University of Pennsylvania **Economics 705, Fall 2016**

Prelim Examination

Friday August 12, 2016 Time limit: 150 minutes

Instructions:

(i) The exam consists of two parts. The total number of points for each part is 50. The number of points for each question is given below.

(ii) The exam is closed book and closed notes.

(iii) To receive full credit for your answers you have to explain your calculations. **You may state additional assumptions.**

Part I

Question 1: Change of Variables (10 Points)

Suppose that $X \sim N(\mu, \sigma^2)$. Let $Y = \exp(X)$.

(i) (6 Points) Suppose that $X \sim N(\mu, \sigma^2)$. Let $Y = \exp(X)$. Compute the expected value $\mathbb{E}[Y]$ and the variance $\mathbb{V}[Y]$.

(ii) (4 Points) Suppose that X_1 and X_2 are independent and have $N(0,1)$ distributions. Define $Y_1 = X_1 + X_2$ and $Y_2 = X_1 - X_2$. Use a change-of-variables argument to obtain the joint probability density for (Y_1, Y_2). Then compute the pdf for the marginal distribution of Y_1. What is the distribution of $X_1 + X_2$?

Question 2: Inference with Two Observations (23 Points)

Consider the following experiment:

$$X_1, X_2 \sim iidN(\theta, 1)$$

(i) (3 Points) Derive the likelihood function and the maximum likelihood estimator for this experiment. What is the sampling distribution of the maximum likelihood estimator?

(ii) (3 Points) Derive a likelihood ratio (LR) test for the null hypothesis $\theta = \theta_*$. State the distribution of your test statistic as well as acceptance and rejection regions for your test.

(iii) (2 Points) Provide a definition of a frequentist confidence interval and derive a 95% confidence interval for the above experiment.

(iv) (3 Points) Consider the prior $\theta \sim U[-M, M]$ where M is some large number and $U[\cdot]$ is the uniform distribution. Derive the posterior distribution of θ.

(v) (2 Points) Provide a definition of a Bayesian credible interval and derive a 95% credible interval for the above experiment.

(vi) (10 Points) How do your answers to (i)-(v) change if we impose $\theta \geq 0$ and change the prior to $\theta \sim U[0, 2M]$?

Question 3: Inference for Variance Parameters (17 Points)

Consider the model $Y_i \sim iidN(0, \theta)$, $i = 1, \dots, n$. The goal is to make inference about θ. We denote the "true" value by θ_0.

(i) (2 Points) Derive the maximum likelihood estimator $\hat{\theta}$ for θ.

(ii) (1 Point) Is the maximum likelihood estimator unbiased? Explain.

(iii) (1 Point) Is the maximum likelihood estimator consistent? Explain.

(iv) (2 Points) Derive the score $s(\theta) = \partial \ln p(Y_{1:n}|\theta)/\partial\theta$.

(v) (4 Points) Assume that the "true" value is θ_0 and derive the limit distribution of the (properly normalized) score evaluated at $\theta = \theta_0$.

(vi) (3 Points) Construct the Lagrange multiplier/score test for the hypothesis $H_0 : \theta = \theta_0$ and state 95% critical value as well as the acceptance and rejection region.

(vii) (4 Points) Show that the power of the LM test against any fixed alternative $\theta_1 \neq \theta_0$ converges to one as $n \longrightarrow \infty$.

Part II

Question 4: Inference with Endogeneity (40 Points)

Consider a linear model

$$y_i = x_i'\beta + e_i$$

where $x_i \in R^k$ and $E(x_i e_i) \neq 0$. Suppose you have instrumental variables $z_i \in R^\ell$ with $\ell \geq k$ and $E(z_i e_i) = 0$.

(i) (3 points) Show why the ordinary least squares estimator is inconsistent.

(ii) (5 points) Show how to identify β with z_i and specify conditions for identification.

(iii) (5 points) How to estimate β consistently with z_i?

(iv) (5 points) Derive the asymptotic distribution of your consistent estimator in part (iii).

(v) (5 points) Provide a consistent estimator of the asymptotic covariance in part (iv) and proves its consistency.

(vi) (5 points) How to test the instrumental variables z_i are exogenenous?

(vii) (7 points) Prove the asymptotic distribution of the test statistic in part (vi).

(viii) (5 points) Now suppose you have another instrument $Z_i^* \in R$ and $E(Z_i^* e_i) = 0$. How do you construct your esimator with this additional instrument? Does this additional instrument improve your estimator?

Question 5: Models with Limited Observations (10 Points)

Consider the following model

$$y_i^* = x_i'\beta + u_i, \; u_i|x_i \sim iid \; N(0, \sigma^2),$$

where $x_i \in R^k$ with $k > 1$. Moreover, the x_i's are also independent across i. We do not observe y_i^*, instead we observe

$$y_i = \begin{cases} y_i^* \text{ if } y_i^* \geq \lambda \\ \lambda \text{ if } y_i^* < \lambda, \end{cases}$$

where λ is a known constant.

(i) (5 points) Write down the log-likelihood function for the maximum likelihood estimator $\widehat{\beta}$.

(ii) (3 points) Give the limit distribution of the maximum likelihood estima tor $\widehat{\beta}$.

(iii) (2 points) Estimate the standard error of the maximum likelihood estimator.