

GOOD LUCK! (Kopecky and Suen, 2010) This question tests your ability to write out MATLAB code as you used on your three assignments (and *not* any other computer language). Consider the following AR1 process $z' = \rho z + \varepsilon$, where $\varepsilon \sim N(0, \sigma)$. A macroeconomist wants to approximate the AR1 process using a N -state Markov chain, where $N > 2$. The variable z is constrained to always lie in a time-invariant grid of N *equally* spaced points centered around 0, so that $z \in \{z_1, \dots, z_N\}$ with $-z_1 = z_N = \psi > 0$, where $\psi = \sigma\sqrt{N-1}/\sqrt{(1-\rho^2)}$. The transition matrix, T^N , has the form

$$T^N = \begin{bmatrix} \pi_{11} & \cdots & \pi_{1N} \\ \vdots & \ddots & \vdots \\ \pi_{N1} & \cdots & \pi_{NN} \end{bmatrix}$$

where π_{kl} is the odds of going *from* state k to state l .

1. Write out the MATLAB code for computing this transition matrix, which is generated using the procedure outlined below.

- (a) Generate a sequence of transition matrices, T^j , recursively for $j = 3, \dots, N$ as follows:

$$T^j = p \begin{bmatrix} T^{j-1} & \mathbf{0} \\ \mathbf{0}' & 0 \end{bmatrix} + (1-p) \begin{bmatrix} \mathbf{0} & T^{j-1} \\ 0 & \mathbf{0}' \end{bmatrix} + (1-p) \begin{bmatrix} \mathbf{0}' & 0 \\ T^{j-1} & \mathbf{0} \end{bmatrix} + p \begin{bmatrix} 0 & \mathbf{0}' \\ \mathbf{0} & T^{j-1} \end{bmatrix},$$

where

$$T^2 = \begin{bmatrix} p & 1-p \\ 1-p & p \end{bmatrix},$$

$\mathbf{0}$ is an $(j-1) \times 1$ *column* vector of zeros, and $p = (1 + \rho)/2$.

- (b) At the end of each iteration, *all but* the first and last rows of T^j should be divided by 2.

2. If

$$p = [p_1 \quad p_2 \quad p_3],$$

is a probability vector describing the current odds of being in each state, then how would you compute the odds of being in each state for next period? How would you compute the stationary distribution associated with this Markov chain? Write out the MATLAB code for doing this, as well as the code for the long-mean, long-run standard deviation, and the long-run coefficient of autocorrelation.