

Microeconomic Theory II
Preliminary Examination
June 3, 2013

The exam is worth 120 points in total.

There are 4 questions. Do all questions. Start each question in a new book, clearly labeled. **Fully justify** all answers and show all work (in particular, describing an equilibrium means providing a **full description of the strategy profile** and proving that it has the desired properties). Label all diagrams clearly. Write legibly. If you need to make additional assumptions, state them clearly.

Good luck!

1. **(25 points)** An investor can invest any fraction $\alpha \in [0, 1]$ of her initial wealth $w > 0$ in a risky asset. Her random wealth when she invests αw is

$$\tilde{y} = (1 + \tilde{r}\alpha)w,$$

where \tilde{r} is the return of the asset. Assume the random variable \tilde{r} is not degenerate, and $\mathbb{E}\tilde{r} > 0$. The investor's Bernoulli utility u satisfies $u' > 0$ and $u'' < 0$. Let $\alpha^*(w)$ be the investor's expected utility maximizing α . Assume that for any w of interest in this problem, $\alpha^*(w) < 1$. (In words, the asset is not so surely profitable that the investor would choose to invest all her wealth in it.)

Kenneth Arrow claimed, as an empirical matter, that as an investor becomes wealthier, she invests a smaller proportion of her wealth in risky assets. Give a sufficient condition for this to be true, in this problem, in terms of the function that measures the investor's *relative risk aversion*:

$$R(y) := -\frac{yu''(y)}{u'(y)}.$$

2. **(20 points)** Suppose Bruce and Sheila play the following game:

		Sheila		
		L	C	R
Bruce	T	1, 2	5, -2	1, -1
	B	5, -2	1, 2	1, -1

- (a) What are all the pure and mixed strategy Nash equilibria of this game? [5 points]

Suppose Bruce has the option of choosing not to play the game (denoted N) and instead receive a payoff of 2 (with Sheila receiving a payoff of 0). Denote the option of playing the game P . The result is a dynamic game in which a choice of N by Bruce ends the game and a choice of P leads to the second stage in which the above simultaneous move game is played.

- (b) Describe all the pure strategy Nash equilibria of the dynamic game. Are any of these profiles subgame perfect? Why or why not. [5 points]
- (c) Describe *all* subgame perfect equilibria (these may involve randomization). [5 points]
- (d) If a subgame perfect equilibrium involves randomization, is the resulting profile a mixed strategy profile or a behavior strategy profile? Explain. [5 points]

3. (40 points) Consider a worker with ability $\theta > 0$ who must invest in job skills s before entering the labor market. The value of a worker with skill level $s \in \mathbb{R}_+$ to firms in the job market is s (i.e., all firms value a skill level s at s). Firms compete for the worker in a first price sealed bid auction by offering a wage, and so we assume that the market wage that the worker receives equals the firms' common estimate of the worker's value. The payoff to a worker with ability θ choosing a skill level s and receiving a wage w is

$$w - \frac{s^3}{3\theta^2}.$$

Note that the disutility to a worker of investing in skills is increasing in the skills acquired and decreasing in ability.

Suppose θ is common knowledge.

- (a) Suppose that the level of skills acquired is publicly observed. Describe the unique subgame perfect equilibrium of the model (recall you may assume the worker's wage equals the firms' common estimate of the worker's value, and so the other player in the model is the "market"). [10 points]
- (b) Suppose now that the level of skills acquired is not observed by any firm. Describe the unique Nash equilibrium of the model. [10 points]
- (c) Maintaining the assumption that the level of skills acquired is not observed by any firm, also assume that the worker can choose a costly but unproductive action (signal) $y \in \mathbb{R}_+$. The worker chooses y *before* choosing his skill level. The level of y is *public* (so known by the market). The cost to the worker of choosing the action y is y^2/s , so higher values of y are more costly, but more skilled workers pay a lower cost. If $y > 0$, the worker must choose $s > 0$, while choosing $(y, s) = (0, 0)$ costs 0. The overall payoff to a worker with ability θ , choosing a skill level $s > 0$ and signal y , and receiving a wage w is

$$w - \frac{s^3}{3\theta^2} - \frac{y^2}{s}.$$

What is the subgame perfect equilibrium, and why is it unique? [10 points]

- (d) Suppose now that the value of θ is not common knowledge. While the worker knows his ability, the firms do not. Their common beliefs assign equal probability to $\theta = 4$ and $\theta = 16$. Let y_θ denote the signal chosen by the type θ worker in the equilibrium from part 3(c). Prove that the separating profile in which the type θ worker signals at level y_θ is not an equilibrium, by showing that the low ability worker has a strict incentive to deviate. (Be explicit in your description of out-of-equilibrium behavior and the appropriate notion of equilibrium.) [10 points]

Question 4 is on the next page.

4. **(35 points)** Consider the stage game where player 1 is the row player and 2, the column player (as usual):

	<i>L</i>	<i>R</i>
<i>T</i>	2, 2	6, 1
<i>B</i>	1, 4	4, 5

- (a) Suppose the game is infinitely repeated, with perfect monitoring. Players 1 and 2 are both long-lived, and have the same discount factor, $\delta \in (0, 1)$. Construct a three state automaton that for large δ is a subgame perfect equilibrium, and yields a payoff to player 1 that is close to 5. Prove that the automaton has the desired properties. (**Hint:** One state is only used off the path-of-play.) [20 points]
- (b) Now suppose that player 2 is short-lived (but maintain the assumption of perfect monitoring, so that the short-lived player in period t knows the entire history of actions up to t). Prove that player 1's payoff in any pure strategy subgame perfect equilibrium is no greater than 4. For which values of δ is there a pure strategy subgame perfect equilibrium in which player 1 receives a payoff of precisely 4? [15 points]