704 Part II

In the following there are 3 questions for 100 points. Be as BRIEF as you can and good luck.

1. **Lucas trees**

   Assume there is a representative agent economy. Each agent owns a tree that produces fruit z_t which follows a Markov chain with transition matrix Γ. In additionn, each agent has one (divisible) unit of time that can be transformed into fruit on a one to one basis. The agent has preferences given by

$$E\left\{\sum_{t=0}^{\infty}\beta^t\left[\frac{c_t^{1-\sigma}}{1-\sigma}+\alpha(1-n_t)\right]\right\}$$

   where n_t is the amount of time spent producing additional fruit.

   (a) (10 points) Define equilibria recursively assuming that the tree prices only depend on the current yield and that there is no market for labor (each agent produces with their own time fruit for themselves).

   (b) (10 points) Write a formula for an option to buy land tomorrow at price p_1 and then reselling it at price p_2 the period after.

   (c) (10 points) Make any assumptions that you want to ensure that in equilibrium the amount of consumption is constant.

2. **Parks and Recreation**

   There is an economy with identical agents with preferences given by

$$E\left\{\sum_{t=0}^{\infty}\beta^t\left[\frac{c_t^{1-\sigma}}{1-\sigma}+\alpha(1-n_t)^{\frac{1}{2}}+\gamma\, P_t^{\frac{1}{2}}\right]\right\}$$

   where c_t is their own consumption at time t, n_t is the fraction of their own time worked at time t, and P_t are public parks. Their initial wealth is A.

   The technology to produce output uses capital (that depreciates at rate δ) and labor:

$$Y_t = F(K_t, N_t)$$

   (a) (10 points) What conditions would be satisfied in a Pareto Optimum in steady state?

   Imagine now that the government levies income taxes and issues debt to pay for the parks. Its initial debt is B.

   Imagine now that this is a small open economy and borrowing and lending can occur and sell at the international rate $\bar{r}$.

   (b) (15 points) Define Recursive competitive equilibrium for this case and for the appropriate policies.

   (c) (10 points) Give an expression for the wage, and for the stock of capital.

3. **Industry and Search**

Imagine a one period economy with a measure one of foreign-owned trees or firms in each island $i = \{1, \cdot, I\}$ where each firm has one unit of bananas that cannot travel. Island i produces better bananas than location $i-1$ for all i. There is a measure x of households that differ in their endowment of a numeraire good over an interval $[0, \overline{a}]$, with $x([0, \overline{a}]) = 1$. Households have preferences given by $u(c, \sum_i \alpha_i b_i)$, where c is consumption of the numeraire, b_i is the quantity of bananas of type i. $u(c, 0) = 0$, $u_2(c, 0) = \infty$.

Households choose one island only to go to. Upon entering the island households may find the central location where bananas are traded. The probability that they find such location is decreasing in the measure of households that go to the island and is given by $\Phi(t_i, h_i)$, $\Phi_h(t_i, h_i) < 0$ where t is the measure of trees and h the measure of households in that island. Upon completion of the search of the central location, there is trade among all firms and all households present.

(a) (10 points) Define an allocation for this economy.

(b) (15 points) Define a competitive equilibrium with special attention to the problem of the households.

(c) (10 points) Imagine that magically all islands were to become one big island with only one central location where trade happens. What changes respect to the previous two questions?