Cole's Problem

Consider a simple environment in which a firm can hire a worker to produce output each period. Both the firm and the worker discount the future at rate β. The firm is risk-neutral and the worker has additively separable expected utility preferences over consumption with flow utility function u. We will assume that $u(0) = 0$, that u is strictly concave and strictly increasing in consumption.

Starting in period 1, the worker can produce output according to Al_t with $l_t = 1$ if he is employed by the firm and $l_t = 0$ if not. Also, in each period $t = 1, ...$ the worker receives a call from home which offers him the possibility of moving back home with stochastic continuation utility v_t where $v_t \in V = \{v_1, ..., v_N\}$. We denote the i.i.d. probabilities of these different continuation utilities by Π_i. Moving back home will mean leaving the firm and no longer working for it. We assume that there is no disutility cost to working for the worker.

We will assume that $v_i < v_{i+1}$, and that

$$\frac{u(0)}{1-\beta} = v_1,$$

so the worst home option is like getting 0 wages forever. In addition, there exists an outside option $\bar{n} \leq N$, such that

$$v_{\bar{n}-1} < \frac{u(A)}{1-\beta} < v_{\bar{n}},$$

so that for these options, it would cost the firm more than the worker's worth to dissuade him from moving back home.

To set some notation, assume that the wage in each period w_t and the worker's decision as to whether to stay or go home, δ_t, is a function of the history of call realizations $v^t = \{v_1, ..., v_t\}$. Let $\delta_t = 1$ denote staying with the firm. To be consistent, the set of decision functions must be such that

$$\delta_t(v^t) = 0 \text{ if } \delta_{t-j}(v^{t-j}(v^t)) = 0 \text{ for some } j,$$

where $v^{t-j}(v^t)$ denotes the first $t-j$ elements in v^t.

A) If the worker does not work for the firm, then he/she consumes 0 until they take the home option. Should the worker move home for any offer above v_1 in this case? The payoff from doing this can be recursively defined by

$$\begin{aligned} W_1 &= \Pi_1 \left\{ u(0) + \beta \sum_{i=2} \Pi_i v_i + \beta \Pi_1 W \right\} \\ &\quad + \sum_{i=2} \Pi_i v_i. \end{aligned}$$

More generally, if the worker moved home as soon as he got an offer of v_j or better, then his ex ante payoff could be recursively defined by

$$
\begin{aligned}
W_j &= \sum_{i=1}^{j-1} \Pi_i \left\{ u(0) + \beta \sum_{i=j} \Pi_i v_i + \beta \sum_{i=1}^{j-1} \Pi_i W \right\} \\
&\quad + \sum_{i=j} \Pi_i v_i .
\end{aligned}
$$

B) Let $\bar{W}$ be the unconditional outside option of the worker from solving the problem in (A). Assume that the worker's call offer is *public information* and the worker *can commit* to the circumstances in which he/she will quit the firm and move back home. So, the contracting problem is done under complete commitment in period 0. The firm has all of the bargaining power and can make the worker a take-it or leave-it offer of a wage contract in period 0, the planning period, and that the worker will take any offer that promises him/her at least $\bar{W}$. Note that the firm can ask the worker to promise never to leave if it wants to. Characterize the firm's optimal offer to the worker and explain what the equilibrium outcome will be. Are there any separations in equilibrium, and if so, are they efficient?

C) Assume now that worker *cannot commit* not to leave the firm if he/she prefers the continuation utility offered from home in the period from what he/she would get under the firm's wage contract. Assume that the worker's offer is still *public information*. Redo the firm's contracting problem and the characterization of the equilibrium contract. Are there any separations in equilibrium, and if so, are they efficient?

D) Now assume that the worker *cannot commit* and that the call offer from home is *private information* to the worker. Will the contract in (B) still work if the firm just has the worker tell it v_t in each period?

E) Try and characterize the optimal contract in (C). Discuss whether separations will be efficient or not.