Cohabitation versus marriage: Marriage matching with peer effects

Ismael Mourife and Aloysius Siow

University of Toronto

1 US trends since the seventies

- The marriage rate has fallen significantly. Starting from a low base, the cohabitation rate has increased significantly.

- Cohabitating unions are more unstable than marriage, often leading to separation and not into marriage.

- Women has over taken men in educational attainment.

- There is evidence of an increase in educational positive assortative matching in marriage.

- Earnings inequality has increased significantly.

- The fraction of children living in a single parent (mother) & poor household has risen significantly.

2 How has changes in marital matching affected family earnings inequality?

The authors below argue that increased earnings inequality and changes in marital matching led to increases in family earnings inequality.

- Burtless (1999).

- Greenwood, Jeremy, Nezih Guner, Georgi Kocharkov, and Cezar Santos (2014).

- Carbone and Cahn (2014). Margaret Wente has a column on the book last Saturday.

The objective of this research agenda is to develop a framework and use it to quantitatively evaluate the determinants of changes in family earnings inequality.

3 The empirical framework:

- We want an empirical framework to study marriage matching which allows for:
  - Peer effects in marriage matching.
  - Changes in population supplies.
  - Choice of partners & relationships: marriage, cohabitation, unmatched.
  - Changes in payoffs to different kinds of relationships & partners.
- Today, we present preliminary results:
  - Returns to scale in marriage matching.
  - Are there peer effects in marriage matching?
  - Do variations in sex ratio affect cohabitation versus marriage?

- Consider a marriage market s at time t. There are I, $i = 1, .., I$, types of men and J, $j = 1, .., J$, types of women. Let m_i and f_j be the population supplies of type i men and type j women respectively. Each individual chooses between three types of relationships, unmatched, marriage or cohabitation, $r = [0, \text{m}, c]$, and a partner (by type) of the opposite sex for relationship r. The partner of an unmatched relationship is type 0.

Let M^{st} and F^{st} be the population vectors of men and women respectively. Let θ^{st} be a vector of parameters. A marriage matching function (MMF) is an $2I \times J$ matrix valued function $\mu(M^{st}, F^{st}, \theta^{st})$ whose typical element is μ_{ij}^{rst}, the number of (r, i, j) relationships.

4 The log odds MMF:

$$\ln \frac{\mu_{ij}^{rst}}{(\mu_{i0}^{0st})^{\lambda_r}(\mu_{0j}^{0st})^{\beta_r}} = \gamma_{ij}^{rst} \ \forall \ (r, i, j) \tag{1}$$
$$\lambda_r, \beta_r > 0$$

- This MMF nests several of behavioral MMF.

Empirically, we estimate:

$$\begin{aligned} \ln \mu_{ij}^{rst} &= \lambda_r \ln \mu_{i0}^{0st} + \beta_r \ln \mu_{0j}^{0st} + \widehat{\gamma}_{ij}^{rst} + \varepsilon_{ij}^{rst} \\ \gamma_{ij}^{rst} &= \widehat{\gamma}_{ij}^{rst} + \varepsilon_{ij}^{rst} \end{aligned}$$

where $\widehat{\gamma}_{ij}^{rst}$ is observable to the analyst.

- Since μ_{i0}^{0st} and μ_{0j}^{0st} are endogenous, we instrument them with m_i and f_j.

- What are the interpretations of λ_r, β_r and γ_{ij}^{rst}?

- The above model is not a causal model of $\ln \mu_{ij}^{rst}$.

- Kirsten and I are working on studying how individual earnings affect $\widehat{\gamma}_{ij}^{rst}$.

- When i and j are ordered, the local log odds is a measure of positive assortative matching:

$$\ln \frac{\mu_{ij}^{rst}\mu_{i+1,j+1}^{rst}}{\mu_{i+1,j}^{rst}\mu_{i,j+1}^{rst}} = \gamma_{ij}^{rst} + \gamma_{i+1,j+1}^{rst} - \gamma_{i+1,j}^{rst} - \gamma_{i,j+1}^{rst}$$

The local log odds measures the degree of local complementarity of γ_{ij}^{rst}.

5 Marriage matching with peer effects

We dispense with s and t.

For a type i man to match with a type j woman in relationship r, he must transfer to her a part of his utility that he values as τ_{ij}^r. The woman values the transfer as τ_{ij}^r. τ_{ij}^r may be positive or negative.

Let the utility of male g of type i who matches a female of type j in a relationship r be:

$$U_{ijg}^r = \tilde{u}_{ij}^r + \phi^r \ln \mu_{ij}^r - \tau_{ij}^r + \epsilon_{ijg}^r, \text{ where} \qquad (2)$$

$\tilde{u}_{ij}^r + \phi^r \ln \mu_{ij}^r$: Systematic gross return to a male of type i matching to a female of type j in relationship r.

ϕ^r: Coefficient of peer effect for relationship r. $1 \geq \phi^r \geq 0$.

μ_{ij}^r: Equilibrium number of (r, i, j) relationships.

τ_{ij}^r: Equilibrium transfer made by a male of type i to a female of type j in relationship r.

ϵ_{ijm}^r: i.i.d. random variable distributed according to the Gumbel distribution.

Due to the peer effect, the net systematic return is increased when more type i men are in the same relationships. It is reduced when the equilibrium transfer τ_{ij}^r is increased.

The above empirical model for multinomial choice with peer effects is standard. See Brock Durlauf.

And $\widetilde{u}_{i0} + \phi^0 \ln \mu_{i0}^0$ is the systematic payoff that type i men get from remaining unmatched.

Individual g will choose according to:

$$U_{ig} = \max_{j,r}\{U_{i0g}^0, U_{i1g}^{\mathrm{m}}, ..., U_{ijg}^c, ..., U_{iJg}^c\}$$

Let $(\mu_{ij}^r)^d$ be the number of (r,i,j) matches demanded by i type men and $(\mu_{i0})^d$ be the number of unmatched i type men. Following the well known McFadden result, we have:

$$ln\frac{(\mu_{ij}^r)^d}{(\mu_{i0})^d} = \tilde{u}_{ij}^r - \tilde{u}_{i0} + \phi^r \ln \mu_{ij}^r - \phi^0 \mu_{i0} - \tau_{ij}^r, \quad (3)$$

The above equation is a quasi-demand equation by type i men for (r,i,j) relationships.

The random utility function for women is similar to that for men except that in matching with a type i men in an (r, i, j) relationship, a type j women receives the transfer, τ_{ij}^r.

The quasi-supply equation of type j women for (r, i, j) relationships is given by:

$$\ln \frac{(\mu_{ij}^r)^s}{(\mu_{0j})^s} = \tilde{v}_{ij}^r - \tilde{v}_{0j} + \Phi^r \ln \mu_{ij}^r - \Phi^0 \ln \mu_{0j} + \tau_{ij}^r. \quad (4)$$

The matching market clears when, given equilibrium transfers τ_{ij}^r,

$$(\mu_{ij}^r)^d = (\mu_{ij}^r)^s = \mu_{ij}^r. \tag{5}$$

Then we get a MMF with peer effects:

$$\ln \mu_{ij}^r = \frac{1-\phi^0}{2-\phi^r-\Phi^r} \ln \mu_{i0} + \frac{1-\Phi^0}{2-\phi^r-\Phi^r} \ln \mu_{0j} + \frac{\pi_{ij}^r}{2-\phi^r-\Phi} \tag{6}$$

$$\pi_{ij}^r = \tilde{u}_{ij}^r - \tilde{u}_{i0} + \tilde{v}_{ij}^r - \tilde{v}_{0j}$$

The presence of peer effects in marriage markets do not imply that $\lambda^r + \beta^r > 1$.

You cannot distinguish ϕ^r from Φ^r. On the other hand, you can test whether $\phi^0 = \Phi^0$.

When there is no peer effect or all the peer effect coefficients are the same,

$$\phi^0 = \Phi^0 = \phi^r = \Phi^r$$

we recover the CS MMF:

$$\ln \mu_{ij}^r = \frac{1}{2} \ln \mu_{i0} + \frac{1}{2} \ln \mu_{0j} + \frac{\pi_{ij}^r}{2}$$

When

$$\frac{1 - \phi^0}{2 - \phi^r - \Phi^r} = \frac{1 - \Phi^0}{2 - \phi^r - \Phi^r} = 1$$

we recover the Dagsvik Manziel MMF which is a non-transferable utility model of the marriage market:

$$\ln \mu_{ij}^r = \ln \mu_{i0} + \ln \mu_{0j} + \pi_{ij}^r$$

DM has increasing returns. In this case, we want the peer effect on relationships to be significantly more powerful than that for remaining unmatched.

Also, when

$$\phi^0 + \Phi^0 = \phi^r + \Phi^r = \phi^{r'} + \Phi^{r'},$$

Chiappori, Salanie and Weiss MMF obtains:

$$\ln \mu^r_{ij} = \frac{1-\phi^0}{2-\phi^0-\Phi^0} \ln \mu_{i0} + \frac{\Phi^0}{2-\phi^0-\Phi^0} \ln \mu_{0j} + \frac{\pi^r_{ij}}{2-\phi^0-\Phi^0}$$

And from (6),

$$\ln \frac{\mu^{\mathsf{m}}_{ij}}{\mu^c_{ij}} = \Omega(1-\phi^0) \ln \mu_{i0} - \Omega(1-\Phi^0) \ln \mu_{0j} + \Delta\pi_{ij}$$

As long as $\phi^c + \Phi^c \neq \phi^{\mathsf{m}} + \Phi^{\mathsf{m}}$, the log odds of the number of m to c relationships will not be independent of the sex ratio.

Note also

$$\ln \frac{\mu^r_{ij}\mu^r_{i+1,j+1}}{\mu^r_{i+1,j}\mu^r_{i,j+1}} = \frac{\pi^r_{ij} + \pi^r_{i+1,j+1}\pi^r_{i+1,j} - \pi^r_{i,j+1}}{2-\phi^r-\Phi^r} \quad (7)$$

If the marital output function, $\pi^r_{ij} = \tilde{u}^r_{ij} + \tilde{v}^r_{ij}$, is supermodular in i and j, then the local log odds, $l(r, i, j)$, are positive for all (i, j), or totally positive of order 2 $(TP2)$. So even in the presence of peer effects, we can learn about complementarity of the marital surplus function.

CSPE MMF is a special case of the Log Odds MMF.

It convenient to summarize the different models and some of their properties.

$$\ln \mu_{ij}^{rst} = \lambda_r \ln \mu_{i0}^{0st} + \beta_r \ln \mu_{0j}^{0st} + \gamma_{ij}^{rst}$$

Models and restrictions on λ^r and β^r				
Model	λ^r	β^r	γ_{ij}^r	Restrictions
Log Odds MMF	λ^r	β^r	γ_{ij}^r	$\lambda^r \geq 0, \beta^r \geq 0$
CS	$\frac{1}{2}$	$\frac{1}{2}$	π_{ij}^r	$\lambda^r = \beta^r = \frac{1}{2}$
DM	1	1	π_{ij}^r	$\lambda^r = \beta^r = 1$
CSW	λ^r	1-λ^r	$k\pi_{ij}^r$	$k > 0; \lambda^r = \lambda^{r'} > 0$
CSPE	$\frac{1-\phi^0}{k^r}$	$\frac{1-\Phi^0}{k^r}$	$\frac{\pi_{ij}^r}{k^r}$	$\lambda^r, \beta^r \geq 0, \frac{\lambda^a}{\lambda^b} = \frac{\beta^a}{\beta^b}$

Theorem [Existence and Uniqueness of the Equilibrium matching] For every fixed matrix of relationship gains and coefficients $\beta_r; \lambda_r > 0$ i.e. $\theta \in \Gamma \times (0, \infty)^2$, the equilibrium matching of the log Odds MMF model exists and is unique.

Proposition (constant returns to scale) The equilibrium matching distribution of the log Odds MMF model satisfies the Constant return to scale property if $\beta_r + \lambda_r = 1$ i.e.

$$\beta_r + \lambda_r = 1 \text{ for } r \in \{a, b\} \Rightarrow \sum_{i=1}^{I} \frac{\partial \mu}{\partial m_i} m_i + \sum_{j=1}^{J} \frac{\partial \mu}{\partial f_j} f_j = \mu.$$

Theorem Let μ be the equilibrium matching distribution of the log Odds MMF model. If the coefficients β_r and λ_r respect the restrictions

1. $0 < \beta_r; \lambda_r \leq 1$ for $r \in \{a, b\}$;

2. $\max(\beta_b - \lambda_b, \beta_a - \lambda_a) < \min_{i \in I} \left(\frac{1-\rho_i^m}{\rho_i^m} \right)$;

3. $\min(\beta_b - \lambda_b, \beta_a - \lambda_a) > - \max_{j \in J} \left(\frac{1-\rho_j^f}{\rho_j^f} \right)$;

where ρ_i^m is the rate of matched men of type i and ρ_j^f is the rate of matched women of type j, then:

- Type-specific elasticities of unmatched.
  The following inequalities hold in the neighbourhood of μ^{eq}:
  $$\frac{m_i}{\mu_{k0}} \frac{\partial \mu_{k0}}{\partial m_i} \geq \begin{cases} \frac{1}{m_i^*} \frac{m_k}{m_k^*} \sum_{j=1}^{J} \frac{[\lambda_a \mu_{kj}^a + \lambda_b \mu_{kj}^b][\beta_a \mu_{kj}^a + \beta_b \mu_{kj}^b]}{f_j^*} > 0 \\ \frac{m_i}{m_i^*} [1 + \frac{1}{m_i^*} \sum_{j=1}^{J} \frac{[\lambda_a \mu_{ij}^a + \lambda_b \mu_{ij}^b][\beta_a \mu_{ij}^a + \beta_b \mu_{ij}^b]}{f_j^*}] > \end{cases}$$
  $1 \leq k \leq I$.
  $$\frac{f_j}{\mu_{0k}} \frac{\partial \mu_{0k}}{\partial f_j} \geq \begin{cases} \frac{1}{f_j^*} \frac{f_k}{f_k^*} \sum_{i=1}^{I} \frac{[\lambda_a \mu_{ik}^a + \lambda_b \mu_{ik}^b][\beta_a \mu_{ik}^a + \beta_b \mu_{ik}^b]}{m_i^*} > 0 \\ \frac{f_j}{f_j^*} [1 + \frac{1}{f_j^*} \sum_{i=1}^{I} \frac{[\lambda_a \mu_{ij}^a + \lambda_b \mu_{ij}^b][\beta_a \mu_{ij}^a + \beta_b \mu_{ij}^b]}{m_i^*}] > 1 \end{cases}$$

$1 \leq k \leq J$,

$$\frac{m_i}{\mu_{0j}}\frac{\partial \mu_{0j}}{\partial m_i} \leq -\frac{[\lambda_a \mu_{ij}^a + \lambda_b \mu_{ij}^b]}{m_i^* f_j^*} m_i < 0, \text{ for } 1 \leq i \leq I \text{ and}$$

$$\frac{f_j}{\mu_{i0}}\frac{\partial \mu_{i0}}{\partial f_j} \leq -\frac{[\beta_a \mu_{ij}^a + \beta_b \mu_{ij}^b]}{m_i^* f_j^*} f_j < 0, \text{ for } 1 \leq i \leq I \text{ and } 1$$

where

$$m_i^* \equiv m_i - \sum_{j=1}^{J} [(1-\lambda_a)\mu_{ij}^a + (1-\lambda_b)\mu_{ij}^b], \text{ for } 1 \leq i \leq I,$$

$$f_j^* \equiv f_j - \sum_{i=1}^{I} [(1-\beta_a)\mu_{ij}^a + (1-\beta_b)\mu_{ij}^b], \text{ for } 1 \leq j \leq J.$$

6 Preliminary empirical evidence

- 1990, 2000 US census; 3 years of ACS around 2010?

- Each state year is a separate marriage market.

- Males are between ages 28-32. females 26-30.

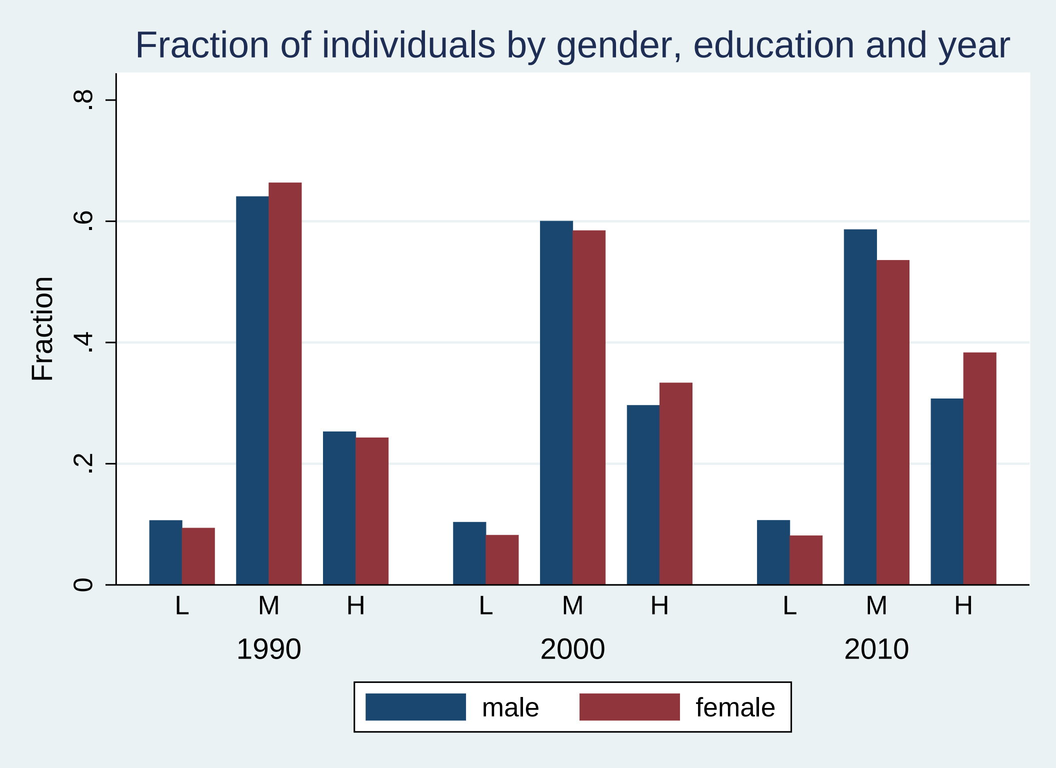
- 3 categories of educational attainment:

  – L: Less that high school graduation.

  – M: High school graduate but not university graduate.

  – H: University graduate and or more.

- Cohabitation: response of "unmarried partner" to relationship to household head.


Fraction of individuals by gender, education and year
Fraction
0
.2
.4
.6
.8
L
M
H
L
M
H
L
M
H
1990
2000
2010
male
female

OLS

	Ln c	Ln m	Ln c	Ln m	Ln c	Ln m
Lu_male	0.453 (0.036)**	0.423 (0.046)**	0.371 (0.076)**	0.198 (0.052)**	0.603 (0.079)**	0.542 (0.033)**
Lu_female	0.564 (0.037)**	0.652 (0.047)**	0.623 (0.077)**	0.649 (0.050)**	0.858 (0.082)**	0.887 (0.035)**
HM			-1.085 (0.079)**	-1.208 (0.054)**	-1.206 (0.074)**	-1.353 (0.035)**
MH			-0.759 (0.084)**	-0.948 (0.070)**	-0.923 (0.076)**	-1.220 (0.045)**
MM			0.446 (0.055)**	0.337 (0.049)**	0.138 (0.072)	-0.099 (0.043)*
ML			-0.730 (0.171)**	-1.386 (0.107)**	-0.657 (0.154)**	-1.443 (0.054)**
LM			-0.861 (0.121)**	-1.861 (0.089)**	-0.829 (0.107)**	-1.804 (0.046)**
LL			-0.422 (0.078)**	-1.620 (0.057)**	-0.027 (0.101)	-1.250 (0.049)**
Y00			-0.017 (0.041)	-0.323 (0.029)**	-0.001 (0.036)	-0.332 (0.016)**
Y10			0.620 (0.059)**	-0.856 (0.041)**	1.348 (0.144)**	0.021 (0.068)
State effects					Y	Y
_cons	-4.043 (0.365)**	-2.192 (0.425)**	-3.409 (0.219)**	1.158 (0.156)**	-8.426 (0.877)**	-3.849 (0.386)**
R^2	0.68	0.66	0.88	0.95	0.91	0.98
N	964	1,034	964	1,034	964	1,034

* p<0.05; ** p<0.01

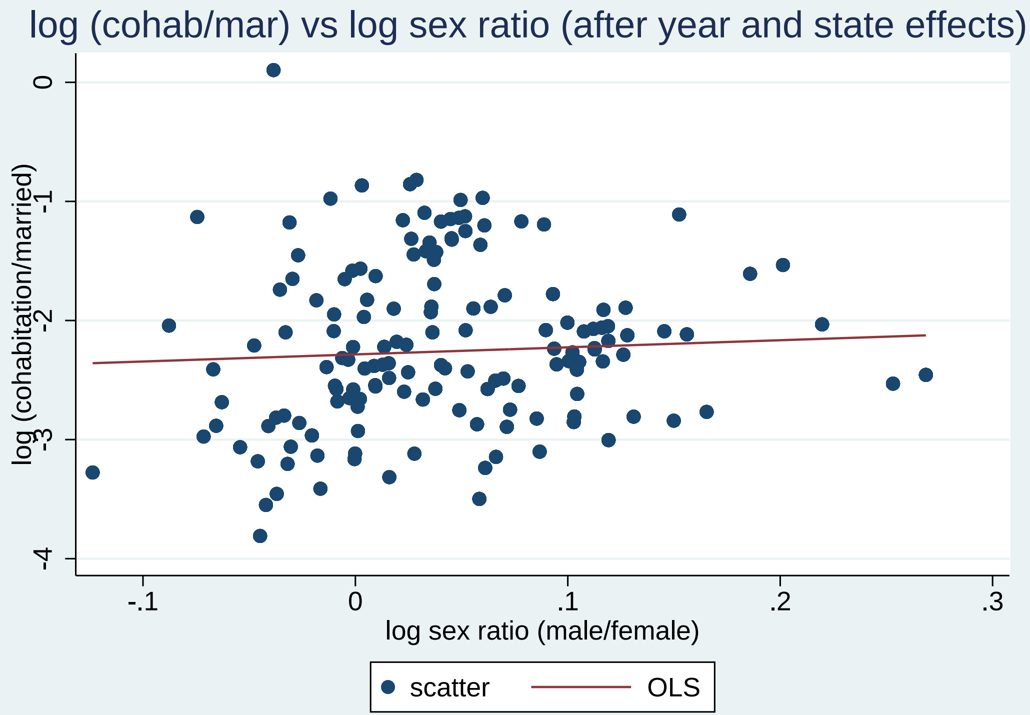
IV: instruments are m_i and f_j

	Ln c	Ln m	Ln c	Ln m	Ln c	Ln m
Lu_male	0.452	0.477	0.322	0.131	0.626	0.601
	(0.036)**	(0.050)**	(0.082)**	(0.054)*	(0.088)**	(0.036)**
Lu_female	0.576	0.694	0.670	0.751	0.953	1.074
	(0.039)**	(0.050)**	(0.083)**	(0.053)**	(0.087)**	(0.037)**
HM			-1.113	-1.267	-1.258	-1.457
			(0.079)**	(0.057)**	(0.074)**	(0.038)**
MH			-0.720	-0.891	-0.936	-1.255
			(0.086)**	(0.075)**	(0.080)**	(0.048)**
MM			0.456	0.331	0.066	-0.251
			(0.056)**	(0.052)**	(0.073)	(0.050)**
ML			-0.646	-1.230	-0.579	-1.299
			(0.181)**	(0.112)**	(0.165)**	(0.059)**
LM			-0.921	-1.962	-0.868	-1.871
			(0.125)**	(0.093)**	(0.115)**	(0.050)**
LL			-0.408	-1.567	0.074	-1.044
			(0.079)**	(0.058)**	(0.104)	(0.054)**
Y00			-0.013	-0.313	0.004	-0.321
			(0.041)	(0.031)**	(0.036)	(0.017)**
Y10			0.614	-0.805	1.527	0.392
			(0.059)**	(0.042)**	(0.150)**	(0.075)**
State effects					Y	Y
_cons	-4.168	-3.180	-3.387	0.783	-9.517	-6.118
	(0.365)**	(0.418)**	(0.216)**	(0.167)**	(0.897)**	(0.427)**
R^2	0.68	0.65	0.88	0.95	0.91	0.98
N	964	1,034	964	1,034	964	1,034

* p<0.05; ** p<0.01

IV with time varying match effects

	Ln c	Ln m	Ln c	Ln m
Lu_male	0.440	0.309	0.608	0.656
	(0.075)**	(0.055)**	(0.076)**	(0.031)**
Lu_female	0.545	0.567	0.756	0.818
	(0.072)**	(0.055)**	(0.077)**	(0.037)**
LL HM	2.28	2.48	2.27	2.47
	(0.138)**	(0.084)**	(0.130)**	(0.033)**
LL ML	1.47	1.89	1.47	1.82
	(0.122)**	(0.110)**	(0.107)**	(0.046)**
LL HM00	0.223	-0.017	0.182	-0.103
	(0.245)	(0.184)	(0.209)	(0.089)
LL ML00	0.203	0.144	0.187	0.146
	(0.198)	(0.154)	(0.175)	(0.087)
LL HM10	1.43	-0.354	1.95	0.474
	(0.264)**	(0.197)	(0.266)**	(0.115)**
LL ML10	0.842	0.167	0.828	0.185
	(0.234)**	(0.223)	(0.232)**	(0.189)
Y00	0.310	-0.007	0.269	-0.090
	(0.095)**	(0.073)	(0.078)**	(0.039)*
Y10	1.185	-0.291	1.710	0.544
	(0.098)**	(0.078)**	(0.147)**	(0.066)**
State effects			Y	Y
R^2	0.89	0.96	0.92	0.99
N	964	1,034	964	1,034


log (cohab/mar) vs log sex ratio (after year and state effects)
log (cohabitation/married)
0
-1
-2
-3
-4
-.1
0
.1
.2
.3
log sex ratio (male/female)
scatter
OLS