Econ 705 : Preliminary Examination

June, 2011

Instructions:

(1) All the answers should be written legibly.

(2) In giving answers, try to mention conditions which justify the derivations.

(3) Even when you are not able to delineate precise conditions, try to provide a sketch of derivations and solutions as clearly as possible.

(1)(a-d) (40 Points) Suppose that a researcher analyzes the labor supply of male workers in Pennsylvania. For $t = 1, 2$, let Y_{it} represent the labor supply in log-hours of the i-th worker in year t, X_{it} be her nonlabor household log-income, and M_t denotes a macroeconomic shock that affects the male worker's labor supply. We assume that for each year t, the observations $\{(Y_{it}, X_{it})\}_{i=1}^n$ have been generated from the following regression models:

$$Y_{it} = \beta_0 + X_{it}\beta_1 + M_t\beta_2 + u_{it},$$

where u_{it} denotes the unobserved variable affecting Y_{it}. For each $t = 1, 2$, the random vectors $\{(Y_{it}, X_{it}, u_{it})\}_{i=1}^n$ are i.i.d., and $\{(Y_{it}, X_{it}, u_{it}) : t = 1, 2\}_{i=1}^n$ are observed.

Assume that $Var(X_{1t}) \in (0, \infty)$ for each $t = 1, 2$,

$$\mathbf{E}[u_{1t}|\{X_{1s}, M_s\}_{s=1}^2] = 0 \text{ and } \mathbf{E}[u_{1t}^2|\{X_{1s}, M_s\}_{s=1}^2] = \sigma^2 > 0.$$

(a)(10 Points) Suppose that M_t is not observed, and that one estimates β_0 and β_1 using OLS with $M_t\beta_2$ term omitted using the observations with $t = 1$. Is the estimator of β_1 unbiased? Explain your answer.

(b)(15 Points) Suppose that we are in the situation of (a). Is the estimator of β_1 in (a) consistent and asymptotically normal? Explain your answer.

(c)(15 Points) Suppose that one observes the macroeconomic shock M_t which is specified as

$$M_t = \frac{1}{n}\sum_{i=1}^n D_{it},$$

where $D_{it} = 1$ if the i-th worker is employed and 0 otherwise. It is assumed that for each t, $\{(Y_{it}, X_{it}, D_{it}, u_{it})\}_{i=1}^n$ are i.i.d. Can one identify β_0, β_1, and β_2 separately using the observations $\{M_t, Y_{it}, X_{it}, D_{it}\}_{i=1}^n$ at $t = 1$? Explain your answer.

(2)(a-e) (60 Points) Suppose that a researcher analyzes the employment status of female workers. Suppose that D_{1i} is 1 if the i-th female worker is employed and 0 otherwise, and that $D_{2i} = 1$ if the i-th female worker has at least one child and 0 otherwise. The variables D_{1i} and D_{2i} are specified as follows:

$$\begin{aligned} D_{1i} &= 1\left\{\beta_0 + X_i\beta_1 + D_{2i}\beta_2 > \varepsilon_i\right\} \text{ and} \\ D_{2i} &= 1\left\{\gamma_0 + X_i\gamma_1 > u_i\right\}, \end{aligned}$$

where the conditional distribution of ε_i given (X_i, u_i) is a standard normal distribution, and the conditional distribution of u_i given X_i follows a standard normal distribution. Let X_i denote the household income of the i-th female worker. The random vectors $\{(D_{1i}, D_{2i}, X_i, D_{2i}, \varepsilon_i, u_i)\}_{i=1}^n$ are i.i.d., and $\{(D_{1i}, D_{2i}, X_i)\}_{i=1}^n$ are observed.

(a)(10 Points) Provide conditions for the identification of β_0, β_1 and β_2. (HINT: Write out $\mathbf{E}[D_{1i}|X_i]$ in terms of β_0, β_1, β_2, and the standard normal CDF. In doing so, argue that D_{2i} is independent of ε_i.)

(b)(10 Points) **(i)** Provide three moment conditions that can be used to estimate β_0, β_1, and β_2.

(ii) Suppose that consistent estimators of β_0, β_1, and β_2 are given. Define GMM estimators of β_0, β_1, and β_2 that use an optimal weighting matrix.

(c)(10 Points) Provide conditions under which the estimators in (b)(ii) are consistent.

(NOTE: Questions (d)-(e) are on the next page.)

(d)(15 Points)(i) Assume that we have estimators $\hat{\beta}_0, \hat{\beta}_1$, and $\hat{\beta}_2$ of β_0, β_1, and β_2 and estimators $\hat{\gamma}_0$ and $\hat{\gamma}_1$ of γ_0 and γ_1. Assume that these estimators are $O_P(1/\sqrt{n})$. Using these estimators, provide a consistent estimator of the interaction average derivative of $P\{D_{1i}D_{2i} = 1|X_i = x\}$, i.e.

$$\mathbf{E}\left[\frac{\partial P\{D_{1i}D_{2i} = 1|X_i = x\}}{\partial x}|_{x=X_i}\right]$$

(HINT: Note that if $D_{1i}D_{2i} = 1$, $D_{2i} = 1$.)

(ii) Show that the estimator is consistent. (For simplicity of derivation, assume for this part (ii) that $\beta_0 = \beta_2 = \gamma_0 = \hat{\beta}_0 = \hat{\beta}_2 = \hat{\gamma}_0 = 0$, and assume that $\hat{\beta}_1$ and $\hat{\gamma}_1$ are $O_P(1/\sqrt{n})$.)

(e)(15 Points) We are interested in testing nonpositivity of average derivative of D_{1i} in D_{2i}, i.e.,

$$\begin{aligned} H_0 &: P\left[D_{1i} = 1|D_{2i} = 1\right] - P\left[D_{1i} = 1|D_{2i} = 0\right] \leq 0 \text{ against} \\ H_1 &: P\left[D_{1i} = 1|D_{2i} = 1\right] - P\left[D_{1i} = 1|D_{2i} = 0\right] > 0. \end{aligned}$$

Provide a reasonable test statistic T_n and critical value c such that $\lim_{n\to\infty} P\left\{T_n > c\right\} \leq 0.05$ under the null hypothesis. You need to show that the test indeed controls the size asymptotically.

End of the Exam