Putting Your Ballot Where Your Mouth Is: An Analysis of Collective Choice with Communication*

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February 2004

Abstract

The goal of this paper is to explore the potential significance of communication to the design of institutions in collective decision making environments. We concentrate on decision panels that are comprised of a collection of agents having a joint task and possessing the ability to communicate at no cost. We observe that communication renders a wide range of voting rules equivalent with respect to the sequential equilibrium outcomes they produce. We show that this equivalence is robust in two dimensions: 1. communication simplicity - the equivalence is maintained even when the communication protocol is limited to one round of public deliberations, and 2. players' strategies - we identify two conditions for the equivalence to hold when players are confined to weakly undominated actions.

Journal of Economic Literature classification numbers: D71, D72, and D78.

Keywords: Communication, Collective Choice, Groups, Strategic Voting.

*We have benefitted from very helpful conversations with David Austen-Smith, Dirk Bergemann, Eddie Dekel, Timothy Feddersen, David Levine, Alessandro Lizzeri, Steven Matthews, Stephen Morris, Nicola Persico, and Bill Zame.

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"What raises us out of nature is the only thing whose nature we can know: language. Through its structure autonomy and responsibility are posited for us. Our first sentence expresses unequivocally the intention of universal and unconstrained consensus."

-Jurgen Habermas.

1 Introduction

Most group decision processes contain some form of communication phase before collective choices are made. For example, trial jurors converse before casting their votes, hiring committees convene before making their final decisions, and top management teams hold meetings before determining their firm's investment strategies.

The current paper explores the potential effects of communication in a variety of collective choice settings. In particular, we point to a wide range of environments in which communication renders a large class of voting institutions equivalent in terms of the sets of sequential equilibrium outcomes they generate.

As an example, consider first a collective choice of one out of two alternatives (such as a jury deciding to acquit or convict a defendant) and the corresponding class of threshold voting rules parametrized by $r = 1, \ldots, n$. Under voting rule r, the first alternative is chosen if and only if at least r agents vote in favor of it (in a jury setting, r could be thought of as the number of votes necessary for conviction). If communication is prohibited, for some information structures, different voting rules may generate different equilibrium outcomes (see Example 1). We start by noting that regardless of the structure of private information, when players can communicate before casting their votes, voting rules $2, \ldots, n-1$ are identical, in the sense that they all yield the same set of sequential equilibrium outcomes.

Indeed, take an outcome (i.e., a mapping from profiles of types to probability distributions over the two alternatives) implementable with communication under voting rule $r = 2, \ldots, n-1$. The revelation principle (see Myerson [1982], Forges [1986]) implies that this equilibrium outcome can be implemented with a communication device in which players truthfully reveal their types to an impartial mediator who disperses recommendations to all players. Each profile of recommended actions corresponds, through r, to one of the two social alternatives. Consider then a modification of this device which prescribes to each profile of private reports a unanimous recommendation to the players matching the social alternative that would have resulted in the original device. Since $1 < r < n$, any unilateral deviation will not alter the outcome, and so equilibrium incentives are maintained. In particular, the modified device generates an implementable outcome coinciding with the one we started with. Moreover, since all recommendations are unanimous, this remains an equilibrium outcome for any voting rule $r' = 2, \ldots, n-1$. The equivalence of all intermediate threshold rules follows.

With voting rules 1 and n (unanimity) it is generally possible to implement only a subset of the outcomes that can be implemented with the "intermediate" voting rules $r = 2, .., n-1$. Intuitively, consider first the unanimity rule (all jurors need to choose conviction for the defendant to be convicted). Any outcome generated with unanimity can be implemented via a mediator who dispels unanimous recommendations as above. Just like all of the intermediate voting rules, when the recommendation is for everyone to vote for the second alternative (acquittal), no one player has an incentive to deviate, since her deviation cannot affect the final outcome. However, when the recommendation is to cast a vote for the first alternative (conviction), a unilateral deviation can in fact alter the final decision under unanimity, and an additional constraint needs to be satisfied for players to obey such a recommendation. This supplementary condition identifies outcomes corresponding to unanimity as a subset of the outcomes generated by any of the intermediate voting rules. Similar intuition holds for the inclusion of outcomes generated by voting rule $r = 1$.

While Section 2 presents the general model and formalizes the intuitive equivalence result described above, the remainder of the paper provides an assortment of conditions under which the results holds when 1. the communication protocol is restricted in that either a mediator is unavailable or the protocol is constrained to only one round, or 2. players are confined to using weakly undominated strategies, or 3. there are more than two possible alternatives and players' action sets are smaller or richer than the set of alternatives.

Focusing on environments in which an impartial mediator is unavailable does not change the implications of the equivalence results. Furthermore, the equivalence of voting rules is maintained when limiting the number of rounds characterizing the communication protocols to one. In fact, restricting the communication protocol to only one round of unmediated public communication does not affect the set of equilibrium outcomes either.

The use of weakly dominated strategies may be important in particular environments. Naturally, if an agent has a preference for one alternative over the other, regardless of the vector of realized types, the modified device we suggest may lead that agent to be using a weakly dominated strategy at times. Namely, she might find herself voting (together with the rest of the agents) for the alternative she least prefers. We confront this issue in Section 4. We show that under rather weak restrictions on agents' preferences, the equivalence result holds even when agents use only weakly undominated strategies. Specifically, we require that for any agent: 1. every alternative is the preferred alternative for some plausible vector of others' types, no matter what her own type is; However, 2. the agent's preferences depend non-trivially on her own type. That is, there is always a situation in which two types of the same player prefer two different alternatives. Intuitively, these two stipulations on preferences make sure that agents are not blind partisans of any one alternative, thereby ruling cases as the one above. Moreover, private types matter to the extent that no agent would be optimizing by blindly voting for either alternative, thereby giving the power of choice to her peers.

As it turns out, the equivalence result extends directly to situations in which there are more than two alternatives. In Section 5 we put no restriction on the number of alternatives (other than finiteness), and consider all the voting structures that are comprised of actions and voting rules. The action sets can be arbitrary (e.g., selected alternatives, ranking orders of all of the alternatives, etc.). The equivalence result holds as long as the voting rules in question are non-dictatorial. That is, for any alternative there is a profile of feasible actions that yields that alternative via the voting rule, and is robust to any one-agent deviation.[1] The class of non-dictatorial voting rules contains most of the voting rules discussed in the literature (see, e.g. Cox [1987]). For example, all intermediate threshold voting rules (with two alternatives), as well as plurality rule, Borda rule, Condorcet winner's selection method (for more than two alternatives) are non-dictatorial. Section 5 illustrates the generalized equivalence result. Namely, for any fixed set of alternatives, the class of non-dictatorial voting structures is an equivalence class with respect to the set of sequential equilibrium outcomes it generates. Furthermore, dictatorial structures yield sets of sequential equilibrium outcomes that are subsets of the set corresponding to the non-dictatorial structures.

Our paper is connected with a few recent attempts to model strategic voting with communication. Austen-Smith and Feddersen [2002a] analyze a model in which a deliberative committee of three agents needs to choose one of two alternatives. Each player has private information on two dimensions: perfect information concerning her preferences and noisy information concerning the state of the world. Austen-Smith and Feddersen model deliberations as a one-round process in which all players simultaneously send public messages. The restriction to this particular form of communication allows them to consider an equilibrium concept (reminiscent of trembling hand perfection) stronger than the notion we use (sequential equilibrium in weakly undominated strategies). They show that when such deliberations precede the voting stage, majority rule induces more information sharing and fewer decision-making errors than unanimity. Coughlan [2000] adds a straw poll preceding the voting stage in a private information, two alternative environment. He

[1] The literature sometimes refers to this notion as "a structure with no veto power." We use the term "non-dictatorial" solely for the sake of expositional brevity.

shows that voters reveal their information truthfully if and only if their preferences are sufficiently close. Austen-Smith and Feddersen [2002b] look at a similar environment in which players can publicly send arbitrary messages before casting their votes. They provide conditions under which unanimity cannot induce full revelation of private information in equilibria comprised of weakly undominated strategies. Furthermore, if full revelation is possible under unanimity, then it is possible under any other rule. Doraszelski, Gerardi and Squintani [2003] study a two-player model with communication and voting. Preferences are heterogenous (not necessarily aligned) and private information. They show that some, but not all, information is transmitted in equilibrium, and that communication is beneficial.

In a similar vein, there has been some experimental work on voting with communication. Guarnaschelli, McKelvey, and Palfrey [2000] constructed an experiment replicating Coughlan's [2000] setup. They noted that during deliberations, voters tend to expose their private information but not to the full extent as predicted by Coughlan's [2000] results.

Blinder and Morgan [2000] conducted a conceptually different experiment in which groups were required to solve two problems - a statistical urn problem and a monetary policy puzzle. The groups could converse before casting their votes using either majority rule or unanimity. They found no significant difference in the decision lag when group decisions were made by majority rule relative to when they were made under a unanimity requirement.

The idea that communication may render a class of institutions equivalent appears in Matthews and Postlewaite [1989] who compare all two-person double auctions and show that they generate the same sets of equilibrium outcomes when the bidders can communicate before submitting their bids.

The importance of communication in political thought has been acknowledged extensively. Habermas was one of the first to lay foundations for a universal theory of pragmatism and direct attention to the importance of communication as foundations for social action. His theory served as a trigger for work on political decisions when communication is possible amongst candidates and electors (for a collection of essays overviewing Habermas' research program, see Habermas [1979]). In fact, the theory of deliberative democracy is a source of an abundance of work in political science on the effects of communication on how institutions function and, consequentially, how they should be designed (see Elster [1998] for a good review of the state of the art of the field). The research presented here provides an initial formal framework to study some of these issues.

The paper is structured as follows. Section 2 describes the general setup of collective choice with communication when there are only two alternatives and provides the comparison between different threshold voting rules. Section 3 restricts the set of allowed communication protocols. Section 4 specifies the robustness of the equivalence results. Section 5 provides the formulation for the generalized equivalence result pertaining to any set of alternatives. Section 6 concludes. Some of the technical analysis is relegated to the Appendix.

2 Deliberative Voting with Two Alternatives

A group of $n \geqslant 3$ individuals has to select one of two alternatives. We use the terminology of jury models and denote the alternatives by A (acquit) and C (convict). Each player i has a type t_i which is private information. We let T_i denote the set of types of player i, and assume that T_i is finite. $T = \prod_{i=1}^{n} T_i$ denotes the set of profiles of types, and p is the probability distribution over T. A player's utility depends on the profile of types and the chosen alternative. Formally, for each player i there exists a function $u_i : \{A, C\} \times T \to \mathbb{R}$.

The existing models of strategic voting assume that there exists an unknown state of the world (for example, the defendant is either guilty or innocent). Each player receives a signal which is correlated with the state. Conditional on the state of the world, signals are independent across players. Moreover, all players have a preference parameter, which may be either common knowledge or private information. The utility of a player is a function of the state of the world, her preference type, and the chosen alternative (see, for instance, Austen-Smith and Banks [1996] or Feddersen and Pesendorfer [1996, 1997]). We consider a more general model than the standard voting setup in that we do not impose any restrictions on the set of possible types. In particular, we allow for correlation of the signals across individuals.

The individuals select an alternative by voting. Each player can vote to acquit, a, or to convict, c.[2] We let $V_i = \{a, c\}$ denote the set of actions available to player i, and $V = \{a, c\}^n$ the set of action profiles. Under the voting rule $r = 1, \ldots, n$, the alternative C is selected if and only if r or more players vote to convict.

Given a voting rule r and a profile of votes v, we let $\psi_r(v)$ denote the group's decision. Formally, $\psi_r : V \rightarrow \{A, C\}$ is defined as follows:

$$\psi_r(v) = \begin{cases} A & \text{if } |\{i : v_i = c\}| < r, \\ C & \text{if } |\{i : v_i = c\}| \geqslant r. \end{cases}$$

The voting rule r defines the following Bayesian game G_r. Nature selects a profile of types in T according to the probability distribution p, then players learn their types, after which they vote simultaneously. If the profiles of types and actions are t and v, respectively, player i obtains $u_i(\psi_r(v), t)$.

In general, the set of equilibrium outcomes corresponding to the game G_r does not coincide with the set of equilibrium outcomes corresponding to the game $G_{r'}$, where $r \neq r'$, as the following example illustrates.

Example 1 *Assume* $n = 4$ *and let* $T = \{(t_1, t_2, t_3, t_4) \mid t_1 = t_2 = t^*, t_3, t_4 \in \{a, c\}\}$ *with* $p(t) = \frac{1}{4}$ *for all* $t \in T$. *Furthermore, assume that:*

$$\begin{array}{l} u_1(A, t) = u_2(A, t) = 1 \quad \forall t, \\ u_3(A, t^*, t^*, a, t_4) = 1, u_3(A, t^*, t^*, c, t_4) = 0 \quad \forall t_4, \\ u_4(A, t^*, t^*, t_3, a) = 1, u_4(A, t^*, t^*, t_3, c) = 0 \quad \forall t_3, \quad \text{and} \\ u_i(C, t) = 1 - u_i(A, t) \quad \forall i, t. \end{array}$$

That is, players 1 *and* 2 *always prefer* A *over* C *and the other two players prefer* A *or* C *with equal probabilities (independent of one another). For voting rule* $r = 2$, *there is an*

[2] The case in which players can also choose to abstain is covered by the analysis provided in Section 5.

equilibrium in which the two A partisans, players 1 and 2, always vote a and the two others vote according to their signal. The induced equilibrium outcome is $\theta_2 : T \to \{A, C\}$:

$$\begin{array}{rcl} t = (t^*, t^*, a, a) & \stackrel{\theta_2}{\to} & A \\ t = (t^*, t^*, a, c) & \stackrel{\theta_2}{\to} & A \\ t = (t^*, t^*, c, a) & \stackrel{\theta_2}{\to} & A \\ t = (t^*, t^*, c, c) & \stackrel{\theta_2}{\to} & C \end{array}.$$

Note that this equilibrium outcome cannot be replicated for the voting rule $r = 3$, since in that case players 1 and 2 would be best responding by choosing a whenever there is a positive probability of less than 3 voters for C. In fact, all of the equilibria of G_3 yield either the certain outcome of A or the certain outcome of C.

We are interested in comparing different voting rules when players are allowed to communicate before casting their votes. We therefore add cheap talk to the game G_r. A cheap talk extension of G_r is an extensive-form game in which the players, after learning their types, exchange messages. At the last stage of the game, the players vote. Payoffs depend on the players' types and votes, but not on their messages. For the moment, we also assume that there exists an impartial and exogenous mediator who helps the players communicate (for a general definition of cheap talk extensions to arbitrary games see Myerson [1991]).

A strategy profile σ of a cheap talk extension of G_r induces an outcome, i.e., a mapping γ_σ from the set of types T into the interval $[0, 1]$. $\gamma_\sigma(t)$ denotes the probability that the defendant is convicted when the profile of types is t (and the players adopt the strategy profile σ). We let Γ_r denote the set of outcomes induced by Bayesian Nash equilibria of cheap talk extensions of G_r. The notion of communication equilibrium (Myerson [1982], Forges [1986]) allows us to characterize the set Γ_r. A mapping μ from T into $\Delta(V)$, the set of probability distributions over V, is a communication equilibrium of G_r if and only if the following inequalities hold:[3]

[3] As usual, T_{-i} denotes the set of types of players other than i.

$$\sum_{t_{-i}\in T_{-i}} p(t_{-i}|t_i) \sum_{v\in V} \mu(v|t)\, u_i(\psi_r(v), t) \geqslant \sum_{t_{-i}\in T_{-i}} p(t_{-i}|t_i) \sum_{v\in V} \mu(v|t_{-i}, t_i')\, u_i(\psi_r(v_{-i}, \delta(v_i)), t)$$

$$\forall i = 1, \ldots, n, \quad \forall (t_i, t_i') \in T_i^2, \quad \forall \delta : \{a, c\} \rightarrow \{a, c\}.$$

The set Γ_r coincides with the set of outcomes induced by communication equilibria of G_r (Γ_r is therefore a convex polyhedron). Let V_r^C denote the set of profiles of votes that lead to conviction under the voting rule r. Formally, $V_r^C = \{v \in V : \psi_r(v) = C\}$. Then we have:

$$\Gamma_r = \{\gamma : T \rightarrow [0,1] \,|\, \exists \text{ a communication equilibrium } \mu \text{ of } G_r$$
$$\text{such that } \gamma(t) = \sum_{v\in V_r^C} \mu(v|t) \text{ for every } t \in T\}.$$

Example 2 *Consider the environment of Example 1. The mapping outcome θ_2 is also an equilibrium mapping of the extended game with communication. That is, $\theta_2 \in \Gamma_2$. Unlike the no communication case, when communication is possible, θ_2 is an equilibrium outcome when the voting rule is $r = 3$ as well, i.e. $\theta_2 \in \Gamma_3$. Indeed, a protocol can be constructed such that all players receive identical voting recommendations that replicate θ_2 as follows:*

Reports		***Recommendations***
(t^*, t^*, a, a)	$\rightarrow$	(a, a, a, a)
(t^*, t^*, a, c)	$\rightarrow$	(a, a, a, a)
(t^*, t^*, c, a)	$\rightarrow$	(a, a, a, a)
(t^*, t^*, c, a)	$\rightarrow$	(c, c, c, c)

In words, the players reveal their types to the mediator. If Players 3 and 4 both announce c, then the mediator recommends action c to all players. Otherwise, the mediator recommends the action a to everyone. This game admits a Bayesian Nash equilibrium in which all players reveal their types truthfully to the mediator and all players follow the mediator's recommendations. The associated outcome is θ_2.

We are now ready to generalize the above example and compare the sets Γ_r and $\Gamma_{r'}$ for two different voting rules r and r'. In Proposition 1 we show that, except for the voting rules $r = 1$ and $r = n$,[4] all other "intermediate" rules are equivalent. If players can communicate, every outcome that can be implemented with a voting rule $r \neq 1, n$ can also be implemented with a different voting rule $r' \neq 1, n$. Furthermore, by adopting an extreme voting rule ($r = 1$ or $r = n$), we cannot enlarge the set of equilibrium outcomes.

Proposition 1 $\Gamma_2 = \ldots = \Gamma_{n-1}$. *Moreover,* $\Gamma_1 \subseteq \Gamma_2$ *and* $\Gamma_n \subseteq \Gamma_2$, *and these inclusions may be strict.*

Proof. We first show that for $r = 1, \ldots, n$, if γ belongs to Γ_r then γ satisfies the following inequality:

$$\sum_{t_{-i} \in T_{-i}} p(t_{-i}|t_i) \left[\gamma(t) u_i(C,t) + (1-\gamma(t)) u_i(A,t)\right] \geqslant$$
$$\sum_{t_{-i} \in T_{-i}} p(t_{-i}|t_i) \left[\gamma(t_{-i}, t_i') u_i(C,t) + (1-\gamma(t_{-i}, t_i')) u_i(A,t)\right] \quad (1)$$
$$\forall i = 1, \ldots, n, \quad \forall (t_i, t_i') \in T_i^2.$$

If γ is in Γ_r, there exists a communication equilibrium μ of G_r that induces γ. For every player i and for every pair (t_i, t_i') we therefore have:

$$\sum_{t_{-i} \in T_{-i}} p(t_{-i}|t_i) \left[\gamma(t) u_i(C,t) + (1-\gamma(t)) u_i(A,t)\right] =$$
$$\sum_{t_{-i} \in T_{-i}} p(t_{-i}|t_i) [(\sum_{v \in V_r^C} \mu(v|t)) u_i(C,t) + (1 - \sum_{v \in V_r^C} \mu(v|t)) u_i(A,t)] =$$
$$\sum_{t_{-i} \in T_{-i}} p(t_{-i}|t_i) \sum_{v \in V} \mu(v|t) u_i(\psi_r(v), t) \geqslant \sum_{t_{-i} \in T_{-i}} p(t_{-i}|t_i) \sum_{v \in V} \mu(v|t_{-i}, t_i') u_i(\psi_r(v), t) =$$
$$\sum_{t_{-i} \in T_{-i}} p(t_{-i}|t_i) [(\sum_{v \in V_r^C} \mu(v|t_{-i}, t_i')) u_i(C,t) + (1 - \sum_{v \in V_r^C} \mu(v|t_{-i}, t_i')) u_i(A,t)] =$$
$$\sum_{t_{-i} \in T_{-i}} p(t_{-i}|t_i) \left[\gamma(t_{-i}, t_i') u_i(C,t) + (1-\gamma(t_{-i}, t_i')) u_i(A,t)\right],$$

[4] The voting rules $r = 1$ and $r = n$ are the only rules which require a unanimous consensus in order to adopt a certain alternative (A if $r = 1$, C if $r = n$).

where the inequality follows from the truth-telling constraint of the communication equilibrium μ.

Intuitively, consider the communication equilibrium μ which induces γ. Suppose that all players are obedient and that all players different from i are also sincere. Let t_{-i} be the profile of types of i's opponents. By reporting the truth, type t_i of i will induce a lottery between the alternatives A and C, with probabilities $1 - \gamma(t_{-i}, t_i)$ and $\gamma(t_{-i}, t_i)$, respectively. If type t_i lies and reports a different message t_i', then the alternative C will be selected with probability $\gamma(t_{-i}, t_i')$. Inequality (1) simply says that every player i has an incentive to report her type truthfully provided that her opponents are sincere and all players (including i) are obedient.

Consider now a voting rule $r = 2, \ldots, n-1$. We now demonstrate that if $\gamma : T \to [0,1]$ satisfies inequality (1), then γ belongs to Γ_r. Given γ, consider the following mapping $\tilde{\mu}$ from T into $\Delta(V)$:

$$\tilde{\mu}(v|t) = \begin{cases} \gamma(t) & \text{if } v = (c, \ldots, c), \\ 1 - \gamma(t) & \text{if } v = (a, \ldots, a), \\ 0 & \text{otherwise.} \end{cases}$$

Obviously, $\tilde{\mu}$ induces γ. It is easy to show that $\tilde{\mu}$ is a communication equilibrium of G_r. First of all, no player has an incentive to disobey the mediator's recommendation. Indeed, when the mediator follows $\tilde{\mu}$, she makes the same recommendation to all players. A player's vote cannot change the final outcome if all her opponents are obedient (notice that we are not considering $r = 1$ and $r = n$). Furthermore, the fact that γ satisfies inequality (1) implies that no player has an incentive to lie to the mediator when her opponents are sincere.

We conclude that $\Gamma_2, \ldots, \Gamma_{n-1}$ coincide with the set of the mappings from T into $[0,1]$ which satisfy inequality (1). Moreover, this set contains Γ_1 and Γ_n.

We now show that the inclusions $\Gamma_1 \subseteq \Gamma_2$ and $\Gamma_n \subseteq \Gamma_2$ may be strict. Indeed, consider first the unanimity rule. Any outcome in Γ_n can be implemented with a communication equilibrium in which the mediator sends the same recommendation to all players. This guarantees that no player has an incentive to disobey recommendation a. Of course, the action profile $(c, \ldots, c)$ is necessary to convict the defendant under the unanimity rule. If a player does not follow recommendation c the final decision will be A (in this case the player's message is irrelevant). Thus, an outcome $\gamma : T \to [0,1]$ satisfies the obedience constraints if and only if the following inequality holds:

$$\sum_{t_{-i} \in T_{-i}} p\left(t_{-i} | t_i\right) \left[u_i\left(C, t\right) - u_i\left(A, t\right)\right] \gamma\left(t\right) \geqslant 0 \qquad (2)$$
$$\forall i = 1, \ldots, n, \quad \forall t_i \in T_i.$$

We conclude that the set Γ_n consists of all mappings from T into $[0,1]$ which satisfy inequalities (1) and (2). Similarly, Γ_1 coincides with the set of mappings from T into $[0,1]$ which satisfy inequality (1) and the following inequality:

$$\sum_{t_{-i} \in T_{-i}} p\left(t_{-i} | t_i\right) \left[u_i\left(A, t\right) - u_i\left(C, t\right)\right] \left(1 - \gamma\left(t\right)\right) \geqslant 0 \qquad (3)$$
$$\forall i = 1, \ldots, n, \quad \forall t_i \in T_i.$$

Clearly, inequality (1) implies neither inequality (2) nor inequality (3). In particular, there may exist outcomes that belong to Γ_2 but do not belong to Γ_1 or to Γ_n. ■

Note that the above proof provides the set of conditions for a mapping $\gamma : T \to [0,1]$ to be in $\Gamma_1, \Gamma_2 = ... = \Gamma_{n-1}$, and Γ_n. In particular, for any specific environment, one can check if there is in fact a difference between the set of outcomes generated by unanimous rules and those generated by intermediate ones.

In what follows we try to identify the robustness of the equivalence observation. We start by considering a stronger equilibrium concept (sequential equilibrium), constraining the communication protocol to be simple, as well as independent of the existence of an impartial mediator. We continue by confining players to play weakly undominated strategies (a common assumption in much of the recent voting literature, see e.g., Feddersen and Pesendorfer [1996, 1997]) and identify the conditions under which the equivalence result holds. In Section 5 we consider generalized environments in which there are possibly more than two alternatives and players' action sets are arbitrary. We demonstrate the equivalence of a wide variety of institutions (such as Borda rule, the Condorcet winner method, and alternative voting).

3 Restricting the Communication Protocols

So far we have assumed that each player can communicate privately with a trustworthy mediator. However, in many instances an exogenous mediator is not available and players can only exchange messages with each other. In addition, there are cases, like jury deliberations, in which a player cannot communicate with a subset of players but has to send her message to *all* her opponents (public communication). We would like to investigate how these restrictions affect our results.

To derive Proposition 1 we have used the Bayesian Nash equilibrium concept. However, cheap talk extensions are extensive-form games, and in a Bayesian Nash equilibrium a player could behave irrationally off the equilibrium path. Another way to check the robustness of our result is to consider a stronger solution concept, such as sequential equilibrium.

Given a voting rule r, we define a cheap talk extension with public communication of G_r as follows. After learning their types, the players undergo a finite number of rounds of communication. In each round one or more individuals send a message to all players. At the end of the communication phase, the players cast their votes simultaneously and the chosen alternative is C (i.e., the defendant is convicted) if r or more players vote c (convict).

Notice that outcomes induced by sequential equilibria of cheap talk extension with public communication of G_r are included in Γ_r, $r = 1, \ldots, n$. In fact, any sequential equilibrium of a cheap talk extension is obviously a Bayesian Nash equilibrium, and any outcome that can be implemented without a mediator can be also implemented with a mediator. Proposition 2 illustrates that the opposite inclusion holds for $r = 2, \ldots, n-1$, even when the public communication phase is restricted to only one round.

Proposition 2 *For every* $r = 2, \ldots, n-1$, *any outcome in* Γ_r *can be implemented (in sequential equilibrium) with one round of public communication.*

Proof. Let γ be an outcome in Γ_r. Consider the following game. In stage 1, all players simultaneously send a public message. The set of messages of player $i = 1, 2$ is $T_i \times [0,1]$ (i.e., players 1 and 2 announce their types and a number in the unit interval). The set of messages of player $i = 3, \ldots, n$ is equal to T_i (i.e., players $3, \ldots, n$ announce their types). In stage 2 the players cast their votes.

Consider the following strategy profile. In stage 1, all players reveal their types truthfully. Furthermore, both player 1 and player 2 randomly select a number in the unit interval, according to the uniform distribution.

Finally, let us describe how the players vote in stage 2. Suppose that the vector of types announced in stage 1 is t. Let ω_i, $i = 1, 2$, denote the number announced by player i. Let $\chi : [0,1]^2 \to [0,1]$ denote the following function of ω_1 and ω_2 :

$$\chi(\omega_1, \omega_2) = \begin{cases} \omega_1 + \omega_2 & \text{if } \omega_1 + \omega_2 \leqslant 1 \\ \omega_1 + \omega_2 - 1 & \text{if } \omega_1 + \omega_2 > 1 \end{cases}.$$

If $\chi(\omega_1, \omega_2) \leqslant \gamma(t)$ all players vote to convict. If $\chi(\omega_1, \omega_2) > \gamma(t)$ all players vote to acquit.

Of course, this strategy profile induces the outcome γ. It is also easy to check that our strategy profile is a sequential equilibrium (consistent beliefs can be derived from any sequence of completely mixed strategy profiles converging to the equilibrium profile). Clearly, a player does not have a profitable deviation in stage 2 since her vote does not affect the final outcome. By announcing two numbers in the unit interval players 1 and 2 perform a jointly controlled lottery which determines how the players will vote. Since ω_1 is independent of ω_2 and uniformly distributed, $\chi(\omega_1, \omega_2)$ is also independent of ω_2 and uniformly distributed. Thus player 2 is indifferent between all numbers in $[0,1]$ (of course, the same argument can be applied to player 1). Finally, the vector of types announced by the players determines which lottery will be used in the second step of the game. Inequality (1) guarantees that each player has an incentive to be sincere provided that all her opponents behave likewise. ■

Notice that the specific communication protocol introduced in the proof of Proposition 2 could be used to implement the entire set of sequential equilibrium outcomes, regardless of the (intermediate) threshold voting rule. This result is reminiscent of the construction introduced by Forges [1990], in which one *universal mechanism* serves to implement the equilibrium outcomes of all noncooperative games with incomplete information and at least four players. This observation is important from the point of view of mechanism design. Indeed, consider a designer who aims at implementing a certain feasible outcome. To accomplish this, the designer should do two things. First, she should find a cheap talk extension with an equilibrium that induces the desired outcome. Second, the designer should induce the players to play that equilibrium. Our analysis shows that, without loss of generality, the designer can use the communication protocol described in the above proof and restrict attention to the problem of inducing players to play the desired equilibrium.

It follows from Proposition 2 that our result on the equivalence of the intermediate rules continues to hold even if we assume that a reliable mediator is not available and require the players to be sequentially rational. In the following section we provide mild restrictions on players' preferences that assure the equivalence result is preserved even when players use weakly undominated strategies.

4 Deliberative Voting with Weakly Undominated Strategies

Our equivalence result does not hold when there are some players who always prefer a certain alternative and the players do not use weakly dominated strategies. For example, consider a committee in which nine members always prefer the first alternative and three members always prefer the second one. Suppose that the members do not use weakly dominated strategies. In this case communication does not play any role and the committee will choose the first alternative under the voting rules $r = 1, \ldots, 9$ and the second alternative under the voting rules $r = 10, 11, 12$.

Certainly, if one is willing to entertain the idea that players care about their vote coinciding with the selected alternative, then our equivalence result holds directly with undominated strategies.[5] In this section we take a different route to ruling out weakly dominated strategies. Namely, we show that under a few weak assumptions on the preference and information structure, it is possible to implement every outcome in Γ_r $(r = 2, \ldots, n-1)$ even without weakly dominated strategies.

In games with incomplete information (as the ones we are considering) there are two different notions of dominance: *ex-ante dominance* and *interim dominance* (see Fudenberg and Tirole [1991], pages 226-229). Ex-ante domination requires that all types of a player have the same beliefs about the play of the other players. In contrast, interim domination allows different types to have different beliefs.

Formally, let $\sigma_i = (\sigma_i(t_i))_{t_i \in T_i}$ denote a (possibly mixed) strategy of player i, where $\sigma_i(t_i)$ is the strategy that player i chooses when her type is t_i. By a slight abuse of notation, we will extend the domain of u_i and let $u_i(\sigma_i(t_i), \sigma_{-i}(t_{-i}), t)$ denote the expected utility of player i when the profile of types is t and the players use the strategies $(\sigma_i(t_i), \sigma_{-i}(t_{-i}))$.

Definition 1

1. *The strategy σ_i is* ex-ante weakly *dominated for player i if there exists a strategy $\hat{\sigma}_i$ such that*

$$\sum_{t_i} p(t_i) \sum_{t_{-i}} p(t_{-i} \mid t_i) \left[u_i(\hat{\sigma}_i(t_i), \sigma_{-i}(t_{-i}), t) - u_i(\sigma_i(t_i), \sigma_{-i}(t_{-i}), t)\right] \geqslant 0,$$

for every strategy profile σ_{-i} and with at least one strict inequality.

[5] For example, this could be captured by each player j's utility taking the form w_j :

$$w_j(v_1, ..., v_n, t_1, ..., t_n) = u_j(\psi_r(v_1, ..., v_n), t_1, ..., t_n) + \varepsilon \mathbf{1}_{v_j}(\psi_r(v_1, ..., v_n))$$

where $\mathbf{1}_x(y) = \begin{cases} 1 & (x = a, y = A) \text{ or } (x = c, y = C) \\ 0 & \text{otherwise} \end{cases}$, and $\varepsilon > 0$, as small as desired.

2. *The strategy* σ_i *is* interim weakly *dominated if there exists a type* t_i *and a strategy* s_i *available to* t_i *such that*

$$\sum_{t_{-i}} p(t_{-i} \mid t_i) \left[u_i(s_i, \sigma_{-i}(t_{-i}), t) - u_i(\sigma_i(t_i), \sigma_{-i}(t_{-i}), t) \right] \geqslant 0.$$

for every strategy profile σ_{-i} *and with at least one strict inequality.*

Clearly, it is easier for a strategy to be interim weakly undominated than ex-ante weakly undominated. In what follows, we provide a set of conditions under which it is possible to implement every outcome in Γ_r $(r = 2, \ldots, n-1)$ with interim undominated strategies and with ex-ante undominated strategies.

We let $\tilde{\Gamma}_r$ and $\hat{\Gamma}_r$ denote the sets of outcomes induced by sequential equilibria of cheap talk extensions of G_r (with mediated communication) in which players do not play interim and ex-ante dominated strategies, respectively.

In order to characterize the sets $\tilde{\Gamma}_2, \ldots, \tilde{\Gamma}_{n-1}, \hat{\Gamma}_2, \ldots, \hat{\Gamma}_{n-1}$, we need to make the following assumptions.

A1 (Full Support) For every t in T, $p(t) > 0$.

A2 (Informational Smallness) For every $i = 1, \ldots, n$ and every $x = A, C$, there exists t_{-i}^x in T_{-i} such that $u_i(x, t_i, t_{-i}^x) > u_i(y, t_i, t_{-i}^x)$,for every $t_i \in T_i, y \neq x$.

A3 (Informational Significance) For every $i = 1, \ldots, n$ and every pair of type t_i, t_i' in T_i, there exists $t_{-i}^{(t_i, t_i')}$ in T_{-i} such that

$$u_i\left(x, t_i, t_{-i}^{(t_i, t_i')}\right) > u_i\left(y, t_i, t_{-i}^{(t_i, t_i')}\right),$$

$$u_i\left(y, t_i', t_{-i}^{(t_i, t_i')}\right) > u_i\left(x, t_i', t_{-i}^{(t_i, t_i')}\right),$$

where $x, y = A, C$ and $x \neq y$.

Assumption A2 guarantees that each player would benefit from the information available to her opponents. In some sense, this assumption implies that each player is "informationally small".[6] A player's information is not sufficient for her to conclude which alternative is optimal.

Intuitively, Assumption A2 is crucial for the equivalence result to hold with undominated strategies (of any type), since it allows us to rule out those situations in which some players always prefer one of the alternatives over the other, regardless of the realized types.

On the other hand, assumption A3 ensures that a player's information is never useless. There is always a situation in which two types of the same player prefer two different alternatives.

Assumption $A2$ and $A3$ are satisfied by most models studied in the literature. The following example illustrates the restrictions they impose on the standard jury setup.

Example 3 *Consider the standard jury setup with n jurors. There are two states, I (innocent) and G (guilty) with prior probabilities $P(I)$ and $P(G)$ respectively. There are two alternatives, A (acquit) and C (convict). Juror j's preferences are given by:*

$$\hat{u}_j(C \mid G) = u_j(A \mid I) = 0$$
$$\hat{u}_j(C \mid I) = -q_j$$
$$\hat{u}_j(A \mid G) = -(1-q_j)$$

where $q_j \in (0,1)$, juror j's preference parameter (capturing her concern for convicting the innocent relative to that for acquitting the innocent). Suppose each agent j observes a conditionally independent signal $t_j \in \{i, g\}$ of accuracy p. That is, $\Pr(t_j = i \mid I) = \Pr(t_j = g \mid G) = p$. Denote by $T = \{i, g\}^n$. Then each juror is identified by her expected utility

[6] Note that A2 is qualitatively different from the concept of informational smallness introduced by McLean and Postlewaite [2002] in that it is not probabilistic. In fact, for any i, the probability the realized types satisfy A2 can be arbitrarily close to 0.

conditional on the entire profile of types (i.e., signals) given by $u_j : \{A, C\} \times T \to \mathbb{R}$*, where*

$$\begin{aligned} u_j(A, t_1, ..., t_n) &= -(1 - q_j) \Pr(G \mid t_1, ..., t_n) \text{ and} \\ u_j(C, t_1, ..., t_n) &= -q_j \Pr(I \mid t_1, ..., t_n). \end{aligned}$$

Denote by $\beta(k)$ *the probability that the state is* C *(the defendant is guilty) when* k *out of the* n *signals are* g*,* $k = 0, ..., n$*. Assumptions A2 and A3 are satisfied as long as* $q_j \in (\beta(1), \beta(n-1))$ *for all* j*. Note that jurors' preferences may differ by quite a margin. For example, for a jury of size* $n = 12$*, prior* $P(I) = P(G) = \frac{1}{2}$*, and signal accuracy of* $p = \frac{2}{3}$*,* $\beta(1) = 0.000976$ *and* $\beta(n-1) = 0.999024$*.*

We are now ready to characterize the sets $\tilde{\Gamma}_2, \ldots, \tilde{\Gamma}_{n-1}, \hat{\Gamma}_2, \ldots, \hat{\Gamma}_{n-1}$ and show that our equivalence result holds even with weakly undominated strategies.

Proposition 3

1. *Suppose that assumptions A1 - A3 hold. Then for every* $r = 2, \ldots, n-1$*,* $\hat{\Gamma}_r = \Gamma_r$*.*

2. *Suppose that assumptions A1 and A2 hold. Then for every* $r = 2, \ldots, n-1$*,* $\tilde{\Gamma}_r = \Gamma_r$*.*

Proof: See Appendix.

Intuitively, assume that A1 and A2 hold. The proof of Proposition 3 specifies a set of messages for each player i of the form $M_i \times T_i$. The sets M_i are determined so that their intersection contains only one word m^*. Roughly speaking, for every player i there is a profile of her opponents's strategies that make her pivotal only when their types are either t^A_{-i} or t^C_{-i} (specified in A2). Moreover, for every player i there is a profile of strategies of her opponents such that i strictly prefers m^* to any other message in M_i. If everyone sends m^*, the mediator transmits a unanimous recommendation to play the action that the equilibrium outcome at hand associates with the vector of types. This specification assures that sending m^* and a truthful type report and then obeying the mediator's recommendation is indeed an undominated strategy, which yields the analyzed outcome. Similar intuition holds for part 2 of the proposition.

Remarks:

1. Assumption A1 is not necessary to derive our results. In fact, we could use a weaker assumption which requires that only some specific profiles of types have positive probability (namely, (t_i, t_{-i}^x), for all $i, t_i \in T_i$, and $x \in \{A, C\}$, for part 1 of Proposition 3, as well as $(t_i, t_{-i}^{(t_i,t_i')})$ and $(t_i', t_{-i}^{(t_i,t_i')})$ for all i, for part 2 of the proposition). To simplify notation, we assumed the environment has full support.

   Moreover, while Assumption A3 is easy to check and makes the proof of Proposition 3 very simple, it could also be weakened. Similar methods to those employed in the proof of Proposition 3 imply that if, in addition to A1 and A2, for every $i = 1, \ldots, n$ there exists a mapping $\gamma_i : T \to [0, 1]$ such that for every pair of types t_i and t_i', the following strict inequality holds:

   $$\sum_{t_{-i} \in T_{-i}} p(t_{-i}|t_i) [u_i(C, t) - u_i(A, t)] (\gamma_i(t) - \gamma_i(t_i', t_{-i})) > 0,$$

   then the equivalence result holds for ex-ante weakly undominated strategies.

2. Propositions 1 and 3 imply that under Assumptions A1 and A2, the sets of equilibrium outcomes corresponding to voting rules 1 and n are included in the sets corresponding to all intermediate voting rules, even when restricting the sets to equilibria comprised of interim weakly undominated strategies. Analogously, if assumptions A1-A3 are satisfied, this conclusion holds even when restricting the sets to equilibria comprised of ex-ante weakly undominated strategies.

In this section, we have assumed that players can communicate via an impartial mediator. This assumption was made only to simplify the exposition of the proofs in the

Appendix. Our equivalence result does not depend on this assumption and holds even when an impartial mediator is not available. In fact, by using a communication protocol similar to the one introduced in Gerardi [2003], it is possible to show that if there are at least five players and Assumptions A1-A3 hold, then any outcome in $\Gamma_r, r = 2, \ldots, n-1$, can be implemented with a sequential equilibrium in ex-ante weakly undominated strategies of a cheap talk extension with direct communication.

In order to test the robustness of the equivalence result, we have considered sequential equilibria in weakly undominated strategies. However, another route would be to adopt a stronger solution concept. Austen-Smith and Feddersen [2002] use a concept which has the flavor of perfect equilibrium. Specifically, Austen-Smith and Feddersen require that the equilibrium voting behavior of each player remains optimal even when there is a small probability that her opponents cast the wrong ballots. Unfortunately, the literature on games with communication has not yet developed enough of a technical apparatus to deal with perfect equilibria of arbitrary cheap talk extended games. In fact, the characterization of the outcomes induced by perfect equilibria of arbitrary cheap talk extensions is still an open question (the research frontier is probably Dhillon and Mertens [1996], who provide an answer only for two-person games with complete information). We are thus less optimistic about finding general results when concentrating on this particular equilibrium notion at this point in time.

5 Deliberative Voting with More than Two Alternatives

This section replicates the construction introduced in Section 2 for a general set of alternatives and a general set of allowable actions for each player.

Consider a group of $n \geqslant 3$ individuals that has to select one of $K \geqslant 2$ alternatives from $\mathcal{A} \equiv \{A_1, A_2, \ldots, A_K\}$. As before, each player i has a type t_i which is private information. We denote by T_i the set of player i's types, and assume that T_i is finite. We let $T = \prod_{i=1}^{n} T_i$

denote the set of type profiles. The prior probability distribution over T is denoted by p. A player's utility depends on the profile of types and the chosen alternative. Formally, for each player i there exists a function $u_i : \mathcal{A} \times T \rightarrow \mathbb{R}$.

A collective choice structure on $\{n, T, \mathcal{A}, \{u_i\}\}$ constitutes of two elements:

- The set of available actions. We denote by V_i the actions available to player i and by $V \equiv V_1 \times V_2 \times \ldots \times V_n$ the set of all possible action profiles.

- A voting rule, which is a mapping $\psi : V \rightarrow \Delta(\mathcal{A})$. Without loss of generality, we assume that $\bigcup_{v \in V} \text{supp } \psi(v) = \mathcal{A}$.

The collective choice structure (V, ψ) defines an analogous Bayesian game $G_{V,\psi}$ to that defined in Section 2. Nature selects a profile of types in T according to the probability distribution p, then players learn their types, after which they vote simultaneously. If the profiles of types and actions are t and v, respectively, player i obtains $\sum_{A_k \in \mathcal{A}} \psi(A_k|v)\, u_i(A_k, t)$.

In order to capture outcomes of the voting procedure with communication, we look at cheap talk extensions of $G_{V,\psi}$. Players exchange messages after learning their types, but before simultaneously casting their votes. We will present the case in which a reliable mediator is handy.[7]

A strategy profile σ of a cheap talk extension of $G_{V,\psi}$ induces an outcome, a mapping γ_σ from the set of types T into the simplex $\Delta(\mathcal{A})$. The vector $\gamma_\sigma(t)$ denotes the probability distribution over collective outcomes when the profile of types is t (and the players adopt the strategy profile σ). We let $\Gamma_{V,\psi}$ denote the set of outcomes induced by Bayesian Nash equilibria of cheap talk extensions of $G_{V,\psi}$.[8] A mapping μ from T into $\Delta(V)$, the set of probability distributions over V, is a communication equilibrium of $G_{V,\psi}$ if and only if the following inequalities hold:

[7] As before, this assumption is made solely for the sake of presentation simplicity, and could be dropped without affecting the reported results.

[8] As in previous sections, all of our results carry through even if we use the stronger notion of sequential equilibrium.

$$\sum_{t_{-i}\in T_{-i}} p\left(t_{-i}|t_i\right) \sum_{v\in V} \mu\left(v|t\right) \sum_{A_k\in \mathcal{A}} \psi\left(A_k|v\right) u_i\left(A_k, t\right) \geqslant$$

$$\sum_{t_{-i}\in T_{-i}} p\left(t_{-i}|t_i\right) \sum_{v\in V} \mu\left(v|t_{-i}, t_i'\right) \sum_{A_k\in \mathcal{A}} \psi\left(A_k|v_{-i}, \delta_i(v_i)\right) u_i\left(A_k, t\right)$$

$$\forall i = 1, \ldots, n, \quad \forall\left(t_i, t_i'\right) \in T_i^2, \quad \forall \delta_i : V_i \to V_i.$$

The set $\Gamma_{V,\psi}$, a polyhedron, coincides with the set of outcomes induced by communication equilibria of $G_{V,\psi}$.

$$\Gamma_{V,\psi} = \{\gamma : T \to \Delta(\mathcal{A}) | \exists \text{ a communication equilibrium } \mu \text{ of } G_{V,\psi}$$
$$\text{such that } \gamma\left(A_k|t\right) = \sum_{v\in V} \mu\left(v|t\right) \psi\left(A_k|v\right) \text{ for every } t \in T, \text{ for every } A_k \in \mathcal{A}\}.$$

In the case of two alternatives, a crucial aspect of the equivalence result was the ability to replicate any equilibrium with profiles that were robust to unilateral deviations (via unanimous profiles). Intuitively, in order to replicate the construction illustrating the equivalence of different voting rules, we will restrict ourselves to the class of *non-dictatorial* collective choice structures in which no one agent has the power to overturn a choice for all circumstances. Formally,

Definition 2 (Non-Dictatorial Structures) *The collective choice structure* (V, ψ) *is* non-dictatorial *if for every* A_k *in* $\mathcal{A}$*, there exists a profile* $v \in V$ *such that for any* $i = 1, \ldots, n$*, and any* $v_i' \in V_i, \psi(A_k|v_{-i}, v_i') = 1$.

That is, the collective choice structure is non-dictatorial if for every alternative, there is a profile of actions that would yield that alternative with probability 1, even if one of the committee members deviates. For example, all of the intermediate threshold voting rules discussed in Section 2 are non-dictatorial. The following examples identify most of the well-known multiple alternative voting rules as part of a non-dictatorial collective choice structure (see, e.g., Cox [1987] and references therein).

Examples (Scoring Rules, Alternative Voting, and Condorcet structures)

1. **Generalized Scoring Rules.** A scoring rule is characterized by a set of scores $\{\omega_k\}_{k=1}^K \subset \mathcal{R}$. Without loss of generality, we will suppose that there exists a $k^* > 1$ such that $\omega_1 > \omega_2 > \ldots > \omega_{k^*} \geqslant \omega_{k^*+1} \geqslant \ldots \geqslant \omega_K$. Each agent i's action set can be written as $V_i = \{(\alpha_1, \alpha_2, \ldots, \alpha_K) : (\alpha_1, \alpha_2, \ldots, \alpha_K)$ is a permutation of $(\omega_1, \omega_2, \ldots, \omega_K)\}$. So an agent reports an allocation of scores to the entire set of candidates. The candidate is then chosen according to:

$$\psi^{score}(A_l|v) = \begin{cases} \frac{1}{|\arg\max_k \sum_{i=1}^n v_i(k)|} & l \in \arg\max_k \sum_{i=1}^n v_i(k) \\ 0 & \text{otherwise} \end{cases}.$$

The scoring rule $\{\omega_k\}_{k=1}^K$ is non-dictatorial if, for example,

$$n\omega_1 - n \; div \; (K-1) \sum_{k=2}^K \omega_k - \sum_{k=2}^{n \bmod (K-1)} \omega_k > 2(\omega_1 - \omega_K)$$

where $n \; div \; k$ denotes the integer part of $\frac{n}{k}$ and $n \bmod k \equiv n - k \times (n \; div \; k)$, the remainder of n when divided by k.

The inequality ensures that for every player it is possible to allocate votes over opponents in a way that the top choice and the next highest scored one differ by more than the maximal individual score difference $\omega_1 - \omega_k$, so that no one individual can overturn the election results.

Indeed, when the above inequality holds, for any $k = 1, \ldots, K$, looking at a profile of actions in which $v_i(k) = \omega_1$ and allocating votes as follows:

$$\begin{aligned} v_1 &= (\omega_2, \ldots, \omega_k, \omega_1, \omega_{k+1}, \ldots, \omega_K); \\ v_2 &= (\omega_3, \ldots, \omega_{k+1}, \omega_1, \omega_{k+2}, \ldots, \omega_K, \omega_2); \\ v_3 &= (\omega_4, \ldots, \omega_{k+2}, \omega_1, \omega_{k+3}, \ldots, \omega_K, \omega_2, \omega_3); \\ &\vdots \end{aligned}$$

ensures robustness to unilateral deviations. Some of the scoring rules that are commonly used are in fact non-dictatorial:

Plurality. When $\omega_1 = 1$ and $\omega_2 = \ldots = \omega_K = 0$, the scoring rule is equivalent to the plurality rule. In particular these scores identify a non-dictatorial system. We will denote the equilibria outcomes of the plurality election with communication by $\Gamma^{Plurality}$.

Borda Rule. The scores $\omega_1 = K-1, \omega_2 = K-2, \ldots, \omega_K = 0$, correspond to the Borda method of electing an alternative. These scores satisfy the condition for a non-dictatorial structure as well, as long as $n \geqslant 4$. We will denote the equilibria outcomes of the Borda election with communication by Γ^{Borda}.

2. **Alternative Voting.**[9] In the Alternative Voting collective choice structure, each voter reports a strict rank order of the alternatives, that is

$$V_i = \{\succ \in \mathcal{A} \times \mathcal{A} : \text{for all } k \neq k', A_k \succ A_{k'} \text{ or } A_{k'} \succ A_k \text{ and } \succ \text{ is transitive}\}.$$

The voting rule $\psi^{AV}(v)$ is defined through a recursive process. Top preference alternatives are tallied. The candidate with lowest count is eliminated and the votes are reconsidered as restricted orderings over the remaining $K-1$ candidates. The process is repeated until one candidate has received half the votes as the most preferred. At each stage, a tie leads to a uniform randomization between the tied candidates.

Alternative Voting is non-dictatorial when $n \geqslant 3$ since for any $k = 1, \ldots, K$, the profile $v = (\succ_1, \succ_2, \ldots, \succ_n)$ that specifies A_k as the top ranked alternative for all agents, i.e., $A_k \succ_i A_{k'}$ for all i and $k' \neq k$ yields $\psi^{AV}(A_k | v_{-i}, v'_i) = 1$ for all $i, v'_i \in V_i$. We will denote the set of equilibria corresponding to Alternative Voting with communication by Γ^{AV}.

[9] Alternative Voting, commonly referred to as instant runoff voting, is rarely used in the U.S., but has actually been adopted as means of electing local candidates in San Francisco. In addition, it is used to elect the House of Representatives in Australia.

3. **Condorcet Winner.** In the Condorcet collective choice structure, each voter reports a strict rank order of the alternative as in Alternative Voting:

$$V_i = \{\succ \in \mathcal{A} \times \mathcal{A} : \text{for all } k \neq k', A_k \succ A_{k'} \text{ or } A_{k'} \succ A_k \text{ and } \succ \text{ is transitive}\}.$$

The voting rule $\psi^{Condorcet}(v)$ is defined as follows. For each pair of candidates, it is resolved how many agents preferred each candidate over the other by counting whether they were higher ranked in the reported preference ordering. If any candidate k is preferred to all other candidates, they are declared the winner and $\psi^{Condorcet}(A_k|v) = 1$. If there is no winner, a *top cycle* is determined. A top cycle is a subset of candidates such that each of the members will beat all candidates outside the top cycle in pair-wise competition, but not all of the candidates within the top cycle. There are several ways that the literature considers for choosing one candidate as the winner from the top cycle: by uniform randomization, by Alternative Voting within the top cycle, or by choosing the candidate who, in the pair-wise competition she does worst in, loses by the least amount (and randomize upon a tie). All these specifications yield a non-dictatorial structure. Indeed, for any $k = 1, \ldots, K$, a profile v ensuring k can be specified such that no run-off is necessary, even upon a unilateral deviation. Namely, since $n \geqslant 3$, $v = (\succ_1, \succ_2, \ldots, \succ_n)$ that specifies A_k as the top ranked alternative for all agents, i.e., $A_k \succ_i A_{k'}$ for all i and $k' \neq k$ implies that the option k gets at least a majority of the counts against any other candidate, even when one of the agents unilaterally deviates. In order for the definition to be complete, we will consider Alternative Voting to take place whenever a runoff vote is necessary and denote the corresponding set of equilibria, when communication is possible, by $\Gamma^{Condorcert}$.

We are now ready to compare the sets $\Gamma_{V,\psi}$ and $\Gamma_{V',\psi'}$ for two different collective choice structures (V, ψ) and (V', ψ'). Proposition 4 shows that all non-dictatorial collective choice structures are equivalent. If players can communicate, every outcome that can be implemented with a non-dictatorial structure (V, ψ) can also be implemented with a different non-dictatorial structure (V', ψ'). Furthermore, by adopting a dictatorial structure, we cannot enlarge the set of equilibrium outcomes.

Proposition 4 *For any non-dictatorial structures* (V, ψ) *and* (V', ψ'), $\Gamma_{V,\psi} = \Gamma_{V',\psi'}$. *Moreover, if* $(\tilde{V}, \tilde{\psi})$ *is dictatorial, then* $\Gamma_{\tilde{V},\tilde{\psi}} \subseteq \Gamma_{V,\psi}$.

The formal proof of Proposition 4 follows the lines of that of Proposition 1, and is thereby omitted. Intuitively, assume (V, ψ) and (V', ψ') are two non-dictatorial collective choice structures. Consider an outcome implementable with communication under a non-dictatorial collective choice structure (V, ψ). The revelation principle implies that this equilibrium outcome can be implemented with a communication device in which players truthfully reveal their types to an impartial mediator who disperses recommendations to all players. Each profile of recommended actions corresponds, through ψ, to one of the possible alternatives. Contemplate a modification of this mapping which prescribes to each profile of private reports a profile of recommendations in V' that corresponds, via ψ', to the social alternative that would have resulted in the original device corresponding to (V, ψ). Moreover, since (V', ψ') is non-dictatorial, the profile of recommendations can be assumed to be robust to unilateral deviations. In particular, the modified device generates an implementable outcome in $\Gamma_{V',\psi'}$ coinciding with the one we started with in $\Gamma_{V,\psi}$. Hence $\Gamma_{V,\psi} \subseteq \Gamma_{V',\psi'}$. The reverse inclusion follows in much the same way. Our generalized equivalence result then follows.

Since Plurality, Borda, Alternative Voting, and Condorcet collective choice structures are all non-dictatorial when $n \geqslant 4$, it follows from Proposition 4 that they all yield they same set of equilibrium outcomes once communication is introduced. Formally, if we assume that the committee is comprised of at least four members,

Corollary $\Gamma^{Plurality} = \Gamma^{Borda} = \Gamma^{AV} = \Gamma^{Condorcet}$.

Note that similar analysis to that provided in Section 2 would assure the generalized equivalence result to hold with unmediated communication and only one round of public communication. Moreover, mild restrictions on agents' preferences, as introduced in Section 4 would provide the equivalence result when players use weakly undominated strategies.

6 Conclusions

The main insight coming out of our current inquiry is that communication between individuals in collective choice scenarios is consequential to the resulting equilibrium outcomes. In particular, communication renders all non-dictatorial rules equivalent with respect to the sequential equilibrium outcomes they generate. This result continues to carry through even when the communication protocols are restricted, and, under a couple of mild assumptions, when players are confined to using weakly undominated strategies.

Our analysis should be viewed as an opening exploration of the effects of communication on the outcomes generated by collective choice institutions. As such, it leaves room for many avenues in which further research is needed, some of which we specify in what follows.

First, the equivalence propositions in the paper rely on the fact that players can send their messages simultaneously. This assumption may be reasonable if players can exchange written messages. Most of the time, however, members of a committee talk. It would therefore be interesting to see what happens if we restrict attention to sequential communication, and consider cheap talk extensions with only one sender in each round of communication. More generally, coming up with a working model of debates may be crucial in identifying the germane outcomes of the type of institutions we are considering.

Second, while we succeeded in identifying conditions for the equivalence result to hold when players are using weakly undominated strategies, it would be interesting to go even further and investigate how institutional outcomes change when players make mistakes, and realize their opponents may be doing so as well. One way to start such an endeavor would be to look at a stronger equilibrium concept, such as trembling hand perfect equilibrium. Unfortunately, as of yet, there are scarcely any general results in the literature characterizing trembling hand perfect equilibria in games with communication.

Third, our equivalence observation may prove particularly important for mechanism design pertaining to collective choice. Indeed, the plausibility of communication makes the problem of a social planner, or a principal choosing a committee, one of equilibrium selection, rather than pure institutional design via the voting rule itself. The analysis in this paper opens a broad set of questions related to the choice of committees and specification of communication protocols once deliberations are taken as part of the mechanism design problem. In Gerardi and Yariv [2002] we attempt to take a first stab at solving a mechanism design problem when agents communicate. More specifically, we consider an environment in which members of a group decide whether to acquire costly information or not (as in Persico [2002]), preceding a communication stage. We find that: 1. Groups producing the optimal collective decisions are bounded in size; and 2. The optimal incentive scheme in such an environment balances a trade-off between inducing players to acquire information and extracting the maximal amount of information from them. In particular, the optimal device may aggregate information suboptimally from a statistical point of view.

7 Appendix

Proof of Proposition 3: We first prove part 1 of the proposition. Part 2 will follow directly. Assume assumptions A1-A3 hold. For $r = 2, \ldots, n-1$, we need to show that $\Gamma_r \subseteq \hat{\Gamma}_r$. Fix $r = 2, \ldots, n-1$ and γ in Γ_r. Consider the following game with mediated communication. In stage 1 all players report their messages simultaneously to the mediator. Each player i sends a message that has two components. The first component is her type. The second component is an element of the set M_i defined by:

$$M_i = \{0, 1a, 1b, \ldots, (i-1)\,a, (i-1)\,b, (i+1)\,a, (i+1)\,b, \ldots, na, nb\}\,.$$

In other words, player i sends a message from the set $T_i \times M_i$. We let $m_i \in M_i$ denote the second component of the message sent by player i.

For each vector of reports, the mediator selects an action profile in $V = \{a, c\}^n$ according to some probability distribution (specified below) and informs each player only of her own action. Finally, players vote and an alternative is selected according to the voting rule r.

Consider the following specification of the mediator's choice of an action profile. We distinguish between three cases:

- Suppose that $m_j = ia$, for some $i = 1, \ldots, n$ and for each player $j \neq i$. Let t_{-i} denote the profile of types reported by the players different from i (t_{-i} is an element of T_{-i}). In this case, the mediator randomly selects an alternative, A or C, with equal probabilities. First, suppose that alternative A is selected. If $t_{-i} = t_{-i}^A$, the mediator recommends action a to player i and to the first $n - r$ players different from i (i.e., the mediator recommends a to $n - r + 1$ players) and action c to every other player. If $t_{-i} \neq t_{-i}^A$, the mediator recommends action a to all players. Suppose now that alternative C is selected. If $t_{-i} = t_{-i}^C$, the mediator recommends action c to player i and to the first $r - 1$ players different from i and action a to every other player. If $t_{-i} \neq t_{-i}^C$, the mediator recommends action c to all players.

- Suppose that for some $i = 1, \ldots, n$, $m_j = ib$, for all players $j \neq i$. Let t_i denote the type reported by player i and t_{-i} the profile of types reported by the players different from i. If $m_i = 0$, the mediator selects (with probability one) the alternative that is optimal for player i when the profile of types is (t_i, t_{-i}) (if both alternatives are optimal, the mediator selects A). Let $h(t_i, t_{-i})$ denote this alternative. If $m_i \neq 0$, the mediator selects the alternative different from $h(t_i, t_{-i})$. In any case, the mediator recommends to every player to vote in favor of the chosen alternative.

- Finally, in all other cases, the mediator selects alternatives A and C with probabilities $1-\gamma(t)$ and $\gamma(t)$, respectively (where t is the profile of types reported by the players). The mediator recommends to every player to vote in favor of the chosen alternative.

Consider now the following strategy for players $i = 1, \ldots, n$. Every type t_i reports the message $(t_i, 0)$ and always obeys the mediator's recommendation (even when she reports a message different from $(t_i, 0)$). Let σ_i^* denote this strategy. Of course, the strategy profile $(\sigma_1^*, \ldots, \sigma_n^*)$ induces the outcome γ. It is also easy to show that we can find a system of beliefs $(\beta_1^*, \ldots, \beta_n^*)$ such that the assessment $((\sigma_1^*, \ldots, \sigma_n^*), (\beta_1^*, \ldots, \beta_n^*))$ constitutes a sequential equilibrium of our game. It is enough to take a sequence of completely mixed strategy profiles which converges to $(\sigma_1^*, \ldots, \sigma_n^*)$ and such that for each player $i = 1, \ldots, n$, deviations to messages with the second component in the set $\{1a, \ldots, (i-1)a, (i+1)a, \ldots, na\}$ are much less likely than other deviations. This implies that in every information set a player assigns probability zero to the event that her vote is pivotal (we omit the details). Sequential rationality of the assessment $((\sigma_1^*, \ldots, \sigma_n^*), (\beta_1^*, \ldots, \beta_n^*))$ trivially follows.

To complete our proof, we need to show that for each player $i = 1, \ldots, n$, the strategy σ_i^* is ex-ante weakly undominated. Of course, a strategy for player i specifies for each type t_i the message that t_i sends and an action for every pair of recommendations and message (even the messages that were not sent). However, for our purposes it is enough to consider the reduced representation and restrict attention to the actions corresponding to the message actually sent.

Let $S(t_i)$ denote the set of pure strategies of type t_i in which t_i does not always obey the mediator's recommendation. Denote by $S'(t_i)$ the set of pure strategies of t_i in which t_i sends a message different from $(t_i, 0)$ and then obeys the mediator's recommendation. Consider any strategy σ_i of player i different from σ_i^*. At least one of the following two alternatives is true: (i) there exists a type $\hat{t}_i$ such that $\sigma_i(\hat{t}_i)$ assigns positive probability to a strategy in the set $S(\hat{t}_i)$; (ii) there exists a type $\hat{t}_i$ such that $\sigma_i(\hat{t}_i)$ assigns positive probability to a strategy in the set $S'(\hat{t}_i)$.

Start with case (i). Consider the strategy profile of players different from i in which every type t_j of player $j \neq i$ sends the message (t_j, ia) and then obeys the mediator's recommendation. It follows from assumption A2 that against this strategy profile, type $\hat{t}_i$ strictly prefers the strategy $\sigma_i^*(\hat{t}_i)$ to the strategy $\sigma_i(\hat{t}_i)$. Moreover, assumption A2 also implies that for every other type t_i the strategy $\sigma_i^*(t_i)$ is weakly better than the strategy

$\sigma_i(t_i)$.

For case (ii), consider the strategy profile in which every type t_j of player $j \neq i$ sends the message (t_j, ib) and then is obedient. Denote this profile by σ'_{-i} and consider type $\hat{t}_i$. It follows from assumption A3 that against σ'_{-i}, the strategy $\sigma_i^*(\hat{t}_i)$ is strictly better than any pure strategy in which $\hat{t}_i$ sends a message $(t_i, 0)$, where $t_i \neq \hat{t}_i$. Furthermore, assumption A2 implies that against σ'_{-i}, $\sigma_i^*(\hat{t}_i)$ is strictly better than any pure strategy in which $\hat{t}_i$ sends a message (t_i, m_i), where $m_i \neq 0$ and $t_i \in T_i$. Of course, against σ'_{-i} disobedience is not beneficial. Thus, $\hat{t}_i$ strictly prefers $\sigma_i^*(\hat{t}_i)$ to $\sigma_i(\hat{t}_i)$. Finally, assumptions A2 and A3 also imply that against σ'_{-i} any other type t_i weakly prefers $\sigma_i^*(t_i)$ to $\sigma_i(t_i)$. This concludes our proof of part 1. Note that the equilibria of the constructed game are played with strategies that are, in particular, not interim weakly dominated, thereby part 2 of the proposition is proven as well.

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