1. A Model of Secular Stagnation

Consider an overlapping generations economy in which one household is born at each period t, $t = 0, 1, 2, \ldots$. The household lives for three periods: t, $t+1$, and $t+2$. Households consume in periods t, $t+1$, and $t+2$, but they only work in period $t+1$. Given a discount factor $\beta < 1$, the utility function for a household born at time t is:

$$\log c_t^t + \beta \left(\log c_{t+1}^t - \frac{\left(l_{t+1}^t\right)^2}{2} \right) + \beta^2 \log c_{t+2}^t.$$

To finance c_t^t, the household born at time t can borrow a quantity b_t^t from a household born at time $t-1$ (and thus currently middle-aged and working) at an interest rate R_t. The household born at time t will pay back its loan at time $t+1$ (when it becomes middle-age) to the household born at time $t-1$. The current old household consumes out of the proceedings of the loan it made to the previous generation. Formally, the budget constraints for the household born at time t are given by:

$$\begin{aligned} c_t^t &= b_t^t \\ c_{t+1}^t + R_t b_t^t + b_{t+1}^t &= w_{t+1} l_{t+1}^t \\ c_{t+2}^t &= R_{t+1} b_{t+1}^t \end{aligned}$$

where w_{t+1} is the wage paid a period $t+1$. The budget constraints for the initial household in their second period of life are given by:

$$\begin{aligned} c_0^{-1} + b_0^{-1} &= w_0 l_0^{-1} \\ c_1^{-1} &= R_0 b_0^{-1}. \end{aligned}$$

The initial old just consumes an exogenously given initial endowment α, $c_0^{-2} = \alpha$.

In addition, there is a representative firm, that operates the production function $y_t = l_t$, under perfect competition, where l_t is the amount of labor hired at time t. There is no money in the economy.

1. Write the aggregate resource constraint of this economy.

2. Define a sequential markets equilibrium for this economy.

3. Characterize the sequential markets equilibrium. In particular, find an expression for consumption for every generation and every period, an expression for l_t, y_t, and w_t, a recursive expression for R_t. All these expression should only depend on parameters (α and β).

4. Assume now that there is an exogenously binding borrowing constraint, that is $c_t^t = \bar{b}$, where $\bar{b}$ is less than the optimal b_t^{t*} that you found in step 3. Recompute consumption, l_t, y_t, and w_t as a function of parameters (α and β) and R_t. You do not need to solve for R_t.

5. Compare the solutions to 3. and 4. Provide intuition (hints: make $\bar{b}$ as small as you need and read the title of the question).