Microeconomic Theory I
Preliminary Examination
University of Pennsylvania

August 8, 2016

Instructions

This exam has 4 questions and a total of 100 points.

Answer each question in a **SEPARATE** exam book.

If you need to make additional assumptions, state them clearly.

Be concise.

Write clearly if you want partial credit.

Good luck!

1. (25 pts) The inverse demand function for oil is given by a continuously differentiable function $P : \mathbb{R}_{++} \to \mathbb{R}_{++}$ satisfying $P' < 0$ and $P(x) \to \infty$ as $x \downarrow 0$. The price elasticity of the demand for oil is defined at any $x > 0$ as

$$e(x) := -\frac{P(x)}{P'(x)x}.$$

The total stock of oil below the ground is $0 < \bar{x} < \infty$. It is all owned by one oil company, which can extract it at zero cost. The firm's profit is zero if it sells no oil, and its profit is px if it sells an amount $x > 0$ at price p.

   (a) (6 pts) Compare the competitive equilibrium (x^c, p^c) to the monopoly outcome (x^m, p^m) under (i) the assumption that $e(x) > 1$ for all $x \in (0, \bar{x}]$, and (ii) under the assumption that $e(\bar{x}) < 1$.

Now suppose there are two periods, $t = 1, 2$, and the firm discounts the second period at rate $r > 0$. The inverse demand function in period t is $P_t(x_t)$, which has the same properties as does the function P above. The firm's discounted payoff if it sells x_t in period t at price p_t is $p_1x_1 + (1 + r)^{-1}p_2x_2$, where (x_1, x_2) must satisfy $x_1 + x_2 \le \bar{x}$. Its profit in period t is 0 if $x_t = 0$.

   (b) (6 pts) Suppose $(p_1^c, x_1^c, p_2^c, x_2^c)$ is a competitive equilibrium satisfying $x_1^c > 0$ and $x_2^c > 0$. Find a system of four equations this equilibrium must satisfy. Then compare x_1^c to x_2^c when $P_1(\cdot) = P_2(\cdot)$.

   (c) (6 pts) Again allowing P_1 and P_2 to be different functions, assume now that for some $\underline{e} > 1$, the elasticities satisfy $e_t(x_t) > \underline{e}$ for all $x_t \in (0, \bar{x}]$ and $t = 1, 2$. Suppose $(p_1^m, x_1^m, p_2^m, x_2^m)$ is a monopoly outcome satisfying $x_1^m > 0$ and $x_2^m > 0$. Find a system of four equations this outcome must satisfy.

   (d) (7 pts) Under the additional assumption that both P_1 and P_2 have constant elasticities, e_1 and e_2, satisfying $e_1 \ge e_2 > 1$, how does $(p_1^m, x_1^m, p_2^m, x_2^m)$ compare to $(p_1^c, x_1^c, p_2^c, x_2^c)$?

2. (25 pts) Consider a Bernoulli utility function $u : \mathbb{R} \to \mathbb{R}$ that has derivatives $u' > 0$ and $u'' < 0$, and exhibits DARA (decreasing absolute risk aversion). Prove each of the following.

   (a) (10 pts) (Lemma) For any $k \in \mathbb{R}$ and any random gamble $\tilde{y}$,

$$\mathbb{E}u(\tilde{y}) = u(k) \;\Rightarrow\; \mathbb{E}u(\tilde{y} + a) > u(k + a) \;\forall a > 0.$$

Even if you were unable to prove the "Lemma" in (a), feel free to use it to prove (b)-(d) below.

   (b) (5 pts) Let $\tilde{x}$ be a random gamble, and let $b(w)$ be the maximum price the agent is willing to pay for $\tilde{x}$ when her wealth is w. Then $b(w)$ increases in w.

   (c) (5 pts) Let $\tilde{x}$ be a random gamble, and let $s(w)$ be the minimum price at which the agent is willing to sell $\tilde{x}$ when her wealth is w. Then $s(w)$ increases in w.

   (d) (5 pts) Now let $\tilde{x}$ be a random gamble that is valuable at wealth w, in the sense that $\mathbb{E}u(w + \tilde{x}) > u(w)$. Then $s(w) > b(w)$, where $b(w)$ and $s(w)$ are defined in (b) and (c) from this $\tilde{x}$ and w.

3. (25 pts) Consider an exchange economy with two consumers and two goods. Good x is a perfectly divisible numeraire. Good y, in contrast, is *indivisible,* that is, consumers can only consume it in nonnegative integer amounts. The utility of consumer $i = 1, 2$ from consuming a bundle (x^i, y^i) of the two goods is given by $u^i(x^i, y^i) = x^i + v^i(y^i)$, where $v^i(\cdot)$ is a function on nonnegative integers. Assume that

$$v^i(2) > v^i(1) = v^i(0) = 0, \text{ and } v^i(y) = v^i(2) \text{ for } y > 2.$$

(Think of good y as chopsticks where the value of only one is 0.) Assume also that

$$v^2(2) \leq v^1(2) \leq 10.$$

The initial endowment of consumer $i = 1, 2$ is (e^i_x, e^i_y). Assume the total endowment of good y is $e^1_y + e^2_y = 2$, and that $e^1_x = e^2_x = 20$.

(a) (4 pts) Describe the Pareto efficient allocations in this economy.

(b) (4 pts) Write conditions for a Walrasian equilibrium for this economy.

(c) (4 pts) Does a Walrasian equilibrium always exist for such an economy? Either prove that it does or give a counterexample.

(d) (4 pts) If a Walrasian equilibrium exists for such an economy, is it Pareto efficient? Either explain why it is or provide a counterexample.

(e) (9 pts) Suppose we replace the assumption $v^i(1) = 0$ with $v^i(1) > 0$, keeping all the other assumptions. Will a Walrasian equilibrium now always exist? Either explain why or give a counterexample.

4. (25 pts) Consider a two-period period GEI model of an exchange economy with a single commodity per state. There are 3 states and 2 assets. The assets pay

$$A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \\ 3 & 1 \end{pmatrix}.$$

(a) (7 pts) If the price of asset 1 is $q_1 = 1$, what prices for asset 2 are consistent with no arbitrage?

(b) (8 pts) Suppose now that the price of each asset is 1. What state prices for state 3 are consistent with these asset prices?

(c) (10 pts) Suppose again that the asset prices are both 1. Suppose also that there is a call option that allows an agent to purchase one unit of asset 1 for 2. What prices for this call option are consistent with these asset prices?