}$. If $\rho < \frac{1}{2}$, then both parents having the attribute results in a more than proportionate increase in the probability that offspring will have the attribute. Consequently, we say that we have *economies of scale* in the transmission of the attribute. Conversely, if $\rho > \frac{1}{2}$, we have *diseconomies of scale*.

An example of an attribute that displays economies of scale is the ability to converse intelligently (or, more generally, being urbane). It certainly seems plausible that if both parents have this attribute, it will certainly be passed on to the children because of the social interactions that occur within the family; further, if only parent has the attribute, then it may be significantly harder for children to acquire the attribute. On the other hand, if the attribute is the ability to play the piano, it may make little difference in the probability that a child acquires the attribute whether one or two parents possess the attribute.

When matching is assortative, either both parents have the attribute, or both parents do not have the attribute. Consequently, the value of ρ is irrelevant, and the fraction of the population that has the attribute is stationary. On the other hand, when matching is mixed, there are many matched pairs in which only one parent has the attribute. Consequently, the fraction of the population with the attribute will vary over time. This in turn has consequences for the stability of the mixed matching. Since there may be different numbers of agents of characteristic Hn and Ly, we assume in the mixed matching that all agents from the smaller group match with agents from the other group, and that the remaining agents from the larger group are matched with agents with the same characteristics.

5.1. Economies of Scale ($\rho < \frac{1}{2}$)

Since $\rho < \frac{1}{2}$, the fraction of the population that has the attribute depends on the matching, and may vary over time. We are primarily interested in the stability of the mixed matching. Since couples in which only one parent has the attribute have a less than even chance of producing offspring with the attribute, it is intuitive that the fraction of the population with the attribute will decline. Moreover, in comparison with genetic transmission, the expected value of the insurance received by matching with an Ly agent is reduced, since offspring have a smaller probability of acquiring the attribute. For simplicity, we continue to assume that the attribute is unproductive.

Proposition 5 *Suppose $k = 0$ and $4-3\beta < 2-\beta$. There exists $\underline{\rho} \in (0, \frac{1}{2})$ and $\bar{\mu} \in (\frac{1}{2}, 1)$ such that if $\rho \in (\underline{\rho}, \frac{1}{2})$ and $\mu_y^0 \in (0, \bar{\mu})$, mixed matching is stable. Moreover, the fraction of the population with the attribute converges to 0 and the utility of the representative agent in this matching converges to that of the representative agent in the assortative matching.*

The analysis is a straightforward variant of that above and we simply outline the argument here. Observe first that agents with the attribute in period t, μ_y^t, are equally likely to have high income, H, or low income, L. In terms of keeping track of μ^t, we can think of each agent being replaced by a single child in the next period. Each agent with the attribute and income H will have a child with the attribute, since the parent is matched with an agent who also has the attribute. Hence, each Hy agent (the proportion of which is $\frac{1}{2}\mu_y^t$) has a child with the attribute. If $\mu_y^t > \frac{1}{2}$, there are more Ly agents than Hn agents; each Hn agent then matches with an Ly agent and has a child with the attribute with probability ρ. The remaining Ly agents match with each other and have children with the attribute. Agents with characteristic Ln match with agents of the same type and have children without the attribute in the next period. Hence the proportion of children with high attribute in period $t+1$, μ_y^{t+1}, is $\frac{1}{2}\mu_y^t + \frac{1}{2}(\mu_y^t - \mu_n^t) + \frac{1}{2}\mu_n^t\rho = \mu_y^t - \frac{1}{2}\mu_n^t(1-\rho) < \mu_y^t$. Thus eventually $\mu_y^t \leq \frac{1}{2}$. So suppose $\mu_y^t \leq \frac{1}{2}$. Each Ly agent then matches with an Hn agent and has a child with the attribute with probability ρ. Hence, the proportion of the population that is Ly, $\frac{1}{2}\mu_y^t$, contributes $\frac{1}{2}\mu_y^t\rho$ children with the attribute in period $t+1$. When $\mu_y^t < \frac{1}{2}$, there are more Hn agents than Ly agents, and so not all Hn agents match with an Ly. Only $\frac{1}{2}\mu_y^t$ will do so, and only these have the possibility of having a child with the attribute. Agents with characteristic Ln match with agents of the same type and contribute no children possessing attribute y in the next period. Hence the proportion of children with the attribute in period $t+1$, μ_y^{t+1}, is $\mu_y^t(\frac{1}{2} + \frac{1}{2}\rho) + \frac{1}{2}\mu_y^t\rho = \mu_y^t(\frac{1}{2} + \rho)$. Hence, $\mu_y^{t+\tau} = \mu_y^t(\frac{1}{2} + \rho)^\tau$, which goes to 0 as $\tau \to \infty$.

Thus, if the probability that for couples in which one parent has high attribute is less than $\frac{1}{2}$, the proportion of agents in the population with the attribute will go to 0 in the mixed attribute matching. Can such a mixed attribute matching be stable however? Clearly if the probability of transmission, ρ, is too small, the mixed attribute matching cannot be stable. An Hn can match with another Hn and get higher utility from consumption, and the continuation payoffs will be nearly the same as if he matches with an Ly agent.

However, similarly to the stochastically increasing income case analyzed in the previous section, when ρ is sufficiently close to $\frac{1}{2}$, the value functions will be nearly the same as in the case with $\rho = \frac{1}{2}$. Thus, since the incentive constraint when $\rho = \frac{1}{2}$ is satisfied with strict inequality (by assumption, (6) holds strictly), the analogous incentive constraint will be satisfied when ρ is close to $\frac{1}{2}$ for the same reason, $\bar{\mu}$ can't be too large.

Note that the proportion of agents with the attribute does not go to 0 under all matchings. In particular, under the assortative matching (which guarantees that attributes match with attributes), all children born to parents with the attribute will have the attribute, and the proportion is unchanged over time.

5.2. Diseconomies of Scale ($\rho > \frac{1}{2}$)

We consider next the case in which $\rho > \frac{1}{2}$, that is, the expected number of children with high attribute coming from mixed matches (matches with exactly one parent with high attribute) is greater than 1. In contrast to the case where $\rho < \frac{1}{2}$, mixed matching is no longer stable.

Proposition 6 *Suppose $k = 0$. If the attribute has diseconomies of scale, then mixed matching is not stable.*

Since the analysis is, like that for Proposition 5, a straightforward variant of that above, we again only outline the argument. We first examine the dynamics of the proportions of agents who possess the attribute, assuming mixed matching. Proceeding as we did in the previous case, we look at a particular period, t, and treat each adult as having a single child, and compute the probability that that child will have the attribute as a function of the proportion of parents that have the attribute.

Since $\rho > \frac{1}{2}$, it is intuitive that, under the mixed matching, the fraction of the population with the attribute is increasing. We make the argument for $\mu_y^t \geq \frac{1}{2}$, obvious modifications show that the same is true if $\mu_y^t < \frac{1}{2}$. As usual, the mixed matching prescribes that an Hy agent match with an agent of the same type, hence this agent will have a child possessing the attribute with probability 1. This gives rise, then, to $\frac{1}{2}\mu_y^t$ children with the attribute in period $t+1$. Similarly, Ln agents match with the same type and contribute no children with the attribute the next period. Since $\mu_y^t \geq \frac{1}{2}$, there are more agents with the attribute than without. Hence, there are fewer Hn agents than Ly agents. Consequently, all Hn agents will be in mixed attribute matches, and this group will contribute $\frac{1}{2}\mu_n^t\rho$ children with the attribute in the next period. The Ly agents who are in mixed matches with an Hn agent also have probability ρ of having a child with the attribute, hence the contribution of this type to the pool of agents with the attribute next period is $\frac{1}{2}\mu_n^t\rho$. Finally, the Ly agents who do not match with Hn agents (there are $\frac{1}{2}(\mu_y^t - \mu_n^t)$ of these) will instead match with other Ly agents and have a child with the attribute with probability 1. Adding these, the proportion of agents with the attribute in period $t+1$ when the proportion in period t is μ_y^t is

$$\begin{aligned}\mu_y^{t+1} &= \frac{1}{2}\mu_y^t + \frac{1}{2}\mu_n^t\rho + \frac{1}{2}\mu_n^t\rho + \frac{1}{2}(\mu_y^t - \mu_n^t) \\ &= \mu_y^t + (\rho - \frac{1}{2})\mu_n^t.\end{aligned}$$

Since $\rho > \frac{1}{2}$, μ_y^t monotonically increases, converging to 1.

Since μ_y^t converges to 1, asymptotically all agents possess the attribute. But when μ_y^t is close to 1, the matching with respect to *income* is almost the same as in assortative matching. That is, nearly all Ly's match with agents of the same type, in particular with low income agents. Hence the difference between the expected utility for a child with the attribute and without is arbitrarily small. Thus, for sufficiently large t, there is little insurance value in having the attribute, and so an Hn agent will prefer to match with another Hn agent to matching with an Ly agent. That is, mixed matching breaks down and Ly's will thereafter match with other Ly's. But then the prescribed matching will "unravel," that is, in the period prior to the period in which the incentive constraint is violated, no Hn will match with an Ly, and hence the same in the period prior to this, and so on. In other words, mixed matching cannot be stable.

It is worth mentioning that it is inconsistent with equilibrium to initially have mixed matching, and then at some time t (when μ_y^t has become sufficiently close to 1) to switch to assortative matching. The large population means that the dynamics on the fraction of the population with the attribute are deterministic, and so the last possible trigger date is common knowledge. But then, as we have just argued, mixed matching will break down in the previous period, and so on.

It is easy to see why mixed matching cannot be stable, since forward looking agents will see that it cannot forever be stable, hence it will unravel. Here, as in most unravelling arguments, the unravelling is highly sensitive to particular features of the model. If it were not common knowledge that the matching would unravel, it may be stable for a long time and eventually break down in a manner analogous to the situation with bubbles in finite horizon rational expectations models. Similarly, we could have mixed matching stable if there was a small stochastic component similar to that introduced in the case of endogenously changing social arrangements above.

6. Endogenous Attribute Choice

We return to the productive attribute case, but allow parents to purchase the attribute for their children if they did not inherit it. We first modify the process by which a new generation inherits the attribute from the previous generation. We assume that if both parents have the attribute, both children inherit the attribute with probability $2p < 1$ and if one parent has the attribute, both children inherit the attribute with probability p. This specification ensures that the proportion of people in any generation who inherit the attribute is $2p$ times the proportion of people in the previous generation that had the attribute.[10] If no couples purchase the attribute for their children, the attribute asymptotically disappears from the population.

In addition to the possibility of inheriting the attribute, we allow parents to make investments that yield positive probability that their children will acquire the attribute in the event that the child does not inherit the attribute. Specifically, we assume that if offspring do not inherit the attribute, they may still obtain the attribute with probability q if parents pay a cost $c(q)$, where $c'(q) \geq 0$ and $c''(q) > 0$, and $c'(0) = 0$. Parents must fund the investments from current income. The choice of expenditure on attribute is made *after* the realization of whether the child has inherited the attribute from his or her parents. As before, agents with the attribute have probability $1/2 + k$ of having high income and those without the attribute have probability $1/2 - k$.

We are interested in the proportion of parents who purchase education in any period in a stationary equilibrium, and in how that proportion is affected by matching. We provide conditions under which the value of the attribute is higher under mixed matching than under assortative matching. When the value of the attribute is higher, all parents whose children did not inherit the attribute will invest more to increase the probability that their offspring will acquire the attribute, and consequently, the proportion with the attribute will be higher. Since the attribute is productive this implies that aggregate income will be higher under mixed matching than under assortative matching.

[10] If $p = \frac{1}{2}$ and a proportion of parents bounded away from 0 purchase the attribute, asymptotically, all agents will have the attribute. Consequently, for reasons analogous to those outlined in the diseconomies-of-scale case, mixed matching is not stable.

We first describe the steady state under assortative matching. In this case, Hn agents do not match with Ly agents; hence, the value of having the attribute is independent of the proportion of people in the population with the attribute, and so of expenditures on the attribute by other agents. As before, we denote the continuation values of children with the attribute and without the attribute (prior to the realization of their income) by V_y^A and V_n^A. The benefit to parents who purchase probability q of acquiring the attribute for their children is then $q(V_y^A - V_n^A)$, and marginal benefit is $(V_y^A - V_n^A)$. For $k > 0$, the marginal benefit is strictly positive. The marginal cost to parents is the marginal utility of forgone consumption. Hence, for a couple with total income $2H$, the marginal cost is $U'(2H - c(q))c'(q)$. This expression is clearly increasing in q, and the couple's optimal q, denoted q_{HH}^A, solves

$$U'(2H - c(q))c'(q) = V_y^A - V_n^A.$$

Similarly, the optimal purchases for families with one high and one low income (if there were any) satisfies

$$U'(H + L - c(q_{HL}^A))c'(q_{HL}^A) = V_y^A - V_n^A,$$

and the choice for families with two low incomes solve

$$U'(2L - c(q_{LL}^A))c'(q_{LL}^A) = V_y^A - V_n^A,$$

where q_{xy}^A denotes the optimal q for a couple whose respective incomes are x and y. It is straightforward to show that, if $k > 0$, $1 > q_{HH}^A > q_{HL}^A > q_{LL}^A > 0$.

These values imply the steady state fraction of the population have the attribute, μ_y^A:

$$\mu_y^A = \frac{q_{HH}^A(1-2k) + q_{LL}^A(1+2k)}{2\left\{(1-2p)\left(1-q_{LL}^A\right) + q_{HH}^A - (1+2k)\left(1-p\right)\left(q_{HH}^A - q_{LL}^A\right)\right\}}.$$

Just as in the productive exogenous attribute case, assortative matching may or may not be stable.

Consider now mixed matching. Under mixed matching, the values of having the attribute are no longer independent of the proportion of people in the population who have the attribute. With mixed matching, an Hn agent matches with an Ly agent *if possible*. The "if possible" modifier is necessary since there may not be equal numbers of the two types. When there are more of one than another, some of those on the long side of the market will not be able to participate in a mixed match, and instead will be matched with others of the same type as themselves.

We are interested in understanding when each of the different matchings is an equilibrium. For mixed matching to be an equilibrium, it must be in the interests of the Hn and Ly agents to match with each other. The incentive constraints for each type of agent to prefer this match to a match with a partner of the same type will depend on the proportion of agents with the attribute, and the expenditures the different matches will make on the attribute should their children *not* inherit the attribute. We first state the following proposition that there are steady state proportions and expenditures; the proof of the proposition is left to the appendix.

Proposition 7 *Let μ_y be the fraction of the population with the attribute, and q_ℓ^M, $\ell \in \{HH, HL, LL\}$, be the probability that income pair ℓ has purchased,* assuming *mixed matching. There exist steady state values of μ_y and $q^M \equiv \left(q_{HH}^M, q_{HL}^M, q_{LL}^M\right)$.*

As for assortative matching, we still need to address the stability of mixed matching. We next present two examples in which mixed matching is stable with productive attributes. In the first example, assortative matching is also stable and gives a lower per capita income than mixed matching, while it is not stable in the second.

Example 1 The cost function is $c(q) = \alpha q^2$, $k = 0$ and $p < 1/2$. Since $k = 0$, under assortative matching, no agents will purchase a positive probability of their children acquiring the attribute. We denote by $\mu_y^A(0)$ the proportion of agents who have the attribute under assortative matching when k equals 0; since $p < 0$, $\mu_y^A(0) = 0$.

If mixed matching is stable, the attribute has value and, because $c''(0) = 0$, all couples whose children have not inherited the attribute will purchase positive probabilities of their children acquiring the attribute. The proportion of agents who have the attribute under mixed matching when k equals 0, $\mu_y^M(0)$, is strictly positive.

Mixed matching may not, however, be stable. For example, if it is very inexpensive for couples to purchase the attribute, an Hn agent will prefer to match with another Hn agent and use the additional family income to purchase high probability of acquiring the attribute to getting probability p of children having the attribute by matching with an Ly agent. However, for sufficiently high α, matching with an Ly agent will be more cost effective for an Hn agent to secure a given probability of offspring with the attribute than relying on the "after market".

To summarize, if α is sufficiently high and $k = 0$, both positive assortative matching and mixed matching will be stable and $\mu_y^M > \mu_y^A = 0$. Furthermore, it is easy to see that μ_y^M is larger for larger p since larger p implies a higher expected number of descendants who will have the attribute.

The value functions V_y^M and V_n^M are continuous in k at $k = 0$. Consequently, if the incentive constraints for mixed matching to be stable are satisfied with strict inequality when $k = 0$, they will still be satisfied for k small enough. Thus, if mixed matching is stable with strict inequalities on matching for $k = 0$, mixed matching will be stable for positive, but small, k. Since V_y^M, V_n^M, V_y^A and V_n^A are continuous in k at $k = 0$, $\mu_y^A(\cdot)$ and $\mu_y^M(\cdot)$ are continuous; hence, for k small, $\mu_y^A(\cdot) < \mu_y^M(\cdot)$. In words, more agents have the productive attribute under mixed matching than under assortative matching. In the case of nonproductive attributes, the matching affected the distribution of income in the society, but not the aggregate income. For the productive example described above, matching affects both the total societal income and its distribution.

It is easy to see why mixed matching leads to greater number of agents with the attribute by looking at the problem facing a couple whose child has not inherited the attribute, $\max_q U(l-c(q))+q(V_y-V_n)$ (l is the pairs combined income). The first order conditions for this problem are

$$U'(l-c(q))c'(q)=V_y^S-V_n^S,\ S\in\{A,M\}.$$

The left hand side is the marginal utility cost of q, while the right hand side is the marginal benefit. The marginal benefit is close to 0 under assortative matching when k is small, but bounded away from 0 for small k under mixed matching due to the "social" benefits of the attribute (i.e., the insurance benefits the attribute provides). The greater marginal value of the attribute under mixed matching naturally leads to higher investment in the attribute.

As noted, this example is driven by the higher marginal value of the attribute in the mixed matching equilibrium than in the assortative matching equilibrium, and the attendant higher investments that result from this. We now present a second example in which mixed matching is stable, while assortative is not.

Example 2 We first consider the incentive constraints describing a mixed pairing under either assortative or mixed matching (these should be satisfied for the mixed matching to be stable, and one must be violated for the assortative matching to be stable). Fix a matching (assortative or mixed) and let V_y (V_n) denoted the expected utility of an agent with (without) the attribute under that matching. Children from an $HnLy$ match acquire the attribute in two ways: they either inherit the attribute with probability p, or failing to inherit, their parents invest $c(q_{HL})$ toward this end. Thus the (unconditional) probability that the offspring of matched pairs $HnLy$ have the attribute is $p+(1-p)\,q_{HL}$, which we denote by p_{HL}. Analogously, we denote by $p_{LL}=2p+(1-2p)\,q_{LL}$ the (unconditional) probability that the offspring of matched pairs $LyLy$ have the attribute. An Hn agent prefers to match with an Ly agent is

$$\begin{aligned} p\{(1-\beta)U(H+L)+\beta V_y\}+(1-p)\{(1-\beta)\,U\,(H+L-c(q_{HL})) \\ +\beta\,[q_{HL}V_y+(1-q_{HL}V_n)]\} \\ \geq (1-\beta)\,U\,(2H-c(q_{HH}))+\beta\,(q_{HH}V_y+(1-q_{HH})\,V_n)\,. \end{aligned} \tag{7}$$

Analogously, an Ly agent prefers matching with an Hn rather than another Ly agent if

$$\begin{aligned} & p\{(1-\beta)U(H+L)+\beta V_y\}+(1-p)\{(1-\beta)U(H+L-c(q_{HL})) \\ & \quad +\beta[q_{HL}V_y+(1-q_{HL}V_n)]\} \\ & \geq 2p\{(1-\beta)U(2L)+\beta V_y\}+(1-2p)\left\{(1-\beta)U\left(2L-c\left(q_{LL}^M\right)\right)\right. \\ & \quad \left.+\beta\left(q_{LL}^M V_y+\left(1-q_{LL}^M\right)V_n\right)\right\}. \end{aligned} \tag{8}$$

Rearranging (7) yields

$$\begin{aligned} \beta(p_{HL}-q_{HH})(V_y-V_n) &\geq (1-\beta)\{U(2H-c(q_{HH})) \\ & -[pU(H+L)+(1-p)U(H+L-c(q_{HL}))]\}, \end{aligned} \tag{9}$$

while (8) yields

$$\begin{aligned} \beta(p_{HL}-p_{LL})(V_y-V_n) &\geq (1-\beta)\{[2pU(2L)+(1-2p)U(2L-c(q_{LL}))] \\ & -[pU(H+L)+(1-p)U(H+L-c(q_{HL}))]\}. \end{aligned} \tag{10}$$

Mixed matching is stable (and assortative matching unstable) if the two inequalities (9) and (10) are satisfied.

For this example, we assume $L=0$, $H>\frac{p}{(1-p)}$, and $U'(0)=\infty$, so that two matched low income agents have no money to purchase the attribute, implying $q_{LL}=0$ and $H-c(q_{HL})>0$. Fix $\varepsilon,\eta>0$ small and assume the cost function satisfies

$$c(q)=\begin{cases} \eta q, & q<p/(1-p), \\ \bar{c}, & q=2p-\varepsilon, \end{cases}$$

with $c'(2p-\varepsilon)=\infty$, $\bar{c}<H$, and c convex on $[0,2p-\varepsilon]$. These assumptions on the cost function assure that (for sufficiently small η, see below) the investment choices for the matched pairs HL and HH satisfy the following inequality:

$$\frac{p}{(1-p)}<q_{HL}<q_{HH}<2p-\varepsilon. \tag{11}$$

Note that these inequalities hold under both mixed and assortative matching. Consequently,

$$\begin{aligned} p_{HL}-p_{LL} &= p+(1-p)q_{HL}-2p \\ &= (1-p)q_{HL}-p>0, \end{aligned}$$

and

$$\begin{aligned} p_{HL}-q_{HH} &= p+(1-p)q_{HL}-q_{HH} \\ &> 2p-q_{HH}>\varepsilon. \end{aligned}$$

Hence, the left hand sides of the inequalities (9) and (10) are positive. Since $L = 0$, and consequently $q_{LL} = 0$, the right hand side of (10) equals (ignoring the $(1-\beta)$ term)

$$U(0) - [pU(H) + (1-p)U(H - c(q_{HL}))],$$

which is negative, since $U(H) > 0$ and $U(\cdot) \geq 0$. This immediately gives us (10).

Next consider (9). We first obtain a lower bound on $V_y - V_n$ that is independent of the behavior of the utility function above $H - \bar{c}$. Note that every paired term in the expression

$$\begin{aligned} V_y - V_n &= \frac{1}{2}\{(V_{Hy} - V_{Hn}) + (V_{Ly} - V_{Ln})\} \\ &\quad + k\{(V_{Hy} - V_{Ly}) + (V_{Hn} - V_{Ln})\} \end{aligned}$$

is nonnegative, so that any term can serve as a lower bound for $V_y - V_n$.

Suppose the matching under consideration is the mixed matching. The fraction of the population with the attribute μ_y in steady state is bounded above by $4p(1-p)$ (from (11)). Note that this bound is independent of the utility function. Thus the probability that an Ly agent is matched with a Hn agent is at least

$$\min\left\{1, \frac{1-4p(1-p)}{4p(1-p)}\right\} \equiv \xi,$$

and with probability p the offspring of that match will have the attribute, and so

$$\begin{aligned} V_y - V_n &\geq \frac{1}{2}(V_{Ly} - V_{Ln}) \\ &\geq \frac{1}{2}\xi p(1-\beta)U(H). \end{aligned}$$

Thus, (9) is implied by

$$\beta\varepsilon\frac{1}{2}\xi pU(H) \geq U(2H - c(q_{HH})) - U(H - c(q_{HL})).$$

But this is clearly satisfied by any utility function that displays sufficient risk aversion. It remains to provide the appropriate upper bound on η (to ensure $q_{HL} > p/(1-p)$). The first order condition determining q_{HL} is

$$(1-\beta)c'(q_{HL})U'(H + c(q_{HL})) = \beta(V_y - V_n),$$

so it enough that

$$\eta < \frac{\beta\frac{1}{2}\xi pU(H)}{U'(2H)}.$$

Turning to assortative matching, observe that

$$V_y - V_n \geq k\left(V_{Hy} - V_{Ly}\right) \geq k2p\left(1-\beta\right)U\left(2H\right),$$

and so 9) is satisfied (and η is sufficiently small) when

$$2k > \frac{\xi}{2}.$$

To summarize, mixed matching is stable for this configuration of cost function and incomes in the example if the agents are sufficiently risk averse. At the same time, if the attribute is sufficiently productive, the assortative matching is not stable. The cost function in the second example was deliberately chosen to have a particular "threshold" form: the cost of acquiring the attribute was relatively low until a point at which it increased steeply. These characteristics of the cost function guarantee that HL couples invest nearly as much as HH couples in the event that their offspring do not inherit the attribute. On the other hand, be setting L very low (0 in the extreme case), LL couples can invest little (or none) in the attribute. Under assortative matching, even though there is no social value to the attribute, a mixed matching is profitable because the concavity of the utility function together with the structure of the cost function imply that the short-run cost for an Hn agent of matching with an Ly is dominated by the productiveness of the attribute.

7. Discussion

1. The example we have analyzed in some detail focuses on income uncertainty and insurance as the conduit through which an asset may have social value. Our point is more general, and to illustrate this, we now describe a simple non-matching example in which a nonproductive attribute can have social value.

In the example, there are overlapping generations of lawyers, each lawyer living two periods. There is a single nonstorable good over which lawyers have identical utility functions, $u\left(c_1, c_2\right) = c_1 \cdot c_2$, where c_i is consumption at age i. There are n lawyers born in each period. Each young lawyer generates an output of 3 in his first period and 1 in his second. Each lawyer can go into practice on his own and consume his own output, which generates total utility 3. Alternatively, a new lawyer can apprentice himself to a "white-shoe" lawyer, who in addition to being a lawyer, has social skills. The social skills have no use in and of themselves, but can be transmitted to others. Each white-shoe lawyer can take on at most 1 apprentice.

Two variables are determined in equilibrium, the salary of an apprentice and the number of "white-shoe" lawyers. Clearly, one possibility is a wage of 3, since the social skill is, by assumption, nonproductive and can be ignored. In this case, the number of “white-shoe” lawyers is indeterminate. There is, however, a second possibility. We now argue that a wage of 1 together with a number $m < n$ “white-shoe” lawyers is an equilibrium. (Again, the number of “white-shoe” lawyers is indeterminate.) A new lawyer who apprentices for a wage of 1 with a white-shoe lawyer will acquire the social skills, and will consequently be a white-shoe lawyer himself in the second period of his life. Being a white-shoe lawyer will allow him to take on an apprentice at wage 1, earning him a “profit” on the younger lawyer of 2. When his own output is added in, this gives an income stream of $(1, 3)$, which yields utility 3. Hence, each young lawyer is indifferent between going into practice on his own and apprenticing for a white-shoe lawyer at a wage of 1. Note that in this equilibrium, the social skills have value even though they have no effect on productivity. Note also that in the case that the social skills are valued, any wage above 1 would make all new lawyers strictly prefer the apprenticeship to going into practice on their own.

There are several aspects of the example worth noting. First, this example is not driven by risk aversion: agents are risk neutral. Second, as in the model we analyzed, missing markets are key. The missing market in our leading example is the market to insure against income shocks to future generations; here it is the market to transfer wealth across periods. A major disadvantage of the lawyer example relative to our leading example is that the lawyer example is too simple to capture the economic incentives that could destabilize the equilibrium use of the attribute.

2. The lawyer example illustrates well the similarity of social assets to money.[11] The social asset (the social skills) plays the same role as money for a white-shoe lawyer. He gives up output when young in exchange for the social asset, and when old, uses it to buy output from a new young lawyer.[12] Such an equilibrium is possible here only because of the assumption that agents desire to transfer wealth from their youth into their old age. In the non-productive version of our model, the attribute also bears a resemblance to money. Parents care about their offspring, but there is no vehicle for a couple to improve the welfare of their offspring, as we assumed that the single consumption good was nonstorable. In the mixed matching equilibrium, however, a high income individual can choose to match with a low income individual with the attribute in the hope that children of the match will have the attribute, which is valuable. Ignoring for now the stochastic nature of the intergenerational transfer of the attribute, this might be interpreted as the high income individual purchasing an asset that can be bequeathed to offspring. Although it bears a similarity to fiat money in this sense, there are several important differences. First, the attribute is inalienable. A child who inherits this asset

[11]We thank Randall Wright for bringing this to our attention.

[12]In this sense, the social attribute plays a role of a record device, similar to money as memory as described by Kocherlakota (1998).

cannot dispose of it; the only use the attribute can be put to is the purchase of a higher income mate than would otherwise be the case. Second, the child who inherits the asset cannot capture the full value of the asset, as he or she *must* bequeath the asset on to their offspring. In a sense the individual who inherits the attribute captures the present flow of value from it, but is unable to capture any of the future value.

3. We have analyzed a model in which there is an interaction between the social environment and agents' decisions. Differing social arrangements can lead to differences in important economic decisions, and, conversely, agents decisions have important consequences for the stability of the social arrangements. Many of the insights the model generates stem from the multiplicity of equilibria. This is not the first paper to point to the importance of multiple equilibria characterized by different economic choices by agents. Diamond (1970), for example, demonstrated the link between different equilibria and the level of aggregate economic activity. The nature of multiplicity in this paper, however, differs in an important way. Diamond's multiplicity stems from a complementarity in the production technology: each agent has little incentive to produce when few other agents produce. There is no analogous production complementarity in our model: in the productive attribute case, the productive value of the attribute is independent of the social arrangements. The *social* value, however is not independent of the arrangements. Hence, the economic consequences of the multiplicity in our model result from a change in the social return to the attribute rather than through the technology.

4. This paper is related to Cole, Mailath, and Postlewaite (1992), which analyzes a growth model incorporating matching between men and women. In that model, there are multiple equilibria characterized by different matchings between men and women. The current paper shares with that paper the feature that different matching arrangements lead to different economic choices – attribute choice in this paper and savings/bequests in that paper. There are important differences between the papers, aside from the differences in the economic decisions that are affected by the specification of matching. In the previous work, we analyzed a model in which, as in this paper, there were multiple equilibria, with one equilibrium characterized by there being value to the "name" one inherited. The equilibrium in the present model in which the heritable attribute has value is similar to that equilibrium. However, the present model is more tractable than the model used in that paper. This enables us to expand the analysis of social assets to the case in which they may be productive. This framework also allows for the endogenous acquisition of attributes, something not possible in the earlier work.

In addition to these substantive differences, there are important technical differences. The multiplicity of equilibria in Cole, Mailath, and Postlewaite (1992) relied on targeting agents who deviated from prescribed matching behavior for punishment: if a couple did not match according to prescribed rules, all their descendants were effective precluded from matching well in the future. The social arrangements in the present paper are "anonymous" in the sense that nothing from the past affects how well an individual will match; all that matters is the individual's current income and whether or not he or she possesses the attribute.

5. There is a large literature arguing that institutions such as a functioning legal system and respected property rights can usefully be thought of examples of social capital; more specifically, social capital is typically viewed as a "community-level attribute" (Putnam (1994), Glaeser, Laibson, and Sacerdote (2001)). From a formal perspective, social capital is best viewed as a characteristic of equilibrium in an infinite horizon game. The social norms as described in Okuno-Fujiwara and Postlewaite (1995) and Kandori (1992) are prototypical examples. A common feature is that the incentives in these equilibria require the punishment of deviators. Consequently, equilibria exhibiting social capital will not be "Markov."

In contrast, the equilibria we study are Markov. While the particular matching structure (e.g., mixed or assortative) could be thought as an instance of social capital, we think it is useful to distinguish between social arrangements which are necessarily sustained by sanctions (arrangements that we would term social capital) and those which do not require sanctions. As we noted in the previous point, the stability of both the mixed and assortative matching is driven by anonymous considerations: the attribute has a certain value (which may be zero) and it is irrelevant how that attribute was acquired.

6. As mentioned earlier, the possibility of the attribute having value in equilibrium when it is nonproductive stems from the inability of parents to fully insure against future income shocks to their descendants. If there were storable goods or other instruments through which parents could make bequests to their children, the welfare improvements that are a consequence of valuable social attributes would be diminished. In the absence of markets that permit perfect insurance against future shocks, however, there remains the possibility of equilibria with qualitative characteristics as in our model.

7. Our model treats agents as belonging to a single, all-inclusive society. From the analysis, it is straightforward that if there are two isolated communities (so that there is no cross-community matching), the communities can be governed by different matching arrangements. The consequence is that we might have communities of similar (even identical) fundamentals for which we observe substantial differences in distributions of income or consumption. This discussion, however, neglects the possibility that there may cross-community matching. As we have argued repeatedly, there are fundamental feedbacks between social and economic arrangements. Consequently, simply using the multiplicity of equilibria as a naive model of heterogeneity ignores the ability of agents in one community to interact with those in another.

8. The mixed matching equilibrium is reminiscent of some versions of the theory of sexual selection (Ridley (1993)). These theories have been motivated by the existence of animals such as peacocks. A nontrivial amount of the peacock's biological resources are invested in long and elegant tail feathers, which serve no productive purpose. Since natural selection selects for the fittest peacocks (those with shorter tails), there is a puzzle. Why do peahens prefer less fit males (those who have devoted scarce resources to feathers), as they must in order for the less fit males to dominate the population? The explanation is similar to the logic of the stability of the mixed matching in our model. Suppose peahens prefer long-tailed peacocks; then so long as long tails are genetically transmitted to male offspring, the male offspring of a peahen matched with a long-tailed mate will fare better in the market for mates in the next generation. Hence, there is an advantage to peahens that match with long-tailed peacocks that offsets the resources associated with long tails.

Peacocks' long tail feathers are similar to social assets in our model, but the incentives associated with the matching differ. A peahen that mates with a short-tailed peacock will have more offspring that survive. However, while a peahen that mates with a long-tailed mate may have fewer surviving offspring, the male offspring will themselves have more surviving offspring due to the advantage long tails confer in the matching process. Thus, selection, which is driving the peahen's choice of mates, is essentially balancing between the genetic advantage of more surviving offspring in the next generation and more surviving offspring in the subsequent generation. In our model, by contrast, a rich person without an attribute considering matching with a poor person with the attribute is balancing current consumption against descendants' consumption.

8. Appendix

Proof of Proposition 7

Proof. We will define a mapping $\Phi : [0,1]^4 \rightarrow [0,1]^4$ with the property that stationary values of $\left(\mu_y, q^M\right)$ are precisely the fixed points of $\Phi \equiv (\Phi_1, \Phi_2)$. The function $\Phi_1\left(\mu_y, q^M\right)$ is the fraction of the population that will have the attribute next period if the current population has fraction μ_y and parents purchase according to q^m: There are two cases we need to deal with, $\mu_y > \frac{1}{2}$ and $\mu_y \leq \frac{1}{2}$. Suppose that $\mu_y \geq \frac{1}{2}$, so that there are at least as many Ly as Hn agents. The fraction of the population with the attribute in the next period is then

$$
\begin{aligned}
&\mu_y \left\{ (\frac{1}{2}+k) \left(2p+(1-2p)\, q^M_{HH}\right) \right. \\
&\left. +(\frac{1}{2}-k) \left(\frac{(1-\mu_y)}{\mu_y} \left(p+(1-p)\, q^M_{HL}\right) + \frac{(2\mu_y-1)}{\mu_y} \left(2p+(1-2p)\, q^M_{LL}\right) \right) \right\} \\
&+ \left(1-\mu_y\right) \left\{ (\frac{1}{2}-k) \left(p+(1-p)\, q^M_{HL}\right) + (\frac{1}{2}+k) q^M_{LL} \right\} \\
&= \mu_y (\frac{1}{2}+k) \left(2p+(1-2p)\, q^M_{HH}\right) + (1-2k)(1-\mu_y) \left(p+(1-p)\, q^M_{HL}\right) \\
&+(\frac{1}{2}-k) \left(2\mu_y-1\right) \left(2p+(1-2p)\, q^M_{LL}\right) + \left(1-\mu_y\right) (\frac{1}{2}+k) q^M_{LL}.
\end{aligned}
$$

On the other hand, if $\mu_y < \frac{1}{2}$, there are more Hn than Ly agents, and so

$$
\begin{aligned}
&\mu_y \left\{ (\frac{1}{2}+k) \left(2p+(1-2p)\, q^M_{HH}\right) + (\frac{1}{2}-k) \left(\left(p+(1-p)\, q^M_{HL}\right) \right) \right\} \\
&+ \left(1-\mu_y\right) \left\{ (\frac{1}{2}-k) \left(\frac{\mu_y}{\left(1-\mu_y\right)} \left(p+(1-p)\, q^M_{HL}\right) + \frac{\left(1-2\mu_y\right)}{\left(1-\mu_y\right)} q^M_{HH} \right) \right. \\
&\left. + (\frac{1}{2}+k) q^M_{LL} \right\} \\
&= \mu_y (\frac{1}{2}+k) \left(2p+(1-2p)\, q^M_{HH}\right) + (1-2k)\mu_y \left(p+(1-p)\, q^M_{HL}\right) \\
&+(\frac{1}{2}-k) \left(1-2\mu_y\right) q^M_{HH} + \left(1-\mu_y\right) (\frac{1}{2}+k) q^M_{LL}.
\end{aligned}
$$

Clearly, Φ_1 is continuous.

In order to define Φ_2, we first need to a calculate a value for the attribute, given μ_y and q^M. In this calculation, it is important to note that we are not requiring that μ_y be consistent with q^M (though at the fixed point, they will be consistent). Denote by $\tilde{V}_i$ the expected value to agent $i \in \{Hy, Ly, Hn, Ln\}$ of being in a situation characterized by $\left(\mu_y, q^M\right)$ *in each period*. Then, setting

$$\tilde{V}_y \equiv (\frac{1}{2}+k)\tilde{V}_{Hy} + (\frac{1}{2}-k)\tilde{V}_{Ly} \tag{12}$$

and

$$\tilde{V}_n \equiv (\frac{1}{2}-k)\tilde{V}_{Hn} + (\frac{1}{2}+k)\tilde{V}_{Ln}, \tag{13}$$

we have

$$\tilde{V}_{Hy} = 2p\left\{(1-\beta)\,U\,(2H) + \beta\tilde{V}_y\right\} \tag{14}$$
$$+ (1-2p)\left\{(1-\beta)\,U\left(2H - c\left(q^M_{HH}\right)\right) + \beta\left(q^M_{HH}\tilde{V}_y + \left(1-q^M_{HH}\right)\tilde{V}_n\right)\right\}$$

and

$$\tilde{V}_{Ln} = (1-\beta)\,U\left(2L - c\left(q^M_{LL}\right)\right) + \beta\left(q^M_{LL}\tilde{V}_y + \left(1-q^M_{LL}\right)\tilde{V}_n\right). \tag{15}$$

Moreover, if $\mu_y \geq \frac{1}{2}$ (the reverse inequality is an obvious modification), we have

$$\tilde{V}_{Hn} = p\left\{(1-\beta)\,U\,(H+L) + \beta\tilde{V}_y\right\} \tag{16}$$
$$+ (1-p)\left\{(1-\beta)\,U\left(H+L - c\left(q^M_{HL}\right)\right) + \beta\left(q^M_{HL}\tilde{V}_y + \left(1-q^M_{HL}\right)\tilde{V}_n\right)\right\}$$

and

$$\tilde{V}_{Ly} = \frac{(1-\mu_y)}{\mu_y}\left[p\left\{(1-\beta)\,U\,(H+L) + \beta\tilde{V}_y\right\}\right.$$
$$\left.+ (1-p)\left\{(1-\beta)\,U\left(H+L - c\left(q^M_{HL}\right)\right) + \beta\left(q^M_{HL}\tilde{V}_y + \left(1-q^M_{HL}\right)\tilde{V}_n\right)\right\}\right]$$
$$+\frac{(2\mu_y - 1)}{\mu_y}\left[2p\left\{(1-\beta)\,U\,(2L) + \beta\tilde{V}_y\right\}\right. \tag{17}$$
$$\left.+ (1-2p)\left\{(1-\beta)\,U\left(2L - c\left(q^M_{LL}\right)\right) + \beta\left(q^M_{LL}\tilde{V}_y + \left(1-q^M_{LL}\right)\tilde{V}_n\right)\right\}\right].$$

The equations (12-17) have a unique solution. Moreover, this solution is continuous in μ_y and q^M.

Note that at this point there is no reason to expect $\tilde{V}_y > \tilde{V}_n$. We define $\Phi_2\left(\mu_y, q^M\right) = \left(\tilde{q}^M_{HH}, \tilde{q}^M_{HL}, \tilde{q}^M_{LL}\right)$ by

$$\tilde{q}^M_\ell = \underset{q\in[0,c^{-1}(\ell)]}{\operatorname{argmax}}\,(1-\beta)\,U\,(\ell - c\,(q)) + \beta\left(q\tilde{V}_y + (1-q)\,\tilde{V}_n\right). \tag{18}$$

The maximizer is unique since U is concave and c is convex. Since the maximizer is unique, it is a continuous function of $\left(\mu_y, q^M\right)$, through the continuous on $\tilde{V}_y$ and $\tilde{V}_n$.

Since Φ is a continuous function on a compact convex subset of $\Re^4$, there is a fixed point by Brouwer. Note also that in the fixed point, $\tilde{V}_y > \tilde{V}_n$: Suppose that $\tilde{V}_y \leq \tilde{V}_n$. Optimization then implies $q^M_\ell = 0$ for all ℓ. But then $\mu_y = 0$ (since $p < \frac{1}{2}$), and so $\tilde{V}_n = \frac{1}{2} - k$ (recall our normalization $U(2L) = 0$). Moreover, if $\tilde{V}_y \leq \tilde{V}_n$, $\tilde{V}_y > (1-\beta)\{\frac{1}{2} + k + (\frac{1}{2} - k)u\} + \beta\tilde{V}_y$ (where $U(H+L) = u$), contradicting $\tilde{V}_y \leq \tilde{V}_n$. Since we have assumed $c'(0) = 0$, $q^M_\ell > 0$ for all ℓ and so $\mu_y > 0$. Note also that

$\mu_y < 1$: For suppose $\mu_y = 1$, then a fraction $\frac{1}{2} - k$ of agents are Ly's and since $2p + (1 - 2p)\, q_{LL}^M < 1$, not all the population in the next period can have the attribute. ■

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