Political Mergers as Coalition Formation: Evidence from Japanese Municipal Amalgamations*

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Abstract

Political coalition formation games can describe the formation and dissolution of nations, as well as the creation of coalition governments, the establishment of political parties, and other similar phenomena. These games have been studied from a theoretical perspective, but the models have not been used extensively in empirical work. This paper presents a method of estimating political coalition formation models with many-player coalitions, and then applies this method to the recent *heisei* municipal amalgamations in Japan to estimate structural coefficients that describe the behaviour of municipalities. The method enables counterfactual analysis, which in the Japanese case shows that the national government could increase welfare via a counter-intuitive policy involving transfers to richer municipalities conditional on their participation in a merger.

Keywords: Coalition Formation, Municipal Mergers, Japan

JEL codes: C71, H77

1 Introduction

In recent years, issues surrounding political coalition formation have attracted considerable interest from both theorists and policy makers. For example, Alesina and Spolaore [1997] examine from a theoretical perspective what size of countries will form under different conditions. The formation and dissolution of countries can be seen as a political coalition formation game, with coalitions consisting of residents of a geographic are. This coalition formation game has obvious practical relevance: the dissolution of Yugoslavia, current conflicts

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in Georgia, and possible *de facto* reunification of the island of Cyprus all involve decisions about how many countries ought to exist and where borders should be drawn. Similarly, the formation of a government also corresponds to a political coalition formation game, with the political parties being the players. Moreover, the parties themselves can be viewed as resulting from an underlying political coalition formation game, this time with individual legislators as the players forming the coalitions. Once again the practical importance of understanding these types of political coalition formation games is borne out by recent news: in the 2007 Belgian national elections it was not obvious even long after the election which parties would form a coalition government, and in Canada in 2005 the defection of members of parliament temporarily saved the government from collapse.

In some of these cases it is possible to change the rules governing the coalition formation game, with new rules leading to a different and more efficient coalition structure. Any analysis of how changes in the rules will affect the coalition structure requires knowledge of the underlying structural parameters and an understanding of the process of coalition formation given various possible sets of rules. For example, a recent proposal in Canada was that members of parliament should be required to stand for a by-election if changing their party affiliation between general elections. Had this rule been in force during recent parliaments, different coalition structures might have resulted, leading to different governments and different policy outcomes. Similarly, different laws regarding how municipalities can cooperate to provide public goods, or how farmers can establish agricultural cooperatives, could lead to very different coalition structures with very different welfare implications. If so, then it is important to make sure that the "right" law is in place. In order to predict the results of different laws, however, it is first necessary to develop a model of the behaviour of the players participating in the coalition formation game, and then use this model to predict the changes in behaviour that would result from the imposition of a different set of laws. Although models of coalition formation date back at least to von Neumann and Morgenstern [1944], relatively few empirical papers have made use of such models, and in general these papers have not examined the effect of possible changes in the rules of the coalition formation game being studied.[1] There is no immediately obvious estimation strategy for these coalition formation models, since neither existence nor uniqueness of a stable coalition structure is guaranteed.

This paper presents a method of estimating the structural parameters of a political coalition formation model. The method is then applied to a recent set of Japanese municipal

[1] Desirable properties of some specific forms of coalition formation games, such as two-sided matching games, have led to extensive empirical study of those game forms [Roth, 2008]. Empirical research on general coalition formation models has also been limited by computational feasibility: the number of coalition structures increases exponentially with the number of players.

mergers (the *heisei daigappei*), where the national government fixed a set of transfer policies and individual municipalities chose, given these policies, what merger if any they wished to participate in. The parameters that determine municipal preferences over mergers are estimated, and these estimates are then used to predict the effect of alternative national government transfer policies. The *heisei* mergers are particularly attractive from a modelling perspective, as government policy allowed mergers to occur only during 1996-2006, and thus the resulting coalition structure can plausibly be treated as the outcome of a single period coalition formation game.[2] Furthermore, the mergers are of interest from a policy perspective, since due to efficiencies of scale the smaller municipalities spend over $10,000 per capita providing the same services that larger municipalities provide for slightly over $1,000, and almost all of this difference was being subsidized by the national government. Overall, then, the paper makes two contributions: first, the method of analysing political coalition formation games, and second, the specific results of this analysis in the Japanese case.

The methodological contribution consists of the use of simulated maximum likelihood estimation to obtain structural parameters describing players' preferences over coalitions when the observed coalition structure can be treated as the outcome of a cooperative form hedonic coalition formation game with non-transferable utility. Two ways of overcoming problems related to non-existence or mulitiplicity of stable coalition structures are presented. First, all players are assumed to have the same preferences over coalitions, resulting in the existence of a unique stable coalition structure [Farrell and Scotchmer, 1988].[3] A second and distinct strategy is to allow players' preferences over coalitions to differ, but restrict the types of blocking coalitions that can form. This guarantees existence but not uniqueness of a stable coalition structure [Ray and Vohra, 1997], and thus estimation requires an additional assumption regarding which one of the set of stable coalition structures is actually selected. The advantages of this approach, however, are that the distributional assumption required on idiosyncratic preferences is less restrictive and a wider variety of covariates can be included

[2] In general, a problem with applying political coalition formation models to observed data is that political coalitions once formed tend to persist, and changes that do occur are often separated by large time periods. The extremely high cost of any realignment means that the stability of existing borders does not provide much information, and it is not clear what it means for there to be a "stable" coalition structure, if changes to this structure occur over time at a slow but constant rate. The Japanese data used in this paper mostly avoids this problem. The fiscal crisis of the 1990s precipitated such significant changes in intergovernmental transfers that in many cases the old municipal borders were effectively untenable, thus leading to a very large number of mergers during the window when mergers were allowed. Furthermore, during the 1970-1995 period, national policy had made municipal mergers extremely unattractive, and thus boundaries remained effectively unchanged even though demographic changes were rendering these boundaries increasingly inefficient.

[3] More specifically, players differ only in that different potential coalitions contain different players, and the players may have different baseline utility levels. This technique has been used previously by Gordon and Knight [2006].

in the specification.

This method is then applied to the case of Japanese municipal mergers. Following Alesina and Spolaore [1997], there are economies of scale in the production of public goods, but also benefits to having local policies specifically tailored to match local preferences. This tradeoff creates an optimal size for municipalities, but pre-existing borders may not create municipalities of this size. These preferences over municipal characteristics imply preferences over coalitions, and the parameters determining these preferences are estimated by applying the method just described to data on the mergers that actually occurred, with the functional form and some parameters for the cost of providing services derived from existing national government estimates. Geographical features of the data allow the set of possible coalitions to be reduced to the point where the model is computationally tractable.

The estimated parameters show that, as expected, municipalities prefer to be in coalitions that offer higher levels of public goods and have lower population. There are some differences between the two methods of estimation: the estimated magnitude of the aversion to amalgamation is higher when all municipalities are assumed to have the same preferences over coalitions, and the preference for high income fellow residents lower. The methods of estimation yield estimates for the optimal population of municipalities of 75,000 or 150,000. Recent Japanese estimates place the lower bound on the efficient population of a municipality at 100,000 [Ministry of Internal Affairs 2003] or 120,000 [Hayashi 2002] and the ability of the model to predict an optimal size close to this, despite using a very different technique, suggests that the magnitudes of the estimated coefficients are reasonable. If, across Japan, the average level of spending on municipal services is optimal, then a resident is willing to pay about 0.5% of income to cut the population of their municipality in half.

In non-transferable utility coalition formation games, there are often coalitions that, if formed, would increase the utility of some players by large amounts, but these coalitions do not form because some other participants in the coalition would end up with slightly lower utility. Thus, national government intervention could lead to different and better coalition structures forming. The structural parameters that have been estimated are used to examine the effects of two counterfactual policies. First, the possibility of national government enforcement of transfers is considered, where the national government allows decentralized negotiations over these transfers to take place between municipalities. In this case, where the game is converted into a transferable-utility game, the outcome depends on the bargaining power of different types of municipalities. While this policy increases the number mergers that occur, it also leads to potentially very large transfers from poor municipalities to richer ones, and the exact amount of the transfers cannot be known in advance without knowing the bargaining method by which municipalities divide the benefits of a merger. Even under the most optimistic assumptions regarding bargaining power, the poorest municipalities end up worse off than under the original policy.

Next, an alternative is considered where the national government provides a financial incentive for municipalities to participate in mergers. This policy results in higher utility for both poor and rich municipalities. In fact, even if incentives to participate in mergers are only offered to richer municipalities, a budget-balanced conditional transfer policy results in higher utility, equivalent on average to an increase in income of 0.3%, at the 5th through 95th percentiles. This result is somewhat counter-intuitive as the problem the national government was attempting to solve was the high cost of supporting small, poor municipalities. The result is consistent with theory, however, since a regressive conditional transfer – taxing everyone and transferring money to the residents of richer municipalities that participate in mergers – provides an incentive for richer municipalities to merge with their neighbours, who then benefit from higher levels of public goods. The very poorest municipalities, however, consisting of approximately 5% of the population, are made slightly worse off by this scheme, since they are never considered as potential merger partners by the richer municipalities. The richest municipalities, consisting of a similar fraction of the population, are similarly made worse off because they never participate in mergers but pay the additional tax. Providing an incentive to richer municipalities mimics the transfers that the municipalities themselves offered in the transferable utility game, but with amounts that are not as large. Thus, fewer mergers occur, but the poorer municipalities are on average better off than in the transferable utility case because they do not have to pay huge transfers to richer municipalities.[4]

The major contribution of this paper is to develop an empirical framework for the estimation of political coalition formation models that takes into account theoretical characteristics of solutions and allows for the analysis of counterfactual policies. This is an advance over previous techniques: the closest related work is Gordon and Knight [2006], which uses a method of moments estimator to examine mergers between pairs of school districts. The maximum likelihood estimator presented below, however, has the advantage that the stable coalition structure does not need to be computed repeatedly as part of the estimation process. This makes it possible to consider coalitions much larger than size 2. Another closely related paper is Brasington [2003], which uses a maximum likelihood estimator, but considers each potential pairwise merger in isolation from other potential mergers.[5] In addi-

[4]This result is related to the theory presented by Armstrong and Vickers [2007] regarding anti-trust regulation of corporate mergers; however, in the Armstrong and Vickers model, the cost of allowing certain mergers to happen is that other, better, mergers do not occur, whereas in the model presented below, the primary cost of having more mergers occur is the ever larger transfers to richer municipalities that must be provided. The transferable utility case thus has "too many" mergers, at least for some social welfare functions.

[5]The restriction to pairwise mergers follows from the use of the Poirier [1980] bivariate probit model. In

tion, other recent empirical political coalition formation papers, such as Alesina et al. [2004], focus on describing patterns that are observed in political boundaries, while this paper estimates structural parameters and predicts how counterfactual policies would change the set of boundaries forming.[6] With suitable modifications, the method used in this paper could be applied to other types of coalition formation games, possibly in other fields as well as in political economy.

The rest of the paper has the following structure. The general estimation strategy is presented in Section 2, including both the version imposing a restriction on the form of players' preferences and the version using instead a restriction on the types of blocking coalitions. The use of this strategy in the Japanese case is then described in Section 3, and potential alternative national government policies are analysed using counterfactual simulations in Section 4.

2 Theory

Notation follows that of Banerjee et al. [2001] and Bogomolnaia and Jackson [2002]. Specifically, let N be the set of players, and $S \subset N$ a coalition of these players. Π is the set of all possible coalition structures, where a coalition structure $\pi \in \Pi$ is a set of coalitions $\{S_1, \ldots, S_K\}$ such that every player is in exactly one of these coalitions. Suppose that player $i \in N$ has preferences $\preceq_i$ defined over the set $\{S \subset N | i \in S\}$, with $\prec_i$ indicating a strict preference. The extension of these preferences to partitions is easy: if $\pi(i)$ is the coalition that municipality i belongs to in partition π, then $\pi \preceq_i \pi'$ if $\pi(i) \preceq_i \pi'(i)$. Let $\pi \prec_S \pi'$ for some coalition S if $\forall i \in S, \pi \preceq_i \pi'$ and at least one of these preferences is strict. The observed coalition structure is treated as the result of a "hedonic coalition formation game", where the payoff to each player depends only on the coalition to which it belongs, and not on what other coalitions occur. This is the "hedonic aspect" introduced by Dreze and Greenberg [1980], except without the possibility of transfers. The inability to negotiate transfers prevents some coalitions from forming:

models of the type used by Brasington and others, the probability that players 1 and 2 will form a coalition is unaffected by the other options that 1 or 2 might have. The method presented below and that used by Gordon and Knight appear to be the only ones that take into account that the presence of a player 3 and an attractive {1, 3} coalition may disrupt a {1, 2} coalition that would otherwise form.

[6]Brasington [2003], Alesina et al. [2004], and most of the other existing empirical studies of political mergers focus on American school districts. Miceli [1993], the earliest example yet found, examines the trade-off that Connecticut school districts faced between efficiencies of scale and locally optimal education quality. Alesina et al. [2004] use a much larger dataset, and examine the relationship between county-level heterogeneity and the number of school districts and other local jurisdictions. While the estimates in each of these papers imply a type of coalition formation game, they do not present an explicit coalition formation model.

Example 1. *Let* $N = \{1, 2\}$*, and* u_i *be a utility function describing the preferences of player* i *over coalitions, with*

$$u_1(\{1,2\}) = u_1(\{1\}) + \epsilon_1$$
$$u_2(\{1,2\}) = u_2(\{2\}) + \epsilon_2$$

If $\epsilon_1 > 0$*,* $\epsilon_2 < 0$*,* $|\epsilon_1| > |\epsilon_2|$*, then the stable coalition structure is* $\{\{1,2\}\}$ *if transfers are possible, but* $\{\{1\}, \{2\}\}$ *if they are prohibited.*

Ideally, given a set of preferences, there would exist a unique stable partition: First, the solution set is defined using the von Neumann and Morgenstern [1944] "stable set":

Definition 1. Π^{VNM} *is a stable set with respect to* $(\Pi, <)$ *for some binary operator* $<$ *if*

1. $\nexists \pi, \pi' \in \Pi^{VNM}$ *where* $\pi < \pi'$ *(Internal stability)*
2. $\forall \pi \notin \Pi^{VNM}, \exists \pi' \in \Pi^{VNM}$ *where* $\pi < \pi'$ *(External stability)*

The goal is to define $<$ in a way that is intuitively plausible yet at the same time guarantees that the stable set exists, but this turns out not to be trivial. Consider, for example, the following definition of $<$: $\pi < \pi'$ if $\exists S \in \pi'$ such that $\pi \prec_S \pi'$ and $\forall S' \in (\pi \setminus \pi'), (S' \setminus S) \in \pi'$ or is empty. Unfortunately, with this definition not only is a stable set not guaranteed to exist, but in general it is not possible to devise another plausible method of selecting a single partition as the solution of this type of coalition formation game [Barberà and Gerber, 2007]. The following "roommates problem" illustrates this point:

Example 2 (Gale and Shapley 1962)**.** *Suppose* $N = \{1, 2, 3\}$ *and preferences are*

$$\{1,2,3\} \prec_1 \{1\} \prec_1 \{1,3\} \prec_1 \{1,2\}$$
$$\{1,2,3\} \prec_2 \{2\} \prec_2 \{1,2\} \prec_2 \{2,3\}$$
$$\{1,2,3\} \prec_3 \{3\} \prec_3 \{2,3\} \prec_3 \{1,3\}$$

With these preferences, no stable partition exists.

Nevertheless, when the Japanese municipalities actually played a coalition formation game, an outcome did occur. The problem is then how to treat observed outcomes such as this one when attempting to estimate parameters. There are at least three ways to proceed: to move to a non-cooperative game structure, to restrict preferences, or to relax the requirements for stability.

A non-cooperative game is guaranteed to provide a set of equilibrium outcomes, but it is difficult to use in this case as no information is available about the way in which the municipalities actually negotiated, or who made what offers, and so forth. Thus, the specification of the rules of the game would be essentially arbitrary. If the equilibria did not depend on the rules, then the lack of information about the negotiation process would not be important, but it is fairly easy to see that in this sort of coalition formation game, different rules produce different outcomes. For example, if there are a finite number of periods in which a proposer can propose a coalition or coalition structure, then the probability with which various municipalities are selected to be the proposer will change the types of proposals made and accepted. Radically different parameter estimates could be obtained by using different probabilities of having a municipality selected as proposer, and there is no information available on what reasonable proposer weights would be, or even whether the proposer type framework is appropriate. Thus, non-cooperative form games will not be used as part of the estimation strategy. The other two potential solutions given above will be used, however. First, preferences will be restricted so as to ensure that a unique stable partition exists. Second, a more general utility function will be used, but certain types of deviations will not be allowed. This ensures the existence of a stable partition, but not its uniqueness, and so estimation becomes somewhat more complicated.

2.1 Restricted Preferences Approach

Consider the following restriction on the form of u_i, the utility that player i derives from a coalition:

$$u_i(S) = u(S) + \alpha_i$$
$$u(S) = v(X_S; \theta) + \epsilon_S$$

Here v is a function of characteristics X_S of S, taking parameters θ. The econometrician observes X_S and knows the functional form of v, and the objective is to estimate the parameters θ. The error term ϵ_S is of a known distribution. The important restriction here is that if $S \prec_i S'$ then $\forall j, S \prec_j S'$. That is, all agents have identical preferences over coalitions.

Theorem 1 (Farrell and Scotchmer 1988)**.** *If all agents have identical preferences over coalitions, a generically unique stable partition exists.*

Proof. The unique stable partition can be constructed as follows:

0. Let V^0 be the set of all potential coalitions, and start with $\pi^0 = \emptyset$ and $k = 0$

1. Find $S^k_{\max}$ such that $\forall S \in V^k$, $u(S) < u(S^k_{\max})$
2. Set $\pi^{k+1} = \pi^k \cup \{S^k_{\max}\}$ and $V^{k+1} = \{S | S \in V^k, S \cap S^k_{\max} = \emptyset\}$
3. If $V^{k+1} \neq \emptyset$, repeat from 1.

□

This restriction on the idiosyncratic error term is strong: it implies that all the unobserved characteristics of a coalition are enjoyed equally by all its members. For example, in the case of municipal mergers, it rules out the possibility that a large municipality merging with a smaller neighbour might take advantage of its dominance on the new amalgamated municipal council in order to geographically skew public spending. The major benefit of placing this restriction on the error term is that it guarantees uniqueness, and thus estimation does not require any assumption about an equilibrium selection rule.[7]

Suppose that partition π_0 is actually observed. The parameters θ can be estimated via simulated maximum likelihood. The likelihood of π_0 occurring is

$$\begin{aligned}\mathcal{L}(\pi_0 \text{ stable } |\theta) &= \int_\epsilon I(\pi_0 \text{ stable } |\theta, \epsilon) f_\epsilon(\epsilon) d\epsilon \\ &= \int_{\epsilon_0} P(\pi_0 \text{ stable } |\theta, \epsilon_0) f_{\epsilon_0}(\epsilon_0) d\epsilon_0\end{aligned}$$

where f is the PDF of the idiosyncratic shocks, and ϵ_0 denotes the vector $\{\epsilon_S | S \in \pi_0\}$.[8] This integral can be numerically approximated by taking a set E_0 of random draws of ϵ_0 and calculating

$$\frac{1}{|E_0|} \sum_{\epsilon_0 \in E_0} P(\pi_0 \text{ stable } |\theta, \epsilon_0)$$

[7] It may be possible to weaken the restrictions on the error term somewhat in the future by instead assuming the monotonic median voter property Acemoglu et al. [2008].

[8] More formally, define ϵ_1 as the ϵ shocks not in ϵ_0, and note that $f_\epsilon(\epsilon) = f_\epsilon(\epsilon_1, \epsilon_0) = f_{\epsilon_1|\epsilon_0}(\epsilon_1|\epsilon_0) f_{\epsilon_0}(\epsilon_0)$. Then rewrite

$$\begin{aligned}\mathcal{L}(\pi_0 \text{ stable } |\theta) &= \int_{\epsilon_0} \int_{\epsilon_1} I(\pi_0 \text{ stable } |\theta, \epsilon_1, \epsilon_0) f_\epsilon(\epsilon_1, \epsilon_0) d\epsilon_1 d\epsilon_0 \\ &= \int_{\epsilon_0} \int_{\epsilon_1} I(\pi_0 \text{ stable } |\theta, \epsilon_1, \epsilon_0) f_{\epsilon_1|\epsilon_0}(\epsilon_1|\epsilon_0) f_{\epsilon_0}(\epsilon_0) d\epsilon_1 d\epsilon_0 \\ &= \int_{\epsilon_0} \left[\int_{\epsilon_1} I(\pi_0 \text{ stable } |\theta, \epsilon_1, \epsilon_0) f_{\epsilon_1|\epsilon_0}(\epsilon_1|\epsilon_0) d\epsilon_1 \right] f_{\epsilon_0}(\epsilon_0) d\epsilon_0 \\ &= \int_{\epsilon_0} P(\pi_0 \text{ stable } |\theta, \epsilon_0) f_{\epsilon_0}(\epsilon_0) d\epsilon_0\end{aligned}$$

This exposition is due to Vadim Marmer.

Because of the "convenient error partitioning" [Train, 1995] of the above, the probability can be expanded into a product of independent events.[9] Let V be the set of all potential coalitions. Then

$$P(\pi_0 \text{ stable } |\theta, \epsilon_0) = \prod_{S' \in V} P(u(S') < \max_{S \in perp_{S'}} u(S)|\epsilon_0, \theta)$$

where $\text{perp}_{S'} = \{S|S \in \pi_0, S \cap S' \neq \emptyset\}$ is the set of "perpetrators" necessary to deviate to S'. The likelihood function used for optimization is thus

$$\mathcal{L}(\pi_0 \text{ stable } |\theta) = \frac{1}{|E_0|} \sum_{\epsilon_0 \in E_0} \prod_{S' \in V} P(u(S') < \max_{S \in perp_{S'}} u(S)|\epsilon_0, \theta)$$

2.2 Relaxed stability requirements

Now suppose that a less restrictive form was imposed on preferences:

$$u_i(S) = v(X_i, X_S; \theta) + \epsilon_{iS}$$

Here, the utility a player derives from a coalition can depend on interactions between the player's characteristics and those of the coalition, and similar the ϵ for a given coalition can vary across players. In this case, the existence, but not uniqueness of a stable partition can be guaranteed so long as some restrictions are placed on the types of blocking coalitions that can form. In particular, only two types of potential deviations will be considered when evaluating whether a given partition is stable: refinements, where subcoalition of a single existing coalition breaks off to form a coalition, and coarsenings, where two or more existing coalitions merger in order to form a new coalition.

To solve this problem, Ray and Vohra [1997] only allow deviating coalitions to force refinements of a partition, and Diamantoudi and Xue [2007] show that this creates a stable set. Because hedonic games are simpler than the "equilibrium coalition structures" that Ray and Vohra examine, refinements and coarsenings will be treated identically. Otherwise, the theory follows that presented in Ray and Vohra. Let $\pi \nearrow_S \pi'$ and $\pi \searrow_S \pi'$ mean that S unanimously prefers π' to pi, where π' is a coarsening and a refinement of π, respectively. Using the terminology of Ray and Vohra, π is blocked by π' if either there is a set of coalitions in π that are unanimously in favour of merging to create π', or there is a subset of

[9] That is, once the ϵ_0 have been drawn, and thus the $u(S)$ are known for $S \in \pi_0$, the events $u(S') > u(S)$ and $u(S'') > u(S)$ are independent. This conditional independence allows conditional probabilities to be expressed as products of the relevant independent events.

"perpetrators" in π that are unanimously in favour of deviating from their current coalition. In the former case π' is the coarsening that results from the merger, while in the latter it is a refinement that includes a coalition for these perpetrators and some arrangement of the "residual" left behind when the perpetrators deviated, such that the configuration of perpetrators and residual is stable. More formally, where $\rightarrow$ should be read as "blocked by":

Definition 2. *$\pi \rightarrow \pi'$ if $\exists S$ such that either $\pi \nearrow_S \pi'$ or $\pi \searrow_S \pi'$, where*

1. *$\pi \nearrow_S \pi'$ if $\pi' \setminus \pi = S$ such that $\pi \prec_S \pi'$ and $S = \bigcup Q$ for some $Q \subset \pi$*
2. *$\pi \searrow_S \pi'$ if $\exists S \in \pi'$ such that $\pi \prec_S \pi'$ and*
   a) *$\pi \setminus \pi' = S'$ with $S' = \bigcup Q'$ for some $Q' \subset \pi'$*
   b) *$\nexists \tilde{Q}$ such that $Q' \rightarrow \tilde{Q}$*

The recursion is well defined since Q' is a proper subset of π'.

Theorem 2. *Let $\twoheadrightarrow$ be the transitive closure of $\rightarrow$.*[10] *Then*

1. *$\Pi^* = \{\pi | \nexists \pi'$ such that $\pi \rightarrow \pi'\}$ is a stable set with respect to $(\Pi, \twoheadrightarrow)$.*
2. *Π^* is unique.*
3. *Π^* contains a Pareto optimal partition.*

Proof (existence). By construction, Π^* is internally stable. Now take some $\pi \notin \Pi^*$. Then $\exists \{\pi_1, \ldots, \pi_m\} \subset \Pi$ such that $\pi \rightarrow \pi_1 \rightarrow \cdots \rightarrow \pi_m$ and either $\pi_m \in \Pi^*$ or there is a cycle with $\pi_m = \pi_l$ for some $l < m$. If there is such a cycle, then it must contain both mergers and dissolutions. Suppose that $\pi_k \nearrow_S \pi_{k+1}$, and let $S_1^+ \subset S$ be the set of agents that strictly prefer π_{k+1} to π_k. If $\pi_{k+1} \nearrow \pi_{k+2}$ then $S_2^+ \supset S_1^+$ since no agent can be made worse off by a merger. If $\pi_{k+1} \searrow_{S'} \pi_{k+2}$ then $S_2^+ = (S_1^+ \setminus R) \cup P$ where R is some subset of the residual, and $P \neq \emptyset$ is some subset of the perpetrators, and $(R \cup P) \subset S'$. Since $S_{m-l+1}^+ = \emptyset$, at some point the agents in S_2^+ must be made worse off. This can only happen via refinements, and only if there is a residual smaller than S_2^+. The latter, though, implies that some subset of S_2^+ cannot be made worse off, and thus S^+ can never be empty. Thus a cycle cannot exist, and $\pi_m \in \Pi^*$. □

[10] To see why the transitive closure is used here, consider the case where $\pi_1 \nearrow_S \pi_2 \searrow_{S'} \pi_3$. π_1 and π_2 should not be in the stable set, while π_3 should, but $\{\pi_3\}$ is not a VNM stable set with respect to $\rightarrow$ because $\pi_1 \not\rightarrow \pi_3$.

(uniqueness). Suppose that Π^{**} is also a stable set with respect to $(\Pi, \twoheadrightarrow)$. Consider the bipartite directed graph defined by $\twoheadrightarrow$ with $\Pi^{**} \setminus \Pi^*$ and $\Pi^* \setminus \Pi^{**}$ as the two sets of nodes. Every node must have in-degree of at least one, but there can be no cycles. The only such graph is empty, and thus $\Pi^{**} = \Pi^*$.[11] $\square$

(PO element). Let $\Pi^{\text{PO}} \subset \Pi$ be the set of Pareto optimal partitions, and $\rightsquigarrow$ the Pareto dominance operator. Suppose that $\Pi^{\text{PO}} \cap \Pi^* = \emptyset$ and consider the directed graph defined by $\twoheadrightarrow \cup \rightsquigarrow$ with Π^{PO} and Π^* as two sets of nodes. $\square$

There is no particular reason, however, to believe that this particular partition is more attractive as solution than the other partitions in Π^*. All partitions in Π^*, including those that are not Pareto optimal, will be treated equally, since imposing additional restrictions at this stage would mean that the solution set would no longer be the outcome of the cooperative game coalition formation process described above.[12]

Estimation of this model is similar to that of the restricted preferences model. Following the notation in the restricted preferences section, let ϵ_0 denotes the vector $\epsilon_{iS}, \forall i \in S, \forall S \in \pi_0$. Suppose that partition π_0 is actually observed. If only partitions in the solution set Π^* are observed, and every partition in Π^* is assumed to be selected with equal probability, then the parameters θ can be estimated via maximum likelihood.[13] The likelihood of π_0 occurring is

$$\begin{aligned}
\mathcal{L}(\pi_0|\theta) &= \int_\epsilon \frac{I(\pi_0 \text{ stable } |\theta, \epsilon)}{\int_\Pi I(\pi \text{ stable } |\theta, \epsilon) f_\pi d\pi} f_\epsilon(\epsilon) d\epsilon \\
&= P(\pi_0 \in \Pi^*|\theta) \int_\epsilon \frac{1}{\int_\Pi I(\pi \text{ stable } |\theta, \epsilon) f_\pi d\pi} f_\epsilon(\epsilon|\pi_0 \in \Pi^*, \theta) d\epsilon \\
&= P(\pi_0 \in \Pi^*|\theta) E_{\epsilon|\pi_0 \in \Pi^*, \theta} \left[\frac{1}{Z} \right]
\end{aligned}$$

where Z is the number of stable partitions, that is, $|\Pi^*|$. Since the distribution of ϵ is known by assumption, this could in theory be calculated exactly, but due to computational constraints, both terms in the above likelihood function will be estimated. Assume that π_0 in fact consists of $\pi_{0,1}, \pi_{0,2}, ..., \pi_{0,K}$, the outcomes of K independent coalition formation games.

[11] This does not imply that $|\Pi^*| = 1$.

[12] There may be some "solutions" that seem particularly unattractive: $\{\pi \in \Pi^* | \exists \pi' \in \Pi^*, \pi \rightsquigarrow \pi'\}$. While the theory above could likely be rewritten to shrink the stable set, eliminating these elements, it is unfortunately computationally infeasible to impose any restrictions that require enumerating the entire stable set.

[13] Thanks to Vadim Marmer and Francesco Trebbi for pointing out an error in an earlier version of this section.

Then $Z = Z_1 Z_2 \cdots Z_K$, and take the Kth root of both sides of the above equation. Then

$$\sqrt[K]{\mathcal{L}(\pi_0|\theta)} = \sqrt[K]{P(\pi_0 \in \Pi^*|\theta) E_{\epsilon|\pi_0 \in \Pi^*,\theta}\left[\frac{1}{Z}\right]}$$

is a consistent M-estimator for θ. However, for computational reasons it is not possible to calculate the above, so instead a numerical approximation of the expectation will be used. In particular, consider the case where $K \to \infty$. Then

$$\frac{\log Z - \mu_{\pi_0,\theta}}{\sqrt{K}} \sim N(0, \sigma_{\pi_0,\theta})$$

and

$$\frac{1}{E_{\epsilon|\pi_0 \in \Pi^*,\theta}[Z]} \to_p E_{\epsilon|\pi_0 \in \Pi^*,\theta}\left[\frac{1}{Z}\right]$$

Now let

$$E_{\epsilon|\pi_0 \in \Pi^*,\theta}[Z] = |\Pi| \rho_{\pi_0,\theta}$$

where ρ is the fraction of partitions that are stable.[14] ρ will be estimated by randomly selecting a set of partitions Π_A and calculating the probability that they are stable. This results in the estimator

$$\begin{aligned}
&\sqrt[K]{P(\pi_0 \in \Pi^*|\theta) \frac{1}{|\Pi| \frac{1}{|\Pi_A|} \Sigma_{\pi \in \Pi_A} P(\pi_i \in \Pi^*|\pi_0 \in \Pi^*, \theta)}} \\
= \; &\sqrt[K]{|\Pi|} \sqrt[K]{\frac{P(\pi_0 \in \Pi^*|\theta)|\Pi_A|}{\Sigma_{\pi \in \Pi_A} P(\pi_i \in \Pi^*|\pi_0 \in \Pi^*, \theta)}}
\end{aligned}$$

Since $|\Pi|$ does not depend on θ, an equivalent estimator is

$$\sqrt[K]{\frac{P(\pi_0 \in \Pi^*|\theta)|\Pi_A|}{\Sigma_{\pi \in \Pi_A} P(\pi_i \in \Pi^*|\pi_0 \in \Pi^*, \theta)}}$$

[14] Interchanging the expectation and reciprocal operations is valid because

$$\begin{aligned}
E[\frac{1}{Z}] &= E[\frac{1}{e^{\Sigma_{i=K} \log Z_i}}] \\
&= E[e^{-\Sigma_{i=K} \frac{\log Z_i}{K} K}] \\
&= E[e^{-\mu K + o(\frac{K}{\sqrt{K}})}] \\
&= E[e^{-\mu K + o(\sqrt{K})}] \\
&\simeq E[e^{-\mu K}]
\end{aligned}$$

since $\mu > 1$ because $\pi_0 \in \Pi^*$.

The actual estimation is performed via numerical approximation of the above probabilities. Specifically, if E_0 is a set of draws from the distribution of ϵ_0, then the approximation is

$$\frac{1}{|E_0|}\sum_{\epsilon_0\in E_0} P(\pi_0 \in \Pi^*|\theta,\epsilon_0)\frac{|\Pi_A|}{\sum_{\pi\in\Pi_A} P(\pi \in \Pi^*|\theta,\epsilon_0,\pi_0\in\Pi^*)}$$

Once again because of the convenient error partitioning of the above, the probability can be expanded into a product of independent events. Let $\mathcal{S}_0^{\uparrow}$ be the set of all coalitions that could be formed by mergers of the coalitions in π_0, and let $\mathcal{S}_0^{\downarrow}$ be the set of all coalitions that are a subset of a coalition in π_0. Then

$$P(\pi_0 \in \Pi^*|\theta,\epsilon_0) = \prod_{S\in(\mathcal{S}_0^{\uparrow}\cup\mathcal{S}_0^{\downarrow})} P(\pi_0 \not\prec_S S|\theta,\epsilon_0)$$

Now approximate the denominator by defining ϵ_l and E_l in the same way as ϵ_0 and E_0:

$$P(\pi_l \in \Pi^*|\theta,\epsilon_0,\pi_0\in\Pi^*) = \frac{1}{|E_l|}\sum_{\epsilon_l\in E_l}\prod_{S\in(\mathcal{S}_l^{\uparrow}\cup\mathcal{S}_l^{\downarrow})} P(\pi_l \not\prec_S S|\theta,\epsilon_0,\epsilon_l,\pi_0\in\Pi^*)$$

There are two problems with estimating this numerically. First, draws need to be made from $\epsilon_l|\theta,\epsilon_0,\pi_0\in\Pi^*$, and second, given a draw of ϵ_l from the correct distribution, the required probability needs to be calculated efficiently. Fortunately, in both cases an application of Bayes' Rule is sufficient. To draw from $\epsilon_l|\theta,\epsilon_0,\pi_0\in\Pi^*$, first define ϵ_S to be the idiosyncratic shocks to coalition $S\in\pi_l$. If S is not a potential deviation from π_0, then $\epsilon_S|\theta,\epsilon_0,\pi_0\in\Pi^* = \epsilon_S$ since no additional information is provided by the fact that π_0 is stable. If S is a potential deviation from π_0, then consider the identity

$$f(\epsilon_S|\theta,\epsilon_0) = f(\epsilon_S|\theta,\epsilon_0,\pi_0\not\prec S)P(\pi_0\not\prec_S S|\theta,\epsilon_0) + f(\epsilon_S|\theta,\epsilon_0,\pi_0\prec_S S)P(\pi_0\prec_S S|\theta,\epsilon_0)$$

$f(\epsilon_S|\theta,\epsilon_0)$ is equal to the unconditional density $f(\epsilon_S)$, which is known by assumption. The second distribution on the right hand side is a set of truncated distributions because if $\pi_0 \prec_S S$ then it must be true that $u_i(S) > u_i(\pi_0)$, and thus

$$\epsilon_i > u_i(\pi_0) - v_i(S)$$

and these can be calculated sequentially. Thus, the desired distribution can be drawn by simulating from

$$f(\epsilon_S|\theta,\epsilon_0,\pi_0 \not\prec_S S) = \frac{f(\epsilon_S|\theta,\epsilon_0) - f(\epsilon_S|\theta,\epsilon_0,\pi_0 \prec_S S)P(\pi_0 \prec_S S|\theta,\epsilon_0)}{P(\pi_0 \not\prec_S S|\theta,\epsilon_0)}$$

which can be done sequentially for each member of S. From a computational perspective, it is important to avoid simulation "chatter", which would occur if a new ϵ were drawn for each new proposed $\hat{\theta}$, the simplest way of ensuring that π_0 was always in the stable set. Instead of simulating ϵ_0 directly, then, draw quantile indices q_j, and create ϵ_j from q_j fresh for each iteration of $\hat{\theta}$.

The next problem is using these drawn ϵ_l to calculate the probability

$$P(\pi_l \not\prec_S S|\theta,\epsilon_0,\epsilon_l,\pi_0 \in \Pi^*)$$

where S' is some coalition not in π_l. If S' is not a potential deviation from π_0, then the calculation is identical for those done for π_0, described above. However, if S' is a potential deviation from π_0, then the fact that π_0 is stable provides additional information that needs to be taken into account. Consider

$$P(\pi_l \not\prec_{S'} S'|\theta,\epsilon_0,\epsilon_l) =$$
$$P(\pi_l \not\prec_{S'} S'|\theta,\epsilon_0,\epsilon_l,\pi_0 \not\prec S')P(\pi_0 \not\prec_{S'} S'|\theta,\epsilon_0,\epsilon_l) + P(\pi_l \not\prec_{S'} S'|\theta,\epsilon_0,\epsilon_l,\pi_0 \prec_{S'} S')P(\pi_0 \prec_{S'} S'|\theta,\epsilon_0,\epsilon_1)$$

and since the left hand side can be calculated, and the second term of the right hand side has the same set of truncated distributions described just above with respect to the ϵ_l, then rearrangement once again permits calculation:

$$P(\pi_l \not\prec_{S'} S'|\theta,\epsilon_0,\epsilon_l,\pi_0 \not\prec_{S'} S') =$$
$$\frac{1 - P(\pi_l \prec_{S'} S'|\theta,\epsilon_0,\epsilon_l) - (1 - P(\pi_l \prec_{S'} S'|\theta,\epsilon_0,\epsilon_l,\pi_0 \prec_{S'} S'))P(\pi_0 \prec S'|\theta,\epsilon_0,\epsilon_1)}{1 - P(\pi_0 \prec_{S'} S'|\theta,\epsilon_0,\epsilon_l)}$$

Everything on the right hand side of this equation can be computed quickly, making optimization feasible.

3 Application

Treating municipal mergers as a hedonic coalition formation problem is consistent with anecdotal evidence concerning how mergers are effected. Negotiations regarding compensation seem to be rare, even though controversy is common and the results of unrest sometimes significant. Usually, some of the involved municipalities were in favour of a merger while others were very much opposed, but those in favour did not promise large transfers to those opposed in order to secure their cooperation. This suggests that there is some problem with contractibility in political mergers such that transfers are difficult or impossible, and thus, as in Acemoglu [2003], the "political Coase theorem" does not hold. It thus seems more plausible to model mergers as a coalition formation game without transfers. First, a simple model of public good provisioning is presented, then some of the parameters are estimated from other Japanese data sources. The data is described, and then the remaining parameters are estimated via the methods presented in the preceding section.

3.1 Municipal Public Goods Model

Suppose that at time t, each municipality m provides a public good of level g_{mt} to its residents at a total cost of $c_m(g_{mt})$, with g including public goods such as local roads, elementary education, waste disposal, and some health care. c_m is assumed to be subadditive in population: $c_{m'} < \alpha c_m$ if the population of m' is α times as large as that of municipality m. To pay for this service provision, the municipality levies taxes at rate τ_{mt} (possibly restricted by the national government to $\bar{\tau}$), and receives transfers T_{mt} from the national government such that the intertemporal budget constraint is satisfied:

$$\int_0^\infty e^{-rt} c_m(g_{mt}) dt = \int_0^\infty e^{-rt} (\tau_{mt} Y_m + T_{mt}) dt$$

where Y_m is the tax base of the municipality, and r is the interest rate. If characteristics of the municipality are constant across time, the discount rate is the same as the interest rate, and there is no uncertainty, then the optimal tax rate and level of government services does not change across time, and everything is perfectly smoothed. Thus, the t subscripts are dropped, and any short term transfers received (as in the next section) are assumed to be perfectly smoothed out. Thus, the budget constraint is treated as

$$c_m(g_m) = \tau_m Y_m + r \int_0^\infty e^{-rt} T_{mt} dt$$

In addition to providing general services of level g_m, the municipal council also has control over some (costless) local policies, as in Alesina and Spolaore [1997]. Different individuals have different ideal points regarding these policies, and thus as the population of a municipality grows, so does the utility loss due to having to impose a constant policies over the entire municipality. Thus, there is a tradeoff between providing the public good cheaply, and having government policies that are carefully tailored to residents' desires. A formal model of this tradeoff is not presented here, but instead, controlling for the economies of scale in production of public goods, higher population will enter the utility function negatively.[15]

3.2 Japanese Local Public Finance

Mochida [2006] provides an excellent summary of the development and current state of the Japanese local finance system. Post-war Japanese fiscal policy placed great emphasis on the provision of equal quality public goods across the country, and established a national standards for the general services that were provided by local governments. To ensure that every municipality had sufficient funds to offer the specified services at or above the standard quality level, the national government developed a complicated system of transfers, called the "Local Allocation Tax".[16] The transfer to municipality m is determined by the equation

$$T_m = \max(\tilde{c}_m(\bar{g}) - .75\bar{\tau}Y_m, 0) \tag{1}$$

where $\bar{g}$ is the minimum quality of services the municipality is expected to provide, $\tilde{c}_m(\bar{g})$ the estimated cost to the municipality of providing those services (referred to as "Standard Financial Need" in official documents), $\bar{\tau}$ the fixed tax rate that the municipality is required to charge. $\tilde{c}_m$ varies significantly from municipality to municipality, based on a formula developed by central ministries. The exposition of this formula consists of approximately 280 pages of Japanese legal text; however, as Figures 2 and 3 show, $\tilde{c}_m$ can be approximated quite well by the linear regression

$$\tilde{c}_m(\bar{g}) = \bar{g}(\beta_0 + \beta_1 POP_m) + \upsilon_m \tag{2}$$

These figures and equations effectively duplicate those found in an official Ministry of Finance publications, as the linear approximation is well known and commonly used.[17] The positive and significant intercept shown in Table 1 reflects the fact that the central ministries believed

[15]The Alesina and Spolaore [1997] model can be extended to two dimensions in order to handle actual geographic data, but the reduced form approach is used here because it is difficult to justify any particular functional form assumption on the utility loss due to government policies being set away from a resident's ideal point. Although multi-dimensional policy spaces in general suffer from cycles, if preferences are Euclidean and the number of voters is large, then the plausible set of policies, following a reasonable definition such as the yolk, lies within a small region. With an assumption on how the policy is selected within that region, the remaining difficulty would be how preferences of voters determine preferences over mergers. One possibility here would be to assume that voters will move randomly within whatever merger they join, and thus do not apply their geography-based policy preferences to decisions regarding which coalition to join.

[16]The slightly-confusing name is due to the fact that it is an *allocation* to local governments from *taxes* collected by the national government.

[17]A new formula introduced after the period of interest is explicitly based on a linear function of population and area.

that there were economies of scale in the production of public goods, and thus per capita costs would be higher in municipalities with lower population. At current exchange rates, $\bar{g}\hat{\beta}_0$ is a little more than \$10M. Thus, if the national government estimated costs correctly, with $\tilde{c}(\bar{g}) \simeq c(\bar{g})$, the per capita cost of providing $\bar{g}$ quality public goods in Ashiyasu village (population 567) is roughly \$22,000, compared with roughly \$1,400 in Sakai city (population 790,000).

With almost half of Japanese municipalities having a population less than 10,000, the decision to provide additional subsidies to smaller municipalities due to their size was an expensive one. Although there were provisions for municipalities to merge, there was little incentive for them to do so, because if a coalition S decided to form a new (amalgamated) municipality, T_S would be calculated identically to Equation 1, above:

$$T_S = \max(\tilde{c}_S(\bar{g}) - .75\bar{\tau}Y_S, 0) \tag{3}$$

Thus almost all savings would be passed to the national government, and even a slight preference for smaller population jurisdictions ensured that residents would be opposed to mergers.[18]

During the financial difficulties of the early 1990s, the national government implemented a series of reforms designed to reduce the total transfers provided to municipalities while minimizing the negative effects of this decrease. First, the government substantially reduced transfers to the smallest municipalities by revising the Local Allocation Tax. This can be approximated as

$$T_m = \max(\tilde{c}_m^{\text{new}}(\bar{g}) - .75\bar{\tau}Y_m, 0)$$

$$\tilde{c}_m^{\text{new}}(\bar{g}) = \bar{g}(\beta_0^{\text{new}} + \beta_1 POP_m) + v_m$$

as shown in Table 2, with β_0^{new} being a little more than half the size of β_0.[19] Second, municipalities that merged between 1995 and 2005 would not have their transfers lowered due to the merger for at least ten years starting from the date of the merger. That is,

$$T_S^{\text{new}} = \sum_{m \in S} T_m^{\text{new}}$$

[18] In general, the division of a municipality was prohibited. In one case, such a split did occur, but both of the resulting municipalities were immediately merged with different neighbours.

[19] Previously, mayors were responsible for delivering hundreds of "agency delegated functions" from higher levels of government, making them bureaucrats as well as elected officials, and making it possible (at least in theory) for central ministries to fire a mayor for not performing a delegated function according to specifications. "Agency delegated functions" were also abolished, and responsibility was devolved in many cases to mayors and municipal councils.

would be provided for the decade following the merger. This resulted in a strong financial incentive for municipalities to merge, as shown by the utility functions derived in the appendix. By 2006 there were only 1821 municipalities remaining, down from 3232 at the start of the merger period in 1995. Figure 4 shows the mergers in Shizuoka Prefecture. Mergers were voluntary, and needed to be approved by the municipal council of every participating municipality.[20] The parameters to the utility function can thus be estimated by examining the mergers that actually occurred.

3.3 Data

There were 3382 municipalities in 1995 at the start of the merger period, divided into 47 prefectures (similar to US states). Since mergers do not cross prefectural boundaries, each prefecture is treated as a separate game in the following sections. Surface area data for each municipality is obtained from a 1996 survey conducted by Geographical Survey Institute, an arm of the Japanese national government. Municipal population data comes from the 2000 national census as reported by the Home Affairs Ministry. Taxable income per capita is used as a proxy for income per capita. Taxable income data for 1996, as well as the list of mergers that actually occurred come from the Asahi Shimbun *minryoku*. To construct the set of possible coalitions, as described in more detail later, information on which municipalities share a border is taken from Global Map files for Japan.

The data regarding municipal tax revenue, as well as the financial data used to generate Table 1 is from the *shichouson betsu kessan joukyou shirabe*, which is an official national government report of municipal finances. The 1996-1997 fiscal year data is used as this is the first year available electronically. To determine the new transfer policy, as shown in Table 2, the 2006-2007 fiscal year data is also used. Because of the large transfers from the national treasury to local governments, this data is handled quite carefully by officials in the central ministries and is generally regarded as accurate, particularly the sections produced by the central government itself. The isolated incidents of fraud reported generally relate to variables reported by the municipalities, which are not used in this paper.

There are no missing values in any of the financial data or surface area data. In the population data, approximately 6 values are missing because one merger took place before the data was issued, and thus the old municipalities were not reported. The 23 special wards covering the area of pre-war Tokyo city, although having powers similar to municipalities, are excluded from the analysis because any enlargement of this *sui generis* area would likely involve adding more wards, rather than changing the borders of existing ones. The 12

[20] In about a third of cases, referenda were held. Nominally, these were consultative, but in general the municipal council would not vote opposite to a referendum result

"designated cities", which have some powers normally reserved for prefectures, are omitted from the financial calculations because their additional responsibilities increase their required spending, but they are included in the rest of the analysis as regular cities. The categories of "core city" and "special city" were created after the merger period, and thus do not directly affect the data. Similarly, the policy distinctions between city (*shi*), town (*chou*), and village (*son*) are ignored.Counties (*gun*) are statistical divisions, and have not had any political function since the 1920s.

The initial laws implementing the new incentive scheme were passed in 1995, and thus it would be optimal to use data from before this period. However, some later data is currently used due to data availability issues. Almost all of the merger negotiations and approvals occurred near the end of the 1996-2006 window, with most occurring after 2002, and the latest data used is from 2000. In fact, the important "Trinity" tax reforms were not even finalized until 2002, and this uncertainty provided municipalities with a strong incentive to wait until the end of the period to conduct any mergers. Thus, it seems that using financial data from 1996-1997 and population data from 2000 should not be a huge concern. In particular, the financial data used is calculations by the national government, not actual spending by municipalities, and thus is not vulnerable to "last minute" capital spending seen in some environments. A final issue is that, due to disorganization, coordination failure, or for political reasons, a few mergers occurred after the end of the merger window. These "late" merging municipalities did not benefit from the incentive scheme described above, and are not considered in the estimation described below.

3.4 Estimation

Suppose that for any coalition S, the utility of municipality i in the coalition is given by

$$u_i(S) = \theta_1 \log\left((1-\tau_S)y_i\right) + \theta_2 \log g_S + \theta_3 \log \mathrm{POP}_S + \theta_4 \log \mathrm{AREA}_S + \phi X_S + \epsilon_{iS}$$

where X_S are other fixed characteristics of the coalition.[21] The third term reflects the value that residents place on smaller jurisdictions that can tailor government policies more carefully to reflect local concerns, and the fourth term is included because heterogeneity of policy ideal points may be greater when the same population is spread out over a larger area. Income per capita and a dummy for whether the coalition is actually a merger (i.e. non-singleton)

[21] The existence of a representative agent is not obvious, since if many different potential merger partners exist, and individuals are allowed to have arbitrary preferences, then Arrow's Impossibility Theorem applies. If the only choice that needs to be made is which merger to pick, then "intermediate preferences" would guarantee that the median voter is the representative agent. To have such a voter exist, though, heterogeneity must be one dimensional and have a specific form [Grandmont 1978].

being the only column in X_S. In the future, however, additional interaction terms could be included. For estimation via the restricted preferences method, the restriction $\epsilon_{iS} = \epsilon_{jS} = \epsilon_S$ is required.

While de jure municipalities were given the power to set taxes as part of the reforms, there is a belief that de facto they do not have much authority to change tax rates, and there appear to have been very few significant shifts in tax rates. Thus, estimation is performed assuming that local governments cannot change the tax rate, which is fixed at $\bar{\tau}$. This tax revenue, combined with transfers, then determines the amount of general services provided.[22]

The determination of V, the set of potential mergers that need to be checked during estimation, is slightly more problematic. There are a number of large mergers observed, with the largest involving 15 municipalities. Almost all observed mergers are geographically contiguous; however, even after restricting V to contiguous coalitions of size 15 or less, there are still over 10^{16} possibilities, which is computationally infeasible.[23] Many of these coalitions look very different than the actually observed coalitions, however. In particular, they tend to be a thin line of municipalities, stretching almost all the way across a prefecture. The actually observed coalitions, on the other hand, look like ellipses. Figure 5 shows a merger that actually occurred (Hanamatsu city, in Shizuoka prefecture, involving 12 municipalities), while Figure 6 shows a typical randomly generated contiguous coalition of the same size. The randomly generated coalition in this case suffers from a defect that is not considered in the utility function given above: because of its elongated shape, travel time to a single centrally-located city hall other such facility would be extremely high for residents starting in certain parts of the amalgamated municipality. Similarly, the cost of visiting the more remote parts of the new municipality would likely be excessive for centrally located municipal bureaucrats. These sorts of coalitions are thus ruled out of consideration by the use of a restriction related to the surface area to perimeter ratio of the coalition. Because the RAND-ESU [Wernicke 2006] algorithm used to generate the coalitions is limited in the types of restrictions it can accomodate, this restriction is formulated in terms of graph theory characteristics, an approximation which makes implementation computationally feasible.

The randomly generated potential coalition shown in Figure 6 differs from actually observed coalitions in that it borders over 30 other municipalities, whereas the actually observed

[22]Since the transfers are phased out after ten years, the present value of this amount is used to determine the effective transfer after smoothing.

[23]More specifically, there are approximately a dozen mergers that are not geographically contiguous. Half of these involve municipalities on small islands amalgamating with a nearby municipality separated by water, while the other half are mergers that would have been contiguous, had one of the participants not dropped out late in the merger process. Although the law stated that the mergers were to be geographically contiguous, these exceptions were allowed. No municipalities of either type are generated as comparison coalitions, although the ones that did occur are retained in the observed partitions.

coalitions of this size never border more than 9. A restriction is placed on the number of municipalities that the coalition can border is introduced, with an upper bound based on the maximum in the actually observed mergers. This restriction dramatically reduces the number of large coalitions that need to be considered: with 15-municipality coalitions, only 1 coalition in 10 billion has a small enough number of neighbours. This reduces the total number of alternatives that need to be considered to about 5 million, which is computationally feasible.

Another problem, only relevant to the estimation via relaxed stability requirements, is the estimation of the size of the stable set. Since the number of partitions grows exponentially with the number of municipalities, it is not possible to examine all partitions. The total number of partitions is unknown but bounded above by the Bell numbers (Sloane #A000110), which are greater than 10^{100} for larger prefectures such as Hokkaidou. Thus, instead, a random sample is drawn; however, it is not trivial to randomly sample from a set which is too large to be enumerated. Thus, random draws are obtained using Markov chains. Let the state space $\mathcal{X}$ be the set of all partitions, and the transition matrix P be

$P_{xy} = k$ if y can be created by either breaking apart one coalition in x into subcoalitions, or merging coalitions in x together to create one new coalition

$P_{xy} = 0 \quad \forall y \neq x$ that do not meet the above condition

$P_{xx} = 1 - \sum_{y \neq x} P_{xy}$

Since the state space is finite, this creates a valid transition matrix for sufficiently small k. P describes a reversible Markov chain, since a transition from x to y via merging implies a possible transition from y to x via a breakup, and vice versa. The chain is connected, since any partition can be obtained by first breaking all coalitions down to singletons, and then constructing the desired partition. There is thus a unique stationary distribution, since the chain satisfies the detailed balance condition [Robert and Casella 2004]. Moreover, the stationary distribution gives equal probability to each state, and thus draws from the stationary distribution are equivalent to random draws from the set of all partitions. These draws can be performed using the Metropolis-Hastings algorithm.

Unfortunately, the number of transitions that need to be considered is too large to be computationally feasible. The number of ways a size 15 coalition could be broken down into sub coalitions could be as high as a billion, and thus it is not practical to enumerate all the possible transitions. Instead, the approach introduced by Wernicke [2006] for the RAND-ESU algorithm will be used. Let p be a length 15 vector of "cut probabilities" and let Q be a set of subcoalitions of S that form a partition of S. Then enumerate a member of Q with probability $\prod_{i \in \{1,\ldots,|S|\}} p_i$. This gives each member of Q an equal probability of being enumerated, but allows for a much smaller, randomly selected set to be considered. So long as this randomly selected $\tilde{Q}$ is re-randomized each time a given state is reached, then the properties of the above Markov chain are unchanged.

3.5 Results

The results are shown in Table 3. Results are consistent with the theory, with high government services and low population being preferred, as predicted. Having richer fellow residents also appears to be preferred, but the magnitude and statistical significance of this effect depends on the method used. The dummy for mergers is negative under both estimation methods, but statistically significant only in the restricted preferences estimates, making it unclear whether there was indeed a strong preference for the status quo. The ratio of the coefficients on government services and population imply that a municipality would be willing to accept an increase in population of one log point in exchange for an increase in the level of services of either .06 or .12 log points. This implies that, the optimal population size for a municipality is 75,000 or 150,000, respectively. This accords well with Japanese estimates of the efficient size for a municipality. The Ministry of Internal Affairs' "Standard Municipality" has a size of 100,000, and the Ministry [2003] has estimated that that is the minimum efficient size for a municipality. Furthermore, Hayashi [2002] finds that the smallest city of efficient scale has a population of 120,000. This estimate is particularly interesting, since Hayashi uses third-party ratings of municipal service quality, which is not used in this paper, but recovers roughly the same population target. Total spending on municipal government services accounts for approximately 10% of income, so, if the assumption of Cobb-Douglas utility is correct, and if the tax rate specified by the national government is approximately the efficient level, then the implied willingness to pay for small jurisdiction size is about 0.5% of income to cut municipal population in half.

Although the sign on surface area is not as expected in the relaxed stability requirements column, the magnitude is tiny. The smaller standard deviation of the error term in the relaxed stability requirements estimates is an indication that the variables included in the estimation are relatively more important in determining choices. Additional data that could be used in the future includes commuting patterns, age and education distribution, and possibly industrial sectors. In some cases it might be possible to also replace means with medians, since the variance (but not the precise distribution) of income is known for each municipality. Finally, prefecture fixed effects could be considered, although this presents some computational issues.

4 Counterfactual simulations

A major advantage of having coefficient estimates for a structural model is the ability to conduct counterfactual analysis. Here, two alternative national government policies will be examined: first, an incentive scheme for richer municipalities that participate in mergers; second, national government enforcement of transfers negotiated between municipalities during the coalition formation process.

4.1 Transferable utility

Suppose that the central government offered to enforce whatever transfers resulted from decentralized negotiations amongst municipalities. That is, if municipality m promised to make a certain transfer from its current residents to the current residents of municipality n, the central government would ensure that this was actually carried out. Assume that the transfers negotiated are "small", in the sense that a linear approximation of utility around transfers of zero is reasonable:

$$\begin{aligned} u_i(S,z) &= \theta_1 log((1-\tau_S)y_i + \frac{\sum_{j\in S} z_{ij}}{\text{POP}_i}) + ... + \epsilon_{iS} \\ &\simeq \theta_1 log((1-\tau_S)y_i) + \frac{\theta_1 \sum_{j\in S} z_{ij}}{(1-\tau_S)y_i \cdot \text{POP}_i} + ... + \epsilon_{iS} \end{aligned}$$

Where $\sum z$ is total transfers received from other municipalities. Now define

$$\begin{aligned} \tilde{u}_i(S,z) &= \frac{(1-\tau_S)y_i\text{POP}_i}{\theta_1} \cdot u_i(S) \\ &\simeq \sum_{j\in S} z_{ij} + \theta_1 log((1-\tau_S)y_i) + ... + \epsilon_{iS} \end{aligned}$$

and thus $\tilde{u}$ approximates a standard transferable utility cooperative game. This does not have a unique solution, but is covered by the Ray and Vohra [1997] approach detailed above, and so will result in some sort of stable set. If random ϵ are drawn, then some idea of the efficiency gain of enforcing the TU game, versus simply administering a fixed incentive structure, can be obtained. The exact transfers, however, depend on which stable coalition structure forms and exactly how the surplus from each coalition is divided. For a given coalition S, the possible utilities of the municipalities are determined by the possible values of x, where the following conditions are satisfied:

1. $\sum_{i\in S} x_i = V(S)$
2. $\sum_{i\in S'} x_i \geq V(S') \quad \forall S' \subset S$

where $V(S) = \sum_{i \in S} \tilde{u}(S)$ is the value of coalition S.

It seems extremely unlikely that poor municipalities would have more bargaining power than rich municipalities, and thus an "equitable" distribution of the surplus seems to be the most optimistic scenario. A more pessimistic scenario would give most of the bargaining power to richer municipalities, with a resulting increase in post-merger inequality.

The nucleolus is used as a "best case" equitable division. It is the allocation which maximizes over all potential deviating coalitions S' the smallest difference between the amount allocated to the members of S' and that which they could obtain if they deviated. More specifically, x is determined by

$$\operatorname*{argmin}_{x} \max_{S' \subset S} E(S')$$

where $E(S')$ is the excess of coalition S':

$$E(S') = V(S') - \sum_{i \in S'} x_i$$

note that if S is part of a stable partition, $E(S') < 0 \quad \forall S' \subset S$. There may be more than one x that leads to the same maximum above, but by continuing to minimize excesses lexicographically, a unique value of x is obtained. This is the nucleolus. To estimate the utility obtained in this case, 100 separate draws of ϵ were performed, and the resulting nucleoli were averaged. This was compared to the no-transfer case, using the same 100 draws of ϵ. The results are shown graphically in Figure 7. Poorer municipalities are worse off, relative to the no-transfers case, while richer municipalities are much better off. Thus, making transfers feasible may or may not be optimal for the national government, depending on its social welfare function. A worst-case scenario was also examined, where richer municipalities are assumed to have all of the bargaining power, and thus as much surplus as possible is transferred to the richest municipality, and then the next richest. The results are very similar to those shown in Figure 7, although the regressive nature of the transfers is slightly more pronounced.

4.2 Incentives to merge

Suppose that the national government offered an additional, budget balanced incentive for certain municipalities to merge. The targeted municipalities should be those that are most likely to be opposed to mergers that would benefit other municipalities, and the most likely municipalities to fall into that category are richer municipalities. Consider the policy that offered a transfer equivalent to 0.3% of income, to residents of richer municipalities that participated in a merger, where a "richer" municipality is defined as one that had above average income per capita in more than half of the potential mergers they could have participated in. This transfer would be paid for by an increase in the income tax on everyone. This type of subsidy preserves the existence of a unique stable partition, since it is equivalent to increasing u_m for rich municipalities while at the same time decreasing $u(S)$ for those singleton coalitions that consist of a single rich municipality.

There are two different types of questions that could be asked regarding the effect of this policy. One is whether, conditional on the observed outcome occurring, the counterfactual policy would have yielded a better outcome. The other is whether the counterfactual policy would have yielded a better outcome without any information about what outcome occurs under the actually implemented policy. The difference concerns the way the ϵ are drawn. Both of these will be considered, and the results obtained via the two methods are similar.

First, consider the case where π_0 is the actually observed partition. Given that the observed outcome occurred under the actually implemented policy, and assuming that the true θ are exactly the estimated $\hat{\theta}$, the distribution of ϵ is no longer i.i.d.; however, draws can be made from $\{\epsilon|\pi_0 \text{ stable}, \theta\}$ via Gibbs sampling. Then, for each of these draws of ϵ, the stable partition under the counterfactual policy can be computed via the algorithm given in the proof of Theorem 1. The changes in utility, averaged over several simulations, are shown in Figures 8 and 9. In one representative simulation, out of 3822 municipalities, 784 received the new incentive transfer but also merged under the original policy, and there were 176 municipalities that participated in mergers that did not occur under the original policy. The mean change in utility is equivalent to an increase in income of 0.3%.[24] However, even though almost all quantiles of the utility distribution are shifted upwards, the new policy is not rank preserving, and a significant number of municipalities would be better off under the actual policy. In each of the simulations, there were a few dozen municipalites that were in mergers under the actual policy, but whose merger partners abandoned them for another coalition under the alternative policy. Thus, not all municipalities would be in favour of switching to the alternative policy.

Next, consider the case where the parameters are known, but not the ϵ. Here, the question is what the difference in the expected utility of municipalities is between the actual and alternative policies. To determine this, the ϵ are drawn randomly, and then stable partitions are generated under both policies.[25] The difference between the actual and alternative

[24] The fact that this is the same as the size of the additional transfer offered is purely coincidental.

[25] These partitions are produced by generating random coarsenings of an all-singleton partition and then drawing from this set of partitions after all the duplicates have been removed. This is a consistent method of drawing a sample of stable partitions with uniform selection probability, although it is biased (i.e. the probability of selection is not uniform) when the random coarsenings drawn do not enumerate all the stable partitions. The degree of bias depends on the number of random coarsenings drawn, but this bias does not appear to be important, since changing the number of random coarsenings generated does not change the

policies are shown in Figure 10.

If transfers between players are impossible, but there is a social planner that can provide incentives for coalition formation, then at least in the two player case, the social planner should offer such incentives:

Example 3. *In the setup described in Example 1, if ϵ_1 and ϵ_2 are random variables with density f, then before the ϵ are known, $\exists \tau, \delta > 0$ such that both players would be in favour of a social planner offering an incentive δ for coalition formation:*

$$\tilde{u}_i(\{1\}) = u_i(\{1\}) - \tau$$
$$\tilde{u}_i(\{1,2\}) = \tilde{u}_i(\{1\}) + \epsilon_i + \delta$$
$$E[1(\tilde{u}_1(\{1,2\}) > \tilde{u}_1(\{1\})) \cdot 1(\tilde{u}_2(\{1,2\}) > \tilde{u}_2(\{2\})) \cdot \delta] = \tau$$

Ex ante, both players are in favour of this incentive being offered because

$$E[u_i] = E[u_i(\{i\})] + E[1(u_1(\{1,2\}) > u_1(\{1\})) \cdot 1(u_2(\{1,2\}) > u_2(\{2\})) \cdot (u_i(\{1,2\}) - u_i(\{i\}))]$$
$$\frac{\partial E[u_i]}{\partial \delta}|_{\delta=0} = f(0)(E[\max(u_i(\{1,2\}), u_i(\{i\}))] - u_i(\{i\})) > 0$$

Intuitively, the gains from the incentive are first order, occurring whenever one player is almost indifferent between forming the {1,2} coalition or not, and the other player is very much in favour. The losses, on the other hand, are second order: when both players are almost indifferent, the incentive results in a {1,2} coalition now forming when it wouldn't have previously. Although it is difficult to show analytically, this result appears to hold in the more complicated coalition formation game examined below, where there are many players and many potential coalitions. In a larger game, however, any proof must take into account that a new coalition able to form only because of the incentive might displace an existing non-singleton coalition, thus changing the set of stable coalition structures. This is an additional cost of offering the incentive, and thus any general theorem regarding the efficiency of offering incentives to form (non-singleton) coalitions would have to have restrictions to ensure that the cost of such a disruption of existing coalitions is not too high. Thus, although this result is likely the reason for the increase in utility observed when additional incentives for mergers are provided, no formal proof is offered.

5 Conclusion

results.

This paper estimated the parameters determining preferences in a cooperative form political coalition formation game, using two different sets of assumptions and definitions of the solution set. The results are consistent with intuition, and are used to examine potential alternative national government policies. Counterfactual simulations suggest that an alternative incentive scheme that rewarded relatively rich municipalities for merging would have resulted in welfare improvements under most reasonable social welfare functions. Allowing transfers to be negotiated between municipalities may or may not be superior, depending on the national government's aversion to inequality and the bargaining power of the various municipalities. The latter is likely unknown to the national government, and thus even if transfers between municipalities could be enforced, it may not be beneficial for the national government to do so. Of course, further work could consider other alternative national government policies.[26] Additional covariates could also be added to the model.[27] At least as important as the implications to government policy, however, is the methodology developed. A coalition formation game without transfers accurately describes many real-world phenomena, but it is rarely estimated in the empirical literature. As the price of computing power decreases, however, the number of uses of this sort of model that are feasible should increase. Although the game presented in this paper could be estimated only because the geographical nature of the data permitted a large number of possible coalitions to be discarded, in the future such restrictions should be less necessary. The results given above, then, are hopefully only the first of many applications of coalition formation models of this type to actual data.

[26]For example, one possibility that was not considered in this paper is that of a tax on negotiated transfers. In the simplest price control model, a tax redistributed to the consumer should be able to mimic a price control, but with the assurance that the consumers with the highest willingness to pay obtain the good. To the extent that the inability to make transfers is like a price control at zero, then, it could be that the optimal policy for the government – rather than specifying a fixed incentive scheme to encourage rich municipalities to merge with their neighbours – would be to allow transfers, but tax them heavily and redistribute the revenue obtained to the poorest municipalities. Overall, the problem bears some resemblance to the classic rent control problem.

[27]One potential incentive for mergers that is not considered in this paper is the *gappei tokurei sai*, special bond issues allowed for municipalities planning amalgamation. Currently, the estimation strategy assumes that these bonds, subsidized by the national government, exactly eliminate any direct financial cost of merging, such as the construction of a new city hall. This follows the official government position; however, many believe that these merger bonds allowed significant capital expenditures beyond the actual costs of amalgamation. Another potential incentive would be the additional powers gained by a municipality that is classified as a "special city" or "core city", both of which have cutoffs based on municipal population. Finally, the grouping of municipalities into counties dates back to the Meiji restoration, by some accounts county lines are reflected in the patern of mergers observed.

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Figure 1:

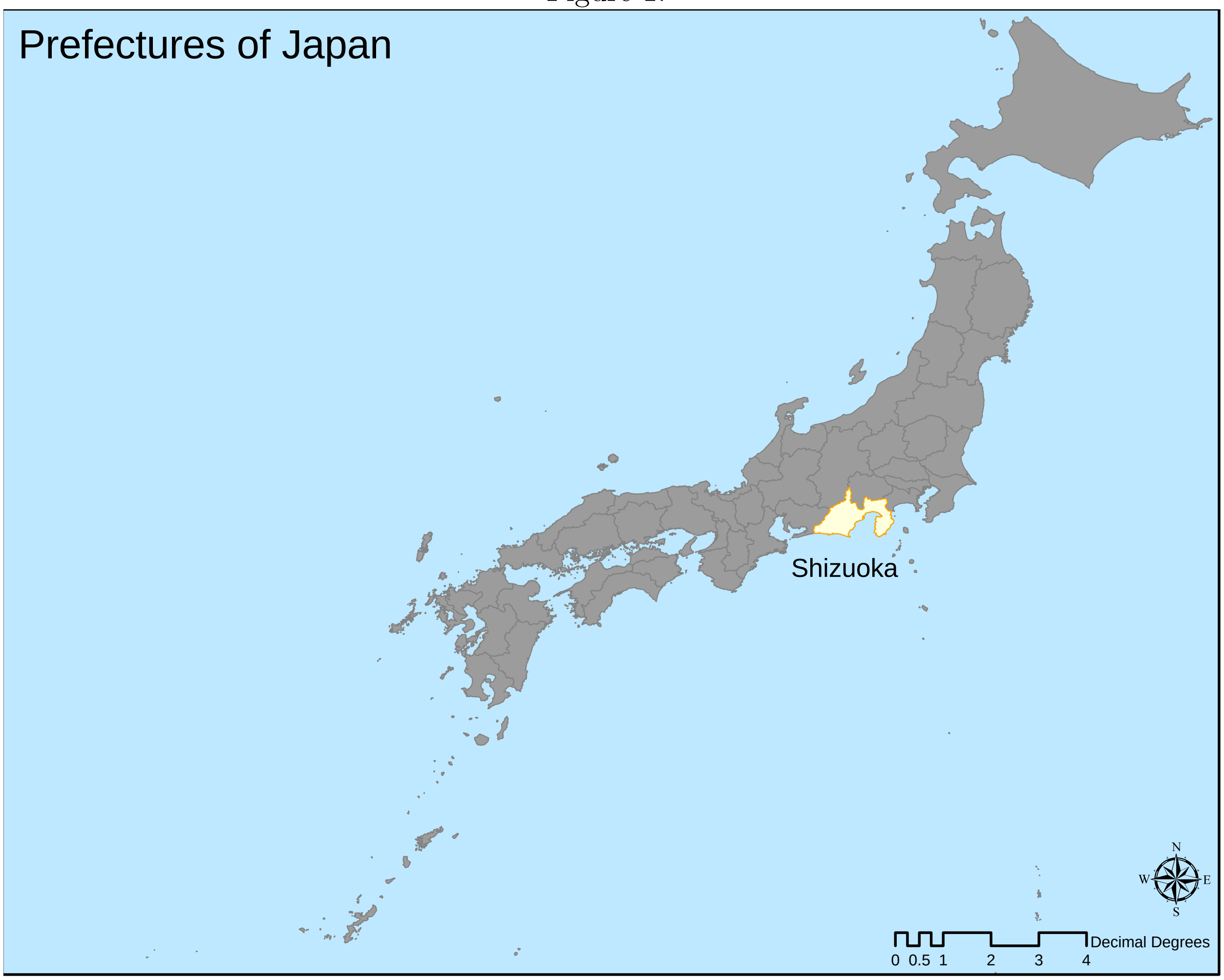

Prefectures of Japan
Shizuoka
N
W
E
S
Decimal Degrees
0 0.5 1 2 3 4

Table 1: Dependent variable is $c(g_S)$, cost of providing general services ('96-'97 fiscal year)

	I	II	III	IV	V
(Intercept)	1294.6	808.4	834.3	792.2	902.7
	(23.0)	(24.4)	(25.2)	(27.1)	(21.2)
POPULATION	136.4	136.0	136.6	142.3	142.5
	(0.3)	(0.3)	(0.3)	(1.7)	(1.3)
AREA		4.3	3.6	3.8	2.9
		(0.1)	(0.1)	(0.1)	(0.1)
INCOME.INEQ		0.4	0.3	− 20.9	− 12.4
		(4.8)	(4.9)	(4.3)	(3.3)
INCOME		−1070.4	−779.8	−164.9	−483.4
		(69.0)	(104.3)	(69.1)	(79.8)
IS.CITY		324.1	369.8	− 16.2	295.4
		(54.9)	(54.2)	(59.2)	(48.1)
POP*INCOME.INEQ				1.1	0.2
				(0.1)	(0.1)
POP*INCOME				− 30.5	− 8.6
				(1.0)	(1.5)
POP*IS.CITY				5.4	− 1.7
				(2.2)	(1.7)
PREFECTURE			*X*		*X*
N	3220	3216	3216	3216	3216

Units: ¥1,000,000 (roughly $10,000) per year. POPULATION is in thousands of residents, AREA is in square kilometers, INCOME is in ¥1,000,000 per capita per year, INCOME.INEQ is the coefficient of variation of income, IS.CITY is a dummy variable coded as 1 if the municipality in question is a city, and zero if it is a village or town. PREFECTURE is a set of dummy variables for each of the 47 prefectures, with the restriction that the sum of the coefficients on these variables must equal zero. Designated cities and special wards are excluded from the regression because they have additional responsibilities devolved from the prefectural governments, and thus have higher (and non-comprable) expenditures per capita.

Table 2: Dependent variable is $c(g_S)$, cost of providing general services

	fiscal year	
	'96-'97	'06-'07
(Intercept)	899.9	582.2
	(43.9)	(59.5)
POPULATION	129.4	131.5
	(0.5)	(0.6)
AREA	4.6	4.6
	(0.2)	(0.2)
N	1194	1194

Units: ¥1,000,000 (roughly $10,000) per year. POPULATION is in thousands of residents, AREA is in square kilometers, designated cities and special wards are excluded as in Table 1. The sample is further restricted to those municipalities that did not participate in a merger in order to have the same sample in both periods. Thus, the change in coefficients represents a change in national government transfer policy on the same group of municipalities during the period in question. Inflation during this period was negligible.

Table 3: Dependent variable is $u_i(S)$, utility to municipality i from merger S

	restricted preferences	relaxed stability requirements
$\log(g_s)$	1.00	1.00
	(0.05)	(0.001)
$\log(\text{POPULATION}_S)$	−0.06	− 0.12
	(0.01)	(0.0004)
$\log(\text{AREA}_S)$	−0.13	0.01
	(0.01)	(0.0004)
$\log(\text{INCOME}_S)$	0.05	1.58
	(0.03)	(0.001)
IS.MERGER_S	− 0.18	− 0.001
	(0.01)	(0.001)
σ	0.35	0.05

The coefficient on government services (g_S) is normalized to 1, with the standard deviation σ of the error term (ϵ_S and ϵ_{iS}, respectively) determined by this normalization. INCOME is income per capita. IS.MERGER is a dummy variable equal to 0 if the coalition S is a singleton (and thus would not imply a merger) and 1 otherwise. The standard errors presented in the second column assume that the error introduced by numerically estimating the size of the solution set is not important.

Figure 2:

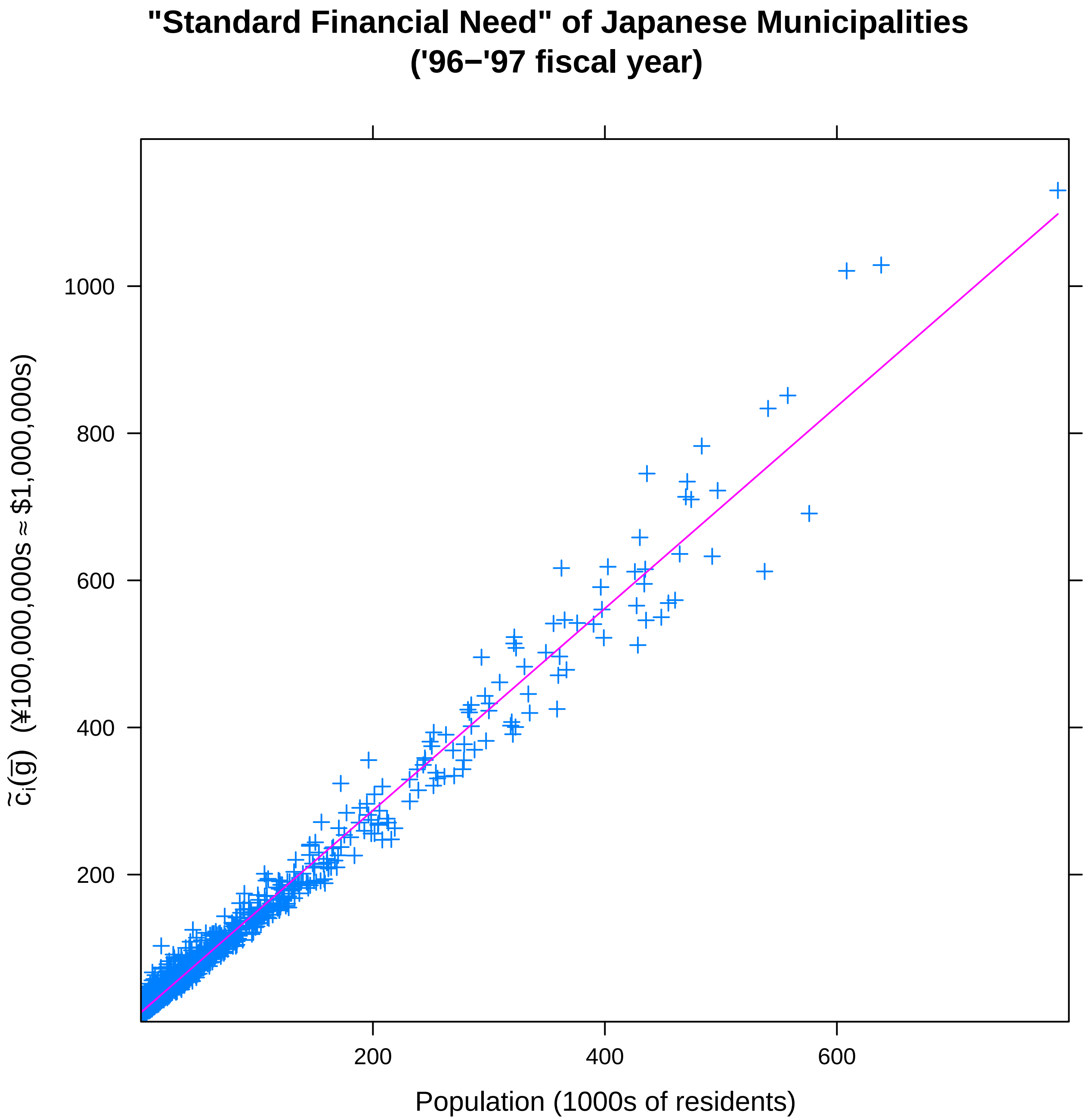

"Standard Financial Need" of Japanese Municipalities
('96−'97 fiscal year)
1000
800
600
400
200
200
400
600
Population (1000s of residents)
$\tilde{c}_i(\bar{g})$ (¥100,000,000s ≈ $1,000,000s)

Figure 3:

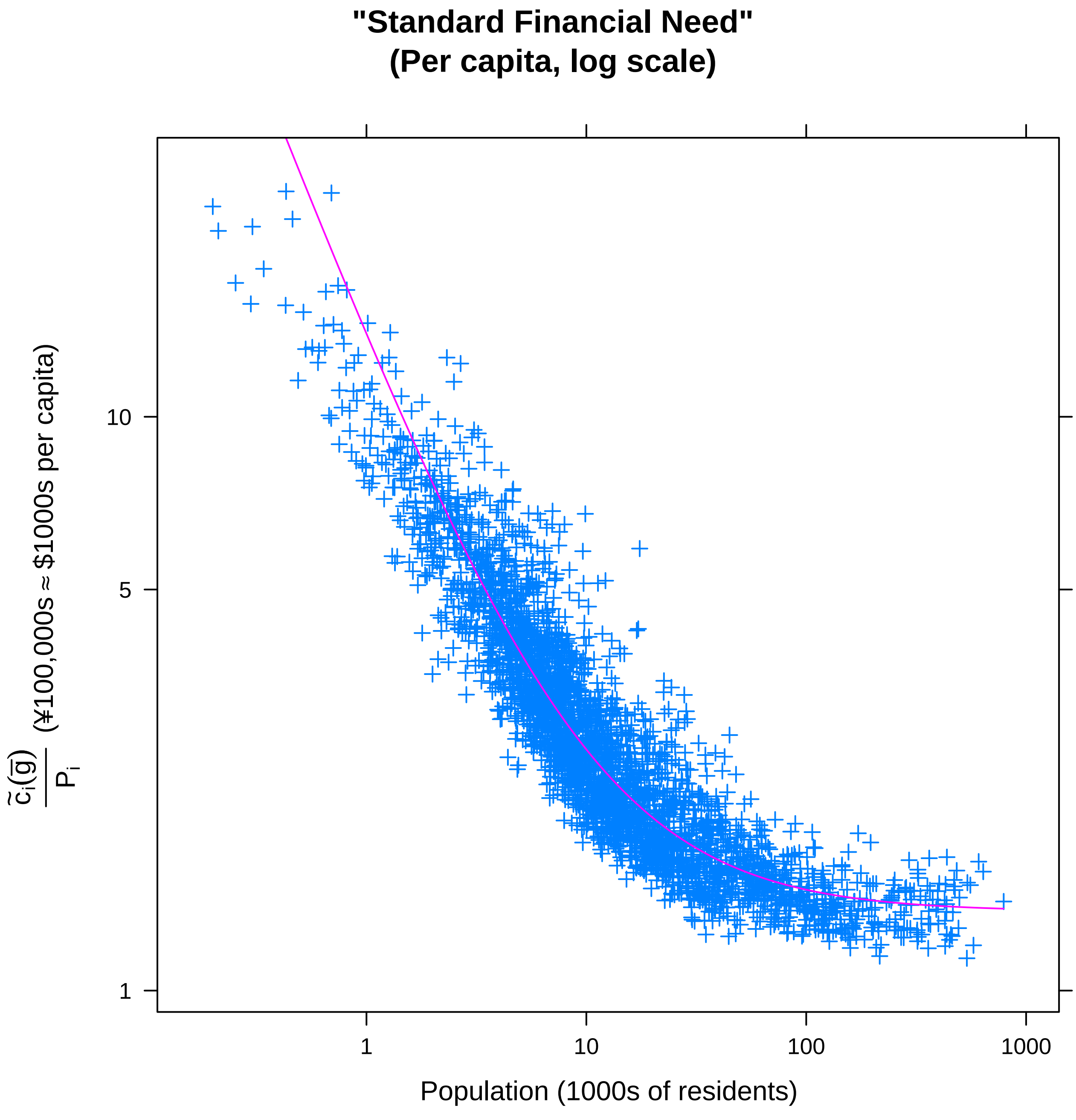

"Standard Financial Need"
(Per capita, log scale)
$\frac{\tilde{c}_i(\bar{g})}{P_i}$ (¥100,000s ≈ $1000s per capita)
10
5
1
1
10
100
1000
Population (1000s of residents)

Figure 4:

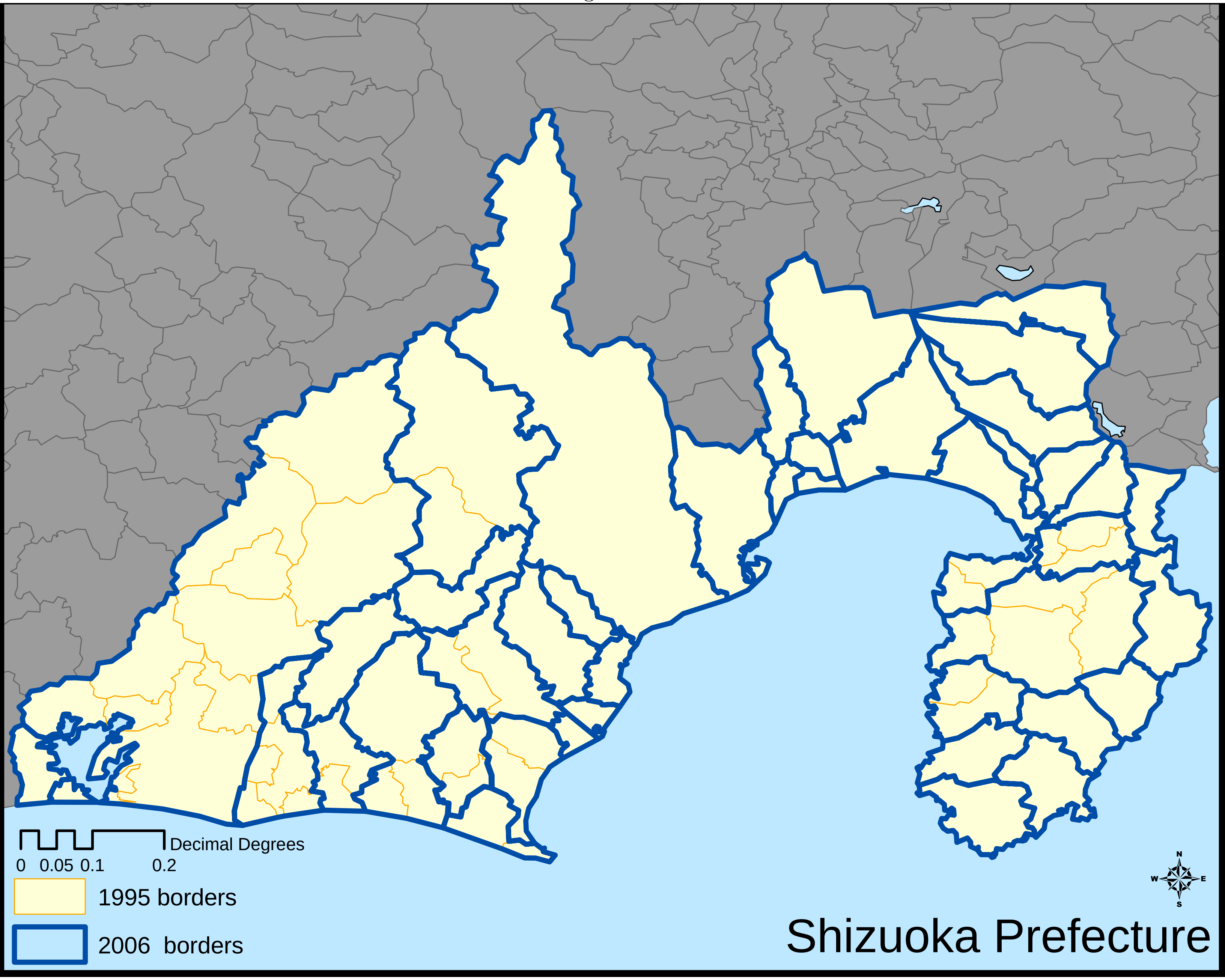
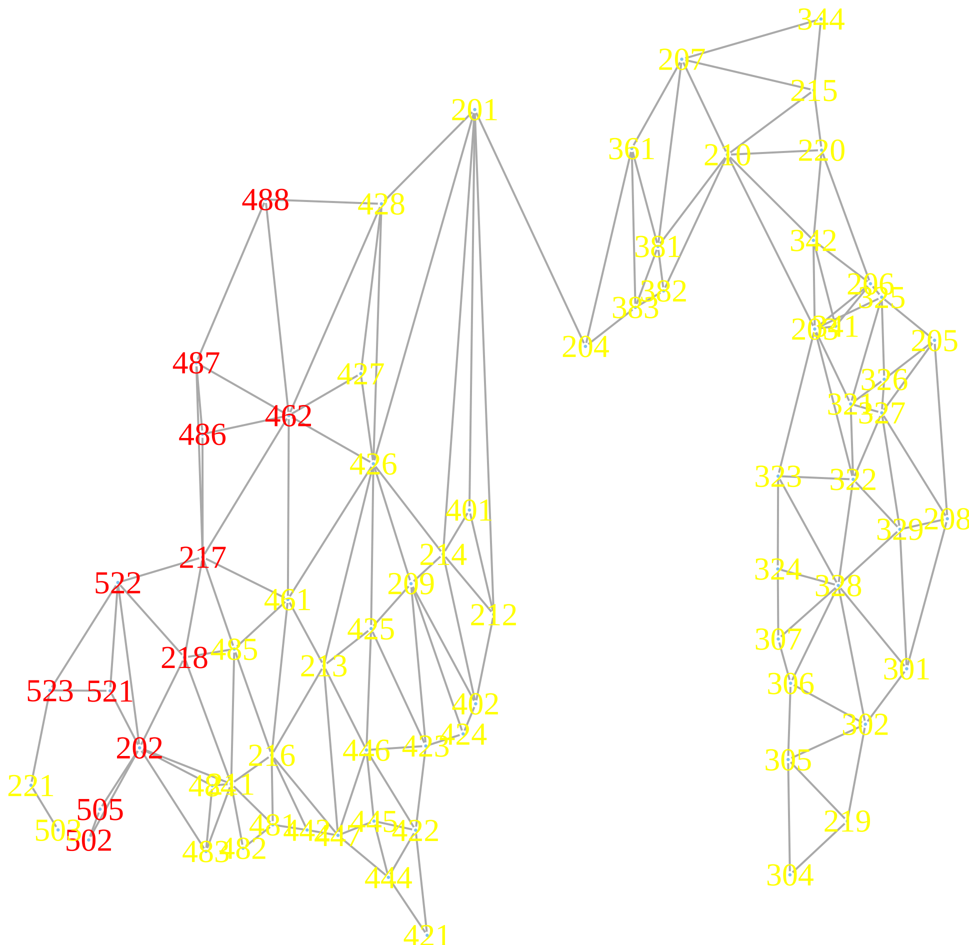
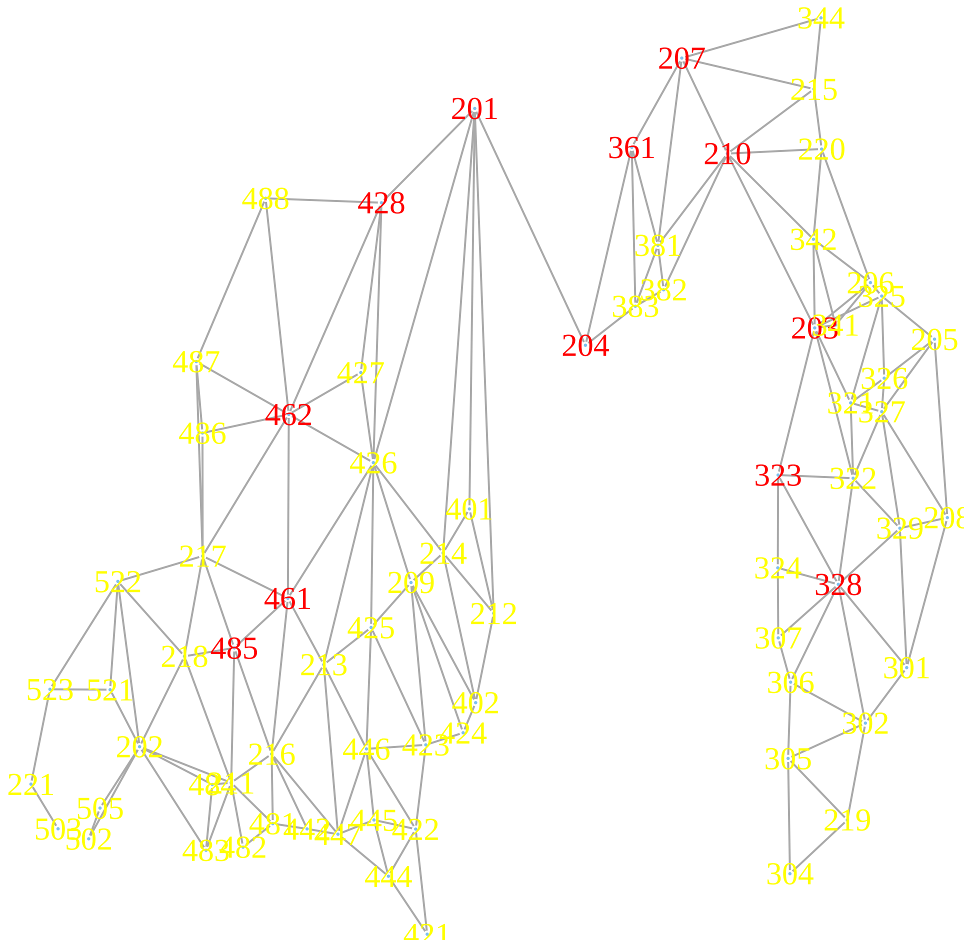

Decimal Degrees
0 0.05 0.1 0.2
1995 borders
2006 borders
Shizuoka Prefecture
N
W
E
S

Figure 5: Hanamatsu City in Red



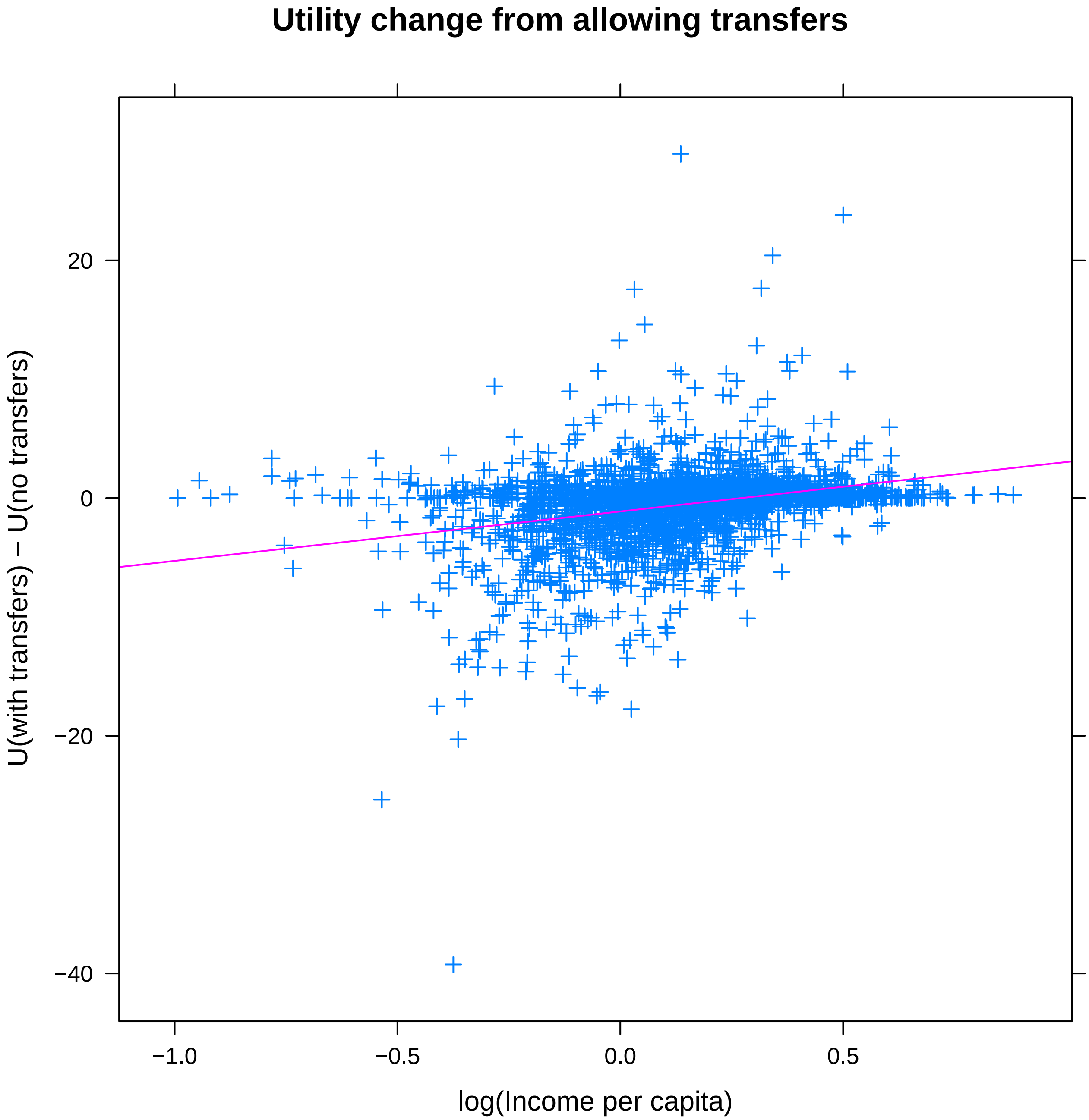
This connected graph represents Shizuoka Prefecture (also shown in Figure 4). "Old" Hanamatsu City is 22202.

Figure 6: Random Contiguous Coalition in Red



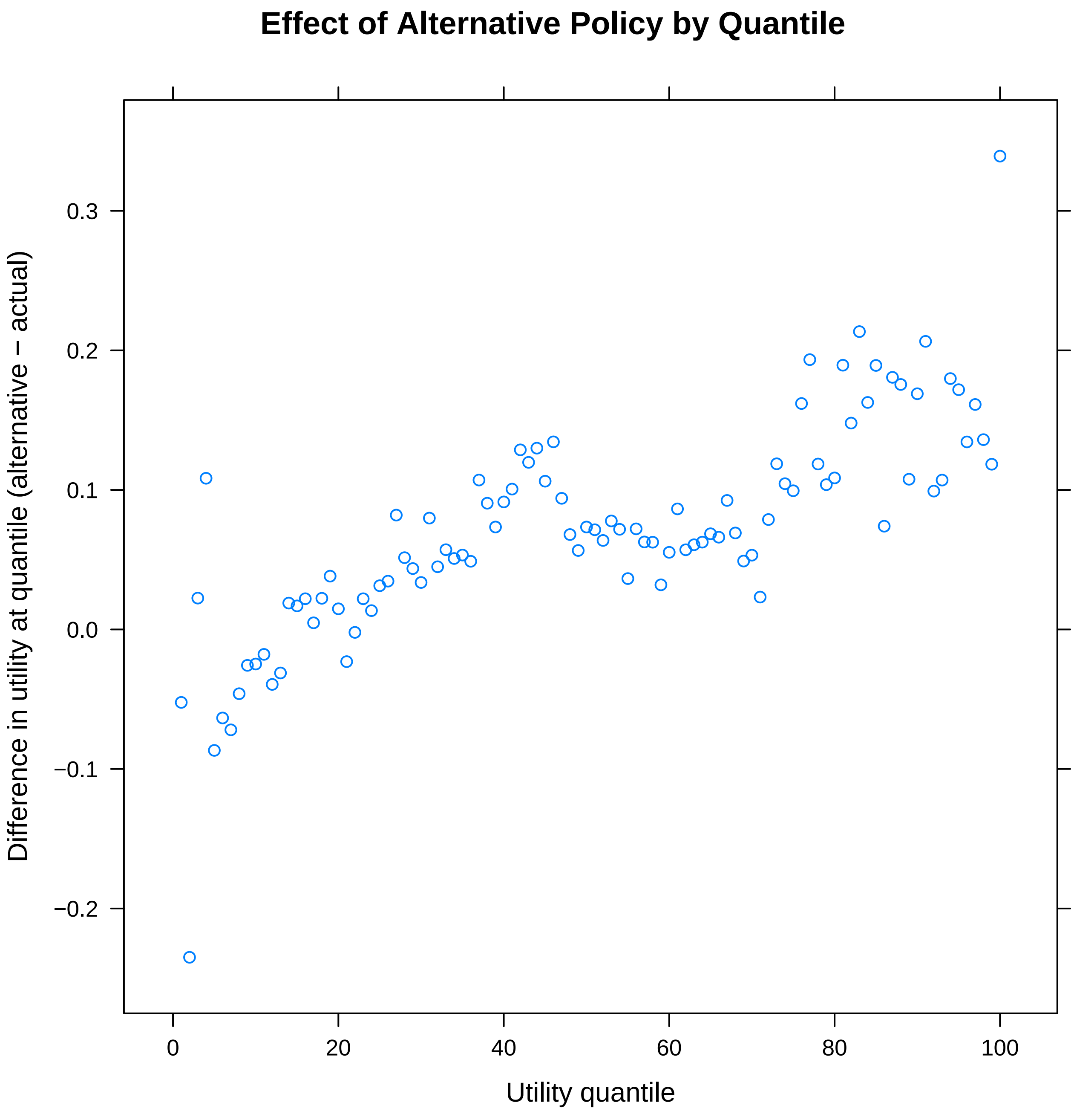
This connected graph represents Shizuoka Prefecture (also shown in Figure 4).

Figure 7:



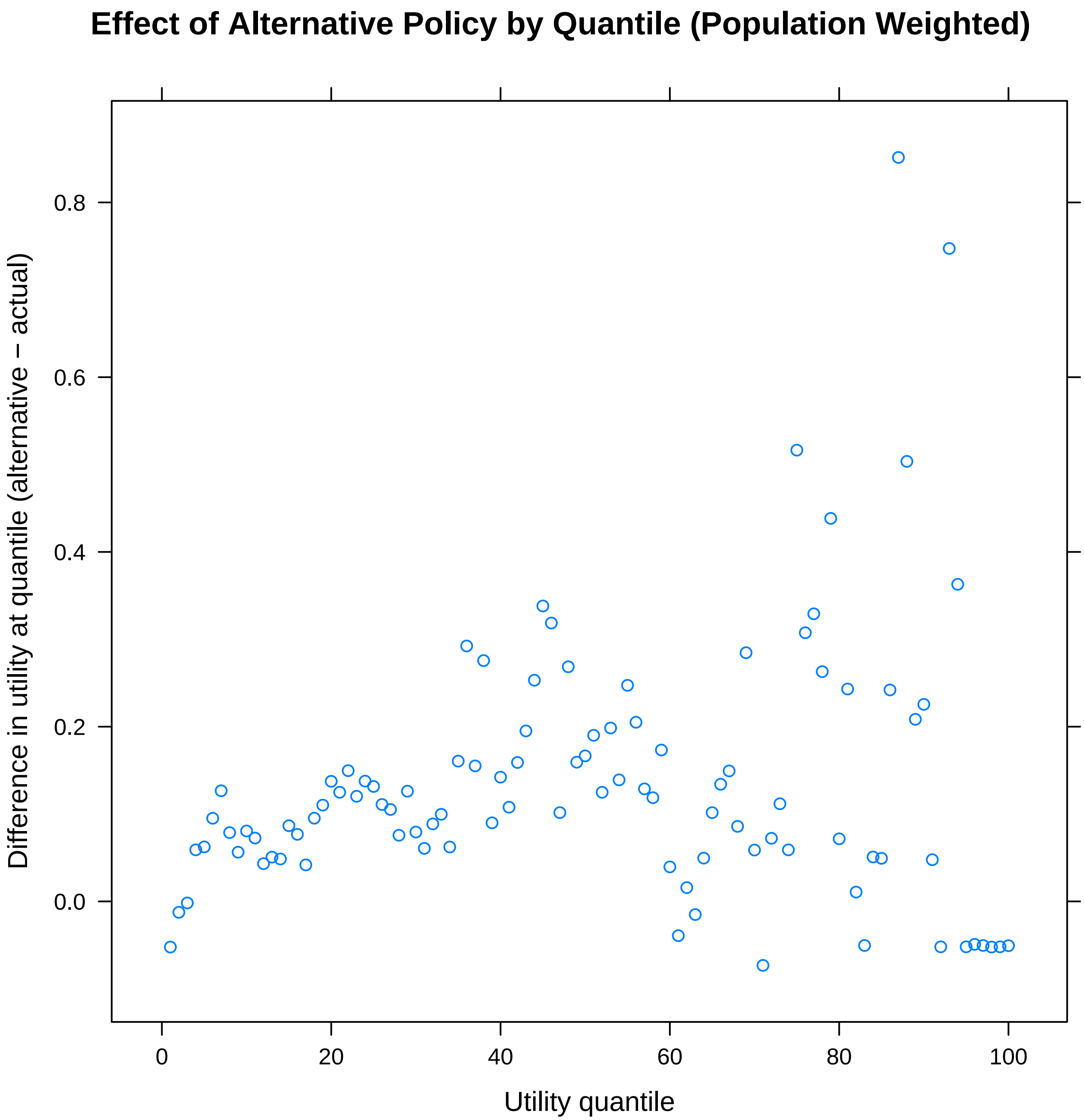
The slope is statistically significant (t=4), although this is not taking into account that the data is generated from a simulation and the simulation itself is based on estimated coefficients.

Figure 8:



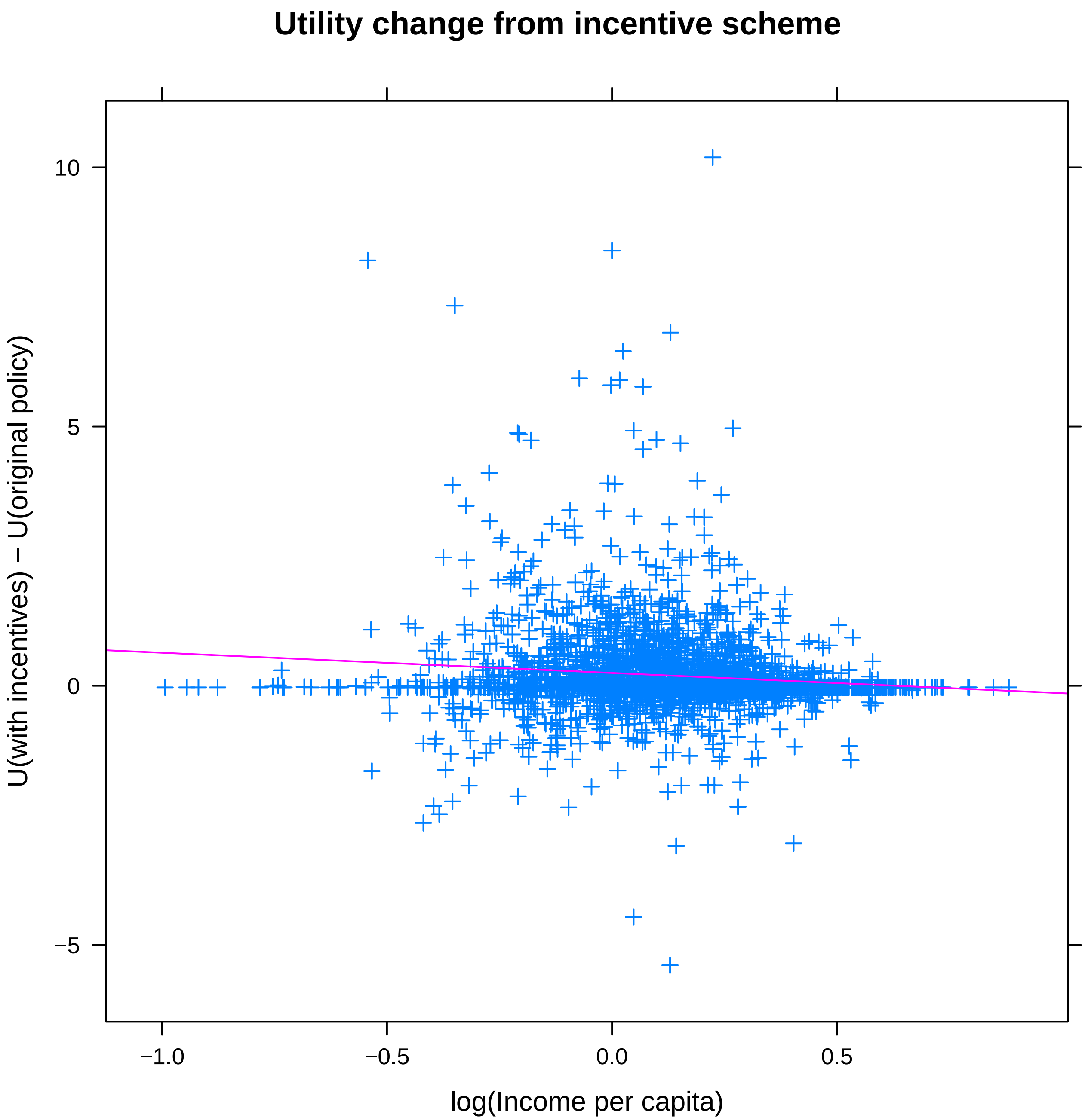
Change in distribution of utility, weighting all municipalities equally.

Figure 9:



Change in distribution of utility, weighting municipalities by population.

Figure 10:



The regression line is not weighted by municipal population; on a population weighted basis, utility is increased for richer municipalities.