Microeconomic Theory II
Preliminary Examination
June 6, 2011

The exam is worth 120 points in total.

There are 4 questions. Do all questions. Start each question in a new book, clearly labeled. **Fully justify** all answers and show all work (in particular, describing an equilibrium means proving that it has the desired properties). Label all diagrams clearly. Write legibly. If you need to make additional assumptions, state them clearly.

1. **(20 points)** Let $u : \mathbb{R}_+ \to \mathbb{R}$ be a strictly increasing twice continuously differentiable Bernoulli utility function.

   (a) Show that if u exhibits constant relative risk aversion equal to $\rho \neq 1$, then $u(x) = \beta x^{1-\rho} + \gamma$, where $\beta, \gamma \in \mathbb{R}$. How is the sign of β determined? [5 points]

   (b) Show that u exhibits constant relative risk aversion equal to 1 if and only if $u(x) = \beta \ln x + \gamma$, where $\beta > 0$ and $\gamma \in \mathbb{R}$. [5 points]

   (c) Suppose an expected utility maximizer's Bernoulli utility function u exhibits constant relative risk aversion equal to $\rho \neq 1$. Consider a simple lottery that pays $z_n \in \mathbb{R}_+$ with probability p_n $(n = 1, \ldots, N)$, and another lottery that pays $\alpha z_n \in \mathbb{R}_+$ with probability p_n, where $\alpha > 0$ is a constant. What is the relationship between the certainty equivalent for those two lotteries (assuming the agent's initial wealth is 0)? Explain your finding. [10 points]

2. **(35 points)** There are two firms, $i = 1, 2$, each of whom has a constant marginal cost $c > 0$ and each posts a price $p_i \in \mathbb{R}$. There is a mass $m > 1$ of consumers, each of whom has a willingness to pay $v > c$. A fraction $\mu \in [0, 1]$ of those consumers observe one and only one of the two prices posted by the two firms, and will buy at the observed price if and only if it is less than or equal to v. Assume each of those consumers observes one of the two prices with equal probabilities independent of the prices posted by the two firms. The remaining $1 - \mu$ fraction of consumers observe both prices and buy at the cheaper price if the price is not greater than v; if $p_1 = p_2 \leq v$, a consumer will buy from each of the two firms with equal probability. Write $F_i(p_i, p_{-i})$ as the probability that each of those $(1 - \mu)$ fraction of consumers will buy from firm i when its price is p_i and firm $-i$'s price is p_{-i}. Firm i's demand can be written as

$$D_i(p_i) = \begin{cases} \frac{\mu}{2} m + (1 - \mu) m F_i(p_i, p_{-i}), & p_i \leq v, \\ 0, & p_i > v, \end{cases}$$

   and firm i's profit is $\Pi_i(p_i) = (p_i - c) D_i(p_i)$. The two firms set their prices simultaneously.

   (a) Show that this pricing game has a unique Nash equilibrium if $\mu = 1$. Find this equilibrium. [5 points]

   (b) Restrict attention to pure strategies. Show that the game has a unique pure strategy Nash equilibrium if $\mu = 0$. Find this equilibrium. [5 points]

   (c) Show that there is no pure strategy equilibrium if $\mu \in (0, 1)$. [10 points]

   (d) Suppose $\mu \in (0, 1)$. A symmetric mixed strategy equilibrium takes the following form: firm i chooses p from a continuously differentiable distribution function G with a support $[\underline{p}, \overline{p}] \subset \mathbb{R}$. Show that $\overline{p} = v$ in any such a mixed strategy equilibrium. [5 points]

   (e) Find a symmetric mixed strategy Nash equilibrium of the above form when $\mu \in (0, 1)$, and verify that what you find is indeed an equilibrium. [10 points]

3. **(35 points)** Players I and II are bargaining over a surplus of size $S > 0$, using alternating offer bargaining, with player I making the initial offer. Suppose both players are risk neutral and discount at the same rate δ. Suppose also that the set of allowable offers is $\left\{0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1\right\}$.

   (a) Suppose the alternating offer bargaining game last three rounds with both players receiving a payoff of zero if no agreement is reached by the last round.

      i. Suppose $\delta \in \left(\frac{2}{3}, \frac{3}{4}\right)$. Describe the set of pure strategy subgame perfect equilibria. [10 points]

      ii. Suppose $\delta \in \left(\frac{3}{4}, 1\right)$. Describe a pure strategy subgame perfect equilibrium with delay. [10 points]

   (b) Suppose now the alternating offer bargaining game has an infinite horizon. Suppose $\delta < \frac{3}{4}$.

      i. Prove that player 1's payoff in any subgame perfect equilibrium cannot be larger than $\frac{3}{4}$. [5 points]

      ii. For what values of $\delta \in \left(\frac{1}{2}, \frac{3}{4}\right)$ is there a pure strategy subgame perfect equilibrium in which 1's payoff is $\frac{3}{4}$? Justify your answer. [10 points]

4. **(30 points)** Consider the following version of a reputation game: There are two periods and two players: a "long-lived" row player (he chooses in both periods and total payoffs are the sum of payoffs from each period) and a column player choosing in the second period. The row player is either an aggressive type (denoted t_A), in which case payoffs are as described in the first pair of payoff matrices, or a passive type (t_P), in which case payoffs are as described in the second pair of payoff matrices. The column player receives a payoff of 0 in the first period, irrespective of the choice of the row player.

T_1	$4, 0$
B_1	$3, 0$

Period 1

	L	R
T_2	$4, 1$	$2, 0$
B_2	$3, 1$	$1, 2$

Period 2

row player is t_A

T_1	$3, 0$
B_1	$4, 0$

Period 1

	L	R
T_2	$3, 1$	$1, 0$
B_2	$4, 1$	$2, 2$

Period 2

row player is t_P

The prior probability that the column player assigns to the row player being aggressive is p. Assume $p \in (\frac{1}{2}, 1)$. In the second period, the column player chooses her action simultaneously with the row player, not knowing his type, but having observed the choice of the row player in period 1.

   (a) What restrictions on second-period behavior of the row player are implied by sequential rationality? (**Hint:** These restrictions are also implied by perfect Bayesian and sequential equilibrium.) Draw the extensive form of the signaling game (representing the original game) implied by these restrictions on the row player's second-period behavior. [10 points]

   (b) Prove that there is no separating perfect Bayesian equilibrium of the induced signaling game. [5 points]

   (c) Describe a pooling perfect Bayesian equilibrium of the induced signaling game in which both types of row player choose T_1 in period 1. [5 points]

   (d) Describe a pooling perfect Bayesian equilibrium of the induced signaling game in which both types of row player choose B_1 in period 1. [5 points]

   (e) One of the pooling equilibria of the signaling game is more plausible than the other. Which one and why? [5 points]