Microeconomic Theory I
Preliminary Examination
University of Pennsylvania

June 3, 2013

Instructions

This exam has **5** questions and a total of **100** points.

Each question is worth **20** points.

Answer each question in a **SEPARATE** exam book.

If you need to make additional assumptions, state them clearly.

Be concise.

Write clearly if you want credit.

GOOD LUCK!!

Question 1

Consider an economy with two agents and three commodities. The individual endowment vectors are

$$e^1 = (e^1_1, e^1_2, e^1_3) = (1, 1, 1), \quad e^2 = (0, 2, 0).$$

Consider three different specifications of utility functions:

a) Specification 1:

$$u^1(c_1, c_2, c_3) = \min(c_1, c_2, c_3), \quad u^2(c_1, c_2, c_3) = \min(2c_1, c_2, c_3)$$

b) Specification 2:

$$u^1(c_1, c_2, c_3) = \sqrt{c_1} + c_3, \quad u^2(c_1, c_2, c_3) = c_1 + c_2$$

c) Specification 3:

$$u^1(c_1, c_2, c_3) = c_1 + c_2, \quad u^2(c_1, c_2, c_3) = c_1 + c_2 + c_3$$

For each of the three specifications of utility functions, determine whether a Walrasian equilibrium exists. If one exists, compute it. If one does not exist, carefully explain why not.

If an equilibrium does not exist, either find a core allocation or explain why there is no core allocation. (Assume a coalition can block an allocation only if all agents in the coalition are strictly better off.)

Question 2

Consider our standard model of trading under uncertainty, with all consumption at date 1 after uncertainty is resolved. The model has one good, four states of the world denoted $s_1, \ldots, s_4$, and four assets denoted $a_1, \ldots, a_4$. The asset return matrix R is shown below (column j is the return vector of asset a_j) :

$$R \;=\; \begin{array}{cc} & \begin{array}{cccc} a_1 & a_2 & a_3 & a_4 \end{array} \\ & \left(\begin{array}{cccc} 1 & 0 & 1 & 0 \\ 1 & 3 & 0 & 1 \\ 1 & 2 & 0 & 1 \\ 1 & 1 & 0 & 1 \end{array} \right) \begin{array}{c} s_1 \\ s_2 \\ s_3 \\ s_4 \end{array} \end{array}$$

The first asset is riskless, paying 1 in every state. The second asset is a stock. Denote the state contingent spot prices as $p = (p_1, p_2, p_3, p_4)$, i.e., p_i is the price of the good in state s_i. Assume the vector of asset prices is $q =$ (1, 1.4, 0.4, 0.6).

a) Do these assets complete the market?

b) Consider a call option that provides the right, but not the obligation, to purchase the stock (asset 2) at a price of one after the state is realized. At what price must this option trade to avoid arbitrage?

c) Find spot prices that are consistent with the asset prices given.

d) Suppose that by investing k, the firm whose stock is asset 2 can change its production from $y = (0, 3, 2, 1)$ to $y + (0, 1, 0, 0)$ prior to the market opening for trade in the assets. (That is, it can increase the state contingent output in state 2 by one unit.) Assume that this does not change the spot prices in period 1. Show that all agents who own asset 2 prefer this change if $k < 0.2$.

Question 3

A seller has 1 unit of an indivisible good. There are two buyers, 1 and 2. Both have independent, private values for the good — buyer 1's value is drawn uniformly from the interval $V_1 = [0, 1]$, while buyer 2's value is drawn uniformly from the interval $V_2 = [0, 2]$.

The seller would like to implement a mechanism such that buyer 1 wins the good whenever her value is more than half as large as buyer 2's value, i.e.,

$$q_1(v_1, v_2) = \begin{cases} 1 & \text{if } 2v_1 \geq v_2, \\ 0 & \text{o.w.} \end{cases}$$
$$q_2(v_1, v_2) = 1 - q_1(v_1, v_2).$$

a) Describe transfer functions, $t_1 : V_1 \times V_2 \to \mathbb{R}$ and $t_2 : V_1 \times V_2 \to \mathbb{R}$, from the buyers to the seller, such that

   i. (q, t) is a dominant strategy incentive compatible mecahnism, and

   ii. A buyer with a value of 0 makes no payment, i.e.,

$$t_1(0, v_2) = t_2(v_1, 0) = 0 \quad \forall v_1 \in V_1,\ v_2 \in V_2.$$

   What is the expected revenue of the seller?

b) Given these allocation rules q_1, q_2, what are the interim allocation rules, i.e., give analytical formulas for $Q_1(v_1) = \mathbb{E}_{v_2}[q_1(v_1, v_2)]$ and $Q_2(v_2) = \mathbb{E}_{v_1}[q_2(v_1, v_2)]$.

c) State the Fundamental IC lemma.

d) Use the Fundamental IC lemma to give formulas for the interim payment rules $T_1 : V_1 \to \mathbb{R}$ and $T_2 : V_2 \to \mathbb{R}$, such that $T_1(0) = T_2(0) = 0$. What is the expected revenue of the seller in this mechanism?

Question 4

There is a public good that costs $k > 0$ to produce. There are n agents. Agent i's value for the public good, v_i, is a random variable drawn uniformly from $[0, \beta_i]$. To 'solve' the public good problem, a principal wants to use the following production rule:

1. Each agent i submits a bid b_i.
2. If $\sum_{i=1}^{n} b_i \geq k$, the good is produced.

In this setting,

a) Describe the pivot mechanism. Show that for any agent i, bidding her true value, $b_i = v_i$, is a dominant strategy.

b) Refer to an agent i as "pivotal" if

$$\sum_{j=1}^{n} b_j \geq k \quad \text{and} \quad \sum_{j \neq i} b_j < k.$$

Show that if all agents are pivotal, the sum of the payments is lower than the cost of the public good.

c) Describe the expected externality mechanism when $\beta_i = 1$ for every agent i. What are its properties (state but do not prove)?

Question 5

A competitive firm uses two inputs to produce one output according to a strictly increasing production function, $y = f(z_1, z_2)$. The input prices, w_1 and w_2, are constant in this problem, and hence we simplify notation by not writing them as arguments of functions.

In the "long-run," the firm chooses z_1 and z_2 to maximize profit. Assume this gives rise to C^2 input demand and supply functions, $z_1^L(p)$, $z_2^L(p)$, and $y^L(p)$, defined on $\mathbb{R}_{++}$.

In the "short-run," the firm chooses only z_2 to maximize profit, because the first input is fixed at some level $\bar{z}_1 > 0$. Assume this gives rise to a C^2 supply function, $y^S(\cdot, \bar{z}_1)$, defined on $\mathbb{R}_{++}$.

Suppose $\bar{p} > 0$ is an output price such that $z_1^L(\bar{p}) = \bar{z}_1$. Show that at $\bar{p}$, the long-run and short-run supply functions specify the same output. Show also that at $\bar{p}$, the price elasticity of supply is larger, at least weakly, for the long-run supply curve than it is for the short-run supply curve.