Economics 706 Preliminary Examination
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There is just one question, with many parts. Good luck!

Consider an N-variable dynamic single-factor model for approximating the dynamics of real macroeconomic activity, as pioneered by Geweke, Sargent and Sims, and significantly extended by Stock and Watson, among others:

$$\begin{pmatrix} y_{1t} \\ \vdots \\ y_{Nt} \end{pmatrix} = \begin{pmatrix} \mu_1 \\ \vdots \\ \mu_N \end{pmatrix} + \begin{pmatrix} \lambda_1 \\ \vdots \\ \lambda_N \end{pmatrix} f_t + \begin{pmatrix} \varepsilon_{1t} \\ \vdots \\ \varepsilon_{Nt} \end{pmatrix}$$

$$f_t = \phi_1 f_{t-1} + \phi_2 f_{t-2} + \eta_t ,$$

where y_{it} is an indicator of real activity (e.g., retail sales growth), f_t is a latent common factor, and all stochastic shocks are white noise with zero mean and constant finite variance, independent at all "own" and "cross" leads and lags.

(1) What is $E(f_t)$? What is $E(y_{it})$?

(2) Under what conditions is f_t covariance stationary? Under what conditions is y_{it} covariance stationary? From this point onward, assume that those conditions are satisfied unless explicitly stated otherwise.

(3) Calculate the cross-covariance function of y_i and y_j, $\gamma_{y_i y_j}(\tau)$. How is it related to the autocovariance function of f_t, $\gamma_f(\tau)$?

(4) Suppose that the polynomial $\Phi(L) = 1 - \phi_1 L - \phi_2 L^2$ has a pair of complex conjugate roots. What does that imply for the behavior of $\gamma_f(\tau)$, as $\tau \to \infty$?

(5) Cast the system in state space form and display its measurement and transition equations. What can you say about the eigenvalues of the coefficient matrix in the transition equation?

(6) Assume now that all stochastic shocks are iid Gaussian. Show concisely how to evaluate the Gaussian log likelihood via a time-domain prediction-error decomposition in conjunction with the Kalman filter.

(7) Continue to assume that all stochastic shocks are iid Gaussian. Show how to do a full Bayesian analysis of the model using a Carter-Kohn multi-move Gibbs sampler.

(8) Assume instead that η_t follows a conditionally Gaussian GARCH(1,1) process, as for example in a typical financial economic, as opposed to macroeconomic, application. Write down an explicit expression for the process. What conditions are needed to ensure positivity of the unconditional variance? Positivity of the conditional variance? Covariance stationarity? Assuming that those conditions are satisfied, is η_t iid? Serially uncorrelated? Serially independent? Unconditionally Gaussian? How does the state space representation of the system change?

(9) Return now to the basic model with strong white noise innovations, and suppose that the polynomial $\Phi(L)$ has two real roots, $r_1 = 1$ and $r_2 = 1.2$. Is F_t I(0) or I(1)?

(10) Show that all observable variables are I(1). Are they also cointegrated? If so, how many cointegrating relationships are there in the system, and how many common trends, and what *are* the cointegrating relationships and common trends? Assuming known system parameters, what can be said about the liner least-squares forecasts $y_{t+h,t}$ calculated via the Kalman filter as $h \to \infty$?