

Preliminary Examination

Econ 702-Macroeconomics
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Instructions: There is one question, divided in several subquestions. The number in brackets represents the number of points awarded for answering correctly the each subquestion. Total available points are 125. If the description of the environment seems incomplete to you, explain why, make the assumptions that you deem necessary to proceed and continue. Good Luck!

1. Vacancies, Unemployment and the Capital Market. (125 points) We develop and characterize the equilibrium of a version of Mortensen and Pissarides (ReStud 1994) in which firms need to find both capital and labor to produce.

Firms maximize the present value of profits discounted at the factor $\beta \in (0, 1)$ (measured in units of output). Firms produce y units of output when they have 1 unit of capital and 1 unit of labor. Firms produce 0 units of output otherwise. Firms enter the economy by paying a cost of $k > 0$ units of output. Firms exit the economy with probability $\delta \in (0, 1)$ per period. Workers maximize the present value of income discounted at the factor β (measured in units of output). Workers earn an income of $h > 0$ units of output when unemployed, and an income of w units of output when employed. Capital sellers maximize the present value of income discounted at the factor β (measured in units of output). Capital sellers pay a cost of $q > 0$ units of output to produce a unit of capital.

After it enters the economy, a firm goes to the capital market. Firms searching in the capital market and capital sellers come together through a CRTS matching function $M_K(b, 1)$, where b is the measure of firms looking to buy capital and 1 is the measure of sellers of capital. We denote as $\lambda_{Kf}(b) = M_K(1, 1/b)$ the capital-finding probability, i.e. the probability that a firm meets a seller of physical capital. We assume $\lambda_{Kf}(0) > 0$, $\lambda_{Kf}(\infty) = 0$ and $\lambda'_{Kf}(b) < 0$. Upon meeting, the firm and the seller Nash bargain over the price p of a unit of capital. The firm's bargaining power is $\sigma \in (0, 1)$, the seller's $1 - \sigma$.

Once the firm has acquired capital, it moves onto the labor market. Firms searching in the labor market and unemployed workers come together through a CRTS matching function $M_L(u, v)$, where u is the measure of unemployed workers and v is the measure of searching firms (i.e. vacancies). We denote as $\lambda_{Lf}(v/u) = M_L(u/v, 1)$ the job-filling probability, i.e. the probability that a firm meets an unemployed worker. We denote as $\lambda_{Lw}(v/u) = M_L(1, v/u)$ the job-finding probability, i.e. the probability that an unemployed worker meets a vacant firm. We assume $\lambda_{Lf}(0) = 1$, $\lambda_{Lf}(\infty) = 0$, $\lambda'_{Lf}(v/u) < 0$, and $\lambda_{Lw}(0) = 0$, $\lambda_{Lw}(\infty) = 1$, $\lambda'_{Lw}(v/u) > 0$. Upon meeting, the firm and the worker Nash bargain over the current wage w . The firm's bargaining power is $\eta \in (0, 1)$, the worker's $1 - \eta$. If the firm and the worker agree on the terms of trade, they produce $y > h$ units of output. The terms of trade are re-bargained in every period. Firm-worker matches are dissolved only when the firm exits the market.

We use the following timing: (i) search takes place; (ii) entry of new firms and bargaining take place; (iii) production and consumption take place. All value functions are measured at the beginning of stage (iii).

- a. (5) Write down the Bellman Equation for C , the lifetime profit for a firm that is seeking capital.
- b. (5) Write down the Bellman Equation for Z_0 , the lifetime profit for a firm that has capital and is seeking labor.

- c. (5) Write down the Bellman Equation for $Z_1(w)$, the lifetime profit for a firm with capital and labor that has agreed to pay its worker a wage of w in the current period.
- d. (5) Write down the Bellman Equation for V_0 , the lifetime utility for an unemployed worker.
- e. (5) Write down the Bellman Equation for $V_1(w)$, the lifetime utility for a worker employed at the wage w .
- f. (5) Formulate the Nash bargaining problem between a firm and a capital seller. The threat point for the seller is the cost of capital q . The threat point for the firm is the value of searching C . Solve for the equilibrium price of capital, p^* , as a function of q , C , Z_0 and the bargaining power of the firm σ .
- g. (5) Formulate the Nash bargaining problem between a vacant firm and an unemployed worker. The threat point for the firm is the value of searching Z_0 . The threat point for the worker is the value of searching V_0 . Using the first-order condition of the Nash problem, express the equilibrium gains from trade accruing to the worker, $V_1(w^*) - V_0$ as a function of the bargaining power of the firm η and the surplus $S \equiv V_1(w) + Z_1(w) - V_0 - Z_0$. Similarly, express the equilibrium gains from trade accruing to the firm, $Z_1(w^*) - Z_0$ as a function of η and S .
- h. (5) Using the Bellman Equations for V_0 , V_1 , Z_0 and Z_1 , write down the Bellman Equation for the surplus S .
- i. (5) Using your findings in (g), solve for S as a function of the job-finding and job-filling probabilities and of fundamentals.
- j. (5) Using your findings in (b), express Z_0 as a function of S and the job-filling probability.
- k. (5) Using your findings in (f), express C as a function of Z_0 and the capital-finding probability.
- l. (5) Write the stationarity condition for unemployed workers, u . Make sure to identify the inflow and the outflow of unemployed workers. We will assume that the solution to the stationarity condition for u is a function of vacancies $u = f(v)$, with $f(0) = 1$, $f(\infty) = \delta/(1 + \delta)$, $f'(v) < 0$.
- m. (5) Write the stationarity condition for vacant jobs, v . Make sure to identify the inflow and the outflow of vacant jobs.

- n.** (5) Using (l) and (m), describe the relationship between the measure of firms seeking capital (b) and the measure of firms seeking labor (v), call it $v = g(b)$.
- o.** (5) Write down the firms' free-entry condition. [Recall that entry takes place after search and before production]. Then, express the benefit of entering as a function of fundamentals of the model and b .
- p.** (10) Prove that there exists a unique steady-state equilibrium value for the measure of firms seeking capital. Use this result, to prove that there exists a unique steady-state equilibrium.
- q.** (10) Suppose that the seller's cost of capital q increases. What is the effect on the steady-state value of b ? Give some economic intuition for your finding.
- r.** (10) What is the steady-state effect on vacancies, the job-finding rate of unemployed workers, the job-loss rate of employed workers and on unemployment? What is the effect on labor productivity? Explain your findings thoroughly.
- s.** (10) Suppose that the firm's bargaining power σ falls. What is the effect on the steady-state value of b ? Explain your findings.
- t.** (10) What is the steady-state effect on vacancies, the job-finding rate of unemployed workers, the job-loss rate of employed workers and on unemployment? What is the effect on labor productivity? Explain your findings thoroughly.