

Microeconomic Theory I
Preliminary Examination
University of Pennsylvania

June 5, 2017

Instructions

This exam has four questions and is worth 100 points.

Answer each question in a **SEPARATE** exam book.

If you need to make additional assumptions, state them clearly.

Be concise.

Write clearly.

Use WORDS to explain your reasoning.

Good luck!

1. (25 pts) *Classical production and demand theory*

- (a) (15 pts) A competitive firm produces one output, q , using two inputs, x_1 and x_2 , which have prices $w = (w_1, w_2)$. The firm has a continuous increasing production function f on \mathbb{R}_+^2 . Find f , given that the firm's cost function is

$$c(q, w) = \begin{cases} 2\sqrt{w_1 w_2 q} - w_2 & \text{if } \frac{w_1}{w_2} \geq \frac{1}{q} \\ w_1 q & \text{if } \frac{w_1}{w_2} \leq \frac{1}{q} \end{cases}.$$

- (b) (10 pts) In a three-good world, suppose a consumer's demands for goods 1 and 2 are given by

$$x_1(p, y) = \frac{p_2}{p_3} \quad \text{and} \quad x_2(p, y) = \frac{p_1}{p_3}$$

on some open subset of \mathbb{R}_{++}^4 . Can these demands arise from the maximization of a continuous utility function representing locally nonsatiated strictly convex preferences on \mathbb{R}_+^3 ? Prove your answer.

2. (25 pts) *Risk aversion and comparative statics*

There are two states of the world, ℓ and n , which will occur with probabilities $\pi \in (0, 1)$ and $1 - \pi$, respectively. In state ℓ , Alex loses $L > 0$ dollars. In state n , he loses 0 dollars. His initial wealth (income) is $w > 0$. Before the state is realized, Alex can purchase insurance: he pays px to obtain a policy that pays x dollars if state ℓ occurs. His state-contingent incomes given x are thus $y_\ell = w - L + x - px$ and $y_n = w - px$. Alex chooses x to maximize his expected utility of income subject to the constraint $x \geq 0$. His Bernoulli utility function u is C^2 and satisfies $u' > 0$ and $u'' < 0$.

Lastly, assume the price of insurance exceeds the actuarially fair rate: $p > \pi$.

- (a) (5 pts) Show that Alex's optimal coverage, x_a , is less than L .
- (b) (10 pts) Assuming u exhibits decreasing absolute risk aversion (DARA), determine whether insurance coverage is a normal or inferior good for Alex.
- (c) (10 pts) Barb is in the same insurance market, and her Bernoulli utility function, v , satisfies the same assumptions as does u . However, Barb is strictly more risk averse than Alex. Determine whether Barb purchases more or less insurance than does Alex. (Assume they both purchase positive amounts.)

3. (25 pts)

- (a) (5 pts) Define the core of a pure exchange economy with l goods and n agents.
- (b) (5 pts) State the core convergence theorem.
- (c) (5 pts) Give a graphic example of a two-person, two-good economy in which there does not exist a competitive equilibrium, but there exists an allocation in the core.
- (d) (10 pts) Consider an exchange economy for which agents have utility functions that are continuous and increasing, but not necessarily concave. Prove the core is nonempty. State clearly any additional assumptions you make.

4. (25 pts) Consider a standard two-period economy, dated $t = 0$ and $t = 1$. Agents consume in both periods. There are three states of nature in the second period. There is a single consumption good, and it is used as a numeraire; hence, the spot price of a unit of consumption at either date is 1. At date 0 agents can trade in two primary securities. Security 1 has the second-period payoff vector $r_1 = (1, 0, 0)$, and security 2 has the second-period payoff vector $r_2 = (1, 2, 3)$. The prices of these securities at date 0 are $q_1 = 0.1$ and $q_2 = 1.1$.
- (a) (10 pts) Suppose there is a *derivative* security denoted security 3. Security 3 is a call option on security 2 with strike price of 1. What are the minimum and maximum possible prices for this security that are consistent with no arbitrage?
 - (b) (5 pts) Suppose that the price of security 3 in part (a) is 1. Show that the system is arbitrage free.
 - (c) (10 pts) Assume these security prices, $q = (.1, 1.1, .6)$, arise in an incomplete markets equilibrium with the specified three securities.
 - i. What would be the market price of a put option on asset 2 with a strike price of 3?
 - ii. What would be the risk-free interest rate on a loan taken at date $t = 0$?