Microeconomic Theory I
Preliminary Examination
University of Pennsylvania

June 6, 2016

Instructions

This exam has 4 questions and a total of 100 points.

Answer each question in a **SEPARATE** exam book.

If you need to make additional assumptions, state them clearly.

Be concise.

Write clearly if you want partial credit.

Good luck!

1. (25 pts) In a two-good world, consider the function $e : \mathbb{R}^2_{++} \times \mathbb{R}_+ \to \mathbb{R}_+$ defined by

$$e(p, U) := U \min\left\{p_1,\ \frac{p_1 + p_2}{3},\ p_2\right\}.$$

   (a) (5 pts) State four properties that an expenditure function must satisfy if it arises from a continuous monotonic utility function, and verify that the given e indeed satisfies them.

   (b) (5 pts) Let $h(p, U)$ denote a Hicksian demand correspondence that gives rise to the expenditure function e. For each $(p, U) \in \mathbb{R}^2_{++} \times \mathbb{R}_+ \to \mathbb{R}_+$, find as many points in the set $h(p, U)$ as you can.

   (c) (15 pts) For each property of a function listed below, state whether there is a utility function satisfying that property which gives rise to the expenditure function e, and sketch proofs of your answers.

      i. (5 pts) strictly quasiconcave.

      ii. (5 pts) quasiconcave.

      iii. (5 pts) non-quasiconcave.

2. (25 pts) A consumer lives for two periods. In period 2 she will purchase a commodity bundle $x = (x_1, x_2)$ to maximize her utility $u(x)$ subject to her budget constraint $p \cdot x \leq y$.

   (a) (5 pts) Let $v(y)$ be the consumers's indirect utility arising from $u(x)$ (we suppress the argument p since it is fixed in this question). Show that v is concave in y if u is concave in x.

   In period 1 the consumer chooses an amount z to invest in a risky asset that returns $(1 + \tilde{r})z$ in period 2. Her initial wealth is $w > 0$, and she is restricted to choosing $z \in [0, w]$. She uses her resulting income in period 2, $\tilde{y} = w + \tilde{r}z$, to purchase x at the prices p. She knows these prices in period 1, and chooses z in order to maximize the expected utility she will ultimately obtain in period 2. For each $w \geq 0$, let $z^*(w)$ be an optimal investment for this consumer.

   Assume the random variable $\tilde{r}$ is continuously distributed on an interval $[\underline{r}, \bar{r}]$, with $\underline{r} > -1$ and $\mathbb{E}\tilde{r} > 0$.

   (b) (10 pts) Suppose v is C^2 and satisfies $v' > 0$, $v'' < 0$, and DARA ($A(y) := -v''(y)/v'(y)$ is a strictly decreasing function). Show that for any $w > 0$, if $z^*(w) < w$ holds, then $z^{*\prime}(w) > 0$.

   (c) (10 pts) Now assume $u(x) = u_1(x_1) + u_2(x_2)$, where each u_i is C^2 with $u_i' > 0$, $u_i'' < 0$, and $u_i'(0) = \infty$. Show that if u_1 and u_2 each satisfy DARA, then v satisfies DARA.

3. (25 pts) Consider a two-person two-period economy with one consumption good. The two people are farmers. Farmer 1 has one unit of the good in period 1, and has already planted a crop that will generate one unit of the good in period 2. Farmer 2 has two units of the good in period 1, and has already planted a crop that will generate two units of the good in period 2. If a farmer consumes x_1 of the good in period 1 and x_2 in period 2, his utility will be $\ln x_1 + \ln x_2$. If a farmer consumes x_1 in period 1 and dies at the end of period 1, his utility will be just $\ln x_1$. Each farmer maximizes expected utility.

   (a) (10 pts) Suppose farmer 1 will die with probability 1/2 at the end of period 1. If he dies his crop will still generate one unit of the good in period 2. Assume there is just one contingent claims market at the beginning of period 1: the only contingent contract that can be traded is for the delivery of the good contingent on farmer 1 dying. The farmers are constrained to not sell more of a contract than they can deliver. What is the competitive equilibrium?

   (b) (5 pts) Is the equilibrium outcome you found in (a) Pareto efficient? Prove your answer, using the following notation: let x_1^i denote farmer i's consumption in period 1, and let x_{2j}^i for $j = 1, 2$ be his consumption in period 2 state s_j.

   (c) (10 pts) Suppose now that it is known that exactly one of the two farmers will die at the end of period 1; it is equally likely to be either one. Assume that the two farmers can trade all state contingent contracts at the beginning of period 1, but that they are constrained not to sell more of a contingent contract than they can deliver. What is the competitive equilibrium?

4. (25 pts) Robinson Crusoe is stranded on an island with 1 unit of wheat. He is sure to be rescued after two periods. The 1 unit of wheat that he has can either be consumed or used as seed to grow more wheat. Crusoe also has one unit of leisure time in the first period. If s, the amount of wheat used as seed, is combined with ℓ units of labor, the wheat output y can be combined in the second period is $2(\ell s)^{.5}$. If Crusoe consumes $x = 1 - \ell$ units of leisure and $z = 1 - s$ units of wheat in the first period, and y units of wheat in the second period, his utility will be $u = xyz$.

   (a) (8 pts) What is Crusoe's optimal allocation in this simple economy?

   (b) (8 pts) Compute the competitive equilibrium prices p of wheat and q of seed, normalizing the price of ℓ to be 1.

   (c) (9 pts) Show that the efficient (x, y, z) you found in (a) maximizes Crusoe's utility subject to his budget constraint given the prices you found in (b), and given that the firm maximizes profit given those prices.