704 Part II

In the following there are 10 questions for 100 points. Be as BRIEF as you can and good luck.

Growth Models

Consider an economy with two regions indexed by $i \in \{M, B\}$ (Madeira and Baleares). Each region is populated by a continuum of infinitely lived identical agents that is taken to be of measure one (this just means that there is a representative agent in each region). Agents cannot change regions.

In Region M there is a constant returns to scale technology given by $f^M(z, k^M, n^M)$, where shocks follow a Markov chain with transition $\Gamma^M_{z,z'}$. Region B has a constant returns to scale technology given by $f(k^B, n^B)$ (no shocks). The only form of wealth is real capital.

Preferences are given by the expected discounted value (both regions' agents have the same discount factor, β) of a region specific strictly concave current utility function. While agents in region M like leisure, those in region B do not.

Assume for the first three questions that output cannot be transferred across regions. Try to be as simple as possible in your answers by exploiting the specific properties that are described.

1. (10 points) Set the value of the shock to its unconditional mean $\bar{z}^M$, and define a sequence of (complete) markets or an Arrow-Debreu (valuation) equilibrium for this economy.
2. (5 points) Define what the steady state of this economy is, and state the conditions that have to be satisfied.
3. (5 points) Would the allocation in this equilibrium be the same than that that would entail if we posed a separate equilibrium for each region?

   Now go back to the stochastic version of this economy and also assume that output can be moved between regions, but that it takes one period to ship output to the other region, i.e. output produced in one region in period t can become either consumption or investment in the other region only in $t + 1$.

4. (15 points) Define Recursive Competitive Equilibrium. Make sure that the market structure that you are imposing implies that equilibria are Pareto Optimal. (No need to prove it).
   Now, assume that the production function of the M-region is given by $f^B(k^B, n^B, N^M)$, Notice that this technology implies the existence of an externality: productivity of the B-region goes up when M-people work harder.
5. (5 points) Define now Recursive Competitive Equilibrium.
6. (10 points) Formulate the equal weight Pareto problem in recursive form (write down the relevant social planner's problem) and discuss the the differences in the implied allocation of this problem with that of the Recursive Competitive Equilibrium.

Incomplete Markets

Consider an economy with many identical consumers, each of them with one efficient unit of labor each period that if used for fishing produces one unit of the good. Preferences are given by

$$E\left\{\sum_{t=0}^{\infty}\beta^t \; s_t \; \frac{c_t^{1-\sigma}}{1-\sigma}\right\}$$

where $\beta < 1$ and where $s_t \in \{s^1, s^2\}$ follows a Markov chain with transition Γ, a shock to preferences.

Each period the agent has access to a storage technology that transforms one unit of the good this period into 1.01 units next period.

In addition to the storage technology, the agent has access to a production technology that produces

$$y_t \;=\; k^{\theta}, \qquad\qquad 0 < \theta < 1$$

if half a unit of time is devoted to operate such technology and zero otherwise.

In the absence of credit markets

7. (15 points) Write the problem of the agent.
8. (15 points) Is there a stationary distribution? Briefly explain.
9. (15 points) Will the poorest people operate the production technology? Will the richest people be using a lot of the storage technology? Discuss the relation between wealth and production.
10. (5 points) Imagine a slightly different world than this one, where agents can borrow up to $\underline{b}$. What is the maximum $\underline{b}$ that makes default impossible.