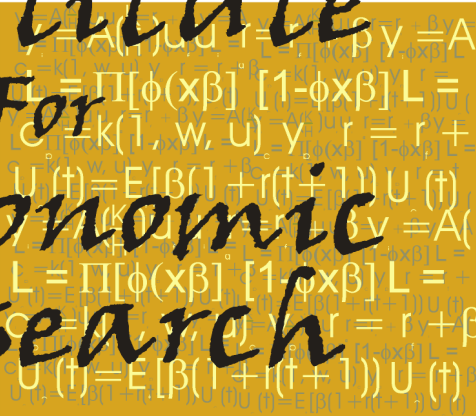


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“Labor Supply Shifts and Economic Fluctuations”

by

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Labor-Supply Shifts and Economic Fluctuations

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Abstract

We investigate the role of labor-supply shifts in economic fluctuations. A new VAR identification scheme for labor supply shocks is proposed. Our method provides an alternative identification scheme, which does not rely on “zero-restrictions”. According to our VAR analysis of post-war U.S. data, labor-supply shifts account for about half the variation in hours and one-fifth of variation in aggregate output. To assess the role of labor-supply shifts in a more structural framework, estimates from a dynamic stochastic general equilibrium (DSGE) model with stochastic variation in home production technology are compared to those from the VAR.

JEL CLASSIFICATION: E32, C52, J22

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Home Production, Bayesian Econometrics

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1 Introduction

A leading question in macroeconomics is the identification of forces that determine the cyclical allocation of time. Modern dynamic stochastic general equilibrium analysis emphasizes random shifts in labor demand due to technological progress. Empirical studies on the decomposition of working hours (e.g., Shapiro and Watson, 1988, and Hall, 1997) have called for an attention to labor-supply movements. For example, Hall (1997) finds a predominant role of labor-supply shifts for fluctuations in hours worked. He suggests non-market activities such as job-search or home production as possible causes for labor-supply shifts.

This paper examines the importance of labor-supply shifts as a source of economic fluctuations by combining two important advances in macroeconomics in recent years: identified vector autoregressions (VAR) and dynamic stochastic general equilibrium (DSGE) models. First, we propose a new VAR identification scheme to decompose the fluctuation of hours and output into disturbances to labor-supply and labor-demand schedule. Impulse responses are computed to assess whether the dynamics of the identified VAR conform with economic intuition on the effects of labor supply and demand shifts. Second, we investigate whether the aggregate dynamics unveiled in the VAR analysis can be reproduced by a DSGE model. In particular, we consider a model that explicitly deals with non-market activity, namely home production, developed by Benhabib, Rogerson, and Wright (1991).¹ The model specification on the labor-supply side is much more general than the utility function commonly used in the literature. In fact, it nests conventional time separable preferences for consumption and leisure as a special case.

Economic fluctuations are viewed as a series of equilibria generated by competitive households and firms whose tastes and technologies are pertubated by stochastic

¹The Beckerian home production models are motivated by the fact that, in any economy, agents spend a significant amount of time on non-market activities. For example, according to the Michigan Time-Use Survey, a typical married couple in the U.S. allocates about 25 percent of its discretionary time to home production activities, while the couple spends about 33 percent of its time for paid compensation (see Hill (1984), or Juster and Stafford (1991)).

disturbances. Our VAR analysis assumes that there are three fundamentals in the labor market. First, labor-supply shocks cause movements of the economy along the short-run labor-demand schedule. Such disturbances generate a negative contemporaneous correlation between labor productivity and hours, which we exploit to identify the temporary labor-supply shock. Second, permanent technological changes affect the long-run level of productivity. A permanent increase in productivity eventually shifts the labor supply as well as labor demand – via wealth effect in a conventional utility or via accumulation of home capital stock in a home production model– leaving hours constant at a higher equilibrium wage. It is possible to identify such shocks through restrictions on their long-run multiplier matrix implied by the VAR representation of the data. The third shock is identified by the assumption that all shocks are orthogonal. Based on the shapes of the impulse responses we will argue that this shock can be broadly interpreted as temporary labor-demand shock. While we interpret it as a temporary shifts in market production function in our DSGE model, its interpretation can be much broader than ours.²

The proposed identification scheme differs from previous approaches. Shapiro and Watson (1988) assume that both hours and aggregate output are non-stationary. Their identification is based on a long-run restriction: labor-supply shocks have a permanent effect on both hours and output, whereas technology shocks only affect output in the long-run. However, the evidence on the non-stationarity of hours is inconclusive. A researcher who believes that hours follow a stationary process will find the data consistent with his belief. Vice versa, there is not much evidence in hours data that would contradict that hours are (locally) non-stationary. Our investigation treats hours as stationary process. This assumption is consistent with a

²Our analysis does not consider other disturbances such as monetary and fiscal policy shocks. For post-war U.S. data, government policy shocks are often considered as a secondary importance in business-cycle analysis. For example, according to King, Plosser, Stock and Watson (1991), permanent nominal shocks identified by imposing long-run neutrality explain little of the variability in real variables. The cyclical components of government spending is not highly correlated with output measures – it is less than 0.2 for Hodrick-Prescott filtered data. Also, expanding the list of shocks often invites arbitrary identifying restrictions in the VAR analysis.

large class of theoretical DSGE models, including the one presented in this paper, in which stochastic growth is induced by a non-stationary labor augmenting technology process and the economy evolves along a balanced growth path.

Hall (1997) identifies the labor-supply or preference shocks by deriving short-run labor supply and demand functions based on assumptions on consumer preferences and the firms' production technology. He expresses the equilibrium hours as a function of the labor-supply shock and several observable variables contained in the first-order condition of utility maximization of households. Based on the labor-market equilibrium the labor supply shocks are calculated as residuals from the first-order conditions of household labor supply decision.³ Similar to Hall's analysis, we also exploit the short-run labor market equilibrium to identify the supply shocks. However, our VAR identification scheme does not rely on a specific form of households' preferences.

Unlike many VAR identification schemes that have been used in the literature, our scheme cannot be implemented solely based on zero-restrictions on the contemporaneous relations among endogenous variables, the long-run multiplier matrix for the structural shocks, and the covariance matrix of the structural shocks. Uhlig (1997) and Canova and De Nicolo (1998) develop identification schemes based on inequality restrictions on the direction of impulse responses. Gordon and Boccanfuso (1998) proposed to express the VAR as a moving average (MA) of structural shocks, specify a proper prior for the MA representation and update the prior based on the sample observations. While the MA representation is not identified in a classical sense, it is still possible to compute a proper posterior distribution of the impulse responses. However, in general the specification of the proper prior distribution is even more demanding than the specification of inequality restrictions. In our scheme, we separate the identifiable reduced form VAR parameters from one non-identifiable parameter, that is, the slope of the labor-demand schedule. Since the reduced form parameters are updated through the sample information, the implied distribution

³The same strategy to identify preference shocks is used in Hall (1986), Parkin (1988) and Baxter and King (1991).

of the impulse response functions is updated with every observation.⁴ To make the VAR analysis consistent with the DSGE model analysis, the same prior distribution for the slope of the labor-demand schedule is used in both specifications.

Our findings can be summarized as follows. According to the variance decomposition from the VAR, consistent with Shapiro and Watson (1988) and Hall (1997), we find an important role of labor-supply shifts in fluctuations of hours. Temporary shifts in labor supply account for about half (46 percent) of the cyclical variation of working hours, whereas temporary labor-demand shifts account for 38 percent. The stochastic trend in productivity account for 16 percent of variation. This decomposition is fairly robust across various business-cycle frequencies. Labor-supply shifts are less important for output fluctuations. They explain 17 percent of the variation in output growth.

When the fully-specified general equilibrium model is fitted to the data, most variation of hours is attributed to the temporary shift in labor-demand, as it accounts for 70 percent of the variation. Labor-supply shifts, postulated by stochastic shifts to home technology, continue to play an important role for hours as they account for 25 percent of hours variation. However, for output fluctuation, only 4 percent of the variation is due to the movements of home production productivity. The equilibrium model reproduces the responses of labor productivity and spending on consumer durable goods in the VAR reasonably well. The response of market hours from the model exhibits a temporal shift compared to the VAR. While the response of hours is immediate in the model, it is delayed by 2-3 quarters in the data.

According to equilibrium models with preference shocks, recessions can occur because agents find it optimal to spend more time in non-market activities. The DSGE model provides estimates of the evolution of market and home technology over time. The latter measures the attractiveness of non-market activities. While there are alternative explanations for recessions that are not captured by the simple DSGE

⁴Poirier (1998) provides a survey and several examples of Bayesian analyses of non-identified econometric models.

model, we find it interesting to compare the estimates of the latent technologies to the NBER business cycle dates. Taken at face value, two out of six business cycle troughs during the period from 1960:I to 1997:IV, namely March 1975 and November 1982, coincide with unusually high productivity of non-market activity.

The paper is organized as follows: In Section 2, we illustrate the economic intuition behind our identification scheme for a vector autoregression. Section 3 presents a general equilibrium model that explicitly considers non-market activity. Section 4 discusses our estimation method for VAR and DSGE model and provides a formal description of the identification scheme. Empirical findings are summarized in Section 5, and Section 6 is conclusion. Data definitions and computational details are collected in the Appendix.

2 Identifying Labor Market Fluctuations

Labor market fluctuations are viewed as a series of equilibria generated by competitive households and firms whose tastes and technologies are pertubated by three types of stochastic disturbances. Figure 1 depicts time series plots of hours, labor productivity and spending on consumer durable goods. For the past several decades, labor productivity and hourly compensation of labor exhibited strong trends, whereas aggregate hours did not show an apparent trend. This observation has led macroeconomists to adopt the notion of a so-called balanced growth path. There are permanent productivity shocks that cause fluctuations in hours in the short-run, but leave hours constant in the long-run as they shift both labor demand and labor supply — via wealth effect in a conventional utility and via accumulation of consumer durable goods in home production models. As in King, Plosser, Stock, and Watson (1991), the permanent productivity shock, which we will denote by $\epsilon_{z,t}$, can be identified by the common trend in labor productivity and spending on consumer durable goods.

In addition to the permanent shock we consider two innovations that cause temporary shifts of labor demand and supply, denoted by $\epsilon_{a,t}$ and $\epsilon_{b,t}$, respectively.

All three fundamental shocks are assumed to be uncorrelated. The identification of the temporary shocks is based on the following assumptions on the aggregate labor market.

The inverse labor demand of a competitive profit-maximizing representative firm can be written in terms of market capital stock $K_{m,t}$ and the state of market technology S_t at time t :

$$W_t = MPL_t = \varphi_t^D(L_{m,t}; K_{m,t}, S_t), \quad (1)$$

where W_t represents real wage rate, MPL_t the marginal product of labor, and $L_{m,t}$ hours employed at time t . The state of market technology is a function of current and past innovations to productivity $\epsilon_{z,t}$ and $\epsilon_{a,t}$, reflecting, respectively, permanent and temporary components: $S_t = S(\{\epsilon_{z,j}, \epsilon_{a,j}\}_{j=0,\dots,t})$.

Similarly, the inverse labor supply by the representative household can be written in a generic form:

$$W_t = \varphi_t^S(L_{m,t}; \Omega_t(S_t, T_t), T_t). \quad (2)$$

where Ω_t represents endogenous variables that influence the labor supply of the household, such as real interest rate, consumption, and wealth. T_t represents the exogenous random shifts in labor supply. T_t is a function of current and past innovations $\epsilon_{b,t}$, which may be called “taste shocks” or “productivity shocks” to non-market activity: $T_t = T(\{\epsilon_{b,j}\}_{j=0,\dots,t})$.

The important distinction is that $\epsilon_{b,t}$ enters the labor-supply function only, as the capital stock is predetermined from period $t - 1$. According to traditional econometric analysis, the slope of labor-demand schedule is identified through an instrument for $\epsilon_{b,t}$. In our VAR analysis, however, we identify $\epsilon_{b,t}$ conditional on the slope of the marginal product of labor.

As the capital stock $K_{m,t}$ is inherited from the previous period, the labor-demand schedule is stable despite exogenous shifts in T_t at time t . This allows us to identify the labor-supply shift that is orthogonal to the labor-demand shifts given the slope of marginal product of labor. For example, with a Cobb-Douglas production

technology, in response to an orthogonal shift in labor supply, real wage and hours must exhibit the following relationship:

$$\frac{\partial \ln W_t}{\partial \epsilon_{b,t}} = (\alpha - 1) \frac{\partial \ln L_{m,t}}{\partial \epsilon_{b,t}}, \quad (3)$$

where α is the labor share parameter in the production function. Unlike in Hall's (1997) analysis, no assumptions with respect to the labor-supply function φ^S have to be made. We will show in Section 4, how the posterior distribution of the VAR based impulse response functions depends on the prior distribution for α .

Even though it is plausible to assume that the capital stock is predetermined, its utilization may fluctuate over the business cycle and lead to shifts in labor demand schedule at impact. However, we demonstrate in Appendix A that even in the presence of variable capital utilization our identification scheme is still valid. In equilibrium allowing for utilization makes the labor demand schedule flatter than the case without utilization reflecting an extra margin for the firms to exploit. In the empirical analysis below, we allow for variation in α through the prior distribution.

3 A Fully Specified Model Economy

A fully-specified-dynamic-general-equilibrium model provides a rigorous interpretation of structural shocks and their propagation. It also helps to understand the economic intuition behind our identification scheme used for the VAR analysis. The model economy consists of identical infinitely lived households who maximize the expected discounted lifetime utility U defined over consumption C_t and pure leisure $1 - L_{m,t} - L_{h,t}$. $L_{m,t}$ is the fraction of time supplied to the representative firm described in the previous section and $L_{h,t}$ is the fraction of hours spent on home production activities (e.g., lawn-mowing, dish-washing, or cooking), which often require the use of consumer durable goods.

$$U = E_t \left[\sum_{s=t}^{\infty} \beta^{s-t} (\log C_s + \kappa \log(1 - L_{m,s} - L_{h,s})) \right] \quad (4)$$

E_t is the expectation operator conditional on information available at time t and β is the discount factor. Consumption is an aggregate of market consumption $C_{m,t}$ and the consumption of home produced goods $C_{h,t}$:

$$C(C_{m,t}, C_{h,t}) = [\chi C_{m,t}^{\frac{v-1}{v}} + (1 - \chi) C_{h,t}^{\frac{v-1}{v}}]^{\frac{v}{v-1}}, \quad (5)$$

where v is the substitution elasticity, reflecting the household's willingness to substitute market and home-produced goods. Output from home production depends on the state of technology and capital stock at home. It is produced according to a constant-returns-to-scale technology with inputs home capital K_{ht} and labor L_{ht}

$$C_{h,t} = [\psi (X_{h,t} L_{h,t})^{\frac{\tau-1}{\tau}} + (1 - \psi) K_{h,t}^{\frac{\tau-1}{\tau}}]^{\frac{\tau}{\tau-1}}, \quad (6)$$

where τ is the substitution elasticity between labor and capital in home production. $X_{h,t}$ is a labor augmenting productivity process that will be specified below. It is important to note that this specification of home production is much more general than the conventional utility with leisure only. In fact, the commonly used separable-in-log utility can be obtained by simply setting $v = \tau = 1$.⁵

The household owns the market capital stock and rents it to the representative firm. The budget constraint is of the form

$$C_{m,t} + I_{m,t} + I_{h,t} = W_t L_{m,t} + R_t K_{m,t}, \quad (7)$$

where $I_{m,t}$ and $I_{h,t}$ are investments on the capital stock in the market $K_{m,t}$, and at home $K_{h,t}$. In each period t , the household chooses $C_{m,t}$, $C_{h,t}$, $I_{m,t}$, $I_{h,t}$, $L_{m,t}$, and $L_{h,t}$. Market capital and home capital accumulate according to:

$$\begin{aligned} K_{m,t+1} &= \phi(I_{m,t}/K_{m,t})K_{m,t} + (1 - \delta)K_{m,t} \\ K_{h,t+1} &= \phi(I_{h,t}/K_{h,t})K_{h,t} + (1 - \delta)K_{h,t}, \end{aligned} \quad (8)$$

where δ is the depreciation rate of capital. The capital accumulation is subject to convex adjustment cost: $\phi' > 0, \phi'' \leq 0$.⁶

⁵Also, one can always write down a utility function with preference shock that is identical to the stochastic shift in home technology.

⁶Unlike one-sector models, in a multi-sector model, the investment in one sector can increase

Output Y_t is produced by a representative firm that operates a Cobb-Douglas technology with the inputs capital $K_{m,t}$ and labor $L_{m,t}$

$$Y_t = K_{m,t}^{1-\alpha} (X_{m,t} L_{m,t})^\alpha. \quad (9)$$

$X_{m,t}$ represents a labor augmenting technology process. The firm solves the one-period problem

$$\max_{L_{m,t}, K_{m,t}} K_{m,t}^{1-\alpha} (X_{m,t} L_{m,t})^\alpha - W_t L_{m,t} - R_t K_{m,t}, \quad (10)$$

which leads to an inverse demand function of the form (1). In equilibrium the output produced by the representative firm is equal to the consumption of market goods and the investment in home and market capital:

$$Y_t = C_{m,t} + I_{m,t} + I_{h,t}. \quad (11)$$

The labor augmenting productivity of the market and home technology are of the form $X_{m,t} = \exp[z_t + a_t]$ and $X_{h,t} = \exp[z_t + b_t]$, respectively. Here z_t represents a common technology process that follows a random walk with drift:

$$z_t = \gamma + z_{t-1} + \epsilon_{z,t}. \quad (12)$$

The processes a_t and b_t capture temporary productivity movements that are modeled as stationary first-order auto-regressions:

$$a_t = \rho_a a_{t-1} + \epsilon_{a,t} \quad (13)$$

$$b_t = \rho_b b_{t-1} + \epsilon_{b,t}. \quad (14)$$

Define $\epsilon_t = [\epsilon_{z,t}, \epsilon_{a,t}, \epsilon_{b,t}]'$. We assume that ϵ_t is serially uncorrelated with diagonal covariance matrix Σ_ϵ . Its diagonal elements will be denoted by σ_z^2 , σ_a^2 , and σ_b^2 , respectively.

enormously at the price of the investment in the other sector, without affecting consumption significantly, resulting in unreasonably volatile investments over time. Adjustment costs of capital accumulation generate a more reasonable behavior of sectoral investment (e.g., Baxter (1996) and Fisher (1997)).

Due to the random walk process z_t the economy evolves along a balanced stochastic growth path. Except for R_t , $L_{m,t}$, and $L_{h,t}$ all endogenous variables exhibit a stochastic trend $\exp[z_t]$. This stochastic trend shifts both the labor supply and demand curves, such that in the long-run a unit shock $\epsilon_{z,t}$ raises the equilibrium wage rate by one percent but does not affect hours worked. The DSGE model is consistent with the identification scheme proposed in Section 2 in the following sense: based on a long series of observations generated from a log-linear approximation of the DSGE model, it is possible to recover the structural shocks ϵ_t through an identified VAR with sufficiently many lags.

4 Econometric Approach

Two specifications are considered: the just-identified VAR, denoted by \mathcal{M}_0 , and the over-identified DSGE model, denoted by \mathcal{M}_1 . The VAR includes hours $L_{m,t}$ and labor productivity P_t and investment on home capital stock (expenditure on consumer durable goods) $I_{h,t}$. According to our home production model, the use of expenditure on consumer durables is obvious as it reflects the productivity in home technology in the long-run, thus allows us to identify the common productivity shocks. Moreover, its use is also justified without the home-production argument. To identify the permanent technological progress which will eventually shift labor supply through wealth effect, one needs a measure that reflects the wealth of the household. The long-run behavior of spending on consumer durables serves as a good proxy for permanent income of households. One expects important technological innovations reflected in productivities of consumer durable goods as well as producers' durable goods.⁷ Under either interpretation, common trends in labor productivity and consumer durable goods identify the permanent component in productivity $\epsilon_{z,t}$.

Both \mathcal{M}_0 and \mathcal{M}_1 generate probability distributions for the data $Y_T = [y_1, \dots, y_T]'$

⁷While the consumption expenditure shares for non-durables and services shifted substantially in the past four decades, the share of durables stayed fairly constant.

where y_t is the 3×1 vector of observables.⁸ Define the cumulative market hours process $\tilde{L}_{m,t} = \sum_{\tau=0}^t L_{m,\tau}$ and let $y_t = [\ln P_t, \ln I_{h,t}, \ln \tilde{L}_{m,t}]'$. In general, we will assume that hours are integrated of order zero, $I(0)$, and productivity and home investment are integrated of order one, $I(1)$. This assumption is consistent with the DSGE model presented in the previous section. Hence, all the elements of the vector $\Delta y_t = [\Delta P_t, \Delta I_{h,t}, L_{m,t}]'$ are $I(0)$.

The VAR and DSGE model parameters, except for α , are stacked in vectors $\theta_{(i)} \in \Theta_{(i)}$, $i = 0, 1$. We assume that both models share the parameter α . The likelihood functions are denoted by $p(Y_T | \theta_{(i)}, \alpha, \mathcal{M}_i)$. In the context of the VAR, α is only needed to identify structural shocks. It does not affect the likelihood function:

$$p(Y_T | \theta_{(0)}, \alpha, \mathcal{M}_0) = p(Y_T | \theta_{(0)}, \mathcal{M}_0). \quad (15)$$

Since our analysis is Bayesian, we place a probability distribution on the two models and their parameters, denoted by the priors $\pi_{i,0}$, $p(\theta_{(i)} | \mathcal{M}_i)$, and $p(\alpha)$. The joint distribution of data and parameters is of the form

$$p(Y, \theta_{(0)}, \theta_{(1)}, \alpha) = p(\alpha) \sum_{i=0,1} \pi_{i,0} p(\theta_{(i)} | \mathcal{M}_i) p(Y_T | \theta_{(i)}, \alpha, \mathcal{M}_i). \quad (16)$$

The use of an informative prior distribution for the DSGE model allows us to incorporate information on structural parameters from microeconomic studies. The next three subsections explain our estimation and evaluation approach, the formal specification and identification of the VAR, and the variance decomposition at business cycle frequencies.

4.1 Estimation and Evaluation

A straightforward application of Bayes Theorem shows that the posterior distribution of the parameters $\theta_{(0)}$, $\theta_{(1)}$ and α is given by:

$$p(\theta_{(0)}, \theta_{(1)}, \alpha | Y_T) = \sum_{i=0,1} \pi_{i,T} p(\theta_{(i)}, \alpha | Y_T, \mathcal{M}_i), \quad (17)$$

⁸Here, we use y_t and Y_T to generically denote the dependent variables, not aggregate market output as in Sections 2 and 3.

where

$$\begin{aligned}\pi_{i,T} &= \frac{\pi_{i,0}p(Y_T|\mathcal{M}_i)}{\sum_{i=0,1}p(Y_T|\mathcal{M}_i)}, \\ p(\theta_{(i)}, \alpha|Y_T, \mathcal{M}_i) &= \frac{p(Y_T|\mathcal{M}_i, \theta_{(i)}, \alpha)p(\theta_{(i)}|\alpha, \mathcal{M}_i)p(\alpha)}{p(Y_T|\mathcal{M}_i)}, \\ p(Y_T|\mathcal{M}_i) &= \int p(\alpha)p(\theta_{(i)}|\mathcal{M}_i)p(Y_T|\theta_{(i)}, \alpha, \mathcal{M}_i)d(\theta_{(i)}, \alpha).\end{aligned}$$

The densities $p(\theta_{(i)}, \alpha|Y_T, \mathcal{M}_i)$ characterize the posterior distribution of $\theta_{(i)}$ and α of an individual who fits model \mathcal{M}_i to the data Y_T . According to the VAR, the data contain no information on α :

$$\begin{aligned}p(\theta_{(0)}, \alpha|Y_T, \mathcal{M}_0) &= \frac{p(Y_T|\theta_{(0)}, \mathcal{M}_0)p(\theta_{(0)}|\mathcal{M}_0)p(\alpha)}{\int \left[p(Y_T|\theta_{(0)}, \mathcal{M}_0)p(\theta_{(0)}|\mathcal{M}_0) \left[\int p(\alpha)d\alpha \right] \right] d\theta_{(0)}} \\ &= p(\theta_{(0)}|Y_T, \mathcal{M}_0)p(\alpha).\end{aligned}\tag{18}$$

Thus, the marginal posterior distribution of α is equal to the prior distribution.

The posterior model probabilities $\pi_{i,T}$ measure the relative time series fit of the two models. Due to the restrictive dynamics of the DSGE model, its posterior probability turns out to be small. However, our empirical analysis does not focus on a comparison of VAR and DSGE model through posterior probabilities. Instead, we will construct an overall posterior distribution for a variance decomposition of aggregate hours and output and examine to what extent these population characteristics can be reproduced by the DSGE model alone. Details of this methodology are provided in Schorfheide (2000).

Let $\varphi \in \mathbb{R}^m$ be an $m \times 1$ vector of population characteristics such as a variance decomposition or a truncated impulse response function. According to model \mathcal{M}_i , the implied population characteristics are functions $\tilde{\varphi}_i(\theta_{(i)}, \alpha)$. The overall posterior distribution of φ is a mixture of two components: with probability $\pi_{0,T}$ the distribution of φ is induced by the function $\tilde{\varphi}_0(\theta_{(0)}, \alpha)$ where $[\theta'_{(0)}, \alpha]'$ has density $p(\theta_{(0)}|\mathcal{M}_0, Y_T)p(\alpha)$. With probability $\pi_{1,T} = 1 - \pi_{0,T}$ the distribution is characterized through $\tilde{\varphi}_1(\theta_{(1)}, \alpha)$ and $p(\theta_{(1)}, \alpha|\mathcal{M}_1, Y_T)$.

Bayesian simulation techniques are used to approximate the posterior model probabilities $\pi_{i,T}$ and to generate draws from the posterior distributions $p(\theta_{(i)}, \alpha|Y_T, \mathcal{M}_i)$

of model parameters. Draws $[\theta'_{(i)}, \alpha]'$ are converted into draws of φ through the mapping $\tilde{\varphi}_i$. A noteworthy feature of our approach is that despite the presence of the DSGE model \mathcal{M}_1 and the informative posterior $p(\alpha|Y_T, \mathcal{M}_1)$ that it generates, the VAR impulse responses have to be identified through the prior $p(\alpha)$, not the DSGE model posterior $p(\alpha|Y_T, \mathcal{M}_1)$, or the overall marginal posterior $p(\alpha|Y_T) = \pi_{0,T}p(\alpha) + \pi_{1,T}p(\alpha|Y_T, \mathcal{M}_1)$.

4.2 VAR and Its Identification

According to the assumptions in Section 2 the fluctuations in the vector y_t of dependent variables are caused by three structural shocks. One of them, $\epsilon_{z,t}$ has a permanent effect on productivity P_t and a transitory effect on market hours $L_{m,t}$. The other two shocks have transitory effects on both hours and productivity. The DSGE model implies that labor productivity and home investment have a common stochastic trend generated by $\epsilon_{z,t}$. The VAR is expressed in vector error correction form

$$\Delta y_t = \Phi_0 + \Phi_{vec} y_{t-1} + \sum_{i=1}^p \Phi_i \Delta y_{t-i} + u_t, \quad u_t \sim iid \mathcal{N}(0, \Sigma_u). \quad (19)$$

The reduced form disturbances u_t are related to the structural disturbances ϵ_t by $u_t = \Phi_* \tilde{\epsilon}_t$, where $\tilde{\epsilon}_t$ is a standardized version of ϵ_t with unit variance.

The DSGE model suggests that Φ_{vec} has rank one and can be expressed as $\phi_{vec} = \mu \lambda'$, where both μ and λ are 3×1 vectors. Strictly speaking, the model implies that $\ln P_t - \ln I_{h,t}$ is stationary, that is, $\lambda = [1, -1, 0]'$. However, rather than imposing this particular co-integration vector, we parametrize λ as $\lambda = [1, -\lambda_{12}, 0]'$ and estimate λ_{12} to allow for a possibly steeper Engel curve for expenditure on consumer durable goods. The VAR specification ensures that productivity and home investment have a common stochastic trend and the cumulative hours process \tilde{L}_{mt} has a second stochastic trend. Let μ_{\perp} be a 3×2 matrix with columns that are orthogonal to μ and define λ_{\perp} as matrix with columns $[\lambda_{12}, 1, 0]'$ and $[0, 0, 1]'$. The stochastic trend in y_t has the form $C_u \sum_{\tau=0}^t u_{t-\tau}$ where

$$C_u = \lambda_{\perp} \left[\mu'_{\perp} \left(I_{3 \times 3} - \sum_{i=1}^p \Phi_i \right) \lambda_{\perp} \right]^{-1} \mu'_{\perp}, \quad (20)$$

see for instance Theorem 4.2 in Johansen (1995). $I_{3 \times 3}$ denotes the 3×3 identity matrix. Due to the composition of λ_{\perp} the first two rows of the matrix C_u are proportional. The factor of proportionality is λ_{12} . In our empirical analysis, we report posterior model probabilities of VAR specifications in which Φ_{vec} has rank one versus specifications in which the matrix has full rank. We also use posterior probabilities to determine the appropriate lag-length p .

The structural shocks $\tilde{\epsilon}_t$ are identified within the VAR specification, if the elements of the 3×3 matrix Φ_* can be uniquely determined based on $\Phi_0, \dots, \Phi_p, \Phi_{vec}$, and Σ_u . Six restrictions can be obtained from the covariance matrix relationship $\Sigma_u = \Phi_* \Phi_*'$. Define $C_{\epsilon} = C_u \Phi_*$. The assumption that the common stochastic trend in productivity and home investment only depends on the disturbances $\epsilon_{z,t}$ generates two additional restrictions: $C_{\epsilon,12} = 0$ and $C_{\epsilon,13} = 0$. Here $C_{\epsilon,ij}$ denotes the element of matrix C_{ϵ} in row i and column j . From the proportionality of the first two rows of C_u it follows that $C_{\epsilon,22} = 0$ and $C_{\epsilon,23} = 0$. Thus, neither $\epsilon_{a,t}$ nor $\epsilon_{b,t}$ have a permanent effect on productivity and home investment. The last restriction is obtained by observing that in response to a temporary home productivity shock, labor productivity and market hours move in opposite directions at impact. Specifically, the restriction $\Phi_{*,13} = (\alpha - 1)\Phi_{*,33}$ obtained from Equation (3) is used.

4.3 Variance Decomposition and Impulse Response Functions

Under the vector autoregression and the log-linear approximation to the DSGE model, the vector process Δy_t has a moving average representation in terms of the standardized shocks $\tilde{\epsilon}_t$:

$$\Delta y_t = \bar{\Delta} y + \sum_{j=0}^{\infty} C_j \tilde{\epsilon}_{t-j}. \quad (21)$$

Define the vectors $M_z = [1, 0, 0]'$, $M_a = [0, 1, 0]'$, and $M_b = [0, 0, 1]'$. The impulse responses to the shock $\tilde{\epsilon}_{s,t}$ are given by

$$\frac{\partial \Delta y_{t+h}}{\partial \tilde{\epsilon}_{s,t}} = C_h M_s, \quad h = 0, 1, \dots, \quad s \in \{z, a, b\}. \quad (22)$$

The h -th order autocovariance matrix of Δy_t is

$$\Gamma_{\Delta y}(h) = \sum_{j=\max\{0,-h\}} C_j C'_{j+h}. \quad (23)$$

The autocovariances can be decomposed according to the contributions of the three structural shocks:

$$\Gamma_{\Delta y}(h) = \Gamma_{\Delta y}^{(z)}(h) + \Gamma_{\Delta y}^{(a)}(h) + \Gamma_{\Delta y}^{(b)}(h), \quad (24)$$

where

$$\Gamma_{\Delta y}^{(s)}(h) = \sum_{j=\max\{0,-h\}} C_j M_k M'_k C'_{j+h}, \quad s \in \{z, a, b\}.$$

Let $\Gamma_{\Delta y, (mj)}(h)$ denote the m 'th row and j 'th column of the matrix $\Gamma_{\Delta y}(h)$. The decomposition of the unconditional variance of the j 'th element of Δy_t is given by the ratios $\Gamma_{\Delta y, (jj)}^{(s)}(h)/\Gamma_{\Delta y, (jj)}(0)$, $s \in \{z, a, b\}$.

The spectrum of the stationary process Δy_t is

$$S_{\Delta y}(\omega) = \sum_{h=-\infty}^{\infty} \Gamma_{\Delta y} e^{-ih\omega}. \quad (25)$$

Just as the autocovariances, the spectrum can be decomposed into the contributions of the three shocks. Let $S_{\Delta y}^{(s)}(\omega)$ denote the contribution of shock s to the spectrum. Since $\Delta y_t = [\Delta \ln P_t, \Delta \ln I_{h,t}, \Delta \ln L_{m,t}]$ is stationary according to \mathcal{M}_0 and \mathcal{M}_1 , the variance decomposition of log hours at frequency ω is given by

$$S_{\Delta y, (33)}^{(s)}(\omega)/S_{\Delta y, (33)}(\omega). \quad (26)$$

Due to the definition of productivity, aggregate output can be recovered from the vectors Δy_t and Δy_{t-1} as $\Delta \ln P_t + \Delta \ln L_{m,t}$. The autocovariance of output growth can be easily obtained from $\Gamma_{\Delta y}(h)$ and its spectrum can be computed according to Equation (25).

Both \mathcal{M}_0 and \mathcal{M}_1 imply that the level of output is integrated of order one. Hence, its autocovariances do not exist and the infinite sum in Equation (25) is not well defined. Let $S_{\Delta \ln Y}^{(s)}(\omega)$ denote the three components of the spectrum of output growth. We define the spectrum of output at business cycle frequencies as

$$S_{\ln Y}^{(s)}(\omega) = \lim_{\phi \rightarrow 1} \frac{S_{\Delta \ln Y}^{(s)}(\omega)}{1 + \phi^2 - 2\phi \cos(\omega)}, \quad \omega > 0. \quad (27)$$

The term $1/[1 + \phi^2 - 2\phi\cos(\omega)]$ is the power transfer function of the AR(1) filter $[1 - \phi L]^{-1}$, where L denotes the temporal lag operator. Equation (27) implies that

$$\frac{S_{\ln Y}^{(s)}(\omega)}{S_{\ln Y}(\omega)} = \frac{S_{\Delta \ln Y}^{(s)}(\omega)}{S_{\Delta \ln Y}(\omega)}. \quad (28)$$

The relative importance of the shocks is not affected by the filter that cumulates the growth rates of output. The filter only alters the relative contribution of different frequencies to the total variance of the filtered process.

4.4 A Small Simulation Experiment

To illustrate the VAR identification procedure and the effect of the non-identifiability of the parameter α , a small simulation experiment is conducted. Data is generated from the DSGE model \mathcal{M}_1 . Posterior mean estimates obtained in the empirical analysis are used to parameterize the DSGE model. In particular, α is set equal to 0.74 (see Table 1). We use sample sizes $T = 20$ and $T = 5000$. The former corresponds to the length of the pre-sample that is used in the empirical analysis to set the prior for the reduced form VAR parameters. The latter sample size, much larger than the typical macroeconomic data set, will highlight the large sample characteristic of our approach. Based on the artificial data we estimate VARs and generate a posterior distributions for the variance decomposition of output at the frequency 1/12 cycles per quarter.

As in the actual empirical analysis, the prior mean of α is chosen to be 0.66. Two different values for the prior standard deviation of α are used. The value $\sigma(\alpha) = 0.02$ implies a 95 percent confidence interval ranging from 0.62 to 0.70. This interval is consistent with a short sample of postwar U.S. labor income shares. The value $\sigma(\alpha) = 0.2$ leads to a confidence interval from 0.46 to 1.06, which covers most plausible as well as many implausible values of α .

Figure 2 visualizes the variance decomposition of output at frequency 1/12 obtained from simulated data. Since the variance decompositions have to sum to one across shocks, they lie in a two dimensional triangular shaped subspace (simplex)

of \mathbb{R}^3 . Each dot in the four panels of Figures 3 corresponds to a draw from the posterior distribution of the variance decomposition based on the VAR. Clusters of points indicate regions of high posterior density. V signifies the posterior mean for the VAR. The three corners z, a, b of the simplexes correspond to decompositions that assign 100 percent of the variation to one structural shock, and 0 percent to the other two shocks.

Informal inspection of the plots suggests that for small samples, such as $T = 20$, the uncertainty with respect to the variance decomposition is dominated by the uncertainty about the reduced form VAR parameters. The dispersion of the posterior draws is quite similar for both choices of $\sigma(\alpha)$. As the sample size is increased to $T = 5000$, the posterior variance of the identifiable VAR parameters decreases substantially. Nevertheless, there remains substantial uncertainty about the role of permanent versus temporary market technology shocks. A drawback of the long-run identification restriction is that it leads to imprecise decompositions.

This paper focuses on the role of the labor demand shock versus the two technology shocks. For $\sigma(\alpha) = 0.02$ the posterior uncertainty, reflected by the vertical spread of the draws, is very small. If $\sigma(\alpha)$ is increased to 0.2 the spread becomes larger. Nevertheless, a comparison of Panel 1 with 3, and Panel 2 with 4 shows that the sample information leads to an update of the beliefs about the relative importance of labor supply shocks. Due to the non-identifiability of α , the proposed identification procedure is not consistent in the sense that the posterior degenerates to the “true” decomposition that corresponds to the parametrized DSGE model as the sample size approaches infinity. Nevertheless, it enables the researcher to extract information from the data and learn about impulse responses and variance decompositions. Under a tight prior for α , e.g. $\sigma(\alpha) = 0.02$, which we think is justified in the subsequent analysis, our procedure will lead to a concentrated posterior in a large sample.

5 Empirical Analysis

The models are fitted to post-war quarterly U.S. data on labor productivity growth, home investment growth and market hours. The construction of the data set is described in Appendix B. Home investment is measured as expenditure on consumer durable goods. The sample period ranges from 1955:I to 1997:IV and the overall sample size is $T = 172$. The first $T_* = 20$ observations are used as training sample to initialize lags and parameterize the prior distributions. The data are plotted in Figure 1. Solid vertical lines correspond to the NBER business-cycle peaks, while dashed lines denote troughs. The peaks coincide with periods in which aggregate hours is high, and troughs coincide with periods in which hours and expenditure on consumer durable goods were at a low. The hours series has no apparent trend, yet its movement is quite persistent.

5.1 Priors

For the DSGE model, we use informative priors for parameters that can be easily inferred (e.g. labor share, average growth rate of productivity), whereas uninformative priors are used for those that cannot be easily observed (e.g. home production technology). The prior distribution used in the estimation is summarized in columns 3 to 5 of Table 1. The shapes of the densities are chosen to match the domain of the structural parameters.

The prior means of labor share in the market production function α and depreciation rate of capital δ are set to 0.666 and 0.025, respectively. The quarterly growth rate of productivity γ is 0.004, and discount factor β is set to 0.993 to yield a 4 percent annual real interest rate *a priori*. These values are commonly used in the literature and can be justified based on a training sample that ranges from 1955:I to 1959:IV. The steady state hours spent for market work L_m and home work L_h are 0.33 and 0.25, respectively, from the Time Use Survey. A larger standard deviation is allowed for L_h , as hours spent on home work may be measured with a greater uncertainty. The prior mean and standard deviation for the steady-state

ratio of home investment to market investment I_h/I_m are obtained from the information in the training sample 1955:I to 1959:IV. For market investment, we use non-residential fixed investment for market investment. The steady-state share of market investment in aggregate output s_{im} is determined by the estimated steady-state real interest rate and capital share in the market production function. The steady-state share of home investment s_{ih} and consumption s_{ic} in output can be calculated by $s_{im} \times I_h/I_m$ and $1 - s_{im} - s_{ih}$, respectively.

We allow for large standard deviations in the prior distributions of home technology parameters as they are not easy to observe. As prior means of the substitution elasticity between market goods and home goods v , and the substitution elasticity between capital and labor in home production τ , we use 1 for both. This case is essentially identical to a conventional utility separable-in-logs in consumption and leisure. The prior mean of the labor share parameter ψ in the home production function is set to 0.666. The weight parameter in the utility χ is determined by other parameters to be consistent with the steady state hours in the market and at home. For the parameters of stochastic process of structural shocks, ρ_a , ρ_b , σ_z , σ_a , and σ_b , we use very diffuse priors. Prior means of persistence parameters for temporary shocks are set to 0.9.

The adjustment cost function is parameterized as follows. First, there is no adjustment cost incurred maintaining the steady-state level of capital. That is, Tobin's q is one: $\phi'(I^*/K^*) = 1$ and $\phi(I^*/K^*) = I^*/K^*$. The elasticity of the investment/capital ratio with respect to Tobin's q , $\eta = (|(I^*/K^*)\phi''/\phi'|^{-1})$ is to be estimated. With no available prior estimate, the prior mean is set to 100 implying small adjustment costs, with a large standard deviation of 100. In the empirical specification of the home production model, we introduce two additional parameters ξ_1 and ξ_2 to adjust the normalization of total hours to one in the data and to capture the average growth rate differential between labor productivity and home investment in the data.

It is assumed that the structural parameters are *a priori* independent of each other. Thus, the joint prior density is simply the product of the marginal densities.

Since all the marginal densities integrate to unity, it follows that the joint prior distribution is proper. The prior distribution for the VAR parameters is described in Appendix C.

5.2 Estimation

Draws from the posterior distributions $p(\theta_{(i)}|Y_T, \mathcal{M}_i)$ cannot be generated directly because in our setup the posteriors do not belong to well-known classes of probability distributions. Draws $\theta_{(0)}^{(s)}$, $s = 1 \dots 90,000$ from the posterior distribution of the VAR parameters are obtained by Gibbs sampling, described in Appendix B. For the parameters of the home production model a random walk Metropolis algorithm, discussed in Schorfheide (2000), is used to obtain a sequence of draws $\theta_{(0)}^{(s)}$, $s = 1 \dots 90,000$. Posterior means, standard errors and confidence intervals are calculated from the sequences of parameter draws. We estimated the VAR model, Equation (19), for different choices of the lag-length p as well as with and without the rank restriction on Φ_{vec} .

Except for λ_{21} , the VAR parameters themselves are not of primary interest. For $p = 1$ and $\Phi_{vec} = \mu\lambda'$ the 90 percent posterior confidence interval for λ_{21} ranges from 0.80 to 1.15. The posterior mean of λ_{21} is 0.97. While the model implies a cointegrating vector of $\lambda = [1, -1, 0]'$ among productivity, home investment and hours, our estimate of $\hat{\lambda} = [1, -0.97, 0]'$ reflects a steeper Engel Curve for consumer durable goods. According to consumer demand analysis (e.g. Houthakker and Taylor (1970) and Bils and Klenow (1998)), most consumer durable goods exhibit income elasticities greater than one. Our estimate of $1/0.97$ corresponds to the income elasticity of expenditure on consumer durable goods.

Columns 6 and 7 of Table 1 contain posterior means and standard errors for the parameters of the home production model.⁹ We will not discuss the parameters of

⁹While McGrattan, Rogerson and Wright (1995) also estimate home production models based on aggregate time series, our analysis distinguishes itself from theirs in several dimensions. First, our approach enables us to compare the predictions from the model to those from the VAR driven by the same set of structural shocks. Second, our approach makes use of indirect evidence from

market labor share, discount factor, average growth rate of productivity, depreciation rate and market hours in detail as they are very standard in the literature. The estimates are $\hat{\alpha} = 0.740$, $\hat{\beta} = 0.978$, $\hat{\gamma} = 0.004$, $\hat{\delta} = 0.016$, and $\hat{L}_m = 0.343$.

Estimates of home technology and structural shocks are as follows. The substitution elasticity between market goods and home goods ν is 2.376. This is slightly higher than the estimates of Rupert, Rogerson and Wright (1995) and McGrattan, Rogerson and Wright (1997). The substitution elasticity between capital and labor in home production τ is 2.568 implying that goods and time are substitutes in home production activity. The labor share in home technology ψ is 0.778 which is slightly higher than that in the market technology. Hours spent on home production activity L_h is 0.165. Temporary home production shock is somewhat more persistent than that of market: $\hat{\rho}_a = 0.774$ and $\hat{\rho}_b = 0.869$. The nature of stochastic variation of home technology $X_{h,t}$, in particular, its relative magnitude and correlation with market productivity shock $X_{m,t}$, is important for business-cycle analysis. Once we identify the underlying innovation to three structural shocks, conditional on time $t-1$ information, the correlation between the market and home productivity $\ln X_{m,t}$ and $\ln X_{h,t}$ can be obtained:

$$\text{corr}_{t-1}[\ln X_{m,t}, \ln X_{h,t}] = \left([1 + (\sigma_a/\sigma_z)^2][1 + (\sigma_b/\sigma_z)^2] \right)^{-1/2} \quad (29)$$

The posterior mean correlation between innovations to market and home productivity $\ln X_{m,t}$ and $\ln X_{h,t}$ is 0.22. The 90-percent posterior confidence interval ranges from 0.16 to 0.27. The estimates are somewhat lower than the values that have been used in the literature. Finally, the adjustment cost parameter η is 23.72 implying a small adjustment cost in capital accumulation.

To obtain posterior model probabilities $\pi_{i,T} = \frac{\pi_{i,0} p(Y_T|\mathcal{M}_i)}{\sum_{i=0,1} \pi_{i,0} p(Y_T|\mathcal{M}_i)}$ conditional on the training sample 1955:I to 1959:IV, one has to evaluate the marginal data densities $p(Y_T|\mathcal{M}_i) = \int p(Y_T|\theta_{(i)}, \mathcal{M}_i) p(\theta_{(i)}|\mathcal{M}_i) d\theta_{(i)}$. Since for both models this micro studies as a prior in our Bayesian estimation. Third, unlike their study, we uncover the comovement of innovation to market and home productivity. This is an important distinction as the relative magnitude of shifts in home technology and its comovement with market technology plays an important role in business cycle analysis.

integral cannot be solved analytically, we use a numerical approximation, known as modified harmonic mean estimator, and described in Geweke (1999). The log marginal data densities are summarized in Table 2.

The results imply that the VAR specification with the highest posterior probability has two lags, that is, $p = 1$ in the notation of Equation (19), and reduced rank $\Phi_{vec} = \mu\lambda'$. The posterior odds of the DSGE model versus the preferred VAR specification are essentially zero. The likelihood based fit of the DSGE model is poor, compared to a simple just-identified reduced rank VAR.¹⁰ Consequently, the overall posterior distribution of population characteristics φ

$$p(\varphi|Y_T) = \pi_{0,T}p(\varphi|Y_T, \mathcal{M}_0) + \pi_{1,T}p(\varphi|Y_T, \mathcal{M}_1) \quad (30)$$

is dominated by the first term ($\pi_{1,T} \approx 0$), which reflects the contribution of the VAR. Here $p(\varphi|Y_T, \mathcal{M}_i)$ denotes the density of φ induced by the mapping $\tilde{\varphi}_i$ and the posterior distribution of $[\theta'_{(i)}, \alpha]'$ conditional on model \mathcal{M}_i .

5.3 Variance Decompositions and Impulse Responses

Our primary interest is to unveil the sources of cyclical variation in hours and output. Table 3 presents the variance decomposition of hours, from both VAR and DSGE, into three structural innovations $\epsilon_{z,t}$, $\epsilon_{a,t}$, and $\epsilon_{b,t}$. It contains posterior means and confidence intervals for the decomposition of the unconditional variance and the

¹⁰To check whether the assumption that hours is stationary is consistent with the estimated model we examine the largest eigenvalue of the autoregressive representation for $(\lambda' y_t, \lambda'_\perp \Delta y_t)$. The 90-percent posterior confidence interval ranges from 0.895 to 0.972. None of the posterior draws of $\theta_{(0)}$ implied an eigenvalue greater or equal to one.

Furthermore, we generate posterior predictive distributions $p(T_j(Y^{rep})|Y_t)$, and examine how far the transformation of the observed data $T_j(Y_T)$ lies in the tails of this predictive distribution. We define T_1 and T_2 as OLS estimator $\hat{\rho}$ and t -statistic ($H_0 : \rho = 1$) for the regression $\ln L_{m,t} = \beta_0 + \rho \ln L_{m,t-1} + \nu_t$. Moreover, T_3 and T_4 are OLS estimator and t -statistic for the regression $\ln L_{m,t} = \beta_0 + \beta_1 t + \rho \ln L_{m,t-1} + \nu_t$. The Bayesian p -values are 0.47, 0.24, 0.49, and 0.15, respectively. This indicates that the autocorrelation observed in the actual hours series $\ln L_{m,t}$ is consistent with the predictive distribution of the model.

variance at several business cycle frequencies: $1/32$, $1/20$, $1/12$, and $1/6$ cycles per quarter.

According to the VAR based decomposition, the temporary labor-supply shifts play a very important role as a source of fluctuations in hours. Looking at the last row of the table, the posterior mean of unconditional variance decomposition indicates that the labor-supply shifts account for about half (46 percent) of the variation. Temporary labor-demand shifts and permanent technology account for 38 percent and 16 percent, respectively. The decomposition is fairly robust across various business-cycle frequencies. Our finding of important labor-supply shifts is comparable to Shapiro and Watson as they assign about 60% of cyclical variation in hours to the stochastic trend component in labor supply. Almost the entire cyclical variation of hours is attributed to preference shocks in Hall (1997).

According to the estimated DSGE model much of the variation of hours (about 70 percent) is caused by the temporary labor-demand shifts due to market productivity fluctuations. Labor-supply shifts caused by home production shocks continue to play a significant role as they account for 25 percent of the variance. The stochastic trend in productivity contributes almost negligible variation in hours, less than 5 percent. This is due to the so-called balanced growth path property. A permanent common productivity shock shifts both labor demand and supply in a similar magnitude at impact. The same is true for the model with conventional utility in consumption and pure leisure, as the income and substitution effect are likely to offset each other in response to a permanent increase in productivity.

Figure 3 visualizes the variance decompositions. The three corners z , a , b of the simplex correspond to decompositions that assign 100 percent of the variation to one structural shock, and 0 percent to the other two shocks. The plots indicate that the VAR decompositions have considerable posterior uncertainty. As described above, while the posterior mean VAR decomposition, V , lies around the center of the simplex, slightly toward b , the posterior mean of the DSGE D lies at the lower right corner, a , as most of variation is caused by temporary demand shifts in the DSGE models.

The decomposition of output variation is reported in Table 4. Compared to the hours series, the role of labor supply shifts for output fluctuations is smaller, as they account for about 17 percent of the variation according to the decomposition of unconditional variance. The most important driving force behind output fluctuations are permanent shifts in technology, as they account for more than half the variation. Temporary shifts in labor demand account for about 36 percent of output variation. Again, the decomposition is robust across various business-cycle frequencies. The DSGE model generates most of output variation from permanent productivity shifts, as they account for almost 90 percent of variation. The contribution of home production shocks is negligible (less than 4 percent). Figure 4 clearly shows that most variation in output is created by either permanent or temporary productivity shifts according to the VAR, as the dots are concentrated at the bottom of the simplex.

We next examine the impulse response functions of the DSGE model and VAR to see if the structural shocks identified from the VAR conform to our economic interpretation. Figure 5 depicts the impulse responses of labor productivity, home investment and market hours to three structural shocks. It shows the DSGE model responses (solid line) and those from the VAR along with the 90 percent confidence interval (dotted lines). In response to a permanent common productivity increase, labor productivity both in the model and data approach the new steady state at a similar pace. Home investment and hours increase immediately in the model, whereas they exhibit somewhat delayed responses, especially for hours, in the VAR. Model responses to a temporary market productivity increase closely trace those from the VAR confirming our interpretation of temporary labor-demand shifts. The response of hours in the VAR is again delayed for about 2 quarters. Finally, in response to a temporary increase in home productivity, while the responses of labor productivity is within the 90 percent confidence interval, it shows a very persistent response in the data, whereas it decays rapidly in the model. Home investment initially decreases and moves above the steady state after 12 quarters in the data, whereas it increases immediately and decays at a much higher pace in the model. Again, hours exhibits somewhat delayed response in the data. Overall, the model,

by and large, reproduces the impulse response in the VAR. Yet the response of hours is delayed for about 2-3 quarters in the data suggesting frictions in the labor market.

According to models with preference shifts, recessions can occur because agents find it optimal to allocate more time in non-market activities. In our DSGE model the attractiveness of non-market activity, or labor-supply shifts in general, is measured by the home technology process. For each draw from the posterior distribution of DSGE model parameters $[\theta'_{(1)}, \alpha]'$ a smoothing algorithm is applied to compute expected values for the technology sequences $\{a_t\}_{t=1}^T$, $\{b_t\}_{t=1}^T$, and $\{z_t\}_{t=1}^T$ conditional on \mathcal{M}_1 and the sample of observations Y_T . These sequences are averaged across the parameter draws and plotted in Figure 6 together with the NBER business cycle peaks and troughs. All six recessions during the sample period are associated with low levels of market productivity. Two business cycle troughs, in March 1975 and November 1982, coincide with unusually high productivity of non-market activities. The strong interpretation of this finding is that an aggregate preference shift contributed to low market employment and output. A weaker interpretation is, that in March 1975 and November 1982 the economic downturn cannot solely be explained by an adverse technology shock in the market. The other four recessions are associated with low productivity in home technology as well as market technology.

6 Conclusion

We investigate the sources of economic fluctuations in the context of a dynamic general equilibrium. A new VAR identification scheme is proposed that identifies three types of underlying disturbances in the aggregate labor market equilibrium: temporary labor-supply shifts, temporary labor demand shocks, and permanent productivity shifts, that eventually move both demand and supply. According to the variance decomposition from the VAR, the labor-supply shift is the most important driving force for the cyclical fluctuation of hours, as they account for about half the variation. However, for output fluctuations, the role of labor-supply shifts is modest. Either permanent or temporary shifts in labor demand, interpreted as permanent

and temporary productivity shifts, respectively, explain more than three-quarters of the variation.

To assess the role of labor-supply shifts in an equilibrium model, a home production model with stochastic variation in non-market technology is estimated, and its predictions are compared to those from the VAR. When the equilibrium model is estimated with the same set of structural shocks, most of the variation of hours is still attributed to the temporary labor-demand shifts. However, the temporary labor-supply shifts play a significant role as they account for 25 percent variation of hours.

In order to make the VAR and DSGE model analysis comparable, it is desirable to use an identification scheme for the VAR that correctly identifies the structural shocks, if the data were in fact generated from the DSGE model. This has also been widely pointed out in the literature on the identification of monetary shocks. However, for many DSGE models the correct identification cannot be achieved based on simple “zero-restrictions” (Canova and Pina, 2000). To overcome this problem, the DSGE model could be re-specified to make it consistent with the “zero-restrictions”, e.g., Rotemberg and Woodford (1998). On the other hand, one could employ an identification scheme that does not solely rely on these “zero-restrictions”. We followed the second path. Unlike in recent papers by Canova and DeNicolò (1998) and Uhlig (1997), who achieve identification based on inequality restrictions, we develop a scheme conditional on one non-identifiable parameter. For our analysis, we find it justifiable to specify a tight prior on this non-identifiable parameter. We view this approach as a promising alternative that has potentially a wide application in macroeconomics and time series analysis.

Parameters		Prior			Posterior	
Name	Range	Density	Mean	S.E.	Mean	S.E.
α	[0,1]	Beta	0.666	0.020	0.740	0.020
β	[0,1]	Beta	0.993	0.001	0.978	0.003
γ	\mathcal{R}	Normal	0.004	0.0005	0.004	0.0004
δ	[0,1]	Beta	0.025	0.001	0.016	0.003
L_m	[0,1]	Beta	0.330	0.020	0.343	0.020
ρ_a	[0,1]	Beta	0.900	0.200	0.774	0.035
ρ_b	[0,1]	Beta	0.900	0.200	0.869	0.038
L_h	[0,1]	Beta	0.280	0.100	0.165	0.040
η	\mathcal{R}^+	Gamma	100.0	100.0	23.72	3.727
ψ	[0,1]	Beta	0.666	0.100	0.766	0.072
ν	\mathcal{R}^+	Gamma	1.000	2.000	2.376	0.325
I_h/I_m	\mathcal{R}^+	Gamma	0.700	0.020	0.684	0.020
τ	\mathcal{R}^+	Gamma	1.000	2.000	2.568	0.475
ξ_1	\mathcal{R}	Normal	2.960	1.000	3.151	0.005
ξ_2	\mathcal{R}	Normal	0.000	0.020	0.005	0.0004
σ_z	\mathcal{R}^+	InvGamma	0.01*	2.000*	0.009	0.0008
σ_u	\mathcal{R}^+	InvGamma	0.01*	2.000*	0.009	0.0008
σ_b	\mathcal{R}^+	InvGamma	0.01*	2.000*	0.021	0.0049

Table 1: Prior and posterior distribution for DSGE model parameters. S.E. denotes standard error. For the Inverse Gamma (u, s) priors we report the parameters u and s . For $u = 2$ the standard error is infinite. The posterior moments are calculated from the output of the Metropolis algorithm.

	VAR(p) rank(Φ_{vec}) = 1	VAR(p) rank(Φ_{vec}) = 3	DSGE
$p = 0$	1230.98	1218.54	
$p = 1$	1413.57	1390.89	
$p = 2$	1408.28	1378.21	
$p = 3$	1395.00	1355.45	
			1304.11

Table 2: Log marginal data densities $\ln p(Y_T|\mathcal{M}_i)$ for VAR specifications and DSGE model. Under equal prior probabilities, posterior odds of specification i versus specification j are given by $\exp[\ln p(Y_T|\mathcal{M}_i) - \ln p(Y_T|\mathcal{M}_j)]$.

Cycles		Vector Autoregression			DSGE Model
Per Quarter	Shock	CI(low)	Mean	CI(high)	Mean
1/32	ϵ_z	0.000	0.211	0.547	0.012
	ϵ_a	0.000	0.300	0.654	0.752
	ϵ_b	0.117	0.490	0.803	0.236
1/20	ϵ_z	0.000	0.215	0.554	0.007
	ϵ_a	0.001	0.273	0.608	0.800
	ϵ_b	0.121	0.512	0.831	0.193
1/12	ϵ_z	0.000	0.213	0.547	0.005
	ϵ_a	0.009	0.262	0.579	0.825
	ϵ_b	0.122	0.524	0.845	0.170
1/6	ϵ_z	0.000	0.194	0.494	0.004
	ϵ_a	0.049	0.266	0.532	0.839
	ϵ_b	0.134	0.541	0.846	0.157
Uncond. Variance	ϵ_z	0.000	0.160	0.512	0.048
	ϵ_a	0.000	0.380	0.723	0.701
	ϵ_b	0.107	0.461	0.782	0.251

Table 3: Decomposition of spectral density for market hours $\ln L_{m,t}$ at 32, 20, 12, and 6 quarters per cycle for horizons 1, 4, and decomposition of unconditional variance. *CI(Low)* and *CI(High)* denote the boundaries of the 90 percent highest posterior density intervals (Bayesian confidence intervals). *Mean* denotes the posterior mean.

Cycles		Vector Autoregression			DSGE Model
Per Quarter	Shock	CI(low)	Mean	CI(high)	Mean
1/32	ϵ_z	0.021	0.463	0.855	0.183
	ϵ_a	0.000	0.364	0.742	0.770
	ϵ_b	0.010	0.173	0.291	0.047
1/20	ϵ_z	0.001	0.487	0.853	0.116
	ϵ_a	0.000	0.341	0.732	0.844
	ϵ_b	0.002	0.172	0.281	0.040
1/12	ϵ_z	0.000	0.499	0.860	0.091
	ϵ_a	0.000	0.329	0.721	0.874
	ϵ_b	0.000	0.172	0.277	0.035
1/6	ϵ_z	0.102	0.514	0.947	0.078
	ϵ_a	0.000	0.308	0.688	0.889
	ϵ_b	0.012	0.178	0.280	0.033
Uncond. Variance	ϵ_z	0.177	0.558	0.957	0.085
	ϵ_a	0.001	0.275	0.622	0.882
	ϵ_b	0.043	0.167	0.262	0.033

Table 4: Decomposition of spectral density for output $\ln Y_t$ at 32, 20, 12, and 6 quarters per cycle for horizons 1, 4, and decomposition of unconditional variance of output growth $\Delta \ln Y_t$. *CI(Low)* and *CI(High)* denote the boundaries of the 90 percent highest posterior density intervals (Bayesian confidence intervals). *Mean* denotes the posterior mean.

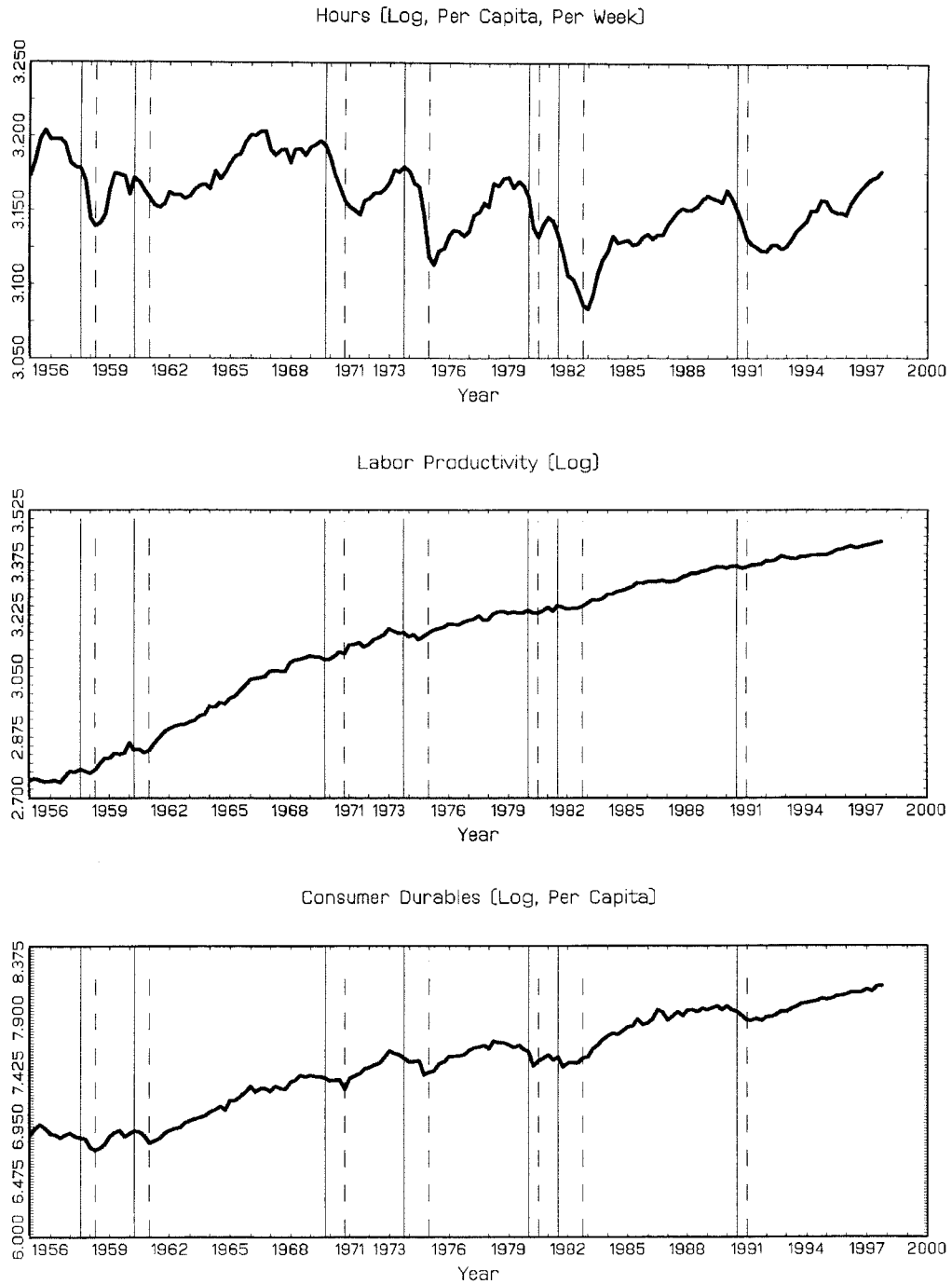


Figure 1: Time series plots of Hours, Labor Productivity, and Consumption Expenditures on Durable Goods (home investment). Solid vertical lines correspond to business cycle peaks, dashed lines denote business cycle troughs (NBER Business Cycle Dating).

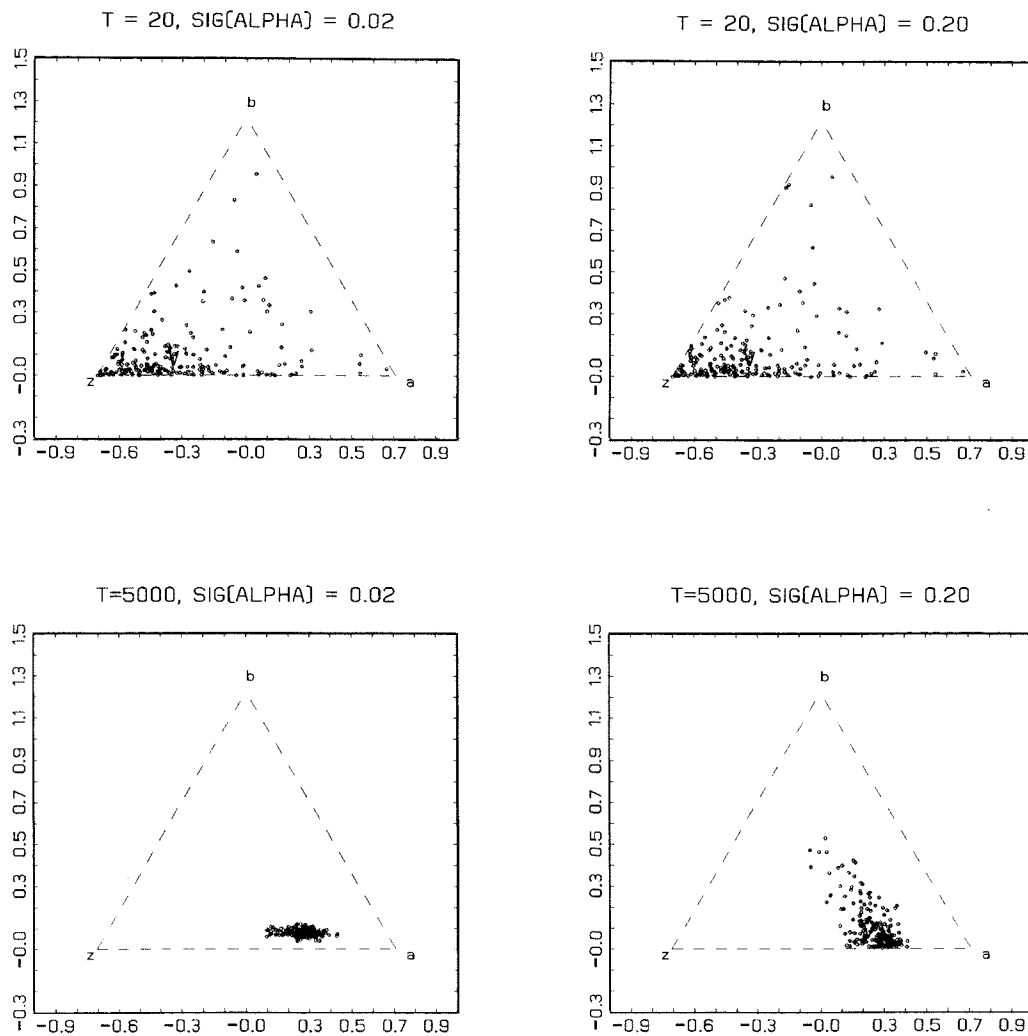


Figure 2: Spectral decomposition of output at frequency $1/12$ cycles per quarter based on artificial observations generated from DSGE model. Sample size: $T = 20$ and $T = 5000$. Prior standard errors of α are 0.02 and 0.20, respectively. Dots correspond to 200 draws from VAR posterior distribution. V indicates posterior mean of VAR.

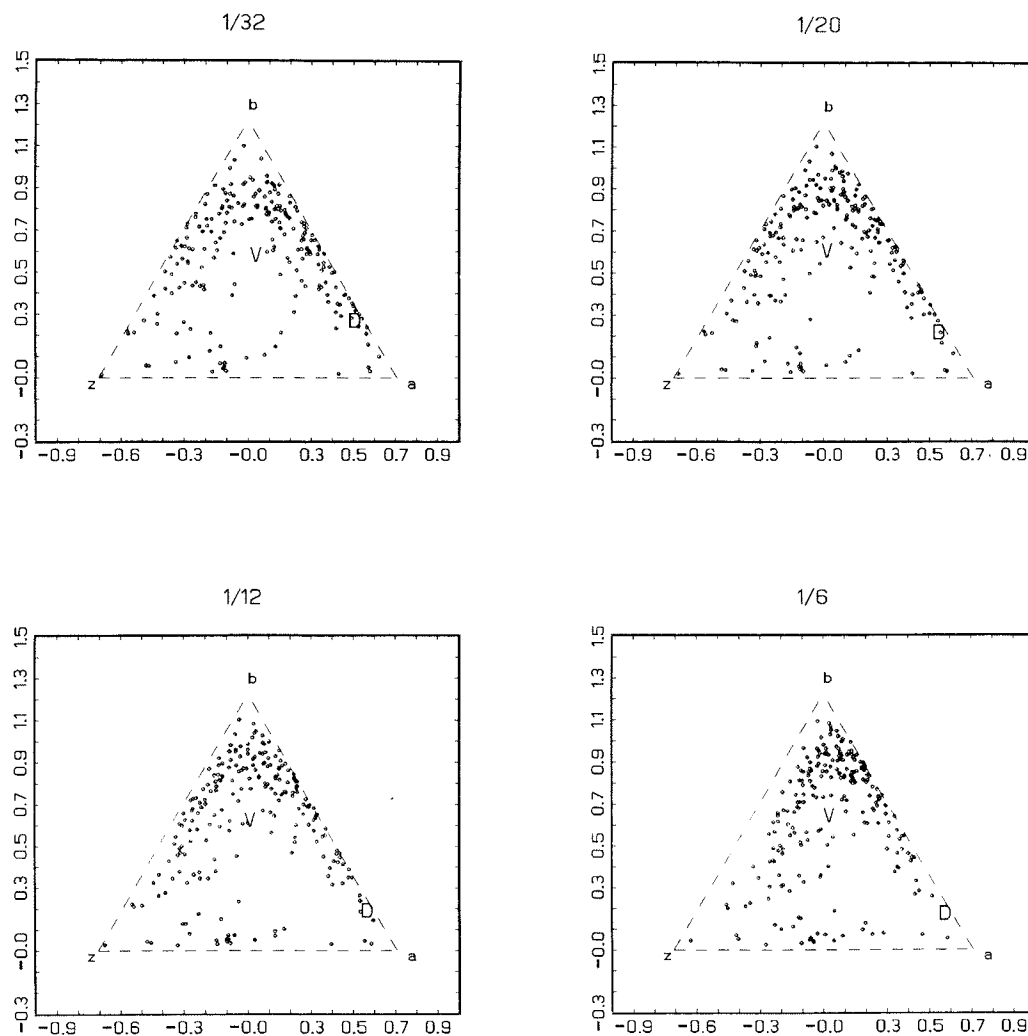


Figure 3: Spectral decomposition of hours at frequencies $1/32$, $1/20$, $1/12$, and $1/6$ cycles per quarter. Dots correspond to 200 draws from posterior distribution. V and D indicate posterior mean of VAR and DSGE models, respectively.

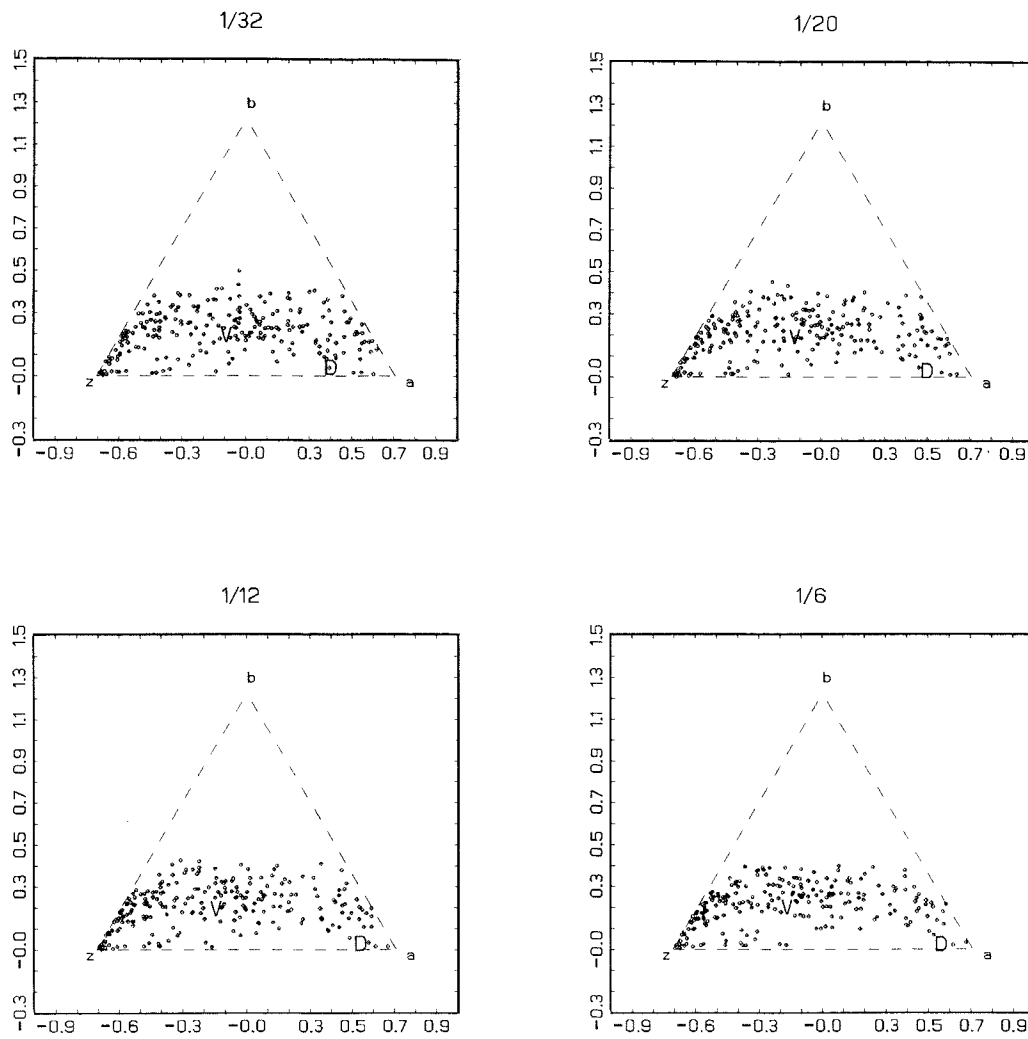


Figure 4: Spectral decomposition of output at frequencies $1/32$, $1/20$, $1/12$, and $1/6$ cycles per quarter. Dots correspond to 200 draws from posterior distribution. V and D indicate posterior mean of VAR and DSGE models, respectively.

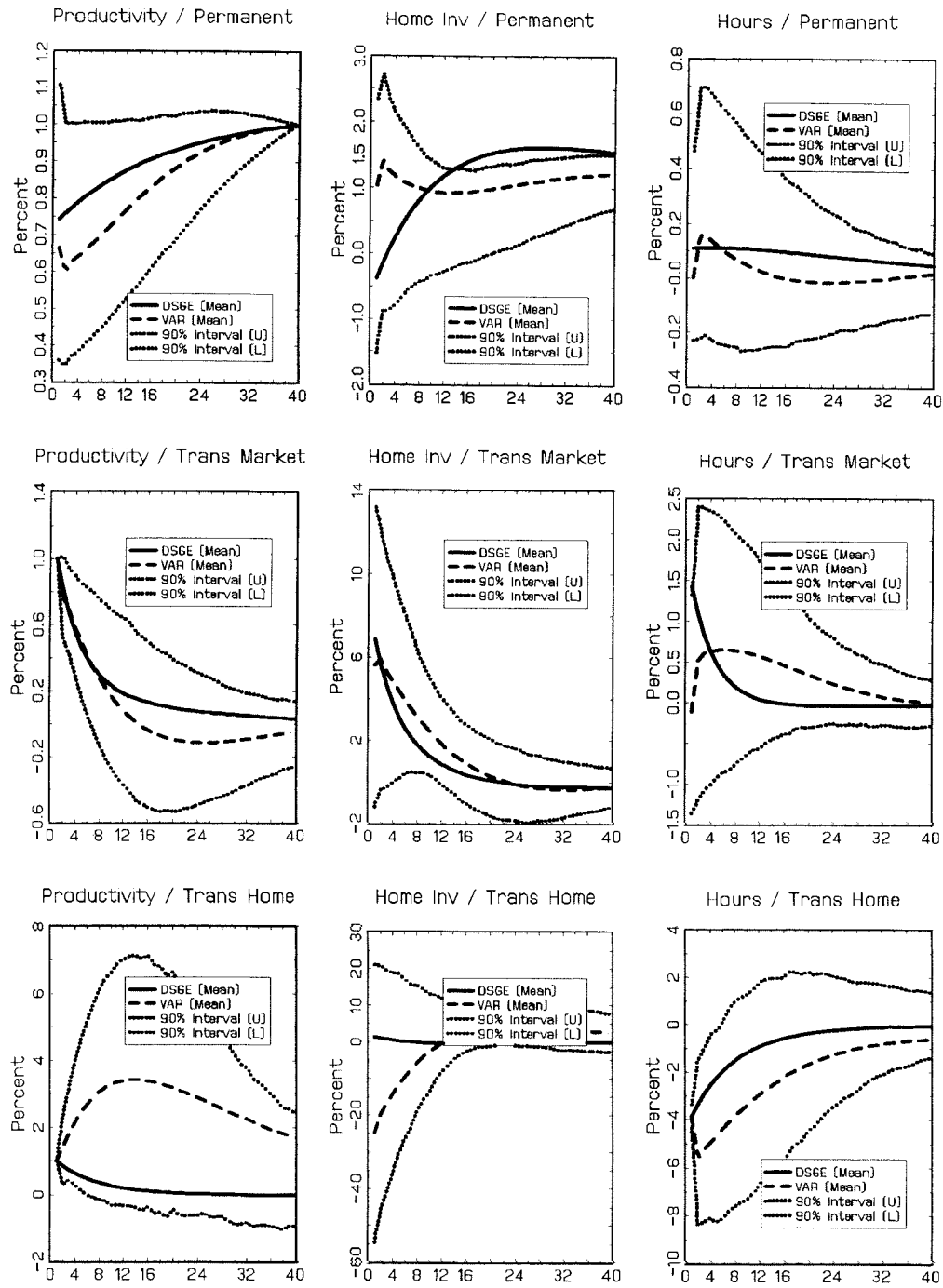


Figure 5: Normalized Impulse-response functions: VAR posterior distribution (mean and 90 percent Bayesian confidence interval) and posterior mean responses of home production model (DSGE).

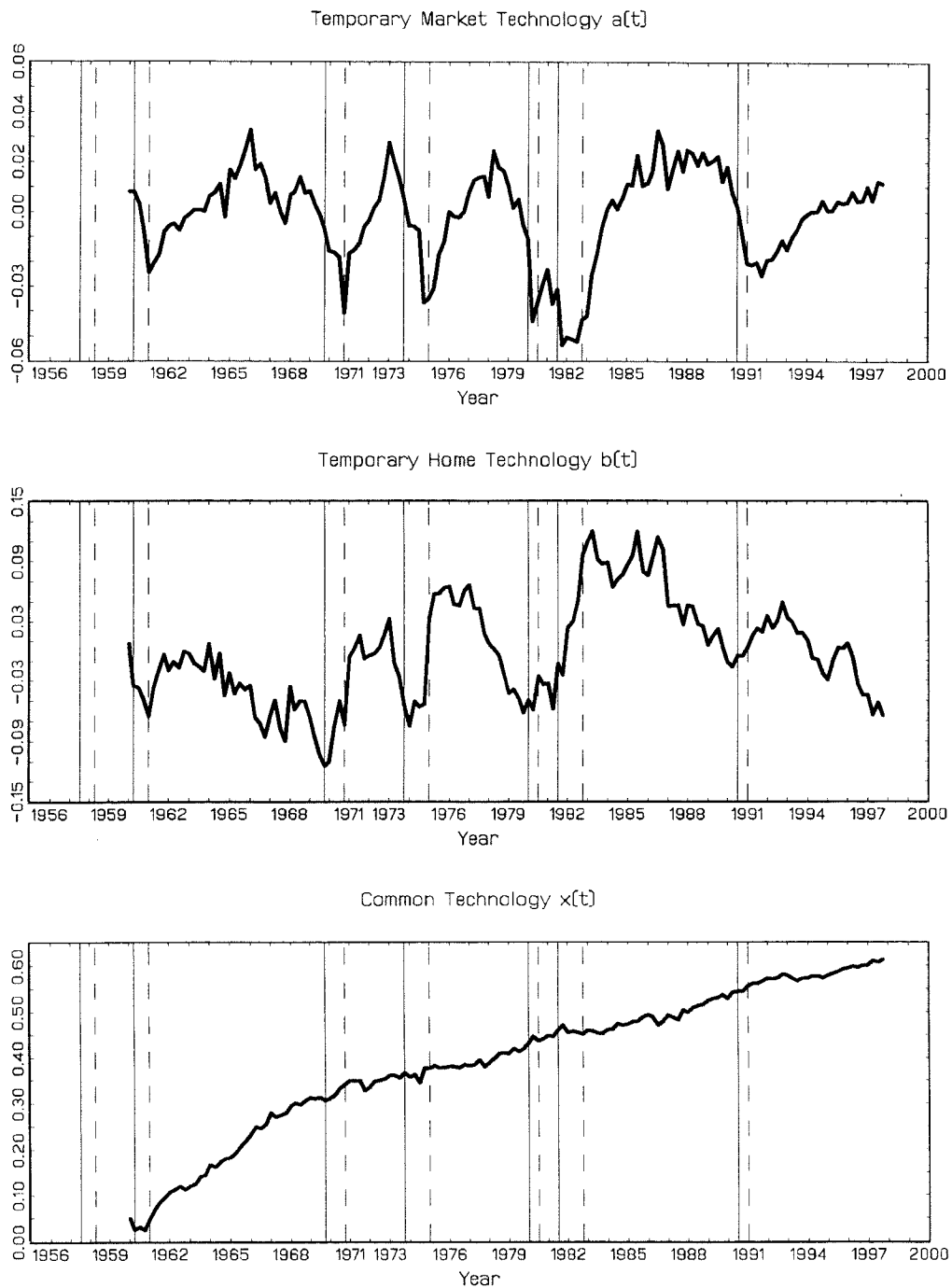


Figure 6: Filtered technology processes a_t , b_t , and x_t , based on posterior estimates of the DSGE model. Solid vertical lines correspond to business cycle peaks, dashed lines denote business cycle troughs (NBER Business Cycle Dating).

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A Labor Demand with Variable Capital Utilization

Consider a Cobb-Douglas production function with inputs in capital services and hours:

$$Y_t = (u_t K_{m,t})^{1-\alpha} (X_{m,t} L_{m,t})^\alpha, \quad (31)$$

where u_t represents the utilization of the capital stock. Suppose the intensive use of capital results in a fast depreciation. As in the main text, the firm solves a profit maximization problem, taking into account the effect of utilization on depreciation:

$$\max_{L_{m,t}, K_{m,t}, u_t} (u_t K_{m,t})^{1-\alpha} (X_{m,t} L_{m,t})^\alpha - W_t L_{m,t} - (R_t + \delta(u_t)) K_{m,t}. \quad (32)$$

For illustrative purposes, assume that the elasticity of depreciation is constant: $\delta(u_t) = \delta_0 \frac{u_t^{\lambda+1}}{\lambda+1}$, where $\lambda > 0$. As $\lambda \rightarrow \infty$, the utilization is held constant and the depreciation rate is fixed. The first order conditions of the profit maximization problem with respect to $L_{m,t}$ and u_t imply that the inverse labor demand schedule still depends on the predetermined capital stock and the market productivity shocks only. However, its slope changes:

$$\frac{\partial \ln W_t}{\partial \epsilon_{b,t}} = \mu(\alpha - 1) \frac{\partial \ln L_{m,t}}{\partial \epsilon_{b,t}}, \quad \mu = \frac{\lambda}{\lambda + \alpha} \leq 1. \quad (33)$$

Therefore, the proposed identification scheme is still valid but the slope of the labor demand schedule is smaller than in the constant utilization case, reflecting an extra margin for the firm to exploit.

B Data Set

The following time series are extracted from DRI: real gross domestic product (GDPQ), consumption of consumer durables (GCDQ), employed civilian labor force (LHEM), civilian noninstitutional population 20 years and older (PM20 and PF20). Population is defined as $POPQ = 1E6 * (PF20 + PM20)$ and used to convert GDPQ and GCDQ into real dollar per capita terms. Thus, $Y_t = GDPQ/POPQ$ and $I_{h,t} = GCDQ/POPQ$.

From the BLS we obtained the series: average weekly hours, private non-agricultural establishments (EEU00500005). Prior to 1963 the BLS series is annual. We used these annual averages as monthly observations without further modification. Our measure of annual hours worked at monthly frequency is $L_{m,t} = 52 * EEU00500005 * LHEM / POPQ$. Hours are converted to quarterly frequency by simple averaging. Our measure of labor productivity is $P_t = Y_t / L_{m,t}$.

C Vector Autoregression

C.1 Prior

Let ΔY_T be the $(T - p) \times n$ matrix with rows $\Delta y'_t$, $t = p + 1, \dots, T$ (the first p observations are used to initialize lags). Let $k = 3 + np$, $X_T(\lambda_{21})$ be the $(T - p) \times k$ matrix with rows $x'_t = [1, t, (1, -\lambda_{21}, 0)y_{t-1}, \Delta y'_{t-1}, \dots, \Delta y'_{t-p}]$, U_T be the matrix with rows u'_t , and $B = [\Phi_0, \Phi_{tr}, \alpha, \Phi_1, \dots, \Phi_p]'$. We include a deterministic trend with coefficient vector Φ_{tr} in the specification of \mathcal{M}_0 to capture long-run shifts in market hours due to structural changes in labor market participation behavior. The reference model can be expressed in matrix form as

$$\Delta Y_T = X_T(\lambda_{21})B + U_T. \quad (34)$$

The home production model implies that $\lambda_{21} = 1$. We relax this implication and use the prior

$$\lambda_{21} \sim \mathcal{N}(1, 0.01). \quad (35)$$

The prior for B and Σ is constructed from a training sample $t = p + 1, \dots, T_*$. Let ΔY_* and $X_*(\lambda_{21})$ be matrices with rows $\Delta y'_t$ and x'_t as defined above, $t = p + 1, \dots, T_*$. Define

$$\hat{B}_* = (X_*'X_*)^{-1}X_*'\Delta Y_*, \quad \hat{\Sigma}_{u,*} = (T_* - p)^{-1}(Y_* - X_*\hat{B}_*)'(Y_* - X_*\hat{B}_*). \quad (36)$$

Then we obtain

$$\begin{aligned} \Sigma_u | \lambda_{21}, Y_* &\sim IW\left((T_* - p)\hat{\Sigma}_{u,*}, T_* - k - p\right) \\ \text{vec}(B) | \Sigma_u, \lambda_{21}, Y_* &\sim \mathcal{N}\left(\text{vec}(\hat{C}_*), \Sigma_u \otimes (X_*'X_*)^{-1}\right), \end{aligned} \quad (37)$$

where IW denotes the Inverted Wishart distribution. In our empirical analysis the size of the training sample is $T_* = 20$ and the lag-length is $p = 2$.

C.2 Posterior Simulation

A Gibbs sampler is used to generate draws from the posterior distribution of the VAR parameters $(B, \Sigma_u, \lambda_{21})$. We draw successively from the conditional posteriors $p(B, \Sigma_u | \lambda_{21}, Y_T, \mathcal{M}_0)$ and $p(\lambda_{21} | B, \Sigma_u, Y_T, \mathcal{M}_0)$. The distribution of $\Sigma_u | \lambda_{21}, Y_T$ is Inverted Wishart and $B | \Sigma_u, \lambda_{21}, Y_T$ is multivariate normal. The parametrization is given by replacing ΔY_* and $X_*(\lambda_{21})$ with ΔY_T and X_T in Equations (36) and (37). To characterize the posterior distribution of λ_{21} , define $\Delta \tilde{Y}_T$ with rows $\Delta \tilde{y}_t' = [\Delta y_t - \Phi_0 - \Phi_{tr}t - \mu(1, 0, 0)y_{t-1} - \sum_{i=1}^p \Phi_i \Delta y_{t-i}]'$ and \tilde{X}_T with rows \tilde{x}_t' , where $\tilde{x}_t = \mu(0, -1, 0)y_{t-1}$. Then one obtains

$$\lambda_{21} | B, \Sigma_u, Y_T \sim \mathcal{N}(m_\lambda, v_\lambda), \quad (38)$$

where $v_\lambda^{-1} = 1/0.01 + tr[\Sigma^{-1} \tilde{X}_T' \tilde{X}_T]$, $m_\lambda = v_\lambda \left(\frac{1}{0.01} + \frac{tr[\Sigma^{-1} \Delta \tilde{Y}_T' \tilde{X}_T]}{tr[\Sigma^{-1} \Delta \tilde{X}_T' \tilde{X}_T]} \right)$, and $tr[\cdot]$ denotes the trace operator.