1 Cole's Problem August 2016

Consider the following private information tax problem. Households are distinguished by their skill level θ_t and produce output according to $y_t = \theta_t l_t$. Skills evolve stochastically over time, and are drawn from a finite set Θ, with some probability Π_Θ on Θ^T. Households learn their skill realization as of time t at the being of period t.

The timing of information is somewhat complicated. For the first $T/2$ periods (yes T is even) a household's current productivity level θ_t and its labor effort l_t are private information, and only it's output y_t is public information. During the remaining $T/2$ periods, the household's output level y_t and productivity θ_t is publicly observable.

An allocation includes mappings (consumption) $c_t(\theta^T)$ and (output) $y_t(\theta^T)$, which map to R_+ and are measurable w.r.t. θ^t (and hence cannot depend upon the future). The output requirement implies the household's labor effort is $y_t(\theta^T)/\theta_t$. All households have the same preferences and their payoff, given this notation, is

$$E\left\{\sum_{t=0}^{T}\beta^t\left[u(c_t(\theta^T)) - v(y_t(\theta^T)/\theta_t(\theta^T))\right]\right\}$$

This is a small open economy which can borrow and lend at gross interest rate R. So the economy faces the *budget constraint*

$$\sum_{\theta^T\in\Theta^T}\sum_{t=0}^{T}R^{-t}\left[c_t(\theta^T) - y_t(\theta^T)\right]\Pi_\Theta(\theta^T).$$

A) Write down the social planning problem and use it to show what a complete information optimal tax program would look like for a utilitarian planner.

B) Given that labor effort and, occasionally, productivity are private information we may need an additional constraint to place on our planning problem to take account of this. In defining this additional constraint, be sure to be explicit about what exactly a mimicking strategy is here.

C) Will the information frictions really bind here, given that the true productivity level can be seen in later periods? Can you think of a stochastic process for which we do not need an additional constraint and the solution is the same as in part A? Can you think of a stochastic process for which you do?

In what follows, assume productivity is i.i.d..

D) Will there be any dispersion in consumption during periods in which productivity was publicly observable? How about when it was not publicly observable?

E) Would you expect the normal Euler condition to hold or the inverse Euler condition? And if so when? Give some intuition for your answer.

F) Sketch how you would prove your claim(s) in part E.