Doubts, Asymmetries, and Insurance

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Introduction

- **Ingredient**: Agents doubt their forecasting models.
- **Question**: Study how these doubts affect risk sharing in economies with aggregate risk.
- **Mechanism**: Heterogeneity in wealth + Doubts
  - New insurance channel
- **Outcomes**: Introducing doubts alters
  - Agents' trading behavior
  - Dynamics of asset prices
  - Evolution of inequality

Sketch of the model

1. **Baseline**
   - Two agents trade in a complete market exchange economy.
   - Fluctuations in aggregate endowment.
   - Two layers of uncertainty
     - **Learning**: Prior over a set of models updated using Bayes rule $\rightarrow$ "approximating" model.
     - **Doubts**: Set of probability distributions statistically close to the approximating model.
2. **Extensions**
   - Publicly observed news shocks
   - Privately observed taste shocks

Key mechanism

- Hansen–Sargent multiplier framework to address doubts.
  - Construct **worst-case beliefs** to obtain decision rules robust to misspecifications
- Worst-case beliefs are
  - **Endogenous**: depend on fluctuations in future utilities
  - **Heterogeneous**: depends on curvature of utility functions
- IES is a key primitive for how agents trade in presence of doubts.

Main results

- **Heterogeneous priors**
  The Friedman conjecture is altered by introducing a small amount of doubts depending on IES.
- **Asset prices**
  Compensation for risk is countercyclical because richer agents have larger belief distortions in recessions.
- **News shocks**
  There is trading on news shocks as agents value resolution of uncertainty through public signals differently.
- **Taste shocks**
  Doubts can generate bounded inequality when insurance is limited by private information.

Literature Review

Heterogeneous beliefs: Harrison–Kreps (1978)

- Exogenous heterogeneity in beliefs $\rightarrow$ trade in financial securities.
- *This paper: heterogeneity in beliefs is endogenously correlated with heterogeneity in wealth.*

Asset pricing: Hansen–Sargent (2010), Miao–Ju (2012)

- Study representative agent economies
- *This paper: wealth inequality affects volume of trade and volatility of asset prices.*

Efficient inequality: Blume–Easley (2006), Atkeson–Lucas (1992)

- Effects of heterogeneous beliefs or heterogeneous information accumulate over time, leading to inequality.
- *This paper: new insurance motives that come from doubts can counter "immiseration" forces.*

Setup

1. **Technology:** Exchange economy with stochastic aggregate endowment $y_t \in \mathcal{Y}$.
2. **Demography:** Two types of agents $\mathcal{I} = \{1, 2\}$.
3. **Endowments:** Both agents have equal shares of aggregate endowment.
4. **Shocks:** Data generating process

$$P^0(y^\infty | y_0) = \prod_{t \geq 0} P_t^0(y_{t+1}).$$

5. **Markets:** Agents trade one-period-ahead Arrow securities.

Doubts and learning

Agents do not know the true data generating process P^0.

1. **Learning**
   - Priors: $\pi_{i,0}(m)$ over a finite set of "parsimonious" specifications

     $$\mathcal{M} = \{m : P_Y(y'|y, m)\}$$

     Use Bayes rule to update $\pi_{i,t}(m)$
   - Approximating model:

     $$P_t^i(y_{t+1}) = \sum_m \pi_{i,t}(m) P_Y(y_{t+1}|y_t, m)$$

2. **Doubts:** A vast set of statistically close alternatives to the approximating model
   Agents use new information to revise where they focus their doubts.

Valuations

Let $V_t^i[\mathbf{c}]$ be Agent i's value of $\mathbf{c} = \{c_t\}_{t\geq 0}$ at history y^t.

1. **Without doubts**

$$V_t^i[\mathbf{c}] = (1-\delta)u[c_t] + \delta \mathbb{E}_t^i V_{t+1}^i[\mathbf{c}]$$

   with $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$ and elasticity of substitution $= \frac{1}{\gamma}$

2. **With doubts**

$$V_t^i[\mathbf{c}] = (1-\delta)u[c_t] + \delta \mathbb{T}_{\theta,t}^i V_{t+1}^i[\mathbf{c}]$$

How are doubts modeled?

Likelihood ratio

$$z_{t,t+1}(y_{t+1}) = \frac{\tilde{P}_t^i(y_{t+1})}{P_t^i(y_{t+1})} \begin{array}{l} \rightarrow \text{Worst-case model} \\ \rightarrow \text{Approx. model} \end{array}$$

$$\mathbb{T}_{\theta,t}^i V_{t+1}^i = \min_{\substack{z_{t,t+1}(y_{t+1}) \\ \mathbb{E}_t^i z_{t,t+1} = 1}} \underbrace{\mathbb{E}_t^i z_{t,t+1} V_{t+1}^i}_{\substack{\text{Expectations} \\ \text{under } \tilde{P}_t^i}} + \theta^{-1} \underbrace{\mathbb{E}_t^i z_{t,t+1} \log(z_{t,t+1})}_{\substack{\text{Relative entropy} \\ \tilde{P}_t^i \text{ w.r.t } P_t^i}}$$

- **Minimizing likelihood ratio**:

$$z_{t,t+1}(y_{t+1}) \propto \exp\left\{-\theta V_{t+1}^i(y_{t+1})\right\}$$

- With $\theta = 0$ we have $\mathbb{T}_{\theta,t}^i = \mathbb{E}_t^i$

Competitive equilibrium

Definition

Given $\{a_{i,0}, \pi_{i,0}\}_i$, and y_0, a competitive equilibrium is a collection of $\{c_{i,t}, a_{i,t}(y_{t+1}), \tilde{P}^i_t(y_{t+1})\}_{i,t\geq 0}$ and Arrow prices $\{q_t(y_{t+1})\}_{t\geq 0}$ such that

- Agents optimize

$$\max_{\{c_{i,t}, a_{i,t}(y_{t+1})\}_{t\geq 0}} V^i_0[c_i]$$

s.t for all t

$$c_{i,t} + \sum_{y_{t+1}} q_t(y_{t+1}) a_{i,t}(y_{t+1}) = y_{i,t} + a_{i,t-1}$$

- Worst-case beliefs are consistent

$$\tilde{P}^i_t(y_{t+1}) \propto P^i_t(y_{t+1}) \exp\left\{-\theta V^i_{t+1}(y_{t+1})\right\}$$

- Goods and asset markets clear

Planner's problem

Use a planner's problem to find competitive allocations.

1. Welfare theorems hold in this environment.
2. Pareto efficient allocations have a recursive structure.

Recursive formulation of planner's problem

$$\mathcal{Q}(\boldsymbol{\pi}_t, v_t, y_t) = \max_{c_1, c_2, \bar{v}(y_{t+1})} (1-\delta) u[c_1] + \delta \mathbb{T}^1_{\theta,t} \mathcal{Q}(\boldsymbol{\pi}_{t+1}, \bar{v}(y_{t+1}), y_{t+1})$$

s.t.

(a) **Promise keeping:**

$$(1-\delta) u[c_2] + \delta \mathbb{T}^2_{\theta,t} \bar{v}(_{t+1}) \geq v_t$$

(b) **Feasibility:**

$$c_1 + c_2 \leq y_t$$

(c) **Bayes Rule:** For all i

$$\pi_{i,t+1}(m) \propto \pi_{i,t}(m) P_Y(y_{t+1} | y_t, m)$$

The multiplier on the promise keeping constraint (λ) is the relative Pareto weight of Agent 2.

Optimal allocation: characterization

The optimal allocation can be represented by

$$c_{i,t} = c_i(\lambda_t, y_t)$$

and a law of motion for λ

$$\frac{\lambda_{t+1}}{\lambda_t} = \frac{\tilde{P}_t^2(y_{t+1})}{\tilde{P}_t^1(y_{t+1})}$$

The allocations are also efficient in an "alternative" economy where agents have no doubts but exogenous heterogeneous beliefs $\{\tilde{P}_t^i\}_{i,t}$.

Endogenous heterogeneity in beliefs

Given the optimal allocation and continuation values V^i_{t+1}

Worst case beliefs for Agent i are

$$\tilde{P}^i_t(y_{t+1}) \propto \underbrace{P^i_t(y_{t+1})}_{\text{Learning}} \underbrace{\exp\left\{-\theta V^i_{t+1}(y_{t+1})\right\}}_{\text{Doubts}}$$

1. **Endogeneity** of beliefs
   - Learning: approximating models are updated using Bayes law.
   - Doubts: agents overweight states where their continuation values are low.
2. **Heterogeneity** of beliefs
   - Initial priors: $\{\pi_{i,0}(m)\}_i$
   - Initial wealth shares: λ_0

Doubts and insurance

Study the consequences of heterogeneity in initial priors

- How are doubts different from learning?
- How is the implied trading behavior altered?

Re-examine the Friedman conjecture

Agents with incorrect priors do worse in the long run.

Long run inequality: no doubts

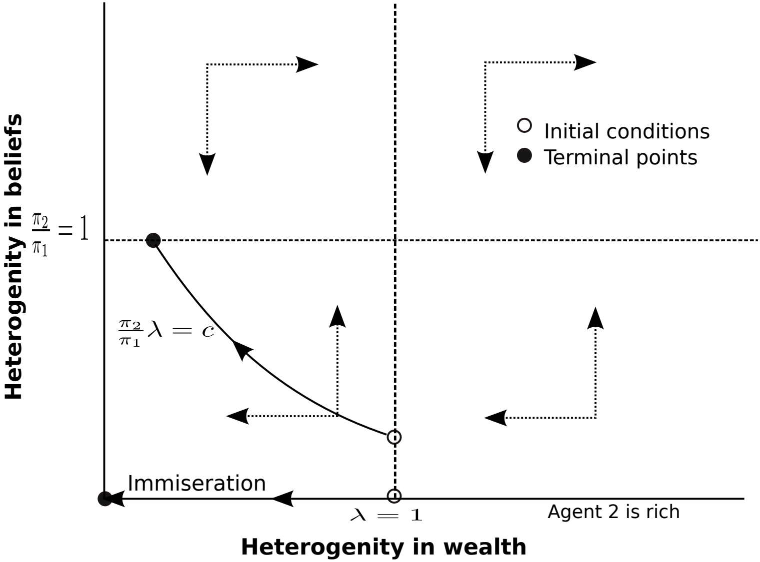
Theorem

For $\theta = 0$, suppose the data generating process is
$P^0_t(y_{t+1}) = P_Y(y_{t+1}|y_t, m^*)$
If $\pi_{1,0}(m^*) > 0$

$$\lambda_t \to \lambda_0 \frac{\pi_{2,0}(m^*)}{\pi_{1,0}(m^*)} \quad P^0 - \textit{almost surely}$$

The ratio $\frac{\pi_{2,0}(m^*)}{\pi_{1,0}(m^*)}$ denotes Agent 2's initial relative "advantage"

Long run inequality: no doubts



Dynamics of Pareto weights with doubts

λ_t is a **martingale** under Agent 1's worst case beliefs.

$$\frac{\lambda_{t+1}}{\lambda_t} = \frac{\tilde{P}_t^2(y_{t+1})}{\tilde{P}_t^1(y_{t+1})} \implies \tilde{E}_t^1 \lambda_{t+1} = \lambda_t$$

"Undoing" Agent 1's distortions we get,

$$\mathbb{E}_t^1 \lambda_{t+1} = \lambda_t - Cov_t^1 \left[\lambda_{t+1}, z_{t,t+1}^1 \right]$$

or

$$\mathbb{E}_t^1 \lambda_{t+1} = \lambda_t - Cov_t^1 \left[\frac{\tilde{P}_t^2(y_{t+1})}{\tilde{P}_t^1(y_{t+1})} \lambda_t, \frac{\tilde{P}_t^1(y_{t+1})}{P_t^1(y_{t+1})} \right]$$

What is the sign of the covariance?

Signing the covariance

Suppose $\pi_{1,0} = \pi_{2,0}$. This shuts off exogenous heterogeneity in beliefs

$$\mathbb{E}_t \lambda_{t+1} = \lambda_t - Cov_t \left[\frac{\tilde{P}_t^2(y_{t+1})}{\tilde{P}_t^1(y_{t+1})} \lambda_t, \frac{\tilde{P}_t^1(y_{t+1})}{P_t^1(y_{t+1})} \right]$$

1. $\frac{\tilde{P}_t^1(y_{t+1})}{P_t^1(y_{t+1})}$: Agent 1's pessimism

   This is countercyclical

2. $\frac{\tilde{P}_t^2(y_{t+1})}{\tilde{P}_t^1(y_{t+1})}$: Agent 2's *relative* pessimism

   Depends on IES and Agent 2's wealth share

Role of IES

- Agents care about fluctuations in utilities relative to costs.
  - Volatile utilities $\implies$ large belief distortions
  - Entropy costs of deviating from the approximating model
- Suppose $c = \eta y$

$$\sigma[u] \approx \sigma[c] u'[\mathbb{E}c]$$

Is $\sigma[u]$ increasing in η?

Role of IES

- When η increases, we have two effects
  1. $\sigma[c] \Uparrow$
  2. $u'[\mathbb{E}c\Uparrow] \Downarrow$
- **Elasticity of marginal utility to changes in consumption** determines which force dominates.
  1. If IES > 1 marginal utility is less sensitive to changes in consumption
     $\rightarrow \quad \sigma[u]$ is increasing in η.
  2. If IES < 1 marginal utility is more sensitive to changes in consumption
     $\rightarrow \quad \sigma[u]$ is decreasing in η.

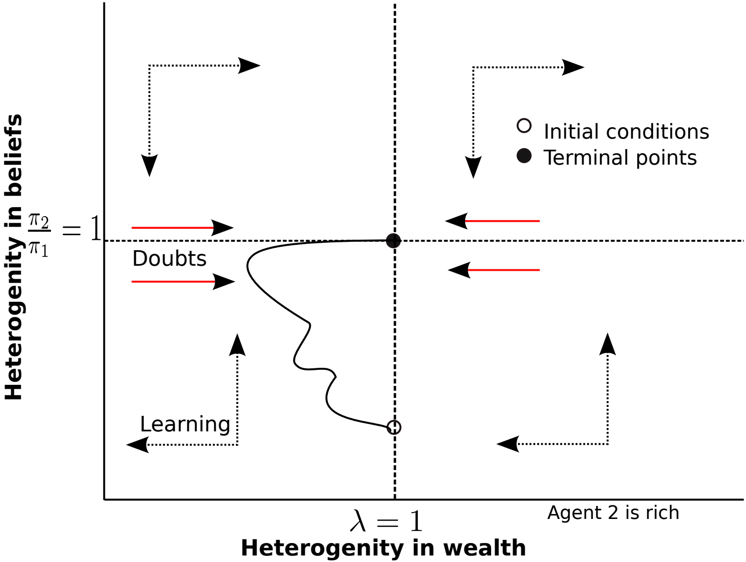
Role of IES

$$\mathbb{E}_t \lambda_{t+1} = \lambda_t - Cov_t \left[\frac{\tilde{P}_t^2(y_{t+1})}{\tilde{P}_t^1(y_{t+1})} \lambda_t, \frac{\tilde{P}_t^1(y_{t+1})}{P_t^1(y_{t+1})} \right]$$

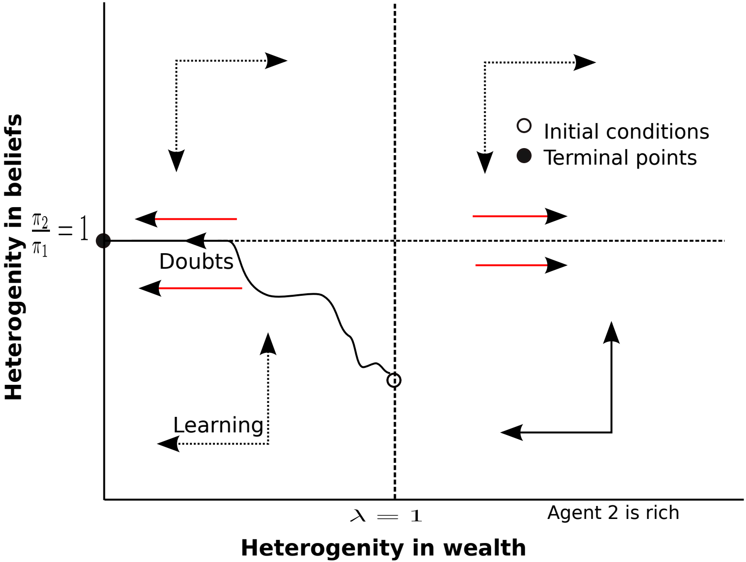
WLOG suppose $\lambda_t > 1$ (Agent 2 is rich)

1. When IES> 1
   - Richer agents have larger belief distortions.
   - Agent 2's *relative* pessimism $\frac{\tilde{P}_t^2(y_{t+1})}{\tilde{P}_t^1(y_{t+1})}$ is countercyclical.
   - **Covariance positive** $\implies$ **negative drift** of λ_t.
2. IES < 1: covariance is negative and λ_t increases.
3. IES $= 1$: homothetic Epstein–Zin preferences

Long-run inequality with doubts: IES > 1



Long run inequality with doubts: IES < 1



Long run inequality with doubts

Theorem

For $\theta > 0$, suppose the data generating process is $P_t^0(y_{t+1}) = P_Y(y_{t+1}|y_t, m^*)$ is i.i.d. over time

[Convergence] If IES>1,

$$\lambda_t \to 1 \quad P^0 - \textit{almost surely}$$

[Divergence] If IES $<$ 1,

$$P^0\{\lambda_t \to 0 \cup \lambda_t \to \infty\} > 0$$

[Homotheticity] If IES $=$ 1

$$\lambda_t \to \lambda_\infty \quad \forall t$$

Remarks

- In absence of doubts, initial heterogeneity in priors have a permanent effect on long run inequality.
- Doubts that are enduring dominate Bayesian learning.
- Even for $\theta \approx 0$, long run outcomes are very different.
- Doubts induce low frequency changes in insurance arrangements whose effects accumulate through time.

Interpreting IES: design of social insurance schemes

Doubts, dogmatism and market selection

Dogmatic beliefs: $\exists\ m \in \mathcal{M}$ such $\pi_i(m) = 1$

1. IES and the 'gap' between approximating models matter for long-run wealth shares.
2. **Main result**

Theorem

Suppose $P^0 = P^1$ and let $\mathbb{I}^{0,2}$ be the relative entropy of Agent 2's approximating model w.r.t the DGP. If $IES > 1$, there exists $\bar{M} > 0$ such that

$$\mathbb{I}^{0,2} < \bar{M}$$

is sufficient for

$$\lambda_t \not\to 0 \quad P^0 - \textit{almost surely}$$

Survival Region: 2 shock case

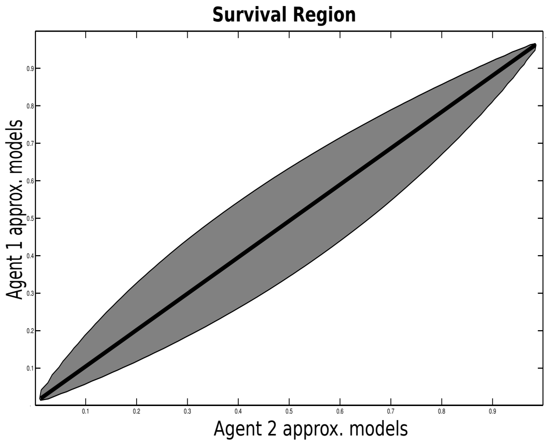


Figure: The shaded region plots the approximating models (Binary-IID) for Agent 1 and Agent 2 for which both agents survive. The DGP $P^0 = P^1$

Asset pricing

So far

- Impact of learning and doubts on long run wealth shares

Next, study how doubts and wealth dynamics generate

- Countercyclical prices of risks
- Motives for trade on news shocks

Market price of risk

- **Common approximating model:** Assume $\pi_{0,1} = \pi_{0,2} = \pi_0$
- **Pricing kernel:** $\rho_t(y_{t+1})$ that prices cash-flows $f(y)$

$$\mathbb{P}_t(f) = \mathbb{E}_t \rho_t(y_{t+1}) f(y_{t+1})$$

It follows that

$$\rho_t(y_{t+1}) = \delta \frac{u_c(c_{i,t+1})}{u_c(c_{i,t})} \left(\frac{\tilde{P}^i_t(y_{t+1})}{\sum_{m \in M} \pi_t(m) P_Y(y_{t+1} | y_t, m)} \right)$$

- **Market price of risk:** the conditional volatility of the (log) pricing kernel

$$\mathrm{MPR}[\boldsymbol{\pi}, v, y] = \mathrm{var}[\log(\rho) | \boldsymbol{\pi}, v, y]$$

This measures quantities from the perspective of an outside econometrician who uses the common approximating model.

Dynamics of MPR

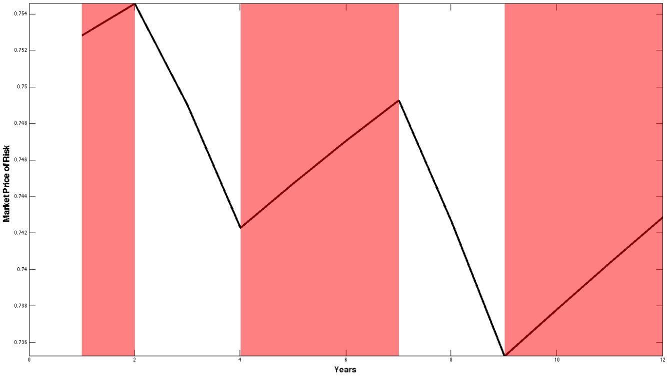


Figure: A sample path of MPR in an economy with $\mathcal{Y} = \{y_l, y_h\}$. Shaded regions denote periods with low aggregate endowment.

Why MPR increases in recessions?

1. IES > 1
   - Belief distortions increase with wealth shares
   - Insurance contracts are resolved in favor of rich agents
   - Their concerns for misspecification are even larger

2. IES < 1
   - Rich agents make insurance payments and lose wealth
   - Concerns for misspecification are again larger due to increase in marginal utilities

In either case, valuations are lower and compensation for risk is higher.

Role of "news" shocks

- Augment economy with "news" shocks

$$\nu_t = y_{t+1} + \epsilon_t, \quad \epsilon_t \text{ i.i.d.}$$

- If agents have identical initial priors and no doubts, news shocks are irrelevant.
    1. Informative public signals only affect information sets.
    2. But these are the *same* across agents, so there is no motive to trade.

News shocks matter

Theorem
For IES $\neq 1$, so long as there is wealth inequality $(\lambda_0 \neq 1)$, there exist $(y^t, \nu^t) \neq (y^t, \tilde{\nu}^t)$ for which

$$c_{i,t}(y^t, \nu^t) \neq c_{i,t}(y^t, \tilde{\nu}^t)$$

1. Heterogeneity in wealth $\implies$ value of resolution of uncertainty differs across agents.
2. Bad news is worse for agents with larger fluctuations in valuations.
3. With complete market, agents trade consumption claims contingent on news.

Extension: asymmetric information

1. Add privately observed i.i.d. taste shocks to Agent 2's utility
   - Efficiency requires insurance arrangements to be incentive compatible.
   - This generates an "immiseration" force as in Atkeson–Lucas or Thomas–Worrall
2. In a simple example, I will contrast how doubts alter these immiseration forces.
3. The planner's problem is modified to incorporate truth telling constraints. ▸ problem

Revisiting the dynamics of Pareto weights

$$\frac{\lambda_{t+1}(y_{t+1}, s_t)}{\lambda_t} = \underbrace{\left[\frac{\tilde{P}_t^2(y_{t+1}, s_t)}{\tilde{P}_t^1(y_{t+1}, s_t)}\right]}_{\text{Heterogeneous beliefs}} \underbrace{\left[1 + \mu_t(s_t) - \mu_t(s_t^{'})\frac{\tilde{P}_t^2(y_{t+1}, s_t^{'})}{\tilde{P}_t^2(y_{t+1}, s_t)}\right]}_{\text{Optimal Incentives}}$$

1. **Heterogeneous Beliefs:** agents disagree on the worst case beliefs about states tomorrow.
2. **Optimal Incentives:** optimal incentives spread promised values.

Immiseration

Theorem

Suppose IES > 1.

- *With* $\theta = 0$, $\lambda_t \to 0$ $\qquad P^0 -$ *almost surely*
- *With* $\theta > 0$, $\lambda_t \not\to 0$ $\qquad P^0 -$ *almost surely*

The force generated by heterogeneity in worst case beliefs dominates the fluctuations due to incentives .

Inspecting the mechanism

1. The bilateral credit market looks like "annuities":
   - High taste shock $\implies$ Agent 2 borrows today and repays by lowering future expected consumption.
2. With $\theta = 0$,
   - Aggregate endowment shocks are immaterial for Pareto weight dynamics.
   - A sequence of high taste shocks drives Agent 2 to immiseration.
3. With $\theta > 0$,
   - As $\lambda \to 0$ agents disagree on likelihoods of y^*.
   - Agent 1 buys "expensive" insurance against bad aggregate outcomes
   - For Agent 2, this income more than offsets the annuities coming from high taste shocks and thus prevents immiseration.

Conclusions

- Theory of endogenous belief distortions
  1. Insurance motives
  2. Trading behavior
  3. Asset pricing
- Implications for how effects of doubts accumulate overtime
  $\rightarrow$Design of social insurance schemes
- **Extensions**:
  1. Role of aggregate risk: study Bewley economies without aggregate fluctuations
  2. Quantitative examination of wealth-driven belief heterogeneity and asset prices and volume
  3. Framework for optimal policy with endogenous belief distortions

Revisiting the planner's problem

$$Q(v, y) = \max_{u_1(s), u_2(s), \bar{v}(s, y^*)} \mathbb{T}_\theta \left[(1-\delta) u_1(s) + \delta \mathbb{T}_{\theta, y} Q(\bar{v}(s, y^*), y^*) \right]$$

subject to

$$\mathbb{T}_\theta \left[(1-\delta) s u_2(s) + \delta \mathbb{T}_{\theta, y} \bar{v}(s, y^*) \right] \geq v$$

$$(1-\delta) u_2(s) + \delta \mathbb{T}_{\theta, y} \bar{v}(s, y^*) \geq (1-\delta) s u_2(s') + \delta \mathbb{T}_{\theta, y} \bar{v}(s', y^*) \quad \forall s, s'$$

$$C(u_1(s)) + C(u_2(s)) \leq y \quad \forall s$$

$$\bar{v}(s, y^*) \leq v^{max}(y^*)$$

back