Microeconomic Theory II
Preliminary Examination
August 15, 2011

The exam is worth 120 points in total.

There are 4 questions. Do all questions. Start each question in a new book, clearly labeled. **Fully justify** all answers and show all work (in particular, describing an equilibrium means proving that it has the desired properties). Label all diagrams clearly. Write legibly. If you need to make additional assumptions, state them clearly.

1. [**20 points**] An agent has $w > 0$ dollars to invest in two assets. The first asset is risk free – for every dollar invested, the agent gets back $r > 1$ dollars. The second asset is risky – for every dollar invested, the agent gets back a gross return of $\theta \in \{\theta_1, \theta_2, ..., \theta_N\}$, where $N > 1$. Assume $\pi(\theta)$ represents the probability that the return is θ. If the agent invests x dollars in the risky asset, his expected utility is

$$U(x) = \sum_\theta u(x\theta + r(w - x))\pi(\theta),$$

where u is the agent's von Neumann-Morgenstern utility function. Assume u is concave, strictly increasing, and differentiable. Assume $U(x)$ is concave in x. The agent's decision problem is

$$\max_{x \geq 0} U(x). \tag{1}$$

We allow the agent to choose any nonnegative investment, including $x > w$, i.e., he can borrow at the risk-free rate r. Write $\mathbf{E}[\theta]$ as the expected value of θ, i.e., $\mathbf{E}[\theta] = \sum_\theta \theta\pi(\theta)$.

   (a) Suppose $\mathbf{E}[\theta] < r$. Solve for the agent's optimal investment decision. [10 points]

   (b) Suppose $\mathbf{E}[\theta] = r$. Show that the investment decision you find in (a) is still an optimal decision. [5 points]

   (c) Suppose $\mathbf{E}[\theta] > r$. Show that if a solution to agent's problem exists, the agent will always invest a positive amount on the risky asset. [5 points]

Question 2 is on the next page.

2. **[30 points]** Two faculty members in the economics department wish to recruit a graduate student to their department. Either faculty member can ensure the student will accept the offer by calling the student and promoting the graduate program. However, it is costly to make this call. Assume the payoffs can be represented as follows:

	C	D
C	$1 - c_1, 1 - c_2$	$1 - c_1, 1$
D	$1, 1 - c_2$	$0, 0$

where C means "Call," D means "Don't call,' and c_i, $i = 1, 2$, is faculty i's cost of making the call. Assume each faculty member chooses his/her action without learning the other's action.

(a) Assume faculty 1's cost c_1 is common knowledge, and $c_1 < \frac{1}{2}$. Faculty 2's cost $c_2 \in \{\underline{c}, \overline{c}\}$ is faculty 2's private information. Faculty 1 assigns probability p to faculty 2's cost being $\underline{c}$ and the remaining probability to $\overline{c}$. This probability assignment is common knowledge. Assume that $0 < \underline{c} < 1 < \overline{c}$ and that $p < \frac{1}{2}$.

i. Show that faculty 2 will not call when his/her cost is $\overline{c}$. [5 points]

ii. Show that faculty 1 will call in any Bayesian Nash equilibrium. [5 points]

iii. Find the unique Bayesian Nash equilibrium of this game. [5 points]

(b) Assume c_1 and c_2 are independently drawn from a uniform distribution on $[0, 2]$. Denote faculty i's pure strategy as a measurable function $s_i : [0, 2] \to \{C, D\}$.

i. Show that the following is true in any pure strategy Bayesian Nash equilibrium (s_1, s_2) : For any $c_i \in (0, 2]$, if $s_i(c_i) = C$, then $s_i(c_i') = C$ for any $c_i' < c_i$. [5 points]

ii. A symmetric pure strategy Bayesian Nash equilibrium (s_1, s_2) takes the following form: for some constant $c^* \in [0, 2]$,

$$s_i(c_i) = \begin{cases} C, & \text{if } c_i \leq c^* \\ D, & \text{otherwise} \end{cases} .$$

Find c^*. [10 points]

Question 3 is on the next page.

3. [**40 points**] A worker is employed by a firm. In each period, the worker decides whether to exert effort (E) or shirk (S). Output is a stochastic function of effort: If the worker exerts effort, an output of Y is produced with probability $p \in (0, 1)$, and no output with complementary probability, $1 - p$. If the worker shirks, then no output is produced for sure (i.e., the probability of output y is 0). Simultaneously with the effort choice, the firm chooses a payment w to make to the workers. The flow cost of effort to the worker is $E > 0$, while shirking is costless. The ex post flow payoff to the worker (player 1) and firm (player 2) are thus given by

$$u_1(e, w) = \begin{cases} w - E, & \text{if } e = E, \\ w, & \text{if } e = S, \end{cases} \quad \text{and} \quad u_2(w, y) = y - w,$$

where $y \in \{0, Y\}$ is the output, and the probability distribution of y is given by $\Pr(y = Y \mid e) = p$ if $e = E$ and 0 if $e = S$. Both firm and worker have a common discount factor of $\delta \in (0, 1)$. For simplicity, assume the employment relationship is cannot be dissolved.

(a) In a grim trigger profile, the first realization of no output triggers Nash reversion. The grim trigger profile is described by a simple two state automaton. What is it? For what parameter restrictions is this profile a perfect public equilibrium? [15 points]

(b) Consider the following modification of the game: In the first period, the firm commits to paying a fixed wage w as long as the output Y has been observed. In other words, in every subsequent period, only the worker has a choice to make. What wage will the firm choose? [5 points]

(c) Suppose $E = 1$, $Y = 24$, and $\delta = \frac{1}{2}$. Suppose that in the game described in part 3(b), while the firm knows p, the worker does not. Instead the worker assigns equal probability to $p = \underline{p} = \frac{1}{3}$ and $p = \overline{p} = \frac{2}{3}$. For simplicity, suppose that the worker in the infinite horizon game has only two strategies: $\widetilde{S}$, which specifies S in every period, and $\widetilde{E}$, which specifies E in the first period and continues with E until the first realization of no output, after which S is played in every period. Define a perfect Bayesian equilibrium of the induced signaling game.

Call the wage obtained in part 3(b) the complete information wage for type p. Prove that the profile in which both types pool on the complete information wage for for $p = \underline{p}$ is consistent, and that pooling on the complete information wage for $p = \overline{p}$ is not consistent with perfect Bayesian equilibrium. [10 points]

(d) Does the game from part 3(c) have a separating equilibrium in which the worker chooses $\widetilde{E}$ after at least one of the wages? [10 points]

Question 4 is on the next page.

4. **[30 points]** Consider the following infinitely repeated game with three players. In each period, player 1 is simultaneously playing a prisoners' dilemma with player 2 and with player 3 (so that within the stage game, all players simultaneously choose actions). The stage games are given by

		player 2	
		E_2	S_2
player 1	E_1	$2, 2$	$-1, 3$
	$S1$	$3, -1$	$0, 0$

		player 3	
		E_3	S_3
player 1	E_1	$2, 2$	$-1, 4$
	S_1	$3, -2$	$0, 0$

(Note that the two games are *not* the same!) All players have a common discount factor $\delta \in (0, 1)$. Player 1's payoffs are the *sum* of payoffs in the two games.

Suppose there is a technological restriction that forces player 1 to choose the same action in the two different games (i.e., player 1 plays E_1 against player 2 if and only if he does so against player 3).

(a) Suppose there is perfect monitoring of all players' past actions. Describe a "grim-trigger" strategy profile that induces the outcome path in which $E_1E_2E_3$ is played in every period, and which is subgame perfect for large δ. Give the bounds on δ for which the profile is a subgame perfect equilibrium. [10 points]

(b) Suppose now that while player 1 observes the past actions of players 2 and 3, players 2 and 3 only observe the past actions of player 1. Players 2 and 3 do not observe each others past actions. Grim trigger is now: player 1 plays E_1 in the first period and then as long as $E_1E_2E_3$ always played, otherwise play S_1; player i plays E_i in the first period and then as long as i observes only E_1E_i, otherwise play S_i, $i = 2, 3$. For what values of δ is this profile a Nash equilibrium? Why is every Nash equilibrium of this game subgame perfect? [10 points]

(c) Continuing with the observability assumption from part 4(b), suppose δ is such that the grim trigger profile from part 4(b) is a Nash equilibrium. For what values of δ is the profile sequentially rational? In your answer, the definition of sequential rationality can be informal, without any mathematics (as long as it is clear). [10 points]

End of the exam!