

Imagine an industry inhabited by a firm and a union. The firm chooses how many workers,  $l$ , to hire at a given wage rate,  $w$ , and a given amount of *known* effort that each worker expends on job  $e$ . The union picks the amount of effort,  $e$ , that each worker puts in on the job and the wage rate,  $w$ . Wages are paid for showing up and not for the amount of on-the-job effort. All workers are unionized and follow union dictates.

The firm produces output,  $o$ , in line with the production function

$$o = l^\alpha e^\beta, \text{ with } 0 < \alpha, \beta, \alpha + \beta < 1;$$

as can be seen, output is a function of both the number of bodies hired and the effort each worker puts in. Production is subject to the fixed cost,  $\phi$ , in terms of output. Output is sold at the fixed price  $p = 1$ . The union has preferences of the following form:

$$w^\omega l^\mu - \varepsilon e, \text{ with } 0 < \lambda/(1 - \alpha) < \omega < 1, \text{ and } \varepsilon > 0.$$

It values the wage,  $w$ , each worker receives, the amount of workers hired,  $l$ , and the effort,  $e$ , that each worker expends. (This question will be graded both upon the economic intuition and the technical ability that you demonstrate. Double space your answer.)

1. Formulate and solve the firm's problem.
2. Formulate and solve the union's problem. Assume that  $(\omega - \lambda)\beta < \alpha$ . (*Hint*: Try to express in the problem in terms of  $e$  only.)
3. How are effort,  $e$ , employment,  $l$ , and wages,  $w$ , linked with  $\phi$ ? What is the intuition here?
4. How are effort,  $e$ , employment,  $l$ , and wages,  $w$ , linked with  $\varepsilon$ ? What is the intuition here?