The Dynamics of Metropolitan Communities*

Dennis Epple
Carnegie Mellon University and NBER

Richard Romano
University of Florida

Holger Sieg
Carnegie Mellon University and NBER

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Abstract

In this paper we study the life cycle locational choices of heterogeneous households and characterize the dynamics of metropolitan areas. We develop an overlapping generations model for households in of a system of multiple jurisdictions. At each point of time households choose among the finite number of jurisdictions, pick consumption and housing plans, and vote over public good and tax policies. A household's composition changes over the life cycle as children grow up and leave the household. These changes in turn impact the household's need for housing and for local public services, particularly education. Households face frictions in relocating. A household may relocate as its needs change, bearing the associated financial and psychic costs of moving. Alternatively, the household may choose to remain in a community that suited its initial needs, finding the costs of relocation to be greater than the potential benefits of moving to a community better suited to its changed housing and public good needs. Our theoretical and quantitative analysis shows that the presence of mobility costs is likely to have a large impact on household sorting patterns. Mobility costs also impact the political decisions that determine local tax and expenditure policies.

1 Introduction

A fundamental premise in modeling local jurisdictions is that households make their location decisions taking account of the public good bundles available in alternative jurisdictions. This hypothesis, first proposed by Tiebout (1956), has been the subject of extensive formal modeling and empirical analysis. Early empirical work, pioneered by Oates (1969), investigated the extent to which differentials in housing prices across jurisdictions reflect differentials in quality of local public goods and property tax rates.[1] Much recent empirical work has focused directly on the extent to which households stratify based on differences in the quality of local public goods.[2] Both research on capitalization and research on stratification of households across jurisdictions supports the hypothesis that households do in fact take account of differences in local public good bundles in making location choices.

In research to date, both theoretical models and empirical research have largely focused on static equilibrium models or cross-sectional empirical studies.[3] However the same logic that suggests households sort based on tastes for local public goods implies that households have incentives to change location over the life cycle. For example, a household's consumption of local public education begins when its first child enters kindergarten and ends when its last child leaves high school. Thus, one would expect that households with school-age children would place weight on the quality of local public schools when considering location choices, but that those same households would place little weight on quality of local public schools when their children have left school. Indeed, one would expect households would tend to

[1] Black (1999), Epple and Romano (2003), Figlio and Lucas (2004) and Bayer, Ferreira, and McMillan (2007) have extended this analysis to investigate whether differences within jurisdictions in the quality of local public goods are capitalized into house prices.

[2] See, for example, Epple and Sieg (1999), Epple, Romer, and Sieg (2001), Bayer, McMillan, and Reuben (2004), Bajari and Kahn (2004), Sieg, Smith, Banzhaf, and Walsh (2004), Ferreyra (2007), and Ferreira (2005).

[3] Examples of theoretical models using static frameworks are, for example, Ellickson (1973), Westhoff (1977), Epple, Filimon, and Romer (1984), Goodspeed (1989), deBartolome (1990), Epple and Romer (1991), Nechyba (1997a, 1997b), Fernandez and Rogerson (1996), Henderson and Thisse (2001), and Rothstein (2006). Exceptions are Benabou (1996a, 1996b), Durlauf (1996), Fernandez and Rogerson (1998), Glomm and Lagunoff (1999), and Ortalo-Magne and Rady (2006).

move to locations with good quality public schools when they have school-age children while moving to locations with lower housing prices and lower quality public schools when their children have left school. Departure of children from high school presages their departure from the household, and the associated decrease in need for housing reinforces the incentives for relocation that accompany the decrease in need for local public services. These incentives for relocation over the life cycle in turn create incentives for young households to make their initial location choices and housing purchases taking account of the likelihood that they will relocate in the future. A dynamic equilibrium model embodying household life cycle choices offers the potential to improve understanding both of community characteristics and of housing markets.

The main contribution of this paper is a new life cycle model of community formation with limited household mobility. Our model captures three important dimensions by which households differ: income, moving cost, and age. Income is clearly a key factor influencing a household's ability and willingness to pay the moving cost and housing price premium to relocate to a community with higher quality public services.[4] Moving costs, both financial and psychic, are important factors in decision process. In addition to transactions costs, relocation often entails costs associated with moving away from friends, neighbors, and familiar surroundings and the associated costs of becoming established in a new neighborhood. While financial costs will typically be roughly proportional to house value, psychic costs are likely to exhibit greater variation across households. Finally, our model also captures the fact that relocation incentives vary over the life cycle. These incentives are largely driven by the presence or absence of children at various points during the life cycle.

We derive the stationary equilibrium of our model and characterize its properties. In our model, adults live for two periods and thus can live in at most two different locations.[5] One important property of equilibrium is that many community pairs are strictly dominated by other pairs in equilibrium. This result is important since it reduces the dimensionality of the choice set. Restricting our attention to community pairs in the effective choice set, we can

[4]Family size is also clearly important. For example, a household with no children would have little incentive to move to a community with high-quality public education.

[5]Children live with their parents until they reach adulthood.

provide conditions that guarantee that households stratify by wealth (conditional on moving costs) in equilibrium. Old households have few incentives to move to a community that has higher levels of public good provision than the community chosen when young. We show that this conjecture is correct if the relative weight placed on the local public good is higher when young than when old.

Equilibria do not have analytical solutions. Nevertheless, we can provide a general characterization of the partition of household types into residential plans. Moreover, we develop an algorithm that can be used to compute equilibria numerically. To illustrate some of the quantitative implications of our model, we calibrate our model and compute equilibria for economies with multiple communities. We find that our model can generate equilibria in which a reasonable fraction of households relocates to a different community when old in equilibrium. This property of equilibrium is consistent with evidence on turnover in local housing markets. This feature of our model has important implications for the political decisions made in the communities. We find that older households are typically in the majority in communities with low levels of public good provision, while young households dominate in communities with high levels of expenditures.

The rest of the paper is organized as follows. Section 2 presents some stylized facts that characterize various dynamic aspects of community formation using Census data for the Boston metropolitan area. Section 3 develops our theoretical model. Section 4 defines equilibrium, discusses its properties and develops an algorithm to compute equilibria. In Section 5 we examine some quantitative properties of our model. Section 6 offers conclusions.

2 Some Stylized Facts

To gain some insights into the quantitative importance of mobility across the life cycle and the persistence of community dynamics, we consider the Boston Metropolitan Area. In Massachusetts, municipal and school district boundaries coincide. This absence of overlapping local jurisdictions facilities characterization of features of interjurisdictional mobility. The first part of the analysis is based on data for the census years 1970, 1980, 1990 and 2000. A distinctive feature of the Boston metropolitan area is that the population was virtually the same in 1970, 1980 and 1990. Thus, the Boston metropolitan area allows us to investigate interjurisdictional mobility in an environment in which the overall community population was unchanging. Of course, real incomes were growing over this period of time and family size was declining.

A striking feature of the data for those three census years is not only that the metropolitan population remained unchanged but that individual community populations remained virtually unchanged as well, despite the growth in income and the decline in family size. For example, the correlation of the logarithms of community populations in 1970 and 2000 was .981, revealing an extraordinary degree of stability of sizes of communities over that thirty-year period. From 1990 to 2000, the Boston metropolitan area population grew by approximately 8%. The comparison of year 2000 to the prior years thus provides a basis for evaluating the impact of metropolitan growth on interjurisdictional mobility.

In addition to stability in population size, the data also reveal strong persistence in community incomes. This is illustrated in Table 1 which shows the correlations of median community incomes for each decade from 1970 to 2000.[6] For example, median community incomes in 1970 and median community incomes in the year 2000 have a correlation of .93. We thus conclude that community level incomes are highly persistent across decades.

Next we analyze the persistence of community compositions by age and family size. We have calculated the correlation between median community income and the fraction of the population aged 19 or younger in each of the four census years that we study. We find the following correlations .33, .42, .22, and .54. These correlations might simply reflect variation in family size with income. However, data for the U.S. population reveal that the proportion of individuals under age 19 is relatively constant across household incomes. This suggests that the positive correlations between income and population below age 19 reported above are a

[6] We consider 119 communities as part of the greater Boston metropolitan area. This list of communities includes almost all communities that were considered to be part of the MSA using the definitions for 1970, 1980, and 1990. The Census drastically changed the definition of the Boston MSA in 2000 and we do not include the communities that were added in 2000. A list of all communities in our sample is available from the authors.

Table 1: Correlations of median community incomes in Boston

	1970	1980	1990	2000
1970	1.00	0.93	0.92	0.93
1980	0.93	1.00	0.97	0.96
1990	0.92	0.97	1.00	0.95
2000	0.93	0.96	0.95	1.00

result of household sorting across communities based on the presence of children. The data also reveal much persistence in the population age distribution over time. This finding is illustrated in Table 2 which correlates the fraction of community populations aged 19 or younger for the four census years.

Table 2: Correlations of fraction of community populations aged 19 or younger

	1970	1980	1990	2000
1970	1.00	0.91	0.70	0.66
1980	0.91	1.00	0.88	0.79
1990	0.70	0.88	1.00	0.84
2000	0.66	0.79	0.84	1.00

We are also interested in characterizing the persistence of public policies. We have assembled a comprehensive data base that contains the main fiscal and tax variables for all communities in the Boston MSA during the past 25 years. We find that the main patterns of expenditures and tax revenues are similar among the communities in our sample. To illustrate some of the main features, we pick two communities, Norton and Woburn, and plot personal property tax rates, total levies, and total educational expenditures. The plots are illustrated in Figure 1. We find that government policies are more volatile than income sorting patterns. For example, we find a decrease in property tax taxes in the first part of the 1980's. This decrease is a direct consequence of Proposition 2 1/2 – a law that restricted property taxation.

This law, which was passed in 1981, limited property tax rates to two-and-a-half percent (after some adjustment period). Since many jurisdictions had property taxes in the period leading up to 1981 that were higher than the limits set in Proposition $2\frac{1}{2}$, the law imposed for all practical purposes a binding constraint on these communities (Calabrese, Epple, and Romano, 2007). Both tax levies and educational expenditures are more stable than tax rates and largely track income increases during that time period.

As we noted above, a further distinctive feature of the Boston metropolitan area is that the population was virtually the same in 1970, 1980 and 1990. In the quantitative analysis of the model that we develop, we consider a stationary equilibrium. Hence, it is instructive to consider an environment with relatively constant population. As we explain in detail later, we calibrate our model treating ages 35 to 49 as typical of young households in our model and ages 55 to 69 as typical of old households. We view the year 1980 as illuminating in that the effects of the baby boom have not yet impacted the sizes of our young and old age cohorts. This and the relatively constant population over the time period in which it is centered make 1980 a plausible candidate to approximate a stationary equilibrium.

Figure 2 plots the ratio of old to young households as a function of median community income for the 92 municipalities in the Boston SMSA in 1980. As is evident from this figure, the proportion of old to young households is inversely related to community income. In our framework, this arises as households locate in communities with high school quality and high housing price premia[7] when they have children of school age, and then relocate to communities with lower school quality and lower housing price premia when their children exit school.[8] It is of interest to note how the pattern observed for 1980 compares to the pattern in a non-stationary environment. With the emergence of the baby boom generation, the cohort aged 35 to 49 in the Boston SMSA was 69% larger than the cohort aged 55 to 69. The ratio of old to

[7]See Calabrese, Epple, Romer, Sieg (2006) for evidence that housing price premia and school quality ascend with community income in the Boston SMSA in 1980.

[8]A natural concern is that the pattern exhibited in Figure 2 might arise because wealthier households exit the metropolitan area when their children complete school. If fact, however, the proportion of old to young households in the Boston metropolitan area in 1980 (93%) was higher than the nationwide ratio (86%), suggesting, if anything, some migration by older households into the metropolitan area.

Figure 1: Tax Rates, Tax Levy, and Educational Expenditures

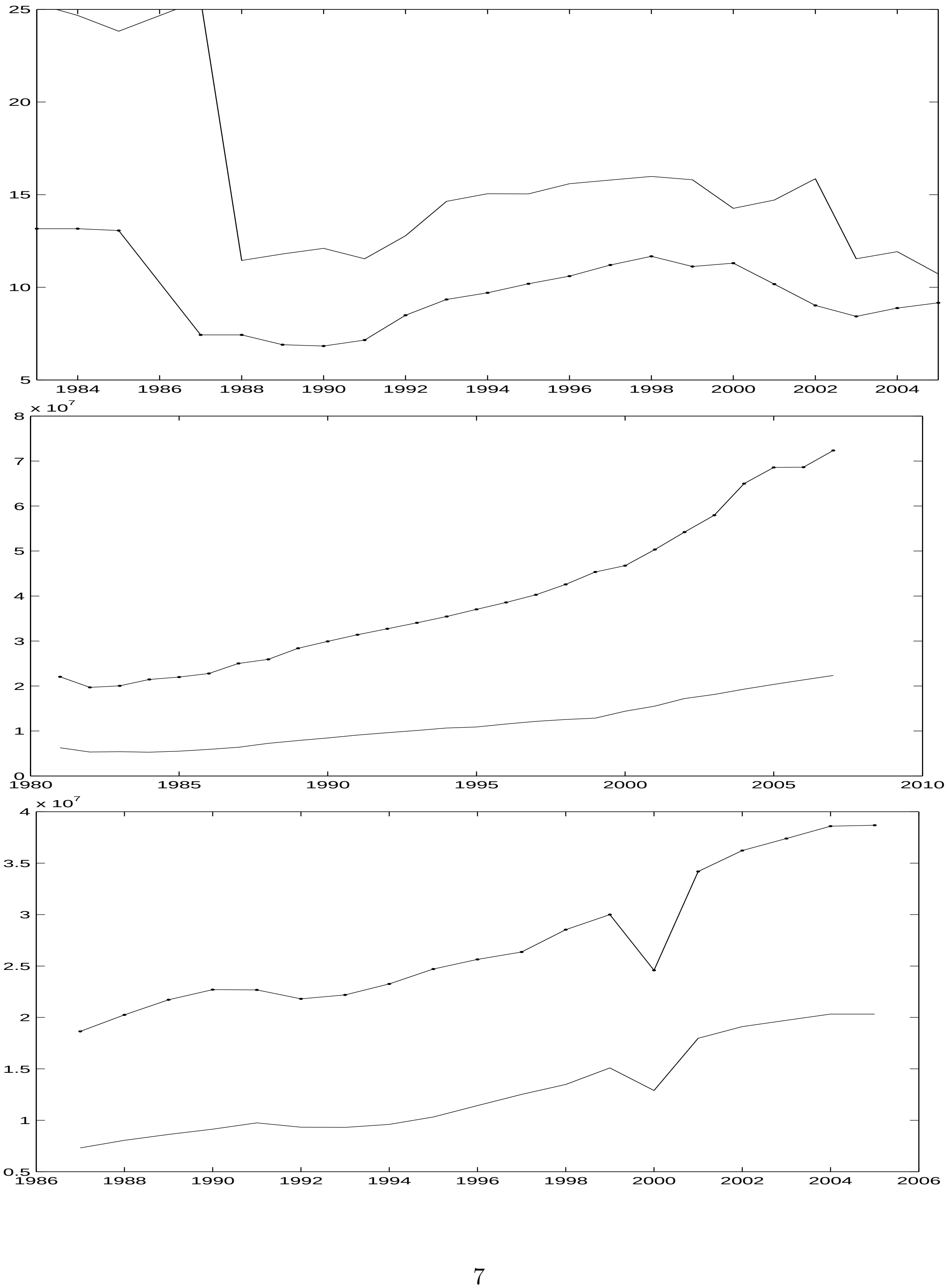
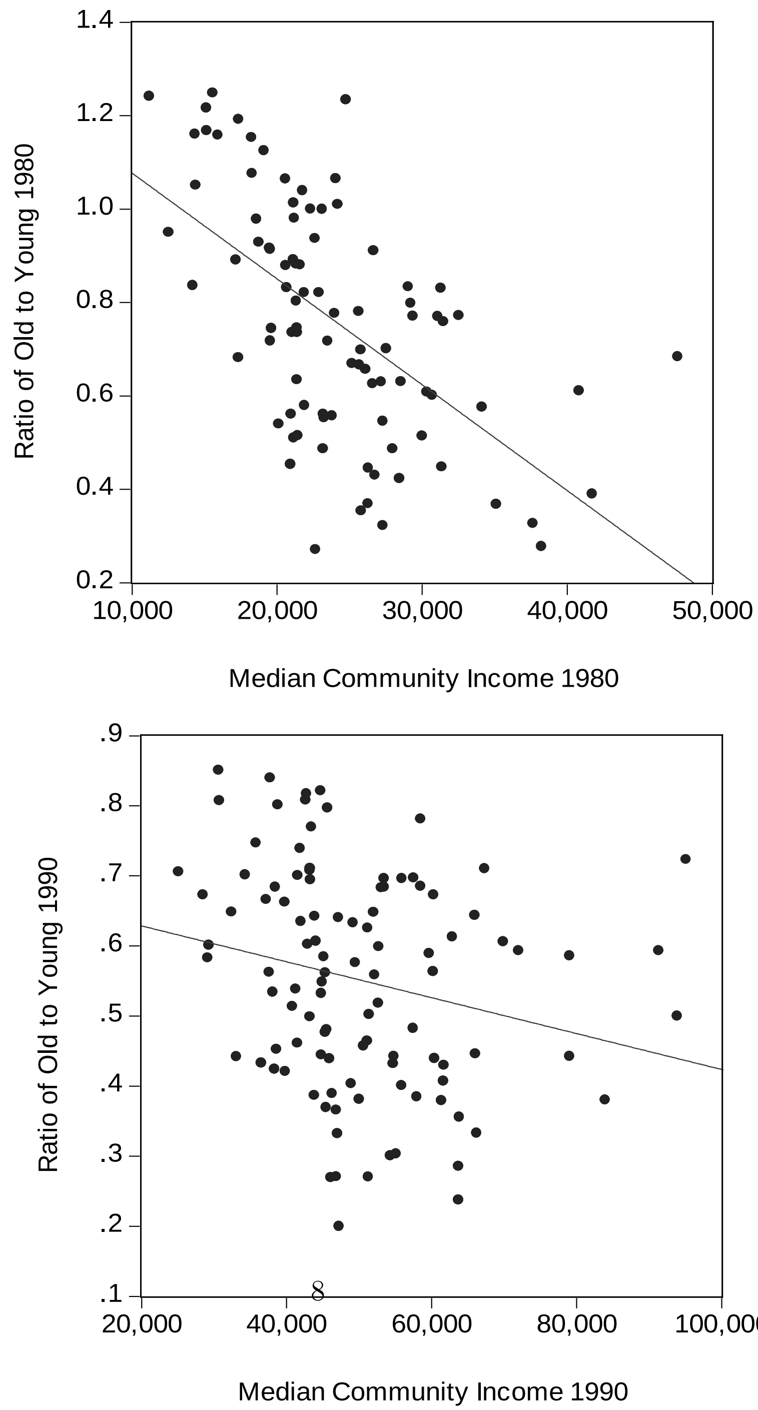


Figure 2

Ratio of Old to Young Households by Community Income in 1980 and 1990



young by community income for 1990, illustrated in the lower panel of Figure 2, exhibits a less pronounced pattern of variation by age, and the pattern for 2000 (not shown) is less pronounced still. In this paper, we develop model of a metropolitan area with intergenerational mobility. In our quantitative analysis we then specialize to a stationary environment, as we have suggested is roughly illustrative of the Boston SMSA in 1980. The impact of significant demographic changes, such as the baby boom, points to the potential value of future investigation of non-stationary environments.

3 An Overlapping Generations Model

We now develop an overlapping generations model to study the life cycle location choices of heterogeneous households, the associated demographic composition and public good provision of communities, and the dynamics of metropolitan areas.

3.1 Communities

Consider a local economy in which activity occurs at discrete points of time $t = 1, 2, ...$ The economy consists of J communities. At each point of time, each community provides a local public good g which is financed by property taxes τ.[9] Each community has a fixed supply of land[10] and thus a housing supply that is not perfectly elastic. Let p^h denote the net of tax price of a unit of housing services, h, in a community and $p = (1 + \tau)p^h$ the gross of tax price.

3.2 Households

There is a continuum of individuals each of whom lives for three periods, one period as a child and two periods as an adult. Thus at each point of time individuals in the economy

[9] We often suppress the time subscripts for notational simplicity. Subscripts have the obvious ranges unless we state otherwise.

[10] Municipal boundaries, once drawn, rarely change. For example, municipal boundaries in Massachusetts were drawn in the the 1930's and have remained unchanged since that time.

consist of three over-lapping generations, denoted child (c), young (y), and old (o). Children live in young households and attend school. There are two generations of adult households each period, young and old. Children become young adults at the same time that young adults transition to old age. Hence, there are no children in old households. Each young adult household is characterized by a lifetime wealth level denoted by w and child achievement denoted $\widetilde{a}$. The achievement of the child, $\widetilde{a}(g, \eta)$, is determined by the child's ability, η, and by government spending in the community occupied by the young household in which the child resides. We adopt the following characterization of achievement:

Assumption 1 *Achievement is additively separable and increasing in both of its arguments:* $\widetilde{a}(g, \eta) = a(g) + \eta$.

Households have additive, time-separable utility. The period utility when young which is defined over housing h, child achievement, and a numeraire good b. The period utility when old is defined over housing, the local public good, and the numeraire. Period utility for a young household, $\widetilde{U}^y(b, h, \widetilde{a}(g, \eta))$, and is assumed to be additively separable in child ability. Hence, $\widetilde{U}^y(b, h, a(g, \eta)) = U^y(b, h, g) + \eta$.[11]

Assumption 2 *The current period utility function of a young household* $U^y(b, h, g) + \eta$ *and the utility function of an old household* $U^o(b, h, g)$ *are increasing, twice differentiable, and concave in* (b, h, g).

Thus, the presence of children in young households that have children induces different preferences for public goods and housing for young than for old households.[12] Lifetime utility is separable over the two periods. Since there is no uncertainty about future equilibrium outcomes in this model, we make the following assumption:

[11] As will become clear, we permit child ability, η, to be correlated with parental wealth, w. This and induced preferences that are non-separable in η add considerable notational burden. There is limited, mixed empirical evidence about the effects of child ability on choice, making the neutral (i.e., additive) specification a natural choice.

[12] An extension of our model incorporates explicit differences in family. An appendix that discusses how to model differences in family size is available upon request from the authors.

Assumption 3 *Households behave as price takers and have perfect foresight about current and future prices, tax rates, and levels of local public goods.*

As will become apparent, household choices do not depend on η; hence choices do not depend on whether η is known at the time decisions are made. It is notationally convenient to assume that η is realized after young households make their housing and location choices.

3.3 Achievement and Wealth

Upon reaching adulthood, the young household has achievement determined by its education when a child. Ability may be correlated with the wealth of the household via home production of education as well as by potential intergenerational genetic correlation of ability. Hence, we assume:

Assumption 4 *Lifetime wealth for a young household having parents with wealth w and education g as a child is $a(g) + \eta$ with $\eta = \gamma_w \ln(w) + \varepsilon$.*[13] *Here γ_w is a parameter and ε is a random variable distributed independently of w. At date t, the distribution of ε, denoted by $F_{\varepsilon,t}(\varepsilon)$, is continuous with support $S_{\varepsilon,t} \subseteq R$ and density $f_{\varepsilon,t}(\varepsilon)$ with $f_{\varepsilon,t}(\cdot)$ everywhere positive on its support.*

3.4 Mobility Costs

When they leave their parents' homes, young households are not endowed with a place of residence. We assume that they can pick any community of residence in the first period without facing mobility costs. Old households have already established a residence when they were young. If they decide to relocate in the second period, they face mobility costs, i.e. mobility costs are only born by old households if they decide to relocate in the second period

[13] This particular functional relationship between η and w is inessential for the theoretical development. It is, however, a natural specification, employed in our quantitative analysis. We avoid further notational burden by introducing the specification here.

of their adult life. Mobility costs are denoted by mc, and mobility costs as a share of income are denoted $z = \frac{mc}{w}$. We assume that households are heterogeneous mobility costs.

Assumption 5 *Mobility costs as a share of wealth are distributed independently of wealth. The distribution of mobility costs as a share of wealth for young households in period t, denoted by $F_{z,t}(z)$, is continuous with support $S_{z,t} \subseteq R_+$ and joint density $f_{z,t}(z)$ with $f_{z,t}(\cdot)$ everywhere positive on its support.*

The assumption that $f_{\varepsilon,t}(\cdot)$ and $f_{z,t}(\cdot)$ are everywhere positive on their supports simplifies some of the arguments, but is not crucial for the main results.

3.5 The Decision Problem of Households

Households are forward-looking and maximize life-time utility which is time separable with constant discount factor β. Households recognize that locational and housing choices in the first period will have an impact on their well-being in the second period. Households must choose a community of residence in each period. Let $d^y_{jt} \in \{0,1\}$ denote an indicator that is equal to one if a young household lives in community j at time t and zero otherwise. Similarly define $d^o_{jt} \in \{0,1\}$ for old households.

Households also determine consumption choices for housing and the composite private good. A young household at date t with characteristics (w_t, mc_t) maximizes lifetime utility:

$$\max_{d^y_t, h^y_t, b^y_t, d^o_{t+1}, h^o_{t+1}, b^o_{t+1}} \sum_{k=1}^{J} d^y_{kt} U^y(b^y_{kt}, h^y_{kt}, g_{kt}) \; + \; \beta \sum_{l=1}^{J} d^o_{lt+1} U^o(b^o_{lt+1}, h^o_{lt+1}, g_{lt+1}) \tag{1}$$

subject to the lifetime budget constraint

$$\sum_{k=1}^{J} d^y_{kt} \; (p_{kt} \; h^y_{kt} + b^y_{kt}) + \sum_{l=1}^{J} d^o_{lt+1}(p_{lt+1} \; h^o_{lt+1} + b^o_{kt+1}) = w_t - \sum_{k=1}^{J} \sum_{l \neq k} 1\{d^y_{kt} = d^o_{lt+1} = 1\} \; mc_t \tag{2}$$

and residential constraints:

$$\begin{aligned} \sum_{k=1}^{J} d^y_{kt} &= 1 \\ \sum_{l=1}^{J} d^o_{lt+1} &= 1 \end{aligned} \tag{3}$$

The last two constraints in (3) impose the requirement that the household lives in one and only one community at each point of time. For simplicity above, we have written household lifetime utility suppressing the expected idiosyncratic ability of the household's child, $E(\eta_t)$, since this term does not affect household choices. There is no other uncertainty in the model. Thus, the planned choices by young households for the future correspond to the optimal future choices in equilibrium. Also, w_t is the present value of lifetime income, thus assuming perfect capital markets.[14] Finally, we have abstracted from discounting of future prices just for simplicity of exposition.

It is often convenient to express this decision problem using a conditional indirect utility (or value) function. Given a household with wealth, w, moving cost, mc, and community choice k when young and l when old, we can solve for the optimal demand for housing and other goods in both periods. Substituting these demand functions into the lifetime utility function yields the conditional indirect utility function which can be written:

$$V_{kl}^{y} = V(w - \delta_{kl} mc, g_k, p_k, g_l, p_l) \tag{4}$$

where $\delta_{kl} = 1$ if $k \neq l$ and zero otherwise. Similarly, the conditional utility function of an old household that occupied community k when young and is occupying community l when old is:

$$V^{o}(w_n^{o}, g_l, p_l) = \max_{h_{lt}} \; U^{o}(w_n^{o} - p_l h_l, h_l, g_l) \tag{5}$$

where $w_n^o = w - \delta_{kl} mc - p_k h_k^y$.

Define the set of young households living in community j at time t as follows:

$$C_{jt}^{y} \;\; = \;\; \left\{ (w_t, mc_t) \middle| \; d_{jt}^{y} = 1 \right\} \tag{6}$$

The number of young households living in community j at time t is given by:

$$n_{jt}^{y} \;\; = \;\; \int \int_{C_{jt}^{y}} f_t(w_t, mc_t) \; dw_t \; dmc_t \tag{7}$$

Similarly define the set of old households living in community j at time t as follows:

$$C_{jt}^{o} \;\; = \;\; \left\{ (w_{t-1}, mc_{t-1}) \middle| \; d_{jt}^{o} = 1 \right\} \tag{8}$$

[14] Abstracting from uncertainties is obviously a strong assumption, and the consequences of uncertainties are of interest to study.

The number of old households living in community j at time t is given by:

$$n^o_{jt} \quad = \quad \int\int_{C^o_{jt}} f_{t-1}(w_{t-1}, mc_{t-1})\; dw_{t-1}\; dmc_{t-1} \tag{9}$$

3.6 Housing Markets

In this model all households are renters.[15] Housing demand functions $h^y_j(\cdot)$ and $h^o_j(\cdot)$ can be derived by solving the decision problems characterized above. Below we introduce subscripts t to indicate the dependence of housing demands on prices young and old households confront at date t. Aggregate housing demand in community j at time t is then defined as the sum of the demand of young and old households:

$$H^d_{jt} = H^y_{jt} \;+\; H^o_{jt} \tag{10}$$

where

$$\begin{aligned} H^y_{jt} &= \int\int_{C^y_{jt}} h^y_{jt}(w_t, mc_t)\; f_t(w_t, mc_t)\; dw_t\; dmc_t \\ H^o_{jt} &= \int\int_{C^o_{jt}} h^o_{jt}(w_{t-1}, mc_{t-1})\; f_{t-1}(w^y_{t-1}, mc_{t-1})\; dw_{t-1}\; dmc_{t-1} \end{aligned}$$

To characterize housing market equilibria, we need to specify housing supply in each community.

Assumption 6 *Housing is owned by absentee landlords. Housing supply is stationary and exogenously given by $H^s_j(p^h_{jt})$.*

This assumption is primarily imposed for convenience. The alternative would be to assign property rights over land. Households would then obtain revenues from rental income. While this extension is feasible, it adds little. We avoid the additional notational complexity by assuming absentee owners of land.

The housing market in community j is in equilibrium at time t if:

$$H^d_{jt} = H^s_j(p^h_{jt}) \tag{11}$$

[15] We discuss the implications of housing ownership in the conclusions.

3.7 Foundation Grants

TO BE WRITTEN

3.8 Community Budget Constraints

We assume that each community provides a congested local public good g_{jt} that is financed by property taxes τ_{jt}. Community budgets must be balanced at each point of time. We assume that the public good primarily reflects expenditures per student on education. Hence we can express the community specific budget constrained as:

$$\tau_{jt}\, p^h_{jt}\, H_{jt} = g_{jt} n^y_{jt} \tag{12}$$

The right hand side of equation (12) equals public expenditure on young households, consistent with young households having one child in public education.

3.9 Voting

Next consider public choice of the tax rate and public good provision in a community. Households have the opportunity to vote twice in their lives, once when they are young and again when they are old. Households vote to maximize their lifetime utility from that point forward. To fully specify a voting model, we need a) to describe the set of alternatives that are considered to be feasible outcomes by the voters; b) define preference orderings over feasible outcomes; and c) define a majority rule equilibrium.

Defining the set of feasible outcomes requires specifying the timing of decisions. We assume the following timing structure.

Assumption 7 *Each young household chooses their initial community of residence and rents a home there. The young household also commits to their old-aged community of residence. Housing markets then clear. Young households then vote taking as given the net housing price, which have already been established, but also g and p in their future community of* *residence. Numeraire and public good consumption take place. The old household then occupies the community planned when young and consumes housing. The old household votes, and, last, old-aged numeraire and public good consumption occur.*

The timing and voter beliefs incorporated in Assumption 5 make the problem tractable. The key simplification is that young voters take their future community choices and the variables that characterize their old-aged community as given when voting. Of course, these variables will be those that arise in equilibrium and the future choice of community will be optimal given the equilibrium values. For example, a household that has committed to move to another community when it becomes old will in fact find it optimal to do so. But voters will not account for changes that would become optimal out of equilibrium, tremendously simplifying the voting problem.

Consider a community j which is characterized by a pair of housing prices and public good provision denoted by (p_{jt}, g_{jt}). Combining the equation relating net and gross housing prices, $p_{jt} = p^h_{jt}(1 + \tau_{jt})$, and the community budget constraint, we obtain:

$$p_{jt} = p^h_{jt} \; + \; \frac{g_{jt}}{H^y_{jt}/n^y_{jt} + (n^o_{jt}/n^y_{jt})\;(H^o_{jt}/n^o_{jt})} \tag{13}$$

Given our timing assumptions, all variables in this expression except (p_{jt}, g_{jt}) have been determined prior to voting. Thus the set of feasible alternatives yields a linear relationship between the choice of g_{jt} and the resulting gross-of-tax housing price p_{jt}.

In each community j, there are two types of voters. The first type of voter is an old household. The second type of voter is a young household. Given the beliefs of each voter about feasible alternatives in equation (13), we can characterize each voters decision problem and thus characterize the voter's behavior.

First consider an old household that has chosen to live in community j after living in community i when young. The household's old age income is given by $w^o_{nt} = w_{t-1} - p_{it-1}h^y_{it-1} - b^y_{it-1} - \delta_{ij}mc_{t-1}$. The household's budget constraint when old is given by: $w^o_{nt} = p_{jt}h^o_{jt} + b^o_{jt}$. Let h^o_{jt} be the amount of housing the household has chosen. The quantity h^o_{jt} is then fixed at the time that voting occurs. Substituting the community budget constraint that prevails at the time of voting into the voter's budget constraint, we obtain:

$$w^o_{nt} \;=\; p^h_{jt}h^o_{jt} \;+ \frac{g_{jt}n^y_{jt}}{H_{jt}}\; h^o_{jt} \;+\; b^o_{jt} \tag{14}$$

The voter's utility function is $U^o(g_{jt}, h^o_{jt}, b^o_{jt})$. At the time of voting, all elements of the preceding budget constraint and utility function have been determined except (g_{jt}, b^o_{jt}). Quasi-concavity of the utility function and convexity of the budget constraint imply that the voter's induced preference over g_{jt} is single-peaked (Denzau and Mackay, 1976).

Next consider a young voter that lives in community j at t and plans to live in community k int $t+1$. The development is analogous to that for old voters, and we thus summarize briefly. At the time of voting in community j, this household will have purchased housing h^y_{jt}. The budget constraint of the young voter is then:

$$\begin{aligned} w_t \;&=\; p^h_{jt}h^y_{jt} \;+\; \frac{g_{jt}}{H^y_{jt}/n^y_{jt} + (n^o_{jt}/n^y_{jt})\;(H^o_{jt}/n^o_{jt})}\; h^y_{jt} \;+ b^y_{jt} \\ &+\; p_{kt+1}h^o_{kt+1} + b^o_{kt+1} + \delta_{jk}\; mc_t \end{aligned} \tag{15}$$

The young voters utility function is: $U^y(b^y_{jt}, h^y_{jt}, g_{jt}) \;+\; \beta U^o(b^o_{kt+1}, h^o_{kt+1}, g_{kt+1})$. At the time of voting, the community tax base, $H^y_{jt}/n^y_{jt} + (n^o_{jt}/n^y_{jt})\;(H^o_{jt}/n^o_{jt})$, and the voter's housing consumption, h^y_{jt}, have been determined. The voter takes current and future prices (p^h_{jt}, p_{kt+1}) and future government provision, g_{kt+1}, as given. Quasi-concavity of the voter's utility function, $U^y + \beta U^o$, and convexity of the budget constraint then imply that induced preferences over g_{jt} are single-peaked (Slutsky, 1975).

Definition 1

A majority voting equilibrium is a public good provision level g_{jt} that defeats all alternative feasible public good provision levels in pairwise majority voting.

We have the following result:

Lemma 1

Voting equilibrium exists in all communities.

Lemma 1 follows from single-peakedness of preferences of all voters. Note that the median voter is not necessarily the household with median income. In general, the identity of the median voter will vary not only with income but also with the age composition of the community.

4 Equilibrium Analysis

4.1 Definition of Equilibrium

Definition 2

An equilibrium for this economy is defined as an allocation that consists of a sequence of joint distributions of income and wealth, $\{F_t(w, mc)\}_{t=1}^{\infty}$*, a vector of prices, taxes, and public goods denoted by* $\{p_{1t}, \tau_{1t}, g_{1t}, ..., p_{Jt}, \tau_{Jt}, g_{Jt}\}_{t=1}^{\infty}$*, consumption plans for each household type, and a distribution of households among communities,* $\{C_{1t}^{y}, ..., C_{Jt}^{y}, C_{1t}^{o}, ..., C_{Jt}^{o}\}_{t=1}^{\infty}$*, such that:*

1. *Households maximize lifetime utility and live in their preferred communities.*
2. *Housing markets clear in every community at each point of time.*
3. *Community budgets are balanced at each point of time.*
4. *There is a majority voting equilibrium in each community at each point of time.*

Recall that achievement of a child of a young household is a function of school quality g_t, household wealth w_t, and an idiosyncratic shock ϵ_t:

$$a_t = a(g_t) + \gamma_w \ln w_t + \epsilon_t \tag{16}$$

A child with achievement a_t then start as a young adult with lifetime wealth w_{t+1}:

$$\ln w_{t+1} = a_t \tag{17}$$

Similarly we can define a law of motion for the distribution of moving costs. We are then in a position to define a stationary equilibrium for our economy:

Definition 3

A stationary equilibrium is an equilibrium that satisfies the following additional conditions:

1. *Constant prices, tax rates and levels of public good provision, i.e. for each community* j, $p_{jt} = p_j$, $\tau_{jt} = \tau_j$, *and* $g_{jt} = g_j$ $\forall t$.

2. *A stationary distribution of households among communities, i.e. for each community* j, *we have* $C^o_{jt} = C^o_j$ *and* $C^y_{jt} = C^y_j$ $\forall t$.

3. *A stationary distribution of household wealth and moving costs, i.e.* $F_t(w, mc) = F(w, mc)$ $\forall t$.

The remainder of the paper focuses on properties of stationary equilibria.

4.2 Equilibrium Residential Choices

Upon entering adulthood, young households choose an initial and an old-age community of residence, correctly anticipating housing prices and local public good provision. Let k and l denote, respectively, the initial and old-age communities, $k, l \in \{1, 2, ..., J\}$. If $k \neq l$, then the household bears moving cost with present value of mc. We adopt the convention of numbering the communities so that $g_{j+1} > g_j$. Since households correctly anticipate g's and p's, gross housing prices will also ascend with the community number.[16]

We now place some restrictions on the form of the household utility function that greatly facilitate the analysis. We continue to suppress the idiosyncratic component of achievement in our analysis of utility and household choices.

[16]We do not examine cases where communities have the same value of g and thus p. If two communities had the same values, then households would be indifferent between them, and we assume they would randomize so that the distributions of (w, mc) would be the same in the two communities. In turn, this would imply there is no difference between the two communities, so they could be treated as one community (with the usual aggregation of housing supplies). Thus there is no loss in generality in requiring that communities be different (as a case with two clone communities is equivalent to another case with one fewer distinct communities).

Assumption 8 *The utility function*

$$U^a(b,h,g) \;=\; u_g^a(g) \,+\, u^a(b,h),\;\; a \,\in\, \{y,o\}, \tag{18}$$

is separable and $u^a(b,y)$ *is homogeneous of degree* ρ.

Let $V^y(g_k, g_l, p_k, p_l, \tilde{w})$ denote indirect lifetime utility of a young household choosing residential plan kl, where $\tilde{w} \,=\, w - \delta_{kl} mc$ is lifetime wealth adjusted for any moving cost. Given the separability assumption, the indirect utility can be written as:

Lemma 2

$$V^y(g_k, g_l, p_k, p_l, \tilde{w}) \;\;=\;\; G(g_k, g_l) \,+\, \tilde{w}^{-\rho} W(p_k, p_l); \tag{19}$$

with G *an increasing function of* (g_k, g_l) *and* W *a decreasing function of* (p_k, p_l).

Proof of Lemma 2: Indirect utility is given by:[17]

$$\begin{aligned} V^y \;\;&=\;\; \underset{h_k,h_l}{Max}\,[u_g^y(g_k) \,+\, u_g^o(g_l) \,+\, u^y(b_k,h_k) \,+\, u^o(b_l,h_l)] \\ &\qquad s.t.\; p_k h_k \,+ b_k \,+\, p_l h_l \,+\, b_l \,\leq\, \tilde{w} \\ &=\;\; G(g_k,g_l) \,+\, \underset{h_k,h_l}{Max}\,[\,u^y(b_k,h_k) \,+\, u^o(b_l,h_l)] \\ &\qquad s.t.\; p_k h_k \,+ b_k \,+\, p_l h_l \,+\, b_l \,\leq\, \tilde{w}\,; \end{aligned} \tag{20}$$

where $G(g_k, g_l) \,\equiv\, u_g^y(g_k) \,+\, u_g^o(g_l)$ is an increasing function of (g_k, g_l). Since $u^a(b,h)$ is homogeneous of degree ρ, it follows from Theorem I in (Lau, 1970) (p. 376) that the maximand in the lower line of (20) equals $\tilde{W}\left(\frac{p_k}{\tilde{w}}, \frac{p_l}{\tilde{w}}\right)$, a function homogeneous of degree $-\rho$ and decreasing in its arguments . Then: $V^y \,=\, G(p_k, p_l) \,+\, \tilde{w}^{-\rho} W(p_k, p_l)$. Q.E.D.

To simplify notation, define $V_{kl}^y = V^y(g_k, g_l, p_k, p_l, \tilde{w})$. The optimal residential choice plan of young adults maximizes V_{kl}^y over (k,l) taking anticipated p's and g's as given. It is also convenient to adopt a notation in which locational choices can be characterized by a single index subscript i. Let $i \,\in\, I_{kl}$, $I_{kl} \,=\, \{kl|\, k,l \,=\, 1,2,...,J\}$, indicate a residential plan. Let

[17] The discount factor β is subsumed in the old age utility function with no loss of generality.

$P_i \equiv -W(p_k, p_l)$ for $i = kl$, which we refer to as the composite price of residential plan i. Note that P_i is increasing in (p_k, p_l). Using this definition, we have that indirect utility from residential plan i is given by:

$$V_i^y \; = \; G_i \, - \, (w - \delta_i mc)^{-\rho} P_i, \tag{21}$$

where $G_i \equiv G(g_k, g_l)$ for $i = kl$. As a final step, let $T \equiv mc/w$ and again rewrite indirect utility using type-dependent price P_i^T.

$$V_i^y \; = \; G_i \, - \, w^{-\rho} P_i^T; \tag{22}$$

where

$$P_i^T \; \equiv \; \begin{cases} P_i \quad \text{if } i \text{ does not move } (k = l) \\ P_i(1-T)^{-\rho} \text{ if } i \text{ moves } (k \neq l) \end{cases} . \tag{23}$$

Household type (w, T) then chooses a residential plan i to maximize V_i^y in (22) taking (G_i, P_i^T), $i \in I_{kl}$, as given.

Immediate properties of the choice problem are summarized in the following lemma:

Lemma 3

The choice problem must satisfy the following conditions:

(P1) Indifference curves V_i^y = const. in the (G_i, P_i^T) plane are linear with slope w^ρ.

(P2) Indifference curve satisfy single crossing, with "slope increasing in income (SII)."

(P3) $dP_i^T/dT > 0$ for $k \neq l$; choices with moving are effectively more expensive as mc rises.

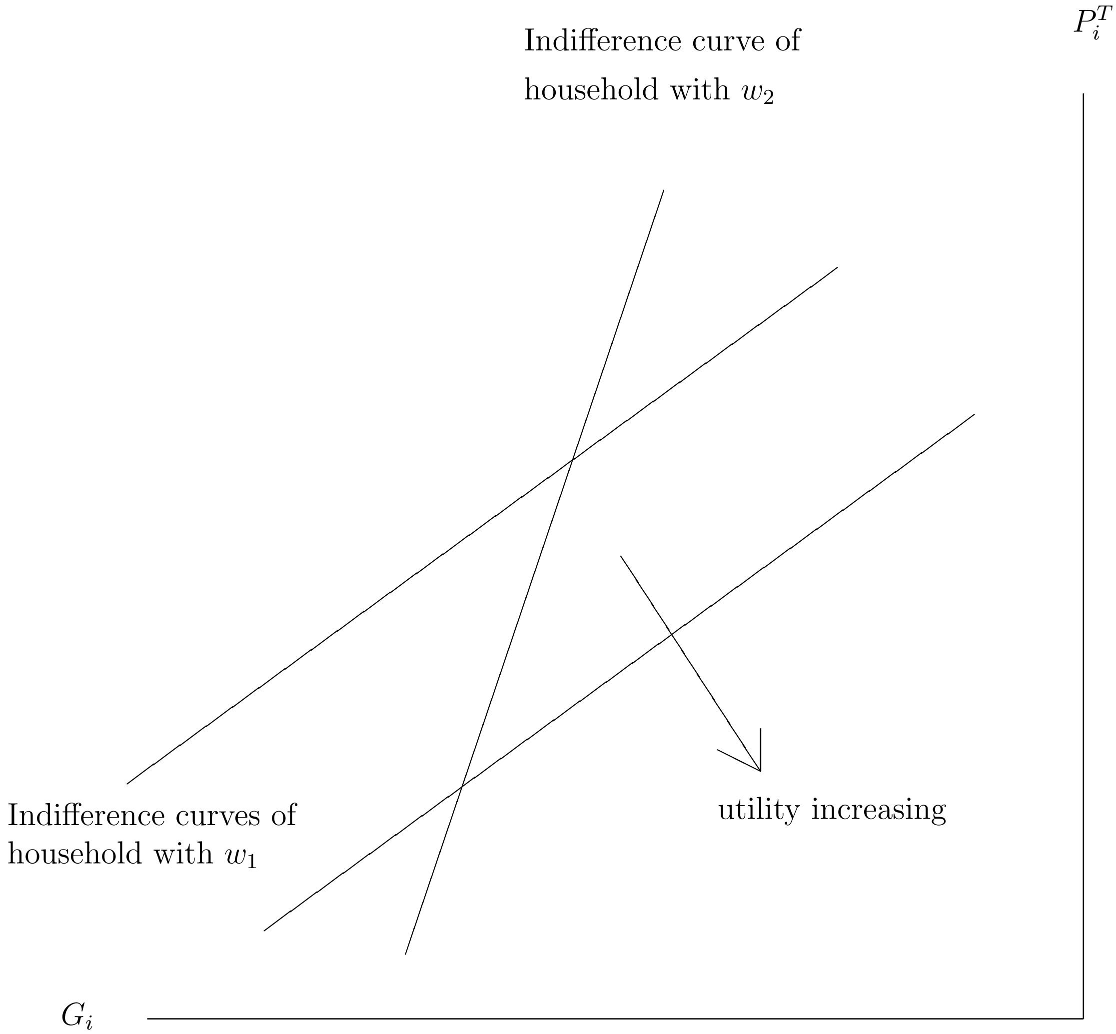
Properties (P1) – (P3) are intuitive and simply confirmed. (P1) will greatly simplify the analysis that follows. (P2) and (P3) are keys to the character of sorting over communities over the life cycle.

An Example.

In our computational analysis below, we adopt the following lifetime utility function:

$$\widetilde{U} \; = \; a + \frac{1}{\rho}[\alpha_h h_k^\rho \, + \, \alpha_b b_k^\rho \, + \, \beta_g g_l^\rho \, + \, \beta_h h_l^\rho \, + \, \beta_b b_l^\rho], \;\; \rho \, < \, 0; \tag{24}$$

Figure 2:



Indifference curve of household with w_2
P_i^T
utility increasing
Indifference curves of household with w_1
G_i

and the following achievement function:

$$a = \frac{\alpha_g}{\rho_a} g_k^{\rho_a} + \eta; \tag{25}$$

Since the idiosyncratic component of achievement, η, does not affect household choices, we continue, as above, to study utility net of the idiosyncratic component, defining $U = \widetilde{U} - \eta$. Hence, substituting the achievement function into the utility function, we obtain:

$$U = [\frac{\alpha_g}{\rho_a} g_k^{\rho_a} + \frac{\beta_g}{\rho} g_l^{\rho}] + \frac{1}{\rho}[\alpha_h h_k^{\rho} + \alpha_b b_k^{\rho} + \beta_h h_l^{\rho} + \beta_b b_l^{\rho}], \ \rho < 0; \tag{26}$$

which satisfies Assumption 6. After some manipulation one obtains indirect utility:[18]

$$V_i^y = G_i - (w - \delta_i mc)^{\rho} P_i; \tag{27}$$

where:

$$\begin{aligned} P_i &= -\frac{1}{\rho} z_{kl}^{-\rho} \left[\alpha_h(\frac{\alpha_b}{\alpha_h} p_k)^{-\frac{\rho}{1-\rho}} + \alpha_b + \beta_h(\frac{\alpha_b}{\beta_h} p_l)^{-\frac{\rho}{1-\rho}} + \beta_b(\frac{\alpha_b}{\beta_b})^{-\frac{\rho}{1-\rho}}\right]; \\ z_{kl} &= [p_k(\frac{\alpha_b}{\alpha_h} p_k)^{-\frac{1}{1-\rho}} + 1 + p_l(\frac{\alpha_b}{\beta_h} p_l)^{-\frac{1}{1-\rho}} + (\frac{\alpha_b}{\beta_b})^{-\frac{1}{1-\rho}}]; \\ G_i &= [\frac{\alpha_g}{\rho_a} g_k^{\rho_a} + \frac{\beta_g}{\rho} g_l^{\rho}]. \end{aligned} \tag{28}$$

and where we have assumed again residential plan $i = kl$. Figure 2 depicts some indifference curves in the (P, G) space for households of two wealth levels $w_2 > w_1$.[19]

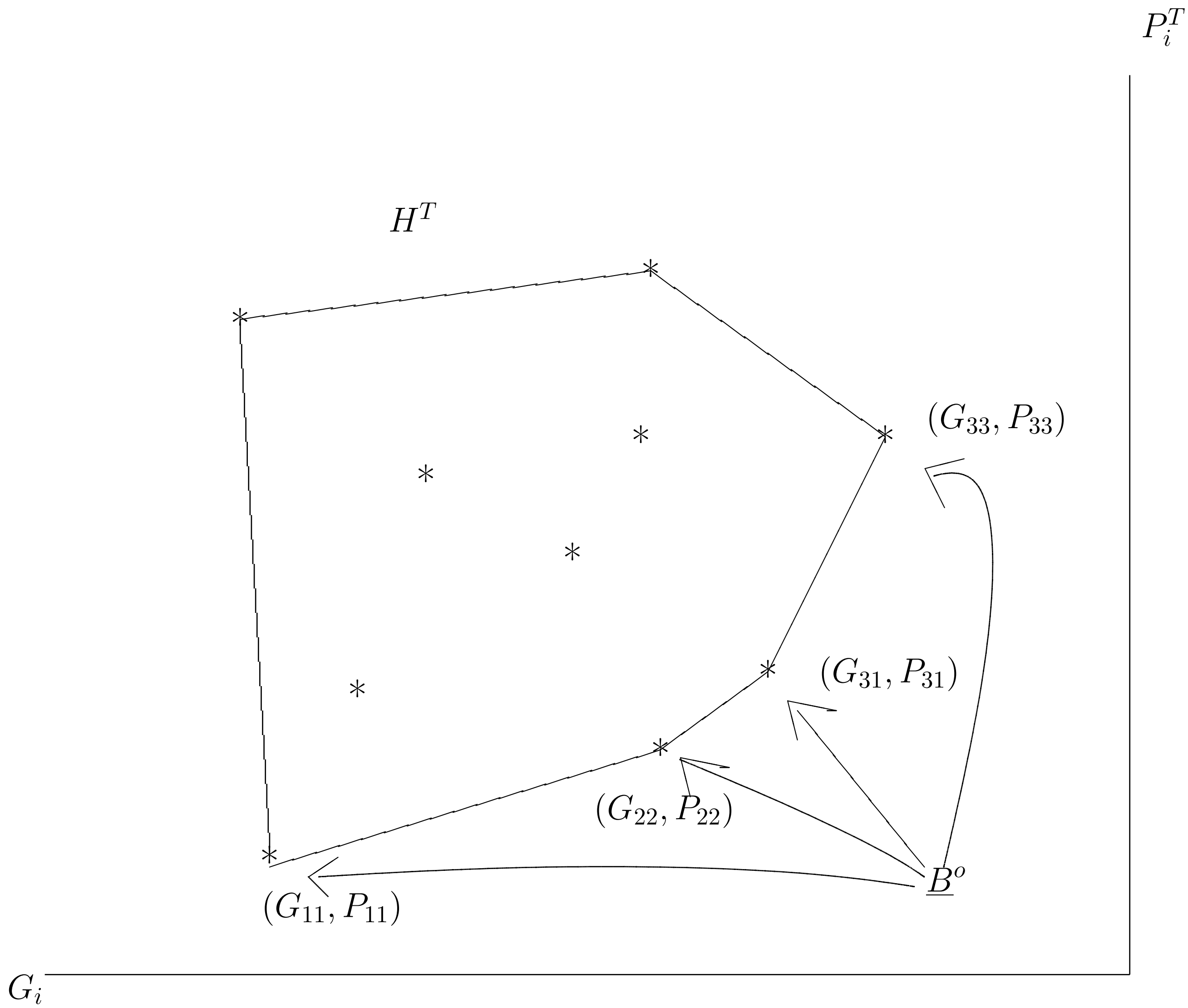
Keeping in mind that $\rho < 0$, one can see that all the properties of the preceding more general case are satisfied. In particular the composite public good G_i is increasing in the g's and the composite price P_i is increasing in the p's.

With J communities, there are J^2 residential plans that could feasibly be chosen. Using properties of the choice problem, we can develop restrictions on the set of plans that are actually chosen and then develop an algorithm for mapping household types into their equilibrium residential plans. Let $B^0 \equiv \{G_i, P_i^T \big| i \in I_{kl}\}$ denote the set of bundles, corresponding to residential plans, that are feasible for households with $T = mc/w$. Let H^T denote the convex

[18] A minus sign does not arise with the exponent ρ in (26), in contrast to the general development leading to (22), because ρ is negative in this CES example.

[19] The underlying CES utility function leads the composite public good G_i to be negative.

Figure 3:



Notation: $*$'s denotes bundles that could be chosen in equilibrium.

hull of B^0 and let $\underline{B}^0(T)$denote the set of residential plans (G_i, P_i^T) on the lower boundary of H^T. Formally, $\underline{B}^0(T)$is defined:

$$\underline{B}^0(T) \equiv \{ (G_i, P_i^T) \in B^T \Big| \text{no distinct} (\tilde{G}_i, \tilde{P}_i^T) \in H^T \text{exist with} \tilde{G}_i \geq G_i \text{ and } \tilde{P}_i^T \leq P_i^T \} \quad (29)$$

Figure 4 shows an example of these concepts for a case with J = 3. We have:

Proposition 1

(i) Any and all non-moving residential plans chosen by households with the maximum T comprise the non-moving residential plans chosen by all households.

(ii) Any and all moving plans chosen by households with the minimum T comprise the moving residential plans chosen by all households.

First, prove Lemma 4.

Lemma 4

Households with relative moving cost T choose in equilibrium any and all residential plans in $\underline{B}^0(T)$.

Proof of Lemma 4: Households with T maximize V_i^y as defined in (22) – (23). Since V_i^y is increasing in G$_i$ and decreasing in P_i^T, households choose among the residential plans in $\underline{B}^0(T)$. Since w ranges from 0 to ∞, the slope of an indifference curve in the (G_i, P_i^T) plane ranges from 0 to ∞ as well, implying all plans in $\underline{B}^0(T)$ are chosen by some households with T. Q.E.D.

Proof of Proposition 1: (i) Obviously all non-moving residential plans chosen by households with the maximum T are in the set of chosen residential plans by all households. To confirm that only these non-moving plans are equilibrium ones, observe from (23) that, since P_i^T is increasing in T for moving plans and independent of T for non-moving plans, lowering T can eliminate but cannot add non-moving plans to $\underline{B}^0(T)$. It follows from Lemma 4 that no households with lower T than the maximum choose a non-moving residential plan not chosen by a household with the maximum T.

(ii) Let T_m denote the minimum T. Obviously all moving plans chosen by such households are in the equilibrium set of moving plans. To confirm only such moving plans are in the equilibrium set of all households, suppose household "2" with (w_2, T_2), $T_2 > T_m$, chooses a moving plan lk in equilibrium that is not chosen by any households with T_m. Consider household "1" with $(w_1, T_1) = (w_2 \frac{1-T_2}{1-T_m}, T_m)$. Note that $w_1 < w_2$. Households 1 and 2 obtain the same level of utility from all moving plans (by (22) – (23)). Household 1 obtains lower utility from all non-moving plans than does household 2, since household 1 has lower wealth (and moving costs are irrelevant). But then household 1 would share household 2's preference for moving plan lk, a contradiction. Q.E.D.

We now show that equilibrium is characterized by a conditional wealth stratification property. Let $J_e \leq J^2$ denote the number of residential plans chosen by any household.[20] Number these plans $1, 2, \ldots, J_e$ such that $G_1 < G_2 < ... < G_{J_e}$. We make the following assumption:

Assumption 9 *The maximum T prohibits moving in equilibrium for all wealth types.*

Then:

Lemma 5

The plan with $G = G_1$ corresponds to $lk = 11$ and the plan with $G = G_{J_e}$ corresponds to $lk = JJ$.

Proof of Lemma 5: The residential plans on the lower boundary of the convex hull of all feasible plans corresponds to just non-moving plans for any types with T that will never move in equilibrium. Plans $lk = 11$ and $lk = JJ$ are the endpoints of the lower boundary of the convex hull for all of these types. The result then follows from Assumption 7 and Lemma 4. Q.E.D.

A simple property of equilibrium residential plans is that there will be "ascending bundles" conditional on type.

[20] Later we show that $J_e < J^2$ under reasonable restrictions.

Lemma 6 *(Ascending bundles)*

Given residential plans chosen in equilibrium by household with T satisfying $G_i > G_j, P_i^T > P_j^T$.

Proof of Lemma 6: If $P_j^T \geq P_i^T$, then choice of plan j would contradict maximization of V^y (recall (22)). Q.E.D.

Note that the hierarchical ordering of residential plans is consistent across types T though the composite prices differ and the subset of the J_e plans chosen by different T types vary.

The conditional wealth stratification property is:

Proposition 2 *(Conditional wealth stratification)*

For given T, if $w_2 > w_1$ and household with wealth w_2 chooses plan with G_i and household with wealth w_1 chooses plan with G_j $(j \neq i)$, then $i > j$.

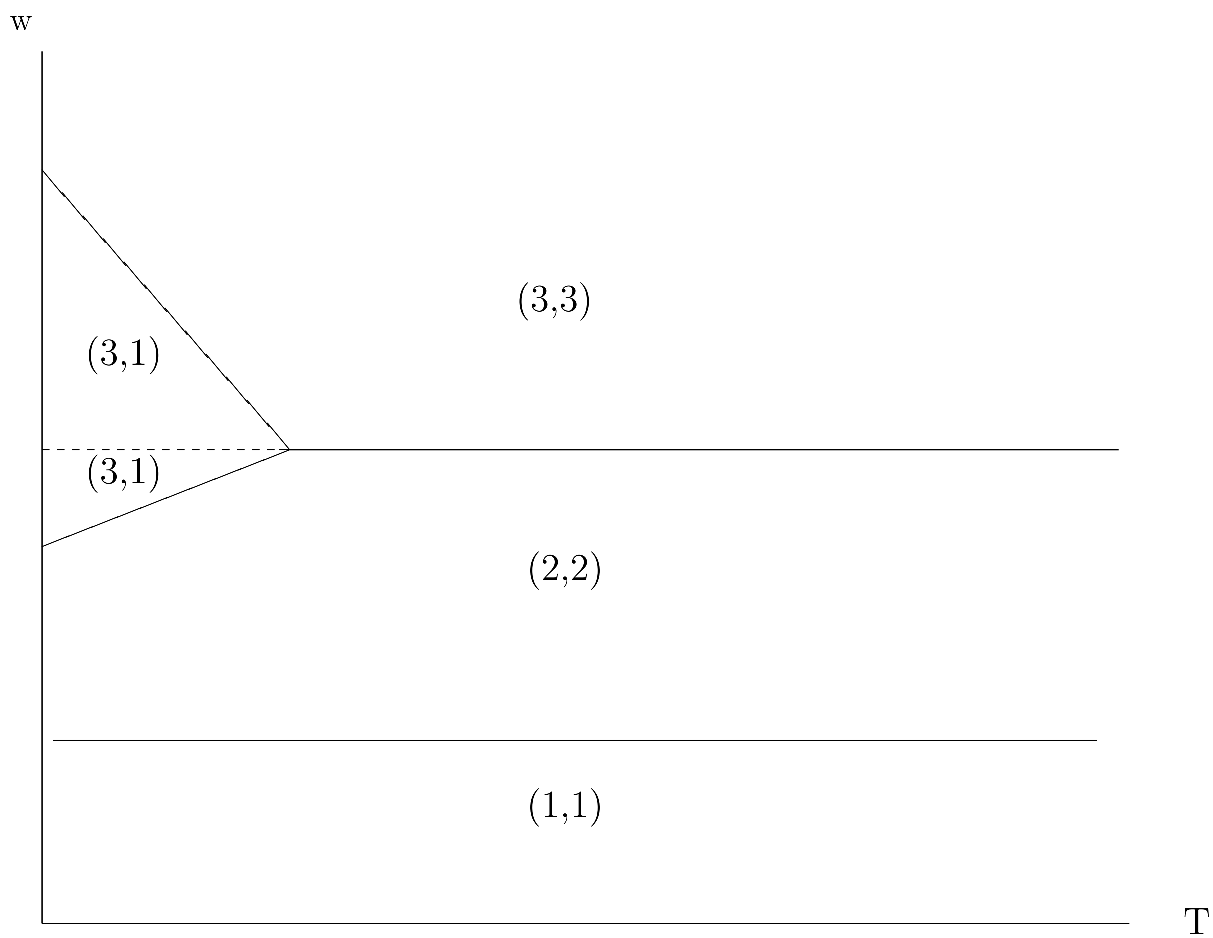
Proof of Proposition 2: Using that households chose residential plans to maximize V^y, wealth stratification follows from the ascending bundles property and SII. Q.E.D.

Figure 4 shows an example with $J = 3$ of the partition of young households by type (w, T) across residential plans kl. In this example, only one residential plan entailing moving arises in equilibrium, with some households choosing the highest g community while young $(k = 3)$ and then the lowest g community when old $(k = 1)$. There are three no-moving plans and thus $J_e = 4$.

While we have established Lemmas 2 through 6 and Propositions 2 and 3 for a stationary equilibrium, it is important to note that, with these propositions extend readily to non-stationary environments.

Another type of restriction on equilibrium residential plans derives from limits on the relative values of the parameters of the utility function. For the preference function used in our computational analysis introduced in equation (26), we provide conditions such that no household will move when old to a community with higher g. We assume that:

Figure 4:



Assumption 10 *The utility function satisfies the following parameter restrictions:*

$$\alpha_g(\alpha_h/\alpha_b)^{1/(\rho-1)} \;>\; \beta_g(\beta_h/\beta_b)^{1/(\rho-1)} \text{ and } \rho_a \geq \rho. \tag{30}$$

Proposition 3

No household will choose a community with higher (p, g) pair when old than when young in a stationary equilibrium.

Proof of Proposition 3: The proof is by contradiction, so suppose a household makes such a choice. Then that choice solves the program:

$$\begin{aligned} \max_{h_k,b_k,h_l,b_l} \quad U \;&=\; [\frac{\alpha_g}{\rho_a}g_k^{\rho_a} + \frac{\beta_g}{\rho}g_l^{\rho}] + \frac{1}{\rho}[\alpha_h h_k^{\rho} + \alpha_b b_k^{\rho} + \beta_h h_l^{\rho} + \beta_b b_l^{\rho}] \\ s.t.\; w - mc \;&=\; p_k h_k + b_k + p_l h_l + b_l \end{aligned} \tag{31}$$

with $(p_k, g_k) < (p_l, g_l)$. Let:

$$\begin{aligned} L^* \;\equiv\; & [\frac{\alpha_g}{\rho_a}g_k^{\rho_a} + \frac{\beta_g}{\rho}g_l^{\rho}] + \frac{1}{\rho}[\alpha_h h_k^{\rho} + \alpha_b b_k^{\rho} + \beta_h h_l^{\rho} + \beta_b b_l^{\rho}] \;+ \\ & \lambda[w - mc \;-\; p_k h_k - b_k - p_l h_l - b_l] \end{aligned} \tag{32}$$

denote the Lagrangian function at the household's optimum, where λ denotes the multiplier on the budget constraint. Thus, $V_{kl}^y(p_k, g_k, p_l, g_l) \;\equiv\; L^*(p_k, g_k, p_l, g_l)$. Using the latter and (32), compute, respectively, slopes of the indifference curves over (p, g) pairs while young and (p, g) pairs while old:

$$\left.\frac{dp_k}{dg_k}\right|_{V_{kl}^y=const.} = -\frac{\partial V_{kl}^y/\partial g_k}{\partial V_{kl}^y/\partial p_k} \;=\; -\frac{\partial L^*/\partial g_k}{\partial L^*/\partial p_k} \;=\; \frac{\alpha_g g_k^{\rho_a - 1}}{\lambda h_k}; \tag{33}$$

and

$$\left.\frac{dp_l}{dg_l}\right|_{V_{kl}^y=const.} = -\frac{\partial V_{kl}^y/\partial g_l}{\partial V_{kl}^y/\partial p_l} \;=\; -\frac{\partial L^*/\partial g_l}{\partial L^*/\partial p_l} \;=\; \frac{\beta_g g_l^{\rho - 1}}{\lambda h_l}; \tag{34}$$

where the last equality in each of (33) and (34) uses the Envelope Theorem. Using the first-order conditions from (31), one obtains:

$$h_k \;=\; \frac{w - mc}{z_{kl}}\left(\frac{\alpha_b}{\alpha_h}p_k\right)^{1/(\rho-1)} \tag{35}$$

and

$$h_l \;=\; \frac{w-mc}{z_{kl}}\left(\frac{\beta_b}{\beta_h}p_l\right)^{1/(\rho-1)}. \tag{36}$$

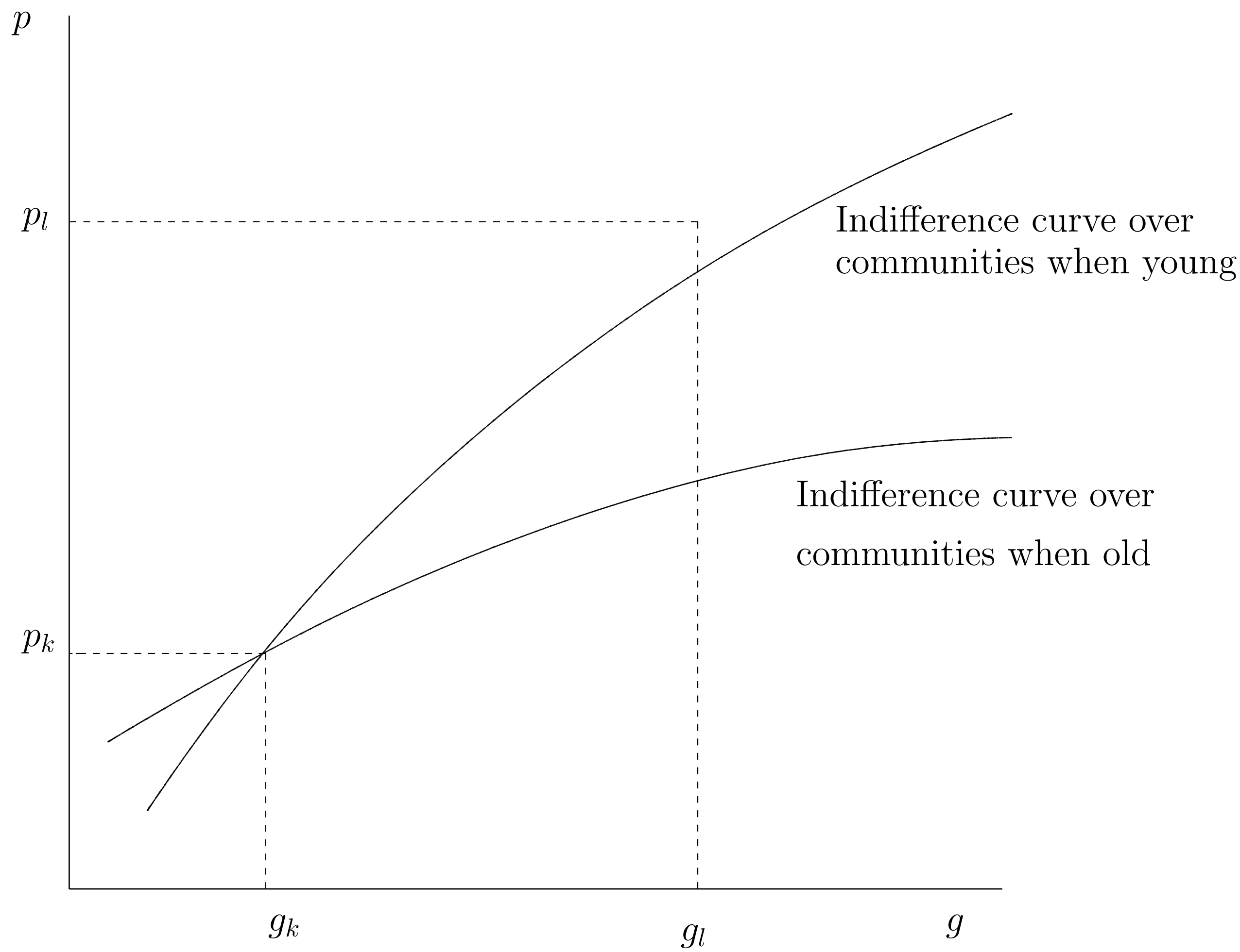
Substituting (36) into (34) and (35) into (33) and evaluating slopes at a common (p,g) point, one finds that the indifference curve over (p,g) pairs while young are everywhere steeper than the indifference curve over (p,g) pairs while old if $\alpha_g(\alpha_h/\alpha_b)^{1/(\rho-1)}g^{\rho_a-\rho} \;>\; \beta_g(\beta_h/\beta_b)^{1/(\rho-1)}$. This condition holds under Assumption 8. In a stationary equilibrium, the (p,g) pairs available in each period of life are the same. The steeper curve in Figure 5 shows the indifference curve of the young household that chooses community k while young given community l is available (with $(p_k,g_k) < (p_l,g_l)$). This curve must pass below the point (p_l,g_l) as shown, or the household would prefer community l while young. (The fact that the household would save moving costs by choosing l while young, while the indifference curves assume moving costs are paid, only reinforces the claim.) The flatter indifference curve shows that of the household through (p_h,g_h) when old, which implies the household would prefer to choose community l when old while paying moving costs. The fact that the household would not have to pay moving costs (since it resides initially in community l) implies a stronger yet preference for community l when old, hence a contradiction. Q.E.D.

The willingness to pay a higher housing price to live in a community with higher g increases with the coefficient on g in the period utility function and decreases with the coefficient on housing. While the presence of children when young indicates that both $\alpha_g > \beta_g$ and $\alpha_h > \beta_h$ are to be expected, the condition of Proposition 3 implies that the relatively stronger preference for g when young outweighs the relatively stronger preference for housing so that moving to a higher (p,g) community when old would not result.

5 Quantitative Analysis

Equilibria of this model can only be computed numerically. We next turn to the quantitative part of the analysis. We first present an algorithm that can be used to compute equilibria. To implement the algorithm, we must fully specify the model choosing functional forms and

Figure 5:


p
p_l
p_k
Indifference curve over
communities when young
Indifference curve over
communities when old
g_k
g_l
g

assigning parameter values. This section specifies a benchmark model and reports some of its quantitative properties.

5.1 Computation of Stationary Equilibria

Given a stationary distribution of wealth and moving costs, a stationary equilibrium in this model is determined fully by a vector $\{p_j, g_j, \tau_j\}_{j=1}^{J}$. Computing an equilibrium is, then, equivalent to finding a root to a system of $3 * J$ nonlinear equations. For each community, the three equations of interest are the housing market equilibrium in (11), the balanced budget requirement in (12), and the majority rule equilibrium requirement.

The full algorithm, therefore consists of an outer loop that searches over admissible distributions of wealth and moving costs and an inner loop that computes a stationary equilibrium holding the joint distribution fixed. The algorithm in the inner loop finds a root of $3 \times J$ dimensional system of linear equations. More specifically, the inner loop algorithm can be describes as follows:

1. Fix the joint distribution of wealth and moving costs.
2. Given a vector (p_j, τ_j, g_j) we can compute p_j^h from the identity $p_j = (1 + \tau_j)p_j^h$.
3. For each young household type (w, mc), we can compute the optimal residential choices for both time periods. Hence we can characterize household sorting across the J communities.
4. Given the residential decisions, we can characterize total housing demand, as well as total government revenues for each community.
5. Given p_j^h, we can compute housing supply for each community, and check whether the housing market clears in each community.
6. Given g_j, we can check whether the budget in each community is balanced.
7. For each young household and each old household living in community j we solve for the bliss point by searching over the set of feasible policies given by equation (13). Given the

bliss points of each voter and the fact that preferences in this model are single-peaked, it is straight-forward to compute the bliss point of the median voter. We can then check whether the bliss point of the median voter is g_j.

8. Given an equilibrium, simulate the model, update the joint distribution of wealth and moving costs and check for convergence.

5.2 Calibration

Our algorithm requires that we specify an initial distribution of household income. We approximate the income distribution using a log-normal. In 2005, U.S. mean and median incomes were \$ 63,344 and \$46,326. These imply that $\mu_{\ln y} = 10.743$ and $\sigma^2_{\ln y} = .626$. We treat each of the two periods of adult life in our model as "representative years." This implies that wealth equals twice annual income, $w = 2y$, and hence $\ln(w) = \ln(2) + \ln(y)$. This and the distribution of $\ln(y)$ imply $\ln(w) \sim N(11.436, .626)$. The mean and standard deviation of w are then \$112,638 and \$78,018. Calibrating wealth as twice annual income is convenient in then permitting us to interpret the equilibrium values of variables as typical annualized values for a young and an old household respectively.

To calibrate moving costs, we take moving costs as a share of income, z, to be log-normally distributed. Ideally, we would like to calibrate our moving cost distribution to data on intra-metropolitan migration. Unfortunately, publicly available Census data provide only migration data at the county or higher level geographic area. We can, however, make some inferences from the publicly available data. We would like to know what fraction of the population changes local jurisdiction in a metropolitan area as they transition from young to old age. We obtained an approximate mobility rate for individuals aged 55 to 74 as follows. We calculated the total number of individuals aged 55 to 74 who moved within the same county between 1995 and 2000 and divided the result by the population aged 55 to 64 in 2000. We find that 24% moved to a different home in the same county. [21] While this gives us a sense of the order

[21] This calculation assumes that those who did not relocate between age 55 to 64 would relocate between age 65 to 74 at the same rate as observed for the cohort of 65 to 74 year-olds in year 2000.

of magnitude of intra-metropolitan mobility of the older population, it does not tell us what proportion of this total is interjurisdictional mobility.

Additional evidence on the latter issue is obtained by using the data presented in the top panel of Figure 2. We define cohorts representative of our young and old households. For the former, we choose age 35 to 49 and, for the latter, age 55 to 69. The metropolitan population in the former cohort is 7% larger than the metropolitan population in the latter. Since our model presumes equal cohort sizes, we increase all community populations in the 55 to 69 cohort by 7%. We combined the 92 communities by income into four groups with population proportions approximately equal to those in our four-community equilibrium. Next, we calculated the ratio of old to young households in each of these groups. The results are in column 2 of Table 4. One might argue that households will typically be in the age range 30 to 44 when their first child enters school. Hence, as a second calculation, we treated the young as cohort 30 to 44. The results are in column 3 of Table 4. It is important to note that the the 30 to 44 cohort is 1980 substantially larger than the 35 to 49 cohort, the former being heavily influenced by the baby boom generation.[22] Thus, while we present it for completeness, the 3rd column is is of questionable value for calibration of our stationary equilibrium. One might also argue that households do not contemplate relocating until their children have completed college. Hence, as a third calculation we defined ages 60 to 74 as the old cohort, with results in column 4 of Table 4.

Comparable calculations for 1970 are presented in the second panel of Table 4. It is useful to note that the cross-sectional comparisons in Table 4 tend to understate the extent of mobility. For example if an old household moves from community 4 to 3, another moves from 3 to 2, and still another from 2 to 1, these three moves would generate a change in the cross-section that would be indistinguishable from movement by a single household from community 4 to community 1. Aggregating into four groups also may lead to understatement of mobility since a relocation from one community to another within a group would not be reflected in the aggregate numbers. The cross-sectional analysis may also understate mobility. As we noted

[22]It is for this reason that we focus on data for 1980. The data for 1990 and 2000 would be even more strongly impacted by the baby boom.

above, the cohort aged 35 to 49 is larger than the cohort aged 55 to 69. If this smaller cohort size reflects migration out of the metropolitan area by wealthier old households, our procedure might then overstate the extent of intra-metropolitan mobility. This latter concern is somewhat ameliorated by our previous calculating showing substantial intra-metropolitan relocation by older households. Despite these limitations, we view the calculations in Table 4 as providing useful guidance for calibrating our model.

Table 3: Cohort Ratios

1970				
1	2	3	4	5
Community	(55-69)/(35-49)	(55-69)/(30-44)	(60-74)/(35-49)	Model
1	1.128	1.025	1.155	1.267
2	1.184	1.181	1.231	1.108
3	0.926	0.956	0.910	0.922
4	0.742	0.804	0.683	0.603
1980				
1	2	3	4	5
Community	(55-69)/(35-49)	(55-69)/(30-44)	(60-74)/(35-49)	Model
1	1.226	1.201	1.281	1.267
2	1.058	1.051	1.051	1.108
3	0.785	0.784	0.749	0.922
4	0.911	0.945	0.894	0.603

We chose parameters of our moving cost distribution to generate an equilibrium with cohort ratios roughly in accord with those summarized in columns 2 through 4 of Table 4. With some experimentation, we settled on $\ln(z) \sim N(.00525, .0026)$ with a correlation of $\ln(z)$ and $\ln(w)$ equal to -.5. This yields the cohort ratios in column 5 of Table 4.

We calibrate parameters of the utility function as follows. We employ data from the Consumer Expenditure Survey (CEX) to obtain expenditure shares, treating the data from the

CEX as if they pertain to a single cohort moving through the life cycle.We take households aged 35-44 as typical of young households in our model, and households aged 65-74 as typical of old households who have relocated.[23] We then find that households spend 60% of lifetime wealth when young and 40% when old. In addition, we find that approximately one third of expenditures at each life stage are for housing services. In obtaining these housing expenditure shares, we adopt the broad CEX definition of housing services which includes shelter, utilities, and furnishings. We view a broad definition as appropriate since since the various ancillary expenses (utilities, furnishings) included in this total are likely to be approximately proportional to the size of the dwelling. These expenditure shares coupled with a value for ρ are sufficient to calibrate the utility function.

We choose $\rho = -.4$. This yields price elasticities between -.7 and -.8 for all goods. Price elasticities in this range would also be obtained for the publicly provided goods if they were instead provided privately. With remaining parameters as calibrated below, we obtain $\alpha_h = 0.13 : \beta_h = 0.07 : \alpha_g = 0.08 : \beta_g = 0.03 : \alpha_b = 0.97 : \beta_b = 0.53$.

While we have emphasized education as a key factor influencing household location choices, we include in local government expenditure other components that potentially influence location choices: expenditures for public safety (police and corrections), fire, sanitation, health, transportation, debt expense, and government administration. These totaled $901.8 billion in 2004. Personal income in 2004 was $9,731 billion, implying local government expenditure equal to of 8.7% of income. Of this total, $474 billion (52.5%) was for education. (Source: Statistical Abstract, 2008, Table 442. Local Governments–Expenditure and Debt by State: 2004)

In calibrating our baseline equilibrium, we assume that the state government provides a foundation grant financed by a flat-rate income tax on all households. In 2006, state and local government revenues for primary and secondary education were approximately equal. Thus, we choose the foundation grant to equalize state and local expenditures on education. As we noted above, education expenditures are 52.5% of local expenditures. With state funding equal to half half this amount in our calibrated equilibrium, we obtain a foundation grant of $2,600

[23]We opt for a somewhat older group of households in calibrating consumption than in calibrating mobility so as to obtain expenditure data typical of households who have completed relocation.

per young household.

We assume that the housing supply has constant elasticity θ and is given by

$$H^s_{jt} = [p^h_{jt}]^{\theta} \tag{37}$$

Note that this assumption implies that the the housing supply function is the same in in each community and set the supply elasticity, θ, equal to 3 which is a conservative estimate (Epple, Gordon, Sieg, 2008).

Our achievement function is:

$$a = (\gamma_q \frac{\alpha_g}{\rho_a} g_k^{\rho_a} + \gamma_w \ln w + \varepsilon) \tag{38}$$

with logarithm of wealth as an adult for a child with achievement a given by:

$$\ln w_c = a \tag{39}$$

To calibrate the achievement function, we proceed as follows. In stationary equilibrium, the distribution of wealth is invariant across generations. Hence the mean and variance of a must equal the mean and variance of $\ln(w)$. The correlation of parent and child earnings is approximately .4 (Solon, Zimmerman). Calibrating the effect of spending on educational outcomes is challenging due to the lack of agreement in the empirical literature about the effects, if any, of spending on outcomes. Fernandez and Rogerson (2003) adopt a utility function that also has education spending entering the utility function is the same way as our function above. They review the evidence, concluding that the exponent on expenditure is in the range from 0 to -3. The value $\rho = . - 4$ that we have chosen for the other components of utility falls within this range. Fernandez and Rogerson (1998) review evidence regarding the elasticity of earnings with respect to education spending, concluding that the evidence suggests a range of 0 to .2. We choose an elasticity, .1, in the middle of this range.

We choose parameters of the achievement function which, in equilibrium, satisfy the conditions in the preceding paragraph. This yields the following parameter values: $\rho_a = -.4$, $\mu_\varepsilon = 6.63$, $\sigma_\varepsilon = .567$, $\gamma_q = 4$, $\gamma_w = .41$.

5.3 Quantitative Properties of Equilibrium

– to be written –

6 Conclusions

In this paper, we provide a new framework for studying the life cycle locational choices of heterogeneous households and the associated dynamics of metropolitan areas. In the context of an overlapping generations framework, we frame the decision problem of households making private market decisions (choosing community of residence and housing consumption over the life cycle), and participating in the collective choice process determining the public education expenditure of their community at each stage of the life cycle. From this characterization of household decisions, we set forth conditions for market equilibrium in each community, and we demonstrate existence of voting equilibrium in all communities. We characterize potential life cycle residential plans of households and the associated patterns of stratification across communities. Finally, we illustrate properties of the model quantitatively, via computed equilibria for two- and three-community settings. These computational results illustrate patterns of household sorting and relocation over the life cycle, as well as the housing price, tax, and public spending levels that emerge within each of the jurisdictions.

Understand household and community dynamics is an important research area, and there is ample scope for future research. One interesting avenue for future research is to analyze the differences between families with and without children. In our model we have implicitly assumed that all families have children when young. However, a substantial fraction of households never have children. The life-cycle incentives of these individuals are different, since they do not have reason to pay the housing price premia to locate in areas with high quality public education. The presence of such households in the model can be expected to affect the age composition of communities as well as the outcomes that arise from voting over public good levels.

A second important generalization is introduction of peer effects. As demonstrated in

Calabrese, Epple, Romer and Sieg (2007), peer effects in education can have very substantial impact on the character of equilibrium outcomes. In particular, econometric evidence in their static framework reveals that income stratification and associated stratification of peers influences expenditure policies, with poorer jurisdictions taxing heavily in order to provide expenditures to compensate for schools that have relatively disadvantaged peer groups.This is clearly an important issue both from the perspective of providing a fuller characterization of equilibrium and from the perspective of policy analysis.

A third task is to extend the quantitative analysis to non-stationary environments. As we note in the paper, our results through Lemma 6 and Proposition 2 can be extended to non-stationary environments. These results in turn provide an important part of the foundation required for extending the computation of equilibrium to study transition dynamics.

A fourth important task for further work is to capture more fully the incentives affecting voting for public services, especially education. For example, households with grown children who plan to move have an incentive to support high provision of education to maintain housing demand and property values. These incentives depend on whether the household owns or rents, and on the household's beliefs about the way in which quality of public services impacts rental prices or the value of the home. Property owners have different preferences over public good provision than renters since owners are affected by capital gains or losses that may arise from changes in public policies. The key complication in such a generalization is in characterizing voting equilibrium. Owner-occupants who anticipate capital gains and losses when voting have been incorporated in static models (Epple and Romer, 1991; Calabrese, Epple, Romano, 2007), and those investigations reveal that ownership substantially affects voter incentives and equilibrium outcomes. Introducing ownership into our dynamic framework is a challenging but important task for future research.

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