Preliminary Examination

Econ 702-Macroeconomics
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Instructions: There is one question, divided in several subquestions. The number in brackets represents the number of points awarded for answering correctly the each subquestion. Total available points are 75. If the description of the environment seems incomplete to you, explain why, make the assumptions that you deem necessary to proceed and continue. Good Luck!

1. Unemployment Benefits and Job Search. Consider the following decision-theoretic problem. A worker has just became unemployed and is entitled to T periods of unemployment benefits, with $T > 2$. While unemployed, the worker searches the labor market and receives job offers with probability $\lambda \in (0, 1)$ per period. If the worker accepts a job offer, he becomes employed and remains employed forever after. If the worker rejects the offer, he keeps on searching. A job offer is a draw $\hat{w}$ from a twice-differentiable cumulative distribution function $F(\hat{w})$ with support $[w_\ell, w_h]$, where $\hat{w}$ denotes the wage that the worker will receive if he accepts the job.

The worker maximizes the present value of income discounted at the factor $\beta \in (0, 1)$. The worker's income is w if the worker is employed at the wage w; z if the worker is unemployed and out of unemployment benefits; $b + z$ if the worker is unemployed and still on unemployment benefits. You can think of z as some basic income that the worker can access and of b as unemployment benefits. We assume that $w_\ell < z < b + z < w_h$.

In every period, events take place as follows: first the worker looks for a job (if unemployed); second the worker produces (if he found a job or had a job already); finally the worker receives his income (z, $z+b$ or w). Let $V(w)$ denote the lifetime utility of a worker employed at the wage w. Let U_0 denote the lifetime utility of a worker who is unemployed and has exhausted his unemployment benefits. Similarly, let U_t denote the lifetime utility of a worker who is unemployed and has t periods of unemployment benefits left, with $t = 1, 2, \ldots .T$. All value functions are measured at the beginning of the consumption stage.

a. (2.5) Write the Bellman equation for $V(w)$ and solve it.

b. (2.5) Write the Bellman equation for U_0.

c. (10) Define the operator T_0 as

$$T_0 U = z + \beta\lambda \int \max\{V(\hat{w}), U\} dF(\hat{w}) + \beta(1 - \lambda)U.$$

Prove that the operator T_0 is a contraction.

d. (5) Prove that a worker who has run out of unemployment benefits follows a reservation strategy such that there exists an R_0 such that he accepts a job if it pays a wage $\hat{w} > R_0$, he rejects a job if it pays a wage $\hat{w} < R_0$ and he is indifferent if $\hat{w} = R_0$.

e. (5) Write down an expression that implicitly defines R_0 in terms of fundamentals.

f. (5) Write the Bellman equation for U_1. Prove that $U_1 > U_0$.

g. (5) Prove that the worker with 1 period of unemployment also follows a reservation strategy. Define his reservation wage R_1 and prove that $R_1 > R_0$.

h. (5) Write down the Bellman equation for U_t, with $t \geq 1$.

i. (5) Define the operator T_1 as

$$T_1 U = b + z + \beta\lambda \int \max\{V(\hat{w}), U\} dF(\hat{w}) + \beta(1-\lambda)U.$$

Prove that the operator T_1 is monotonic.

j. (10) Prove that $U_{t+1} > U_t$.

k. (5) Prove that $R_{t+1} > R_t$.

l. (5) Derive the job-finding probability h_t for an unemployed worker with t periods of unemployment benefits left. Is h_t increasing, decreasing or independent of t. Explain your findings.

m. (5) Suppose that the government extends the unemployment benefits from T to T' periods, with $T' > T$. What happens to the path of job-finding probability for a newly unemployed worker? Explain your findings.

n. (5) Write an expression for the average duration of an unemployment spell. What happens to the average duration of the unemployment spell in response to the increase in the duration of benefits? Explain your findings.