

1. An Endogenous Growth Model

Consider the following model. There is a representative household with a utility function:

$$U(0) = \int_0^{\infty} e^{-\rho t} \left(\log c(t) - \psi \frac{l(t)^{1+\eta}}{1+\eta} \right) dt$$

There is no population growth.

There is a perfectly competitive final good producer with technology:

$$Y(t) = \frac{1}{\alpha} \left(\int_0^{N(t)} x(v, t)^\alpha dv \right) l(t)^{1-\alpha}$$

where $x(v, t)$ is an input, fully depreciated in production and sold at price $p(v, t)$, and $N(t)$ is the total number of existing varieties.

Each input is produced either by the monopolist that invent it. The production of inputs is at (constant) marginal cost ψ .

New varieties are developed according to the innovation function:

$$\dot{N}(t) = \eta Z(t)$$

where $Z(t)$ is investment in R&D and with some initial $N(0)$. There is free entry into the innovation activity.

1. Define an equilibrium for this economy.
2. Characterize the Balanced Growth Path (BGP) of this economy. Among other things, you need to:
 1. Find the price of each variety.
 2. Write the Hamilton-Bellman-Jacobi equation of the monopolist.
 3. Find an expression for the growth rate of the economy along the BGP.
3. Are there transitional dynamics in this economy? Justify your answer.
4. Characterize the behavior of $l(t)$ over time.
5. Solve the social planner's problem. Compare with the solution in steps 2-4.