Microeconomic Theory II
Preliminary Examination
June 5, 2017

The exam is worth 120 points in total.

There are 4 questions. Do all questions. Start each question in a new book, clearly labeled. **Fully justify** all answers and show all work (in particular, describing an equilibrium means providing a **full description of the strategy profile** and **proving** that it has the desired properties). Label all diagrams clearly. Write legibly. If you need to make additional assumptions, state them clearly.

Good luck!

1. **(40 points)** Consider a Cournot duopoly. The market price is given by $1-q_1-q_2$, where q_1 and q_2 are the quantities of output produced by the two firms. There are no costs.

   (a) Find the (Nash) equilibrium quantities of output. **[5 points]**

   (b) Suppose that firm 1's owner first hires a manager, after which the manager of firm 1 and owner of firm two simultaneously choose outputs q_1 and q_2. The manager of firm 1 is paid $\kappa\pi_1(q_1,q_2)+\lambda q_1-B$, where q_1 is the quantity chosen by the manager, $\pi_1(q_1,q_2)$ is the profit earned in the duopoly game (given outputs q_1, and q_2), and κ, λ, and B are nonnegative constants chosen by the owner of firm 1. The outside option for the manager is 0. Assume that firm 2 observes the values of κ, λ, and B before the two firms simultaneously choose their outputs. What is the subgame perfect equilibrium of this game. Compare the result to the outcome of the Stackelberg model (without managers). **[15 points]**

   (c) Now suppose that both owners hire managers, simultaneously making public the terms ((κ_1,λ_1,B_1) and (κ_2,λ_2,B_2)) of the managers' contracts, after which the managers simultaneously choose outputs. Solve for the equilibrium. **[10 points]**

   (d) How do firms' outputs and profits in the previous part compare to the those of the Nash equilibrium without managers? Explain your answer. Notice that your answer to the previous two parts should allow you to make these comparisons without calculating equilibrium outputs and profits. **[5 points]**

   (e) Suppose that a law is proposed making it illegal to disclose the compensation contracts of managers of firms. Given that the owners always have the option of not disclosing such contracts, why would such a law have any effect? Would you expect the owners of the two firms to support this law? **[5 points]**

[Question 2 is on the next page.]

2. This question concerns an asynchronous version of an infinitely repeated game, with stage game given by

	A	B
A	$3,3$	$0,0$
B	$0,0$	$1,1$

.

The discount factor is given by $\delta \in (0,1)$, and time is indexed by $t = 0,1,2,\ldots$. For $t > 0$, player 1 (the row player) chooses in odd periods, and player 2 (the column player) chooses in even periods. So, for $t > 0$, each player's action is fixed for *two* periods. Period 0 is different in that both players choose simultaneously in that period, and player 1 gets to choose again in the next period, period 1.

(a) Prove that the strategy profile in which each player always plays B is not a subgame perfect equilibrium for any δ. **[10 points]**

(b) Prove that there exists a δ^* such that the infinite repetition of BB is a subgame perfect equilibrium outcome if $\delta < \delta^*$. What is the value of δ^*? **[10 points]**

Questions 3 and 4 concern the same setting, but are otherwise independent.

In the common setting, two players jointly own an asset (with equal shares) and are bargaining to dissolve the joint ownership. Suppose v_i is the private value that player i assigns to the good, and suppose v_1 and v_2 are independently drawn from the interval $[0,1]$, according to the distributions F_i, with densities f_i, for $i = 1,2$. Efficiency requires that player i receive the asset if $v_i > v_j$. If the partnership is not dissolved, player i receives a payoff of $\frac{1}{2}v_i$.

3. The players bargain as follows: the two players simultaneously submit bids, with the higher bidder winning the object (with ties resolved by a fair coin flip), and the winner pays the loser the winning bid.

(a) What are the interim payoffs for each bidder? **[5 points]**

(b) Suppose (σ_1, σ_2) is a Bayes-Nash equilibrium of the auction, and assume σ_i is strictly increasing and differentiable function for $i = 1,2$. Describe the pair of differential equations the strategies must satisfy. **[10 points]**

(c) Suppose v_1 and v_2 are uniformly and independently distributed on $[0,1]$. Describe the differential equation a symmetric increasing and differentiable equilibrium bidding strategy must satisfy, and solve it. [**Hint:** conjecture a functional form.] **[10 points]**

(d) Prove the strategy identified in part 3(c) is a symmetric equilibrium strategy. **[5 points]**

4. (a) Describe the class of Groves mechanisms to efficiently dissolve the partnership. What attractive property do Groves mechanisms have in terms of strategic behavior? **[5 points]**

(b) Prove that there is a Groves mechanism satisfying ex ante budget balance and interim individual rationality. **[15 points]**

(c) Is there a Groves mechanism satisfying ex post budget balance? Provide a proof. **[10 points]**