Signaling Character in Electoral Competition*

Navin Kartik†
UC San Diego

R. Preston McAfee‡
CalTech

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Abstract

We study a one dimensional Hotelling-Downs model of electoral competition with the following innovation: a fraction of candidates have "character" and are exogenously committed to a campaign platform; this is unobservable to voters. However, character is desirable, and a voter's utility is a convex combination of standard policy preferences and her assessment of a candidate's character. This structure generates a signaling game between strategic candidates and voters, since a policy platform not only affects voters' utilities directly, but also indirectly through inferences about a candidate's character. The model generates a number of predictions, starting with a failure of the median voter theorem. The results can explain why candidates sometimes choose non-median platforms, and moreover, win elections with such platforms. We discuss a number of extensions, including a model of primaries.

Keywords: electoral competition, signaling, character, integrity, valence median voter theorem, policy divergence, primaries

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†Email: nkartik@ucsd.edu; Web: http://econ.ucsd.edu/∼nkartik; Address: Department of Economics, 9500 Gilman Drive, La Jolla, CA 92093-0508.

‡Email: preston@mcafee.cc; Web: http://mcafee.cc; Address: 100 Baxter Hall, MC 228-77, Humanities and Social Sciences, California Institute of Technology, Pasadena, CA 91125.

"Americans believe Mr. Bush himself honestly believed Saddam was a threat ... [voters] can tell he is not doing it all by polls and focus groups ... You can agree or disagree with him, but it is hard to doubt his guts, his seriousness, and his commitment ... This is why in presidential elections character trumps everything."

— Wall Street Journal Editorial, April 22, 2004

1 Introduction

Throughout the 2000 U.S. presidential race, spokespeople for the George Bush campaign frequently alleged that opponent Al Gore would "say anything to get elected." The implicit suggestion is that George Bush believed in his position and would state it even if it hurt his chances of being elected. The interesting proposition that the Bush campaign took is that voters should vote against politicians who state popular positions, because such politicians lack character.

That career politicians might lack character is not new. Criticism of career politicians and insiders has been a frequent refrain in political campaigns. George Washington said "Few men have virtue to withstand the highest bidder." Calvin Coolidge said "Character is the only secure foundation of the state." Barry Goldwater used character ineffectively against Lyndon Johnson in 1964. In a series of television advertisements, a spokesman said "You must not give power to a man unless, above everything else, he has character. Character is the most important qualification the President of the United States can have." It is a historical irony that Goldwater's spokesman for presidential character was Richard Nixon.

The simple purpose of this paper is to investigate the effects of character—taken to be an exogenous characteristic of people—on policy selection and voting. The idea is that a politician with character prescribes to voters the position that the politician thinks is best. Politicians without character tell voters whatever is most likely to get the politician elected. Voters have preferences both over positions and over character. Voters try to infer character from the positions taken by candidates. Only the candidates without character are strategic, because the candidates with character state their exogenous best position.

One immediate consequence of character is that the median voter theorem cannot hold. To see why, suppose that all politicians without character choose the preferred position of the median voter. Then even the slightest deviation from that position will inform the voters that the candidate has character, and hence insure the election of the candidate, who would be a politician with character almost at the median. This is by itself an important effect of character, because while the median voter theorem is robust in many theoretical models (see, for example, Banks and Duggan, 2003), it does not appear to be an empirically salient characteristic of many elections (Ansolabehere et al., 2001).

The theory we develop produces a unique equilibrium, in which candidates without character randomize. The distribution of their positions is related to the distribution of the positions of candidates with character, with a distortion toward the median. Thus, even though the median voter theorem fails in its strong form, policies are biased towards the center. Most elections result in a tie; only extreme candidates lose, and extremists are candidates with character. Tied elections arise because candidates without character randomize, randomization entails indifference, and in the political context, indifference entails ties.

Why do voters care about character? We appeal to a standard answer given in the literature: voters use it to partially assess the set of actions a candidate may take if elected. Politicians cannot commit to a full set of contingent actions, and so voters are necessarily unsure what an elected official will actually do once in office. At most, the candidate can commit to a handful of stated positions. As voters typically care about subtle and sometimes unobservable behavior by an elected official, voters will also care about attributes of the candidate beyond the candidate's stated policy dimension. The canonical class of such attributes are known as *valence* and are familiar characteristics of political theory (Stokes, 1963).[1] The crucial feature of valence characteristics is that they are valued by all voters—more of the trait is preferred to less—independent of ideological position. It is this interpretation we take when introducing our basic model. The novel aspect of our approach is that the position chosen by the candidate is interpreted by the voters as a signal about character.

In the latter part of the paper, we extend our theory to cover a richer set of voter preferences. In particular, this permits voters with differing ideal policy positions to value a candidate's character differently. Moreover, a voter's preference for character can vary with the stated position of the candidate. This allows us to endogenize the preference for character, stepping beyond mere valence interpretations. Suppose that an elected official takes actions of two kinds: observable or "in plain view", and unobservable or "out of sight". Candidates can credibly commit to what observable actions they will take because voters can punish them should they not keep their promises; on the other hand, unobservable actions cannot be committed to, since voters cannot enforce such promises. A politician who has character says what he will actually do (on both dimensions), whereas a politician without character may do something very different on the unobservable dimension than what he promised—and necessarily lives up to—on the observable dimension. In this sense, it is natural to interpret the character trait here as that of *integrity*.[2] This setting generates an endogenous taste for character among voters, which will depend both on stated position of the candidate and a voter's own ideal position. We show that our main insight extends to

[1] Not all valence attributes are about character, however. Although Stokes's (1963) original discussion of valence comports well with our notion here, more recent literature sometimes uses the term 'valence' to describe attributes such as "handshaking ability", charisma, and so forth. These traits are not what we have in mind.

[2] The Merriam-Webster dictionary definition of 'integrity' is "firm adherence to a code of especially moral or artistic values". Discussion of integrity often arises in politics, and in fact the word was the most-researched word on Merriam-Webster online by Americans in 2005. (Source: CNN `http://www.cnn.com/2005/US/12/10/top.word.ap.ap/`.)

such an environment.

Recent literature, such as Osborne and Slivinski (1996) and Besley and Coate (1997), considers candidates who cannot commit to positions at all. Lack of commitment should tend to make candidate character more important, strengthening the implications of our analysis. Some commitment is plausible, however, based on repeated game arguments of Alesina (1988) and Alesina and Spear (1988); empirical support for commitment appears in Poole and Rosenthal (1997). One implication of our analysis is that a candidate who changes their position toward the median may be perceived as lacking character, thus encouraging commitment to positions. Thus, in a dynamic or repeated election context, the possibility of character itself may enhance commitment.

We formulate our model in the simplest possible structure: the standard one-dimensional spatial model of electoral competition following Downs (1957), adapted from the spatial model of Hotelling (1929). Character itself is taken to be an exogenous characteristic of candidates. We think it is reasonable to consider that character is formed long before individuals choose to enter politics, and that character in the general population is in fact exogenous. However, character may play a role in the selection of candidates who run for office (e.g. Bernheim and Kartik, 2004), and a weakness of our analysis is that the process generating candidates is not modeled. In defense, we analyze the subgame of platform selection given an arbitrary candidate selection mechanism, and analysis of the subgame is necessary to investigate a more general model.

We find that the posterior probability that a candidate has character is higher the further the candidate is from the ideal position of the median voter. This is the feature of the equilibrium that leads to voter indifference. Voters think extremists usually mean what they say, while middle-of-the-road candidates are more likely to have simply said what voters want to hear. The beliefs are constructed to create indifference among candidates without character. A useful aspect of this construction is that it leads to a closed form for the density of positions of candidates without character, up to a single parameter that must be implicitly constructed. This construction would aid an attempt at empirical analysis, although the problems of quantifying the position space and beliefs about character necessary for empirical testing are daunting, indeed.

As the proportion of candidates with character diminishes to zero, the equilibrium platform distribution of those without character collapses on the median voter's ideal point. However, the support of these positions does not collapse. Interestingly, however, if almost all candidates have character, both the distribution and support of positions of candidates without character also collapses to the median. Thus, while there is a sense in which the median voter theorem holds at either extreme of the model, the mere presence of character has an echo on policy platforms even when it is very unlikely.

An important feature of the equilibrium of our model is that it is an ex-post equilibrium, which means that even after the candidates see each other's position, they do not regret their choice. Thus, it does not matter if the game is played sequentially or simultaneously, as the predicted behavior is invariant to timing. We find this a particularly appealing aspect of our mixed-strategy equilibrium.

The theory presented here provides a theoretical but intuitive grounding for a mechanism that is often believed to operate in real elections. As the opening quote from the Wall Street Journal suggests, the perception of not pandering to the public can be valuable to a politician. This provides a novel explanation for why candidates may select non-median platforms, and moreover, why voters may vote for such candidates, rather than simply selecting a centrist candidate. Applications of this principle abound. For example, it is widely agreed that in the 2004 U.S. presidential election, George W. Bush won despite choosing a platform that was well to the right of the center. His victory is often attributed to a belief amongst the public of his "conviction" in his policy position, in contrast to the perception of John Kerry. This is consistent with our theory; moreover, the model delivers this as rational behavior, and in particular, rational inferences by the public. However, our theory also makes the following point: it is not guaranteed that the non-centrist candidate truly possesses the desirable character trait; he could in fact be a strategic politician mimicking the behavior of those with character.

As another example, consider the case of the VLD (Flemish Liberal Party) in Belgium.[3] In 1994, the VLD committed to propose as its platform the policy preferred by the majority of the Belgian public, which it elicited through a public poll. It lost the election by a large margin to a less centrist party. In 1999, on the other hand, the VLD did not pursue this approach, and instead proposed a less centrist platform without polling the public. It won the election, beating more centrist parties. Why? The explanation here is that in simply pandering to the median voter in 1994, the VLD leaders signaled a lack of character. In contrast, in 1999, despite not being as desirable purely on the platform dimension, the new leaders of the VLD signaled that they would choose policies that *they* believed were best, and this trait was valued by voters. Needless to say, this should be viewed as just one component in explaining the reversal in VLD's fortunes.

As a final example, we recall a well-publicized statement made by senator John McCain during a 1999 Republican Primaries debate in Iowa: "I'm here to tell you the things that you dont want to hear..." McCain went on to denounce ethanol subsidies, which are widely popular in Iowa. From the perspective of appealing to voters on policy alone, this is perhaps puzzling; in fact, all the other Republican candidates either supported or expressed neutrality on this issue. However, when a campaign is interpreted as also signaling character, McCain' approach is straightforward to interpret. The quoted statement prefacing his position suggests that his goal was to convince voters that he would not merely choose platforms that appeal to them, but instead choose platforms that he truly believed in—and that such a trait should be appreciated by voters.

There are other papers in the literature that derive non-median-voter results, starting with Calvert (1985) and Wittman (1977). The closest in spirit to ours is the work of Callander (2004). He too presumes that voters value a trait about politicians that is unob-

[3] We owe this example to Carrillo and Castanheira (2002).

servable but may be signaled in their campaign platform. The specific trait he considers is that of effort exerted in implementation of policy. Unlike in our model with character, all candidates in his model are fully strategic: they differ only in whether they are policy motivated (and exert high effort if elected) or office motivated (and exert low office if elected). While there are obvious similarities in motivation and in the results derived, there is an important conceptual difference between the notions of policy motivated candidates versus candidates with character as we define them. The former entails a preference about final policy outcomes; the latter entails a direct preference over one's own campaign platforms. We discuss this in more detail after introducing our model.

Signaling in electoral competition was first considered by Banks (1990). He develops a model where campaign promises are non-binding, and candidates implement their privately known preferred policy if elected. However, candidates who are elected incur a cost of implementing a policy different from their campaign platform. Thus, the non-binding platforms can nevertheless serve as a signal of their preferred policy. Our model is different because we assume that some candidates—those with character—announce their truly preferred platforms, regardless of any considerations of whether they will be elected or not. Callander and Wilkie (2003) extend Banks (1990) by allowing candidates to incur differential costs of implementing a policy distinct from their campaign platforms: some suffer zero disutility, whereas others incur a cost just as in Banks (1990). To some extent, our model can be thought of as a case where some candidates suffer infinite disutility from lying and are therefore simply honest, whereas others suffer none. Both Banks (1990) and Callander and Wilkie (2003) assume no commitment, i.e. that campaign platforms are non-binding; we derive our main results in a model with commitment, but show how our central insights can be extended to the case without commitment as well.

The impact of valence on policy platforms was formally studied by Ansolabehere and Snyder (2000), Aragones and Palfrey (2002), and Groseclose (2001). In a related vein, Londregan and Romer (1993) study an incumbency advantage model where an incumbent has a valence advantage of "ability" over challengers. These papers take a candidate's valence attribute as observable, hence there is no signaling element. Policy divergence stems from asymmetry of valence across candidates. Our divergence result arises even when candidates are completely ex-ante symmetric.

The rest of the paper is structured as follows. We present the basic model in the following Section. In Section 3, we derive the unique equilibrium and study a number of its implications. Section 4 discusses a number of extensions of the theory. We conclude in Section 5. All formal proofs are collected in an Appendix.

2 The Model

The basic element of the model is a standard Hotelling-Downs one-dimensional policy location game. The set of policies is denoted $X = [0, 1]$. There is a continuum of voters (synonymous with citizens), each with single-peaked policy preferences on X. A voter is identified by her ideal point, $v \in X$, and we assume that her[4] policy preferences can be represented by a utility function $u(x, v)$ that is twice continuously differentiable, and satisfies $u_1(v, v) = 0$ and $u_{12}(x, v) > 0$ for all x.[5] A voter's ideal point is drawn from a probability distribution with median $m \in (0, 1)$. There are two candidates (synonymous with politicians), A and B, each of whom must commit to a platform, $x^i \in X$. After observing both candidates' platforms, each voter votes sincerely to maximize her expected utility.[6]

We now depart from the standard model by introducing *character* as follows. Character is a binary variable: candidate $i \in \{A, B\}$ either possess it $\left(c^i = 1\right)$ or does not $\left(c^i = 0\right)$. This is private information and drawn independently from a Bernoulli distribution with $\Pr\left(c^i = 1\right) = b > 0$. If a candidate i has character, his platform choice, x^i, is constrained to be the draw of a random variable that has a differentiable cumulative distribution function F with density $f(x) > 0$ for all $x \in X$.[7] Hence, a candidate with character has no strategic choice to make, and we refer to such types as *non-strategic types*.[8] On the other hand, candidates without character only care about holding office, and hence strategically choose their platform to maximize their probability of being elected.

Voters care about character in addition to policy: a voter v's expected utility from a candidate i with platform x^i is denoted $U(x^i, v)$, where

$$U(x^i, v) \equiv \lambda \Pr\left(c^i = 1 \middle| \, x^i\right) + u\left(x^i, v\right)$$

Thus, $\lambda > 0$ is the relative weight attached to the character of a politician by voters. Note that the inference about a politician's character depends upon his chosen platform. The standard Hotelling-Downs model is a special case of our model when either $b = 0$ or $\lambda = 0$.

Some remarks are in order about our modeling choices, and further literature connections. First, candidates without character in our model are purely office-motivated and fully strategic. Formally, candidates with character are non-strategic and can be thought

[4]Throughout, we use female pronouns for voters and male pronouns for candidates.

[5]This represents single-peaked preferences because for any $x \neq v$,

$$u(v, v) - u(x, v) = \int_x^v \left[u_1(z, v) - u_1(z, z)\right] dz = \int_x^v \int_z^v u_{12}(z, y) dy dz > 0$$

[6]Sincere voting is fully rational in this setting; it only serves to eliminate trivial equilibria such as all voters voting for one candidate, and no-one deviating because they are powerless to change the outcome.

[7]The assumption that F has full support is *not* crucial. Our analysis extends to arbitrary F distributions; failure of the Median Voter Theorem (Corollary 1) only requires that the support of F include an interval containing the median, m.

[8]As will be clear from the analysis, what formally matters is not that there actually be positive probability that a candidate is non-strategic, but only that the electorate believe this to be case.

of as "crazy" types following Kreps et al. (1982). However, this interpretation is strained, and we prefer to think of them as fully rational, despite our choice of terminology. The idea is that candidates with character suffer (infinite) disutility from proposing a platform they do not "believe in". As discussed in the introduction, our assumption that citizens care about a politician's character is meant to capture the idea that citizens have not only policy preferences, but also care about some unobservable characteristic about politicians that is correlated with their willingness to campaign on platforms that are not their true policy preferences. Indeed, our model can be thought of as a reduced form for the following: each candidate i has a policy, p^i, drawn from the cdf F, that he thinks is "right". He then learns whether he has character, in which case he effectively must choose platform $x^i = p^i$, perhaps due to a preference for not pandering. If he does not have character, he strategically chooses platform x^i to maximize probability of being elected. Now, suppose character types will resist special interests if elected, whereas non-character or flexible types will fall prey to them. Voters get a disutility of λ from an elected politician who deals with special interests. In this environment, a voter has an instrumental reason to prefer politicians who are non-strategically choosing their platform, all else equal.

The candidates with character in our model are not policy motivated in the usual sense the term is used in (Wittman, 1977; Calvert, 1985). The utility for policy motivated candidates depends on the final policy outcome and not directly on their individual policy platform; whereas candidates with character in our model care directly about the platform they propose, regardless of the final policy outcome. In a sense, character candidates are *procedurally* motivated. To take another perspective, policy motivated candidates have unlimited ability to compromise in platform, and will do so if it results in a more desirable final policy outcome; character candidates are limited in their ability to compromise on their platform. Fiorina (1999, esp. fn. 10 on p. 9) makes this distinction.[9]

We have assumed that the prior probability of having character is the same for both candidates, and the distribution of platform conditional on character is also the same. Thus, there is no ex-ante asymmetry between the candidates. This contrasts with the literature on observable valence asymmetry between candidates (e.g. Aragones and Palfrey, 2002). In Section 4, we extend our model to cover ex-ante differences between the candidates.

Since candidates with character are non-strategic, we refer to a candidate's strategy as his behavioral rule conditional on *not* having character, i.e. conditional on being strategic. A strategy for candidate $i \in \{A, B\}$ is represented by a cumulative distribution function (cdf), G^i; for technical convenience, we restrict attention to strategies that can be written as the sum of absolutely continuous and discrete distributions.[10] If G^i has a density, we denote it g^i. As this is a signaling game, voter beliefs about a candidate's character are critical. Let $\varphi^i(x)$ be the posterior probability that i has character given his platform

[9]The policy divergence results of Roemer (1994) also assume that parties are policy motivated, not procedurally motivated.

[10]By the Lebesgue decomposition theorem for the Real line, this is only a restriction insofar as it precludes a strategy from having a singular component without mass points.

choice of x.

It is convenient to define $\mu(x) \equiv u(x, m)$, so that $\mu(x)$ is the median voter's policy utility from platform x. Given the posterior belief of character and the platform of candidate i, the median voter's expected utility should this candidate be elected is

$$\alpha^i\left(x|\varphi^i\right) = \lambda\varphi^i(x) + \mu(x)$$

Where there is no risk of confusion, we typically suppress the dependence of α^i on φ^i to reduce notation. Sincere voting implies that candidate i wins and candidate $j \neq i$ loses if $\alpha^i\left(x^i\right) > \alpha^j\left(x^j\right)$. When the two expressions are equal, the election is tied, and each candidate gets elected with probability $\frac{1}{2}$. Given the beliefs φ^A and φ^B, voter behavior is completely pinned down (except perhaps for a measure 0 set of voters), hence we are not explicit about it in what follows.

Our solution concept is that of (weak) perfect Bayesian equilibrium (Fudenberg and Tirole, 1991). This requires that the platform distributions, G^A and G^B respectively, maximize the probability of being elected for each strategic candidate given voter beliefs φ^A and φ^B, and that φ^A and φ^B be consistent with Bayes Rule.[11]

3 Signaling Character

3.1 The Unique Equilibrium

Due to the symmetry in the model, a candidate must win with positive probability if strategic. To see this, observe that a candidate can always play the same strategy as his opponent. If he does so, he loses with probability one only if voters believe that for all platforms in the support of this strategy, he is less likely to have character than his opponent. Given that both candidates have the same prior likelihood of character, $b > 0$, and the same distribution over platforms with character, F, this cannot be the case.

The only way that strategic candidates win with positive probability and yet maximize their probability of winning is if every platform that they choose offers the same value to the median voter (and all platforms not chosen offer no greater value). Call this value α^*. Since there are no mass points in the platforms of the candidates with character, strategic candidates will not use mass points either. To see this, note that the Bayes' update on the probability a candidate has character is zero at any mass point in the strategy of the strategic candidates. Thus at any mass point in the strategy, there is a choice for the strategic candidate with a nearby policy platform, but a positive likelihood of having character. Consequently, the distribution of positions chosen by the candidates will be

[11] More precisely, each φ^i must be a regular conditional distribution derived from the distributional strategy (Milgrom and Weber, 1985) induced by F and G^i.

continuous. This means that we can write the Bayes' update as

$$\varphi^i(x) = \frac{bf(x)}{bf(x) + (1-b)\, g^i(x)}$$

Since within the support of their strategies, both strategic candidates offer value α^*, we conclude that

$$\alpha^* = \mu(x) + \lambda \frac{bf(x)}{bf(x) + (1-b)\, g^i(x)} \tag{1}$$

The above equation says that for any position that may be taken by a strategic candidate, the utility offered to the median voter is a constant. Recall that this utility is the sum of the position's direct value to the median voter and the Bayes' updated beliefs about the candidate's likely character. Equation (1) can be solved for the density of platform choices, which turns out to be

$$g^i(x) = g^*(x) = \max\left\{0, \frac{bf(x)}{1-b}\left[\frac{\lambda}{\alpha^* - \mu(x)} - 1\right]\right\} \tag{2}$$

This is a symmetric expression, so g doesn't depend on i. Moreover, the value of α^* is determined by the requirement that g^* be a density, and hence integrate to one. It is readily shown that $\mu(m) < \alpha^* < \mu(m) + \lambda$, since a candidate can always offer a value of at least $\mu(m)$ by choosing platform m, and can offer value of no greater than $\mu(m) + \lambda$.

An important aspect of this construction is that the median voter is indifferent to every position taken by strategic candidates. Thus, even knowing their opponent's platform, a strategic candidate has no incentive to revise his position. This property means that the equilibrium is an *ex-post equilibrium*, and in this setting, it has two important implications. First, the outcome is not sensitive to the timing of the game: the same behavior remains an equilibrium whether one candidate announces first, or second, or both announce platforms simultaneously. Second, all elections between strategic candidates result in a tie, even though candidates have distinct positions.

It turns out that, with substantially more work, it can be shown that this equilibrium is unique. This conclusion is summarized in Theorem 1, whose formal proof is in the Appendix.

Theorem 1. *There is a unique equilibrium. It is an ex-post equilibrium where both candidates use the same strategy, G^*, with density*

$$g^*(x) = \max\left\{0, \frac{bf(x)}{1-b}\left[\frac{\lambda}{\alpha^* - \mu(x)} - 1\right]\right\}$$

where $\alpha^* \in (\mu(m), \mu(m) + \lambda)$ is the unique constant such that $\int_x g^*(x)dx = 1$.

Therefore, both candidates mix in equilibrium, using a continuous density. Our interpretation of mixed strategies follows the Bayesian view of opponents' conjectures, originating in Harsanyi (1973). That is, a candidate's mixed strategy need not represent him literally randomizing over platforms; instead, it represents the uncertainty that the *other candidate and the electorate* have about his pure strategy choice. Each candidate could be playing a pure strategy which depends upon an auxiliary variable that is his private information.

One implication of Theorem 1 that is worth emphasizing is that the median voter theorem (MVT) does not hold in our model. Note that in the current setting, the appropriate version of the MVT is that ex-ante, each candidate chooses platform m with probability $1 - b > 0$, viz. with the probability of being strategic.

Corollary 1. *The MVT fails. In the unique equilibrium, the ex-ante probability that either candidate chooses platform m is 0.*

3.2 Properties

We now derive various implications of Theorem 1. The first is a simple observation.

Fact 1. *If a candidate is strategic, he wins with probability at least as large as if he had character.*

This is an immediate consequence of the fact that a strategic candidate provides the median voter with utility α^*, whereas a candidate with character provides the same utility if his platform falls within the support of the equilibrium strategy, G^*, and strictly less utility if his platform falls outside the support.

Since α^* is the expected utility of the median voter, comparative statics on α^* immediately become comparative statics on the utility of the median voter. The comparative statics on α^* are readily computed from the equation

$$\int_x \max\left\{0, \frac{bf(x)}{1-b}\left[\frac{\lambda}{\alpha^*(b, f, \mu, \lambda) - \mu(x)} - 1\right]\right\} dx = 1 \tag{3}$$

which implicitly defines α^* as a function of b, f, μ, and λ. We use this and the equilibrium construction from the previous section to discuss various implications of our theory. Denote by $\varphi^*(x)$ the posterior held by voters upon seeing platform x, in equilibrium.

Fact 2. *The posterior belief on character is single-troughed around the median. That is, $\frac{g^*(x)}{f(x)}$ is strictly increasing for $x < m$, and strictly decreasing for $x > m$. $\varphi^*(x)$ is strictly decreasing for $x < m$, and strictly increasing for $x > m$.*

This property of $\frac{g^*(x)}{f(x)}$ is immediate from inspection of (2), because μ is single-peaked around m. That the reverse is true for φ^* then follows because we can write the Bayes' update as

$$\varphi^*(x) = \frac{b}{b + (1-b)\frac{g^*(x)}{f(x)}}$$

Thus, strategic candidates skew their positions toward the median voter's preferred position. Moreover, voters believe that candidates near the median voter's preferred policy are less likely to have character, and the further is the chosen position, the more likely the candidate is to have character. This leaves the median voter indifferent, provided the positions are in the support of g^*. Outside the support of g^*, the voters are certain the candidate has character but the candidate loses nonetheless, because the advantage of a certainty of character is unable to overcome the disadvantage of an extreme position. Thus, the model accommodates extreme positions losing with certainty and distinct moderate positions resulting in ties.

Fact 3. *The median voter's utility from a strategic candidate, α^*, is increasing in the value of character, λ, and the probability of character, b.*

This directly follows from inspection of (3). That ex-ante welfare, α^*, increases in b is relatively trivial, since voters value character, and b is the ex-ante probability of a candidate having character. The rationale behind α^* increasing in the weight placed on character, λ, is more subtle. As λ rises, $\frac{g^*(x)}{f(x)}$ flattens out, and strategic candidates look more like candidates with character. This casts the effect of λ on utility in an interesting light. The flattening of $\frac{g^*(x)}{f(x)}$ means that the value of the platform offered by strategic candidates falls as λ rises. Moreover, the likelihood that a candidate has character hasn't changed. Thus, the value offered by strategic candidates falls and their prevalence remains unchanged. The behavior and prevalence of candidates with character hasn't changed. Thus, the overall value of the system seems to have fallen. The apparent paradox is resolved by noting that the total value of the candidates with character has risen because character is valued more highly.

Fact 4. *The support of a strategic candidate's choices is increasing in the value of character, λ.*

By (2), the support of g^* is given by the solutions to $\lambda + \mu(x) \geq \alpha^*$. Thus, whether the support increases in λ reduces to the question of whether $\frac{\partial \alpha^*}{\partial \lambda} < 1$. This is indeed the case, since by differentiating (3), we see that

$$\int\limits_{\{x|\lambda+\mu(x)\geq\alpha^*\}} \frac{bf(x)}{1-b} \frac{1}{[\alpha^* - \mu(x)]^2} \left[\alpha^* - \mu(x) - \lambda \frac{\partial \alpha^*}{\partial \lambda}\right] dx = 0$$

Since $\alpha^* \leq \lambda + \mu(x)$ for all x in the support of g^*, the above equality cannot hold unless $\frac{\partial \alpha^*}{\partial \lambda} < 1$. Thus, the support of g is indeed increasing in λ. This means that as character becomes more important, the likelihood of extremists, with and without character, being elected rises. The intuition is that when policy becomes less important relative to character, candidates with extreme platforms can still win. If character is sufficiently important, so that is λ is sufficiently large, g^* will have full support. In this case, any election is tied.

Fact 5. *As utility from character becomes infinitely more important than direct policy utility, strategic candidates perfectly mimic the distribution of candidates with character. That is, as $\lambda \to \infty$, $g^*(x) \to f(x)$ for all x.*

We defer the argument to the the Appendix. Intuitively, as the weight on character diverges, the gain from being perceived as having character increases for each candidate. Thus, in the limit, strategic candidates fully mimic the distribution of those with character.

Another comparative static considers a change in the platform of candidates with character. Suppose that f is replaced with h, and that $h(x) - f(x)$ is increasing for $x < m$, and decreasing for $x > m$. That is, candidates with character are more likely to come from the center under h than than under f. In this case, we say that h is a *more central density* than f.

Fact 6. *The median voter's utility from a strategic candidate, α^*, is higher when the distribution of platforms from a candidate with character comes from a more central density.*

The proof is in the Appendix. The idea is simple: strategic candidates mimic the distribution of those with character, but skew their play towards the median voter's preferred policy. If the distribution of platforms of candidates of character is more concentrated towards the center, then both strategic and non-strategic candidates' play become more desirable to the median voter in policy terms. Thus, α^* goes up.

For our last comparative statics, we consider the limits as b converges to 0 or 1.

Fact 7. *As $b \to 1$, Supp$(g^*) \to \{m\}$. As $b \to 0$, Supp$(g^*) \to \{x|\mu(x) + \lambda \geq \mu(m)\}$, but G^* converges to an atom on m.*

The proof is in the Appendix. If most candidates have character, so that b is close to one, there is little advantage to a strategic candidate of signaling character by position since the prior is so strong. Consequently, the support of the distribution of the strategic candidate's strategy collapses to m. This case replicates the standard model and the MVT emerges at this limit. In contrast, if there are few candidates with character, and b is close to zero, the distribution collapses on m but the support of the distribution converges to the set $\{x|\mu(x) + \lambda \geq \mu(m)\}$. This is as large as the support can get for a given λ. Thus, while almost all strategic candidates locate very near the middle, the possibility of strategic candidates a long way from the middle remains in the limit as candidates without character vanish. The possibility of candidates with character then has an echo in the model, even when the probability of such candidates goes to zero. Note that in such a case, ex-post, an elected candidate may have a position far from the middle.

4 Discussion

In this section, we discuss various interpretations and extensions of our theory.

4.1 Tied Elections

As already noted, our simple model results in all elections between strategic candidates being tied. Moreover, if the weight on character, λ, is large enough, then all elections end in ties. Nevertheless, we do not wish to suggest that elections in real life typically end in ties. This is a consequence of the assumption we made that candidates have no uncertainty about the electorate, and in particular, about the median voter's location, m. Consider an extended model where candidates share a common belief about the median voter's location, but the uncertainty is only resolved ex-post after positions have been chosen. Our analysis carries through unchanged, but it would now be the case that after uncertainty has been resolved at the last stage, ties do not generally occur.[12] However, the ex-ante probability of winning when two strategic candidates compete against each other remains one half. Indeed, this is an unavoidable and not unreasonable feature of any (symmetric) model with strategic office-motivated candidates. Note, in particular, that it also applies to the standard Downsian model.

4.2 Broader Interpretations of Character

Under some conditions, our model can be given a richer interpretation.[13] The main hypothesis we posed is that there is an unobservable trait among politicians that is valued by voters and is also negatively correlated with the willingness to pander to the public in order to get elected. Assume instead that while there is an unobservable trait that is valued by voters, it has nothing to do with willingness to pander. For example, the trait may be competence: some candidates are competent, some are not. All candidates are purely office-motivated and fully strategic. Suppose voters conjecture that competent candidates are playing the strategy F, which recall is the (exogenous) distribution chosen by non-strategic types in our model, whereas incompetent candidates are playing the strategy G, which recall is the (endogenous) distribution chosen by strategic types in our model. If the preference weight on competence (λ) is large enough such that the equilibrium of our model has a full support G, then the median voter is indifferent upon observing any platform. But then, candidates are indifferent over platform choices, and it is in fact an equilibrium for the competent ones to play G and the incompetent ones to play F.

The benefit of this perspective is that our policy divergence equilibrium is rationalized under much broader interpretations of unobservable traits, viz. any unobservable but desirable trait, even if it has nothing to do with willingness to pander for office. However, there are two caveats: first, it only applies if the preference weight on the unobservable trait is sufficiently large; second, and perhaps more importantly, unlike in our model with non-

[12] It remains true that neither candidate has an incentive to deviate from his strategy after observing the other candidates' position, but this property would not hold true once the median voter's location is revealed. On a related point, adding private information for the candidates about the location of the median voter would complicate matters (Bernhardt et al., 2003).

[13] We thank Dino Gerardi for this suggestion.

strategic types, uniqueness of equilibrium will not hold here. In particular, a "median voter equilibrium" also exists, where all candidates choose the median voter's platform. This is sustained in equilibrium by the out-of-equilibrium beliefs that when any non-median platform is observed, the candidate must not possess the desirable trait (e.g., must be incompetent).

4.3 Ex-ante Asymmetry

Suppose that the prior likelihood of having character can differ across candidates, and moreover, the distribution of policies conditional on character can also differ. This may be appealing when thinking of candidates as representing different political parties, for example. We now index b and f as b^i and f^i for each $i \in \{A, B\}$; the setup is otherwise unchanged. Call this the *asymmetric model.*

Theorem 2. *In the asymmetric model,*

1. *There is an ex-post equilibrium where candidate $i \in \{A, B\}$ uses the strategy, $\hat{G}^i$, with density*
$$\hat{g}^i(x) = \max\left\{0, \frac{b^i f^i(x)}{1-b^i}\left[\frac{\lambda}{\hat{\alpha}^i - \mu(x)} - 1\right]\right\} \tag{4}$$
where $\hat{\alpha}^i \in (\mu(m), \mu(m) + \lambda)$ is a constant.

2. *If $\hat{\alpha}^A = \hat{\alpha}^B$, then the above is the unique equilibrium.*

3. *If $\hat{\alpha}^i > \hat{\alpha}^j$, then in any equilibrium, candidate i wins with probability 1 when strategic.*

The first part of Theorem 2 is a trivial extension of the existence portion of Theorem 1. As before, the constant $\hat{\alpha}^i$ is unique and determined by the requirement that $\int_x \hat{g}^i(x)\,dx = 1$. As was the case earlier, the median voter's expected utility from electing candidate i is $\hat{\alpha}^i$, for any observed platform that is in the support of strategic i's strategy in this equilibrium. The difference with the base (symmetric) model is that it will no longer generally be true that $\hat{\alpha}^A = \hat{\alpha}^B$. Therefore, one of the candidates may win with probability 1 whenever he is a strategic type. For example, if $f^A = f^B$ but $b^A > b^B$, then when playing the strategies $\hat{G}^A$ and $\hat{G}^B$, candidate A always wins so long as the chosen platform is in the support of G^A.

That one of the candidates may win with probability 1 when there is an ex-ante asymmetry is not all too surprising. Such a property of the Downsian model is well-known when one candidate has an observable valence advantage (Aragones and Palfrey, 2002; Groseclose, 2001). The effect of asymmetric b^i's (or f^i's) is similar in our model, since it endows one candidate with an ex-ante advantage. As is the approach taken in that literature, extending our setting to one where there is uncertainty over the median voter's location—as we have already outlined earlier—would yield the possibility that even with ex-ante asymmetry, neither strategic candidate wins with probability 1.

Part 2 of Theorem 2 says that if the constellation of parameters do in fact generate the same $\hat{\alpha}$, then the equilibrium identified in Part 1 is unique. Thus, the result subsumes Theorem 1. When $\hat{\alpha}^A \neq \hat{\alpha}^B$, then the equilibrium identified in Part 1 of Theorem 2 may not be unique. Intuitively, say that candidate B is at an ex-ante disadvantage relative to candidatex A, and will lose with probability 1 when both are strategic types in the equilibrium of Part 1. Then, candidate B may be indifferent over multiple losing strategies when strategic, and this can lead to a multiplicity of equilibria. However, the multiplicity is inessential in terms of whether candidate A wins when strategic. This is the content of Part 3 of Theorem 2.

4.4 Richer and Endogenous Preferences for Character

We now enrich the preferences for character thus far considered. Let utility for a voter with ideal point v facing a candidate i with policy x^i be given by

$$U(x^i, v) \equiv \lambda(x^i, v) \Pr\left(c^i = 1 \middle| x^i\right) + u\left(x^i, v\right)$$

Here, the weight placed on character need not be constant—which was the assumption before—but instead can depend upon both the candidate's platform and a voter's ideal point. One natural motivation for such a specification is to capture the idea that politicians take actions of two kinds: one observable ("in plain view") and one unobservable ("out of sight"). The $u(\cdot,\cdot)$ component of a voter's utility represents the utility over observable actions that have been committed to by the politician during the electoral process. The $\lambda(\cdot,\cdot)$ component represents the voter's utility over the unobservable actions that the politician will take in office. If the politician has character, then his position on the unobservable dimension will be the same as what he committed to on the observable action, since those with character say what they will actually do. If he does not have character, however, he might do something very different on the unobservable dimension that what he promised (and necessarily lives up to) on the observable dimension. That $\lambda(\cdot,\cdot)$ can vary over policies and over voter ideal points allows for the possibility that different voters care differently about whether a candidate keeps his word behind the scenes, and moreover, this can depend on the position a candidate promised in different ways to voters. For example, a voter with ideal point $v = 1$ may prefer a candidate with platform $x^i = 0$ to *not* have character and thus likely do something different on the unobservable dimension than what was promised. On the other, the same voter may prefer a candidate with $x^i = 1$ to in fact have character, thus guaranteeing that he will take the same policy position on the unobservable dimension. This preference ordering over character can be reversed for a voter with ideal point $v = 0$.

The function $u(x, v)$ is exactly the same as earlier. Following the above discussion, we assume that $\lambda(x, v)$ is twice continuously differentiable, $\lambda(v, v) > 0$ for all v, and $\lambda_{12}(x, v) \geq 0$ for all x, v.[14] In words, preferences for character are smooth; every voter

[14]In fact, the assumption that $\lambda(v, v) > 0$ for all v is stronger than necessary. All that is needed for the

values character positively at least when a candidate's platform is her most-preferred policy; and as the platform of a candidate shifts to the right, the marginal increase in the preference for character is weakly higher for voters with ideal points further to the right. The assumptions on candidates are as before. Call this the *rich preferences model.* The following result extends our main insight to this setting, under a further assumption on the value for character.

Theorem 3. *If* $\max_{x,v} |\lambda_2(x,v)|$ *is sufficiently small,*[15] *the rich preferences model has an ex-post equilibrium where both candidates use the same strategy,* $\overline{G}$*, with density*

$$\overline{g}(x) = \max\left\{0, \frac{bf(x)}{1-b}\left[\frac{\lambda(x,m)}{\overline{\alpha} - u(x,m)} - 1\right]\right\}$$

where $\overline{\alpha} > u(m,m)$ *is the unique constant such that* $\int_x \overline{g}(x)dx = 1$.

The restriction on $|\lambda_2(\cdot,\cdot)|$ requires that the value on a candidate's character for any particular policy platform not change too sharply with voter ideal points. The logic of the equilibrium parallels Theorem 1. By construction of the equilibrium strategy, $\overline{G}$, voter m is indifferent over all platforms in the support of $\overline{G}$. The assumption on $|\lambda_2(\cdot,\cdot)|$ ensures that voter m remains a well-defined median voter in the space of campaign platforms, taking into account both direct policy utility and inferred character utility. This guarantees that indifference of m is sufficient for elections to end in ties among all platforms in the support of $\overline{G}$, which implies that it is optimal for strategic candidates to play $\overline{G}$. Plainly, the basic case we considered in the main part of the paper, that $\lambda(x,v)$ is a strictly positive constant, is covered by the Theorem.

4.5 No Commitment

Suppose that campaign statements are pure cheap talk. If elected, a politician need not necessarily implement his campaign platform, and can instead choose any policy in the policy space. We suppose that the policy a candidate will actually implement if elected is private information, and drawn from the density $f(x)$. A candidate also has character with probability b; as usual, this is also private information. Candidates with character announce precisely what they will do if elected, whereas candidates without character will say anything to get elected. Voters have preferences *only* over final policy, $u(x,v)$, and not directly over character at all. All the notation follows our standard use.

Suppose that a politician without character plays the same distribution over platforms independent of his ideal point. It follows that a voter with ideal point v has expected

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ensuing result is that $\lambda(x,v) > 0$ for some x. So long as the median voter values character positively at some policy platform, it's not necessary that $\lambda(\cdot,v)$ attain its maximum at v, nor that every voter value character positively for some platform.

[15] More precisely, write $\lambda(x,v) = \tilde{\lambda}(x) + \theta\hat{\lambda}(x,v)$. We require that $\tilde{\lambda}(x) > 0$ for all x. The conclusion of the Theorem is true for all θ smaller than some threshold $\overline{\theta} > 0$.

utility from a platform of x^i $(i = A, B)$:

$$U\left(x^i, v\right) = \varphi^i\left(x^i\right) u\left(x^i, v\right) + \left(1 - \varphi^i\left(x^i\right)\right) \mathbb{E}_x\left[u\left(x, v\right)\right]$$

where $\varphi^i(x^i)$ is the Bayes update about character of candidate i given his platform x^i, and the expectation term is taken with respect to the prior density f, without conditioning on the announced platform (since by hypothesis strategic politicians do not vary their strategy based on their ideal points). Rewriting gives

$$U\left(x^i, v\right) = \mathbb{E}_x\left[u\left(x, v\right)\right] + \varphi^i\left(x^i\right)\left[u\left(x^i, v\right) - \mathbb{E}_x\left[u\left(x, v\right)\right]\right]$$

Analogous to the construction (2), define the strategic candidate's density by

$$\tilde{g}\left(x\right) = \max\left\{0, \frac{bf\left(x\right)}{1-b}\left[\frac{u\left(x, m\right) - \mathbb{E}_x\left[u\left(x, m\right)\right]}{\tilde{\alpha} - \mathbb{E}_x\left[u\left(x, m\right)\right]} - 1\right]\right\} \tag{5}$$

There is a unique constant $\tilde{\alpha} \in \left(\mathbb{E}_x\left[u\left(x, m\right)\right], u\left(m, m\right)\right)$ such that $\int_x \tilde{g}(x) = 1$, by the usual argument. This formula in (5) has an intuitive interpretation: a strategic candidate can offer utility of at least $\mathbb{E}_x\left[u\left(x, m\right)\right]$ and at most $u\left(m, m\right)$.

Theorem 4. *If* $\max\limits_{x,v}\left|u_2\left(x, v\right) - \frac{d}{dv}\mathbb{E}_x\left[u\left(x, v\right)\right]\right|$ *is sufficiently small,*[16] *the model without policy commitment has an ex-post equilibrium where both candidates use the strategy given by* (5).

The requirement that $\max\limits_{x,v}\left|u_2\left(x, v\right) - \frac{d}{dv}\mathbb{E}_x\left[u\left(x, v\right)\right]\right|$ be sufficiently small plays a similar role to the restriction on λ_2 in Theorem 3: it is necessary to ensure that the voter m remains a well-defined median voter in the sense that her indifference—which is by construction of the density in (5)—is sufficient for elections to end in ties. It has the interpretation that for any x, the benefit (or loss) of having that policy with certainty over a random policy draw must not change by much for a small change in voter ideal points.

4.6 Two-Stage Elections

In many electoral systems, such as the U.S. presidential process, a candidate must win a party nomination before competing in a general election. Roughly speaking, a party nomination is won by garnering the majority of votes amongst the subset of all voters who are affiliated with the party. This implies that the set of voters a candidate must appeal to is different in the within-party primary vote (first stage) versus the general election vote (second stage). For instance, in the U.S., the median voter in the Republican (resp. Democratic) party's primary is to the right (resp. left) of the median voter in the general election. In this section, we study a simplified version of our model extended to such a

[16] More precisely, the statement is analogous to fn. 15.

two-stage electoral process. Rather than presenting an exhaustive analysis, our goal is to only suggest how the central theme of the paper can be extended to such settings, with interesting implications.

For tractability, we simplify the policy space to the discrete set $X = \{-1, 0, 1\}$. There are two political parties, denoted L and R. Within each party, there are two candidates for office. The winner of the election is decided through a two-stage process. In the first stage, the *primary election*, each candidate from each party starts by simultaneously committing to a policy position that is either 0 or his own party's extreme policy: 1 for party R and -1 for party L. Then, each party determines its nominee through a vote between its two candidates by the citizens affiliated with the party. In the second stage, the *general election*, all citizens get to vote for one of the two nominees (one nominee from each party); the winner of this general election is elected into office, and implements the policy he committed in the first stage. That is, candidates are committed to the same platform in the general election as they chose in their party primary.

As usual, denote a voter's policy utility by $u(x, v)$ where x is the policy and v is her ideal point. For convenience, assume there is a *distinct, single* (and thus median) voter in each of the three elections: the two party primaries and the general election.[17] The voter in party L's primary has ideal point -1, the voter in party R's primary has ideal point 1, and the voter in the general election has ideal point 0. Furthermore, we restrict attention to a symmetric setup across parties and policies such that $u(-1, 0) = u(1, 0) < u(0, 0)$, $u(0, 1) = u(0, -1)$, $u(1, -1) = u(-1, 1)$, and $u(-1, -1) = u(1, 1)$. If a candidate has character, then any voter gets an added utility of $\lambda > 0$ if that candidate is elected. Each candidate has probability $b \in (0, 1)$ of having character, and if so, is committed to an exogenously drawn platform. A candidate with character from the R party draws his platform from a distribution that places probability $f_0 > 0$ on platform 0 and probability $f_1 \equiv 1 - f_0 > 0$ on platform 1. Symmetrically, a candidate with character from the L chooses platform 0 with probability f_0 and platform -1 with probability $f_{-1} \equiv 1 - f_0$. Candidates without character aim to maximize their probability of being elected to office (i.e. win the general election).

Notice a key feature of the setting we have described: *ceteris paribus*, an extreme candidate is preferred in the party primaries, whereas a centrist candidate is preferred in the general election. However, we assume that primary voters are forward-looking insofar as they don't myopically vote for the candidate they just prefer in the primary, but rather, they take into the account the prospects of each candidate in the general election, and vote for the primary candidate who maximizes their overall expected utility, which depends only on the general election winner's platform and character. In particular, the primary median of each party would prefer a centrist (from either party) over an extremist from the other party, character considerations aside.

[17]This can be interpreted as representing median voters amongst a large set of voters in each election, with one caveat in footnote 19.

We seek symmetric equilibria where strategic candidates from party R (resp. L) play platform 0 with probability g_0 and platform 1 (resp. -1) with probability $g_1 \equiv 1 - g_0$ (resp. $g_{-1} \equiv 1 - g_0$). By Bayes' rule, the belief about character as function of platform, $\mu(x)$, is the same for all candidates in a symmetric equilibrium, and is given by $\mu(x) = \frac{bf_x}{bf_x+(1-b)g_x}$ for $x = -1, 0, 1$.

Even within this simple setting, a variety of symmetric equilibria can emerge, even sometimes for the same set of parameters. The reason for multiplicity is a kind of "self-fulling prophecy" across primaries and the general election. Intuitively, the candidate generation process at the primaries influences the success rate at the general election: extreme candidates who emerge from the primary are less likely to do well in the general election, all else equal. However, the potential for success in the general election influences the voter preferences in the primary: if a candidate with a particular platform is is believed to win the general election, that candidate is more preferred in the primary.

Three kinds of equilibria are of particular interest. The first kind, *Naive Policy Preference* equilibrium, has the property that in the general election, centrists are preferred over extremists, whereas in the party primaries, extremists are preferred over centrists;[18] In the second kind, *General Election Indifference* equilibrium, the general election voter is indifferent between centrists and extremists (and randomizes in voting), whereas in the party primaries, extremists are preferred over centrists. Finally, in the third kind, *Centrist Dominant* equilibrium, both the general election and the primary voters prefer centrists over extremists.

If the probability of character, b, were zero—strictly speaking, not allowed by the model, but a benchmark case to understand the effect of introducing character—it would be an equilibrium for all candidates to choose the centrist platform, and both primary and general election voters to prefer a centrist candidate to extremists, i.e. Centrist Dominant equilibrium necessarily exists. It is clear that the general election voter must have this preference when $b = 0$; the reason the primary voter prefers centrists over its own extremist is because she realizes that an extremist would have no chance on winning the general election. On the other hand, the following result shows that when character exists ($b > 0$), under some conditions, Naive Policy Preference and General Election Indifference equilibria exist, whereas Centrist Dominant equilibrium do not. We say that u is *convex* if $2u(0, 1) < u(1, 1) + u(-1, 1)$, i.e. the party voters prefer a uniform lottery among the two extreme policies over the certain center policy.

Theorem 5. *In the two-stage election model, assume u is convex. Then, Centrist Dominant equilibrium does not exist. Moreover, Naive Policy Preference and General Election Indifference equilibria exist for (possibly distinct) open parameter sets of positive measure.*

In both the Naive Policy Preference equilibrium and the General Election Indiffer-

[18] Note that regardless of how strategic candidates behave, there is always positive probability that both candidates in a party's primary have centrist or extreme platforms. Thus, in the general election, there is positive probability on all possible platform pairs by the two nominated candidates.

ence equilibrium we construct, strategic candidates randomize between their own party's extreme platform and the center platform. It is important to note that in the General Election Indifference equilibrium, even though the general election voter is indifferent across all candidates, her indifference cannot be broken by randomizing uniformly across centrists and extremists. Instead, since extremists defeat centrists in the party primary, the probability that a centrist beats an extremist in the general election needs to be such that strategic candidates are indeed indifferent between choosing extreme and center platforms at the outset.[19]

That character matters even in two-stage elections is seen most clearly in two facts. First, Centrist Dominant equilibrium always exists when $b = 0$ but does not exist under convexity of preferences when $b > 0$. Second, a General Election Indifference equilibrium can exist for some parameters when $b > 0$, while it never can when $b = 0$, because without character, the general election voter is never indifferent between centrist and extreme policies. The indifference when $b > 0$ arises precisely because even though the general election voter prefers centrist platforms over extreme ones in direct policy utility, the behavior of strategic candidates is biased towards the center, so that extremists are more likely to have character.

5 Conclusion

"In a president, character is everything." — Peggy Noonan

This paper develops a theory of character in elections. The two key assumptions we make are that some candidates may have character and do not strategically choose policy platforms to simply maximize the probability of getting elected, and voters value character in addition to campaign promises. Character quashes the median voter theorem, as strategic candidates pretend to have the positions of candidates with character. Elections between strategic candidates are tied in the symmetric version of our model. As character becomes more important to voters, the behavior of strategic candidates grows further away from the ideal policy of the median voters and closer to the behavior of candidates with character.

We have illustrated a number of extensions of the basic idea. In discussing extensions to richer preferences, we sketched a model of endogenous preference for character, where character entails following the promised policy in all circumstances, even when the choice is unobservable to voters. Voters who care about the unmonitored behavior will naturally care about the character of candidates. However, our treatment of this setting is

[19]With the assumed single voter in the general election, this presents no conceptual issue. If instead this were a median voter representing a large number of voters with heterogeneous ideal points, we take the interpretation that the non-uniform tie-breaking results from uncertainty about the median that is resolved in between the primary and general election.

very stylized, and it would be useful to model this in a more detailed way. We also believe that the model of two-stage elections developed here is worthy of further study.

On the politicians' side, certainly our assumption that character is "all or nothing" is stark. While we do think that a population of agents should include both extremes of the character spectrum we have modeled, it would also be interesting to consider adding intermediate character types, such as those who are willing to pander but face a cost of doing so, where the cost is private information.[20] In addition, the introduction of policy motivated candidates with commitment would enrich the theory, if policy motivated candidates with character do what they promised even out of sight of voters, whereas policy motivated candidates without character follow their own preferences whenever possible.

We have modeled the proportion of candidates possessing character as an exogenous parameter, but in fact candidates usually run for higher office only after a complex winnowing process that includes serving for lower offices. Does this process tend to favor candidates with character or strategic candidates? In the model, the probability that a candidate with character is elected is no more, and maybe strictly less, than the probability that a strategic candidate is (see Fact 1). This suggests that the winnowing process may favor strategic candidates. However, if candidates with character are more likely to be re-elected or advance to higher office—perhaps because a candidate's type may be discovered with some probability once in office—then character may actually increase through the political hierarchy. A richer analysis is needed to illuminate the dynamics of candidate selection, but we view our model as a useful starting point.

[20] This is reminiscent of Banks (1990) and Callander and Wilkie (2003), but those are models with commitment and without our extreme "honest at all costs" types.

Appendix: Proofs

To prove Theorem 1, some preliminaries are needed. Note that due to the continuum of policy locations, various statements about optimal strategies will be subject to "almost all" qualifiers; we suppress such caveats unless essential.

The first Lemma shows that a candidate must win with positive probability when strategic.

Lemma A.1. *In any equilibrium, the strategic type of a candidate wins with positive probability; thus, ex-ante, both candidates win with positive probability.*

Proof. Without loss of generality, it suffices to show that for some set of Y of positive G^A-measure, $\varphi^A(x) \leq \varphi^B(x)$ for G^A-a.e. $x \in Y$. (Because then by concentrating mass on Y, strategic B can win with positive probability.) This is immediate if G^A has atoms because $\varphi^A(x) = 0$ for any x that G^A has an atom at, so suppose that G^A is atomless, hence absolutely continuous with density g^A. Let X^B be the set of all non-atomic points of G^B; since G^B can have only a countable number of atoms, X^B has full G^A-measure and there is a density of G^B, denoted g^B, on X^B. Then for a.e. $x \in X^B$, $\varphi^i(x) = \frac{bf(x)}{bf(x)+(1-b)g^i(x)}$, and there must be a set $Y \subseteq X^B$ with the desired properties; otherwise $1 = \int_{X^B} g^A(x)dx < \int_{X^B} g^B(x)dx \leq 1$, a contradiction. □

Next, we show that both candidates must play mixed strategies.

Lemma A.2. *In any equilibrium, neither candidate plays a pure strategy.*

Proof. By way of contradiction, suppose there is an equilibrium where candidate j is playing a pure strategy of choosing platform $\hat{x}$. This implies that $\varphi^j(\hat{x}) = 0$ and $\varphi^j(x) = 1$ for all $x \neq \hat{x}$. It suffices to argue that j must win probability 1 when strategic, since this is a contradiction with Lemma A.1. First note that if G^i has an atom on x, then $\alpha^i(x) = \mu(x) < \lambda + \mu(m)$. Moreover, for any $x \neq m$, $\alpha^i(x) < \lambda + \mu(m)$. Now consider j choosing a platform $m+\varepsilon$ $(m+\varepsilon \neq \hat{x})$ with small $\varepsilon > 0$. Then $\alpha^j(m+\varepsilon) = \lambda + \mu(m+\varepsilon) > \alpha^i(x)$ for all x outside an ε-neighborhood of m and those x inside the neighborhood that G^i has atoms at. It follows that by picking ε arbitrarily small, j can win with probability arbitrarily close to 1. Thus, if j does not win with probability 1 when strategic, he has a profitable deviation. □

We now define an **ex-post** equilibrium, which is an equilibrium such that even if a candidate observed his opponent's platform before choosing his own, he would have no incentive to deviate from his prescribed strategy.

Definition 1. An equilibrium with cdf's G^A and G^B is an *ex-post equilibrium* if the probability that candidate $i \in \{A, B\}$ wins is the same for all realizations of x^A and x^B in the support of G^A and G^B respectively.

Given Lemma A.1, it is straightforward that a pair of cdf's $\left(G^A, G^B\right)$ constitutes an *ex-post equilibrium* strategy profile if and only if for each $i \in \{A, B\}$:

$$\alpha^i\left(x^i\right) = \alpha^j\left(x^j\right) \text{ for all } x^i \in Supp[G^i] \text{ and } x^j \in Supp[G^j] \quad \text{(A-1)}$$
$$\alpha^i(y) \leq \alpha^i(x) \text{ for all } x \in Supp[G^i] \text{ and } y \notin Supp[G^i] \quad \text{(A-2)}$$

The important result is that every equilibrium is an ex-post equilibrium.

Lemma A.3. *Any equilibrium is an ex-post equilibrium.*

Proof. The following notation will be used:

$$\begin{aligned}
\overline{x^i} &\in \arg\max_x \alpha^i(x|\varphi^i) \\
\overline{\alpha^i} &\equiv \max_x \alpha^i(x|\varphi^i) \\
\underline{x^i} &\in \arg\min_x \alpha^i(x|\varphi^i) \\
\underline{\alpha^i} &\equiv \min_x \alpha^i(x|\varphi^i)
\end{aligned}$$

First, we prove that condition (A-1) must hold in any equilibrium. It suffices to show that for $i \in \{A, B\}$, $\overline{\alpha^i} = \underline{\alpha^i}$, and $\overline{\alpha^A} = \overline{\alpha^B}$. The latter is straightforward: if not, wlog say $\overline{\alpha^A} > \overline{\alpha^B}$, then playing $\overline{x^A}$ guarantees election for A, contradicting equilibrium mixing and Lemma A.1. Similarly, $\underline{\alpha^A} = \underline{\alpha^B}$. Now, if $\overline{\alpha^i} > \underline{\alpha^i}$, then $\underline{x^i}$ loses at least against $\overline{x^j}$, whereas $\overline{x^i}$ would win with probability $\frac{1}{2}$ against $\overline{x^j}$ and moreover against anything that $\underline{x^i}$ either ties or beats, so it is not optimal for $\underline{x^i}$ to be in the support of i's strategy.

Now, we prove necessity of condition (A-2). If condition (A-2) does not hold, then given condition (A-1), one of the candidates can profitably deviate to winning the election with probability 1, since he only wins with lower probability in equilibrium by Lemma A.1. □

The following Lemma formally shows that strategic candidates do not use mass points in their strategies.

Lemma A.4. *In any equilibrium, both G^A and G^B are atomless.*

Proof. Suppose that G^i has an atom on $\hat{x}$. Then $\alpha^i(\hat{x}) = \mu(\hat{x})$. Since there can be only a countable number of atoms, we can find a small $\varepsilon \neq 0$ such that

$$\alpha^i(\hat{x} + \varepsilon) = \lambda\varphi^i(\hat{x} + \varepsilon) + \mu(\hat{x} + \varepsilon) > \mu(\hat{x})$$

If $\hat{x} + \varepsilon \in Supp[G^i]$, condition (A-1) is violated; if $\hat{x} + \varepsilon \notin Supp[G^i]$, condition (A-2) is violated. Either way, G^i cannot be an equilibrium strategy. □

Accordingly, any equilibrium strategy, G^i, has a density, g^i, and by Bayes rule,

$$\varphi^i(x) = \frac{bf(x)}{bf(x) + (1-b)\, g^i(x)}$$

The following Lemma shows that the support of a candidate's strategy must be in an interval containing the median voter's ideal policy.

Lemma A.5. *In any equilibrium, for $i \in \{A, B\}$, Supp$[G^i]$ is an interval that contains m.*

Proof. First we argue that the support is an interval. Suppose not for player i. Then there exist $x > y > z$ such that $g^i(x) > 0$, $g^i(y) = 0$, and $g^i(z) > 0$. This implies that $\varphi^i(x) < 1$, $\varphi^i(y) = 1$, and $\varphi^i(z) < 1$. If $y \geq m$, it follows that $\alpha^i(y) > \alpha^i(z)$; if $y \leq m$ then $\alpha^i(y) > \alpha^i(x)$. Either case contradicts condition (A-2) for an ex-post equilibrium.

Next, we show that the interval must contain m. If it didn't, wlog say it is $[l, h]$ with $h < m$, then $\varphi^i(m) = 1$, hence for any $x \in [l, h]$,

$$\alpha^i(m) = \lambda + \mu(m) > \lambda\varphi^i(x) + \mu(x) = \alpha^i(x)$$

contradicting condition (A-2) for an ex-post equilibrium. $\square$

The above lemmata in hand, we can now proceed with the proof of Theorem 1.

Proof of Theorem 1 on page 9.

(Existence) We first prove that both players playing G^* is an ex-post equilibrium for some constant α^*. It is easy to verify that given G^*, the posterior belief is the following function φ^* for both candidates (so we drop the superscripts indexing candidates):

$$\varphi^*(x) = \begin{cases} \frac{\alpha^* - \mu(x)}{\lambda} & \text{if } g^*(x) > 0 \\ 1 & \text{if } g^*(x) = 0 \end{cases}$$

Accordingly,

$$\alpha(x|\varphi^*) = \lambda\varphi^*(x) + \mu(x) = \begin{cases} \alpha^* & \text{if } g^*(x) > 0 \\ \lambda + \mu(x) & \text{if } g^*(x) = 0 \end{cases}$$

Noting from (2) that $g^*(x) = 0$ requires $\alpha^* - \mu(x) \geq \lambda$, one sees that conditions (A-1) and (A-2) for an ex-post equilibrium are indeed satisfied. It only remains to verify that there is a constant α^* which makes g^* a density. Since $g^*(x) \geq 0$ for all $x \in X$, we need to only check that $\int_X g^*(x)\, dx = 1$ for some α^*.

Define

$$\gamma(x, \alpha) = \max\left\{0, \frac{bf(x)}{1-b}\left[\frac{\lambda}{\alpha - \mu(x)} - 1\right]\right\}$$

so that $g^*(x) = \gamma(x, \alpha^*)$. We now prove that there is a unique constant α^* such that $\int_X \gamma(x, \alpha^*)\, dx = 1$. First observe that γ is continuous in α. Next, note that if $\alpha \geq \lambda + \mu(m)$, then $\lambda \leq \alpha - \mu(x)$ for all $x \in X$, hence $\gamma(x, \alpha) = 0$ for all $x \in X$ and $\int_X \gamma(x, \alpha)\, dx = 0 < 1$. On the other hand, for $\alpha = \mu(m)$ and small $\varepsilon > 0$,

$$\begin{aligned}
\int_X \gamma(x, \mu(m))\, dx &\geq \int_{m-\varepsilon}^{m+\varepsilon} \gamma(x, \mu(m))\, dx \\
&= \int_{m-\varepsilon}^{m+\varepsilon} \max\left\{0, \frac{bf(x)}{1-b}\left[\frac{\lambda}{\mu(m) - \mu(x)} - 1\right]\right\} dx \\
&\approx \frac{bf(m)}{1-b}\left[\int_{m-\varepsilon}^{m+\varepsilon} \frac{\lambda}{\mu(m) - \mu(x)} dx - 2\varepsilon\right] \\
&\approx \frac{bf(m)}{1-b}\left[\int_{m-\varepsilon}^{m+\varepsilon} \frac{\lambda}{-\frac{1}{2}\mu''(m)(x-m)^2} dx - 2\varepsilon\right] \\
&\approx \infty
\end{aligned}$$

where the penultimate line follows from $\mu'(m) = u_1(m, m) = 0$ and Taylor expansion, and the last line from the fact that $\mu''(m) = u_{11}(m, m) \leq 0$. By the Intermediate Value Theorem, there is a value of α, call it $\alpha^* \in (\mu(m), \mu(m) + \lambda)$, that satisfies $\int_X \gamma(x, \alpha^*)\, dx = 1$. It is straightforward that there cannot be any other value of α such that $\int_X \gamma(x, \alpha)\, dx = 1$ since $\gamma(x, \alpha)$ is strictly decreasing in α for any x such that $\gamma(x, \alpha) > 0$.

(Uniqueness) Now we prove that both players playing G^* is the unique equilibrium. Denote the support of G^* by $[l^*, h^*] \supsetneq \{m\}$. Suppose there is another (necessarily ex-post, by Lemma A.3) equilibrium where a candidate i plays $G^i \neq G^*$. By Lemmas A.4 and A.5, G^i has a density g^i with support $[l, h] \supsetneq \{m\}$. Since $G^i \neq G^*$ and their supports have non-empty intersection, there must be some non-degenerate interval, $Y \subseteq [l, h]$, such that $g^i(x) > g^*(x)$ for all $x \in Y$. Then $\alpha^i(x) > \alpha^*(x)$ for all $x \in Y$, and ex-postness implies $\alpha^i(x) > \alpha^*(x)$ for all $x \in [l, h]$. Consequently,

$$g^i(x) > g^*(x) \text{ for all } x \in [l, h] \tag{A-3}$$

Since g^i and g^* are both densities, either $l > l^*$ or $h < h^*$ (or both). Suppose $l > l^*$ (the argument is analogous for $h < h^*$). By its construction, $g^*(x) > 0$ for all x in a small neighborhood of l. A contradiction ensues with (A-3) if we show that $g^i(x) \to 0$ as $x \to l$. To prove this, observe that $\varphi^i(x) = 1$ for all $x < l$, and hence $\alpha^i(x|\varphi^i) = \lambda + \mu(x)$ for all $x < l$. By condition (A-2) for ex-postness, and using Bayes rule, it must be that for any $\varepsilon > 0$, there is a $\delta > 0$ such that $g^i(x) < \varepsilon$ for all $x \in (l, l + \delta)$. $\square$

Proof of Fact 5 on page 12. Observe that for any $x \in Supp\,[G^*]$, when $\lambda \approx \infty$,

$$\begin{aligned} g^*(x) &= \frac{bf(x)}{1-b}\left[\frac{\lambda}{\alpha^*-\mu(x)}-1\right] \\ &= \frac{bf(x)}{1-b}\left[\frac{1}{\frac{\alpha^*}{\lambda}-\frac{\mu(x)}{\lambda}}-1\right] \\ &\approx \frac{bf(x)}{1-b}\left[\frac{\lambda}{\alpha^*}-1\right] \end{aligned}$$

The proof is completed by noting from (2) that $\frac{\lambda}{\alpha^*} \to \frac{1}{b}$ as $\lambda \to \infty$. □

Proof of Fact 6 on page 12. Let the value α_f^* refer to the the utility generated by f and α_h^* refer to that generated by h. Since $\frac{\lambda}{\alpha^*-\mu(x)}-1$ is decreasing as x moves away from the median, m, $h(x)-f(x)$ and $\frac{\lambda}{\alpha^*-\mu(x)}-1$ are positively correlated. If the support of g^* is the whole policy space,

$$\int \frac{b(h(x)-f(x))}{1-b}\left[\frac{\lambda}{\alpha_f^*-\mu(x)}-1\right]dx > \int \frac{b(h(x)-f(x))}{1-b}dx \int \left[\frac{\lambda}{\alpha_f^*-\mu(x)}-1\right]dx = 0$$

and thus

$$\int_x \frac{bh(x)}{1-b}\left[\frac{\lambda}{\alpha_f^*-\mu(x)}-1\right]dx > 1$$

It follows that $\alpha_h^* > \alpha_f^*$. □

Proof of Fact 7 on page 12. For the case of $b \to 1$, inspection of (2) shows that α^* increases in b without no bound. Since $Supp(g^*) = \{x|\lambda+\mu(x) \geq \alpha^*\}$, as α^* increases, the support shrinks to $\{m\}$. (Note that m is always in the support; see Lemma A.5.)

As $b \to 0$, inspection of (2) shows that $\alpha^* \to \mu(m)$. Thus indeed $Supp(g^*) \to \{x|\mu(x)+\lambda \geq \mu(m)\}$. To see that G^* converges in distribution to point-mass on m, observe that for as $b \to 0$, for any $\varepsilon > 0$, both $\int_0^{m-\varepsilon} \max\left\{0, \frac{bf(x)}{1-b}\left[\frac{\lambda}{\alpha^*-\mu(x)}-1\right]\right\}dx \to 0$ and $\int_{m+\varepsilon}^1 \max\left\{0, \frac{bf(x)}{1-b}\left[\frac{\lambda}{\alpha^*-\mu(x)}-1\right]\right\}dx \to 0$. □

Proof of Theorem 2 on page 14. The first part is almost identical to the existence portion of Theorem 1, hence omitted. To prove the second and third parts, two intermediate claims are needed. For $i \in \{A, B\}$, let $\hat{\alpha}^i$ denote the constant defined by $\int_X \hat{g}^i(x)\,dx = 1$.

Claim 1: $\hat{G}^i$ is the unique solution to the following program:

$$\max_{G^i,\varphi^i}\left[\min_{x\in Supp(G^i)}\lambda\varphi^i(x)+\mu(x)\right]\text{ s.t. }\varphi^i\text{ being a posterior given }G^i \tag{A-4}$$

Proof: Let $\tilde{G}^i$ be a solution to program (A-4). We argue that $\tilde{G}^i=\hat{G}^i$. The support of $\tilde{G}^i$ must be contained in the support of $\hat{G}^i$; otherwise by the construction of $\hat{G}^i$, it is immediate that $\tilde{G}^i$ cannot be a solution to (A-4). Clearly then $\tilde{G}^i$ has no atoms; hence it has a density. But then if there is an interval on which $\tilde{g}^i(x)>\hat{g}^i(x)$, there must be an interval on which $\tilde{g}^i(x)<\hat{g}^i(x)$, for the densities to integrate to 1. This contradicts $\tilde{G}^i$ being a solution to (A-4). ‖

Claim 2: $\hat{G}^i$ is the unique solution to the following program:

$$\min_{G^i,\varphi^i}\left[\max_x\lambda\varphi^i(x)+\mu(x)\right]\text{ s.t. }\varphi^i\text{ being a posterior given }G^i \tag{A-5}$$

Proof: Let $\breve{G}^i$ be a solution to program (A-5). We argue that $\breve{G}^i=\hat{G}^i$. Suppose not. Let $\breve{\alpha}^i\equiv\max_x\lambda\breve{\varphi}^i(x)+\mu(x)$, where $\breve{\varphi}^i$ is a posterior that is a solution to program (A-5). We have $\breve{\alpha}^i<\hat{\alpha}^i$. If $\breve{G}^i$ has mass outside the support of $\hat{G}^i$, then there must be an interval inside the support of $\hat{G}^i$ on which $\breve{G}^i$ has a density $\breve{g}^i(x)<\hat{g}^i(x)$, which contradicts $\breve{\alpha}^i<\hat{\alpha}^i$. So the support of $\breve{G}^i$ is within that of $\hat{G}^i$. But then, since the distributions are not the same by hypothesis, there must be an interval on which $\breve{G}^i$ has a density $\breve{g}^i(x)<\hat{g}^i(x)$, which contradicts $\breve{\alpha}^i<\hat{\alpha}^i$. ‖

Proof of Part (2) of the Theorem: Suppose that $\hat{\alpha}^A=\hat{\alpha}^B$ and let (G^A,G^B) be an equilibrium. By Claim 2, some platform for A provides a utility of at least $\hat{\alpha}^A$; by Claim 1, some platform in the support of G^A provides a utility weakly less than $\hat{\alpha}^A$. The same applies to player B. Since $\hat{\alpha}^A=\hat{\alpha}^B$, we conclude that both players win with positive probability when strategic. By the same logic as Lemma A.3, it follows that the equilibrium must be ex-post, and all platforms in the support of each G^i must provide the same utility. So now suppose towards contradiction that without loss of generality, $G^A\neq\hat{G}^A$. Claim 1 implies that some platform in the support of G^A provides utility strictly less than $\hat{\alpha}^A$, which by ex-postness extends to all platforms in the support. But Claim 2 implies than B has a platform that provides utility at least that of $\hat{\alpha}^B=\hat{\alpha}^A$, implying that B must win with probability 1 if strategic, a contradiction.

Proof of Part (3) of the Theorem: If $\hat{\alpha}^i>\hat{\alpha}^j$ then for any equilibrium (G^i,G^j), some platform in the support of G^j gives weakly less utility than $\hat{\alpha}^j$ (Claim 1) and some platform for i gives gives weakly more utility than $\hat{\alpha}^i$ (Claim 2). So i wins with positive probability when strategic. If j wins with positive probability when strategic, then by the same logic as Lemma A.3, it follows that the equilibrium must be ex-post, and all platforms in the support of both distributions G^i and G^j must provide the same utility. But then, i has a profitable deviation to some platform outside the support of G^i, a contradiction. $\square$

Proof of Theorem 3 on page 16. The same logic as in the proof of Theorem 1 proves that there must exist a unique $\overline{\alpha} \in (u(m,m), u(m,m) + \max_x \lambda(x,m))$ such that $\int_x \overline{g}(x) = 1$. We need only prove that both strategic candidates playing this density constitutes an equilibrium. It is easily verified that the Bayes update about character for a candidate who chooses platform $x \in Supp[\overline{G}]$ is $\overline{\varphi}(x) = \frac{\overline{\alpha} - u(x,m)}{\lambda(x,m)}$. Thus, the expected utility to a voter, v, from electing a candidate with platform $x \in Supp[\overline{G}]$ is

$$\overline{U}(x,v) = u(x,v) + \lambda(x,v)\left[\frac{\overline{\alpha} - u(x,m)}{\lambda(x,m)}\right]$$

Clearly, the median voter is indifferent over all platforms in the support of $\overline{G}$, i.e. $\overline{U}_1(x,m) = 0$ for all $x \in Supp[\overline{G}]$. It follows that if we prove that $\overline{U}_{12}(x,v) > 0$ for all v and all $x \in Supp[\overline{G}]$, strategic candidates are indifferent over all platforms in the support of $\overline{G}$. Taking the cross-partial derivative yields

$$\overline{U}_{12}(x,v) = u_{12}(x,v) + \lambda_{12}(x,v)\frac{\overline{\alpha} - u(x,m)}{\lambda(x,m)} - \lambda_2(x,v)\frac{\lambda(x,m)u_1(x,m) + (\overline{\alpha} - u(x,m))\,\lambda_1(x,m)}{(\lambda(x,m))^2}$$

Since $u_{12} > 0$, $\lambda_{12} \geq 0$, and $\overline{\alpha} > u(x,m)$, the sum of the first two terms in the right hand side above is strictly positive. It follows that when $\max_{x,v} |\lambda_2(x,v)|$ is sufficiently small (see fn. 15), the right hand side above is strictly positive, proving that $\overline{U}_{12}(x,v) > 0$ for all v and all $x \in Supp[\overline{G}]$ as desired.

Now, consider platforms outside the support of $\overline{G}$. We claim that strategic candidates have a strict incentive to not choose such platforms. To see this, note that by construction of the density $\overline{g}$, voter m strictly prefers any platform in the support of $\overline{G}$ to any platform outside the support (because $\overline{\alpha} > u(x,m) + \lambda(x,m)$ for any $x \notin Supp[\overline{G}]$). Since $\overline{\varphi}(x) = 1$ for all $x \notin Supp[\overline{G}]$, and $u_{12} + \lambda_{12} > 0$, this in turn implies that all voters $v < m$ also strictly prefer any platform in the support of $\overline{G}$ to any platform outside the support of $\overline{G}$. Therefore, any platform outside the support of $\overline{G}$ loses to a platform in the support of $\overline{G}$, and we conclude that that is optimal for strategic candidates to play $\overline{G}$. $\square$

Proof of Theorem 4 on page 17. By construction, voter m is indifferent over all platforms in the support of $\tilde{g}$, and strictly prefers these to all platforms outside the support. It remains to show that voter m really is a median voter under this construction — similar to Theorem 3. Computing expected utility in this putative equilibrium to voter v gives

$$W(x,v) = \frac{\tilde{\alpha} - \mathbb{E}_x[u(x,m)]}{u(x,m) - \mathbb{E}_x[u(x,m)]}\left(u(x,v) - \mathbb{E}_x[u(x,v)]\right) + \mathbb{E}_x[u(x,v)]$$

Taking the cross-partial and writing $\varphi(x) = \frac{\tilde{\alpha} - \mathbb{E}_x[u(x,m)]}{u(x,m) - \mathbb{E}_x[u(x,m)]}$ gives

$$W_{12}(x,v) = \varphi(x) u_{12}(x,v) - \left(u_2(x,v) - \frac{d}{dv}\mathbb{E}_x[u(x,v)] \right) \frac{(\tilde{\alpha} - \mathbb{E}_x[u(x,m)]) u_1(x,v)}{(u(x,m) - \mathbb{E}_x[u(x,m)])^2}$$

Since $u_{12} > 0$, this will be positive if $\left| u_2(x,v) - \frac{d}{dv}\mathbb{E}_x[u(x,v)] \right|$ is sufficiently small for all $x, v \in [0,1]$. □

Proof of Theorem 5 on page 19. Throughout, assume that u is convex, i.e $2u(0,1) < u(1,1) + u(-1,1)$.

Step 1: First, we show that a Centrist Dominant equilibrium does not exist. As shorthand, let $z \equiv bf_0 + 1 - b$. In a Centrist Dominant Equilibrium, since both primary voters and the general election voter (strictly) prefer centrists to extremists, strategic candidates must choose the centrist platform, 0, with probability one, i.e. $g_0 = 1$. Thus, the Bayes update about character given platform 0 is $\varphi(0) = \frac{bf_0}{z}$, whereas given platform 1 or -1 it is $\varphi(-1) = \varphi(1) = 1$. Note that given this behavior, the probability that a party's nominee is extreme is $(1-z)^2$, i.e. it is the probability that both of the party's candidates have character and extreme position. The probability that a party's nominee is centrist is $1 - (1-z)^2$.

We argue that party R's voter strictly prefers to nominate an extreme candidate with platform 1 rather than a centrist, which contradicts equilibrium behavior. From the perspective for the R party primary voter, if she nominates a centrist candidate, then the general election winner will be centrist, and regardless of which party the general election winner is from, her utility from that candidate is $u(0,1) + \lambda$ (using the fact that the Bayes update about character from centrist candidates of both parties is the same). On the other hand, if she nominates an extreme candidate from her party (if one exists, which is a positive probability event), then there are two possibilities: either party L's nominee is extreme (platform -1), which has probability $(1-z)^2$; or party L's nominee is centrist, which has probability $1-(1-z)^2$. In the first case, each of the two extreme nominees wins with probability $\frac{1}{2}$;[21] in the second case, the centrist from L wins with certainty. Thus, the R primary voter's expected utility from nominating an extremist is

$$\frac{1}{2}(1-z)^2 \left((u(1,1) + \lambda) + (u(-1,1) + \lambda) \right) + (1 - (1-z)^2) \left(u(0,1) + \lambda \frac{bf_0}{z} \right)$$

Therefore, the primary voter has a profitable deviation to nominating an extremist over a centrist if the above expression is strictly greater that the utility from nominating a centrist,

[21] Strictly speaking, the general election voter is indifferent between nominees with platforms -1 and 1, and need not necessarily randomize uniformly across them. However, the ensuing argument extends to cover this case because if the randomization favors -1 over 1 to such an extent that the strategic candidate from party R does not have an incentive to deviate to platform 1, then a strategic candidate from party L will have an incentive to deviate to platform -1.

$u(0,1) + \lambda$. Some algebra reveals that this is equivalent to

$$u(0,1) < \frac{1}{2}\left(u(-1,1) + u(1,1)\right) + \lambda\frac{1-b}{z}$$

which is true since u is convex.

<u>*Step 2:*</u> Next, we show that for an open set of positive measure, a General Election Indifference equilibrium exists. To construct such an equilibrium, we need indifference between centrists and extremists of the general election voter; strict preference for extremists over centrists for party voters; and indifference between platforms for the strategic candidates. Assume that party voters randomize equally over two candidates with the same platform. We only consider party R, by symmetry, the logic extends to party L. Let $z_0 \equiv bf_0 + (1-b)g_0$.

For the general election voter to be indifferent between centrists and extremists, we must have

$$u(0,0) + \lambda\frac{bf_0}{z} = u(0,1) + \lambda\frac{b(1-f_0)}{1-z}$$

or

$$u(0,0) - u(1,0) = \lambda\left(\frac{b(1-f_0)}{1-z} - \frac{bf_0}{z}\right) \tag{A-6}$$

The left-hand side of (A-6) is strictly positive. The right-hand side is continuous and strictly increasing in z, where z can range from bf_0 (if $g_0 = 0$) to $bf_0+(1-b)$ (if $g_0 = 1$). It can be verified that the right-hand side is strictly negative if $z = bf_0$ and strictly positive but less than λ if $z = bf_0 + (1-b)$. Thus, so long as $u(0,0) - u(0,1) < \lambda$, there is a unique solution z to (A-6). Note that the solution z pins down g_0, i.e. the behavior of strategic candidates.

Suppose that if faced with two nominees with platforms -1 and 1, the general election voter randomizes uniformly over the two; if faced with one centrist and one extremist, she votes for the extremist with probability p. Then, the probability that a candidate from party R wins the party nomination and the general election with platform 0 is $\frac{1}{2}z\left(\frac{1}{2}z^2 + (1-z^2)(1-p)\right)$, where $\frac{1}{2}z$ is the probability that the other candidate within the party is also a centrist, and z^2 is the probability that the nominee from the other party, L, is a centrist. Similarly, the probability of winning with platform 1 is $\left(z + \frac{1}{2}(1-z)\right)\left(pz^2 + (1-z^2)\frac{1}{2}\right)$. These two probabilities of winning must be equal for strategic candidates to randomize between platforms, which solves for $p = \frac{1}{2}\left(1 - \frac{1}{z+z^2}\right)$. Plainly, $p < \frac{1}{2}$, and moreover, for $p \geq 0$ requires that $z + z^2 \geq 1$, or $z \geq \frac{\sqrt{5}-1}{2}$. This places a restriction on the set of parameters (recall that z must solve (A-6)), but clearly there is an open and positive measure set of parameters for which $z \geq \frac{\sqrt{5}-1}{2}$.

It remains only to verify that the R primary voter strictly prefers a candidate with platform 1 to one with platform 0. Computing the probabilities of various events, this incentive compatibility constraint can be written as

$$
\begin{aligned}
& \left(u(0,1)+\lambda\frac{bf_0}{z}\right)\left(z^2+\left(1-z^2\right)(1-p)\right)+\left(u\left(-1,1\right)+\lambda\frac{b\left(1-f_0\right)}{1-z}\right)\left(1-z^2\right)p \\
< \; & \left(u(1,1)+\lambda\frac{b\left(1-f_0\right)}{1-z}\right)\left(\frac{1}{2}\left(1-z^2\right)+z^2p\right) \\
& +\left(u(0,1)+\lambda\frac{bf_0}{z}\right)z^2\left(1-p\right)+\left(u\left(-1,1\right)+\lambda\frac{b\left(1-f_0\right)}{1-z}\right)\frac{1}{2}\left(1-z^2\right)
\end{aligned}
$$

Substituting $p=\frac{1}{2}\left(1-\frac{1}{z+z^2}\right)$ and the symmetry assumption $u(-1,0)=u(1,0)$ into the above inequality, tedious but straightforward simplification shows that the above inequality is equivalent to

$$u\left(0,1\right)-u\left(-1,1\right)-\left(u\left(0,0\right)-u\left(1,0\right)\right)<\left(u\left(0,0\right)-u\left(0,1\right)+u\left(1,1\right)-u\left(0,1\right)\right)\frac{z}{1-z^2}$$

Since $z\geq 1-z^2$ (because $p\geq 0$), the above inequality holds if

$$u(0,0)-u(1,0)+u(1,1)-u(0,1)>u(0,1)-u(-1,1)-u(0,0)+u(1,0)$$

which is equivalent to

$$u(1,1)-u(-1,1)>2u(0,1)-2(u(0,0)-u(1,0))$$

which holds because u is convex and $u(0,0>u(1,0)$.

Step 3: To show that a Naive Policy Preference equilibrium exists for an open parameter set of positive measure, one proceeds similarly to the logic of Step 2, but using the appropriate equilibrium conditions: primary voters strictly prefer party extremists over centrists, the general election voter strictly prefers centrists over extremists, and strategic candidates randomize over the center platform and their party extreme. We omit the argument to conserve space; details are available from the authors. □

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